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Eleftheriou, Konstantinos and Michelacakis, Nickolas

University of Piraeus

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A comment on ‘Sequential spatial competition in vertically related industries with different product varieties’

Konstantinos Eleftheriou^{*,†} and Nickolas J. Michelacakis^{*}

Abstract

The aim of this paper is to revise and correct the results obtained in Beladi et al. [Beladi, H., Chakrabarti, A., Marjit, S., 2010. Sequential spatial competition in vertically related industries with different product varieties. *Economics Letters* 106, 112-114]. Specifically, we prove that following a vertical merger, the downstream firms will locate away from the social optimum in the following manner: to the direction of the un-integrated follower or to a direction determined by the wholesale price charged to the un-integrated leader.

JEL classification: L13, L42, D43, R32

Keywords: Product differentiation; Spatial price discrimination; Sequential competition; Merger

1 Introduction

In their paper Beladi et al. (2010) attempt to extend the work of Braid (2008) by examining the effect of a vertical merger on the equilibrium locations of two downstream firms, which are engaged in sequential spatial competition and sell different varieties of a product. Their main findings can be summarized as follows: (a) In the pre-merger case, both downstream firms will locate away from the socially optimal location towards opposite directions (i.e., they will diverge from the middle of the linear market) and (b) A merger between the upstream monopolist and either of the downstream competitors, will induce both downstream firms to locate further away (compared to the pre-merger case) from the socially optimal location towards opposite directions.

^{*}Department of Economics, University of Piraeus, 80 Karaoli & Dimitriou Street, Piraeus 185 34, Greece. E-mail: kostasel@otenet.gr (Eleftheriou); njm@unipi.gr (Michelacakis).

[†]Corresponding author. Tel: +30 210 4142282; Fax: +30 210 4142346.

The precision of the above results has been compromised by both technical and conceptual flaws. Specifically, Beladi et al. (2010) fail to appropriately account for the wholesale price effects.

We show that:

(i) in the absence of any vertical merger, downstream firms will locate away from the social optimum

(ii) a merger between the upstream monopolist and the downstream leader will locate the firms away from the socially optimal location, to the direction of the un-integrated firm while

(iii) a merger between the upstream monopolist and the downstream follower will locate the firms away from the socially optimal location, to a direction depending on the wholesale price the upstream monopolist is charging the un-integrated leader.

2 Model and Results

The notation follows that in Beladi et al. (2010). Two downstream firms, R_i , $i = 1, 2$, compete in a vertically related industry where the only input (intermediate good) required for downstream production is provided by an upstream supplier, M . The intermediate good is transformed (on a one-to-one basis) by R_1 and R_2 into the differentiated final goods they sell to uniformly distributed consumers with unit density on a uni-dimensional (linear) market interval with support $[0, 1]$. The location of R_1 and R_2 is denoted by x and y , respectively, with $x < y$ in $[0, 1]$. Consumers are offered three varieties of a differentiated product: U and W from firm R_1 and V and W from firm R_2 . We assume that the fraction of consumers buying only good U is equal to the fraction of consumers buying good V , and both equal to c . A fraction b of consumers buys W .¹ Our assumptions about the provision of downstream goods imply that R_1 and R_2 enjoy monopoly power over the goods (or varieties) U and V while they compete for market share regarding the good (or variety) W . Transportation costs are equal to td , where t is a positive scalar and d is the distance

¹If a denotes the fraction of consumers deciding to buy no good, then it is clear that $b + 2c + a = 1$. Fractions b , c and a are constant throughout.

shipped. The maximum reservation price a consumer is willing to pay for any product is denoted by k . This price is sufficiently large and becomes relevant only for product varieties where there is no competition between the two firms. Downstream firms bear transportation costs, with their marginal delivered cost for selling at location z being equal to the marginal production costs plus the transportation cost for shipping the good to the consumer's location z . The pricing of downstream goods is as follows. For goods U and V , the downstream firms take advantage of their monopoly power and charge all consumers infinitesimally less than k . For good W , there is Bertrand competition à la Hoover (1937) and Lerner and Singer (1937). Specifically, the price charged for W by the firm that is closer to the consumer is equal to (or infinitesimally less than) the delivered cost of the firm that is further away.

The structure of the game follows Beladi et al. (2008) with the exception that R_1 and R_2 choose their locations sequentially (with R_1 being the leader and R_2 the follower) and not simultaneously. In particular, firms and consumers are engaged in a six stage game of complete information. In the first stage, M makes a take-it-or-leave-it two-part tariff offer, (w_i, F_i) , $i = 1, 2$, to firm R_i , where w_i denotes the marginal wholesale price and F_i is a fixed fee. In the second stage, R_1 and R_2 simultaneously decide whether or not to accept or decline the two-part tariff contract offered by the upstream monopolist, M . In the third stage, R_1 chooses its location in the market. Having observed the location decision of R_1 , R_2 chooses its location in the fourth stage. In stage five of the game downstream firms engage in spatial price discrimination by choosing delivered price schedules. In the final stage, consumers make their purchasing choices to clear the market.

The solution of the game is given by backward induction and a comparison is being made between three distinct cases: the case where none of the two firms merges upstream and the case where either the downstream leader (R_1) or the downstream follower (R_2) merges with M . Let $r = \frac{b}{4(b+c)}$ and $\Delta = 5b^2 + 16c^2 + 20bc$.

Absent a vertical merger, the profit function of R_2 is given by

$$\begin{aligned} \Pi_2(x, y) = & \left(c(k - w_2) - \frac{ct}{2} [y^2 + (1 - y)^2] \right) \\ & + \left(\int_{\left(\frac{x+y}{2} + \frac{w_2-w_1}{2t}\right)}^y b[t(2z - x - y) + w_1 - w_2] dz \right. \\ & \left. + \int_y^1 b[t(y - x) + w_1 - w_2] dz \right) - F_2 \end{aligned} \quad (1)$$

where $\frac{x+y}{2} + \frac{w_2-w_1}{2t}$ indicates the location of the indifferent consumer and $\frac{w_2-w_1}{2t} \leq \frac{y-x}{2}$.² More specifically, the location s of the indifferent consumer for good W is determined by equating the two delivered costs in regard to the common good W : $t(y - s) + w_2 = t(s - x) + w_1 \implies s = \frac{x+y}{2} + \frac{w_2-w_1}{2t}$. Solving the first order condition for profit maximization we obtain R_2 's reaction function:

$$y(x) = \frac{4t(b + c) + 2tbx + 2b(w_2 - w_1)}{2t(4c + 3b)} \quad (2)$$

Notice that $y(x)$ is expectedly a function of both w_1 and w_2 as opposed to the corresponding result in Beladi et al. (2010), equation (2) where $y(x)$ depends only on w_2 .

The profit function of R_1 is

$$\begin{aligned} \Pi_1(x, y(x)) = & \left(c(k - w_1) - \frac{ct}{2} [x^2 + (1 - x)^2] \right) \\ & + \left(\int_0^x b[t(y(x) - x) + w_2 - w_1] dz \right. \\ & \left. + \int_x^{\left(\frac{x+y(x)}{2} + \frac{w_2-w_1}{2t}\right)} b[t(x + y(x) - 2z) + w_2 - w_1] dz \right) - F_1 \end{aligned} \quad (3)$$

The above equations serve to correct equations (1), (2) and (3) respectively in Beladi et al. (2010). Beladi et al. (2010) (incorrectly) do not account for the fact that the delivered cost for good W of the firm that is located further away, which is equal to the sum of its transportation cost and marginal cost, is equal to the price charged to consumers by its rival [see Braid, 2008, p. 345]. Since the marginal cost of R_i is equal to the wholesale price w_i ,

²If $\frac{w_2-w_1}{2t} > \frac{y-x}{2}$, both firms are reduced to spatial-price discriminating monopolists where the common good W is now provided only by firm R . We consider this case trivial and focus only on the case where $\frac{w_2-w_1}{2t} \leq \frac{y-x}{2}$. Moreover, $\frac{w_2-w_1}{2t} \leq \frac{y-x}{2}$ guarantees the existence of non-negative profits for the downstream firms.

the profits of R_1 realizing a sale of good W to a customer located at place z , will be equal to $w_2 + t(y - z) - t|z - x| - w_1 = t(y - z) - t|z - x| + w_2 - w_1$. The corresponding profits for R_2 can be defined in a similar way. Further, the misconception regarding the delivered costs by Beladi et al. (2010) leads to the incorrect definition of the threshold determining the segment of the market captured by each firm concerning the common good W . This threshold, s , defines the limit of integration and its correct derivation is mentioned above.

Solving the first order condition for profit maximization of R_1 and substituting the solution into (2), we get

$$x = \left(\frac{1}{2} - r\right) + \frac{b[bt(3b + 4c) + (4b + 4c)^2(w_2 - w_1)]}{4t(b + c)\Delta} \quad (4)$$

$$y = \left(\frac{1}{2} + r\right) + \frac{b[tb^2 + 4(3b + 4c)(b + c)(w_2 - w_1)]}{4t(b + c)\Delta} \quad (5)$$

This leads to the following proposition.³

Proposition 1 *Absent the possibility of any merger, the Nash equilibrium locations of the two downstream firms, engaged in spatial competition in a vertically related industry, are $\left[\left(\frac{1}{2} - r\right) + \frac{b[bt(3b+4c)+(4b+4c)^2(w_2-w_1)]}{4t(b+c)\Delta}\right]$ for the leader and $\left[\left(\frac{1}{2} + r\right) + \frac{b[tb^2+4(3b+4c)(b+c)(w_2-w_1)]}{4t(b+c)\Delta}\right]$ for the follower. These locations are different from the social optimum, $(x^*, y^*) = \left(\frac{1}{2} - r, \frac{1}{2} + r\right)$.*⁴

The results in Proposition 1 are different than (and serve to correct) the results in Beladi et al. (2010).

In case of a merger taking place between the upstream firm M and R_1 , then $(w_1, F_1) = (0, 0)$ and the profits of the downstream firms R_1 and R_2 , respectively become

$$\begin{aligned} \Pi_1(x, y) = & \left(ck - \frac{ct}{2} [x^2 + (1 - x)^2] \right) \\ & + \left(\int_0^x b[t(y - x) + w_2] dz \right. \\ & \left. + \int_x^{\left(\frac{x+y}{2} + \frac{w_2}{2t}\right)} b[t(x + y - 2z) + w_2] dz \right) \end{aligned} \quad (6)$$

³Throughout the paper, for brevity, we omit the details of the calculations which are available upon request.

⁴The total welfare is defined as the sum of the total profits (upstream and downstream) and total consumer surplus. The socially optimal location can be easily derived by maximizing total welfare with respect to x and y and solving the system of equations.

$$\begin{aligned} \Pi_2(x, y) = & \left(c(k - w_2) - \frac{ct}{2} [y^2 + (1 - y)^2] \right) \\ & + \left(\int_{\left(\frac{x+y}{2} + \frac{w_2}{2t}\right)}^y b[t(2z - x - y) - w_2] dz \right. \\ & \left. + \int_y^1 b[t(y - x) - w_2] dz \right) - F_2 \end{aligned} \quad (7)$$

Solving the first order condition for R_2 's profit maximization we obtain R_2 's reaction function:

$$y(x) = \frac{2t(b + c) + b(w_2 + tx)}{t(3b + 4c)} \quad (8)$$

Either substituting (8) into (6) and solving the first order condition for R_1 's profit-maximization or setting $w_1 = 0$ into (4) and (5) we get the following Nash equilibrium locations respectively

$$x = \left(\frac{1}{2} - r \right) + \frac{b[bt(3b + 4c) + (4b + 4c)^2 w_2]}{4t(b + c)\Delta} \quad (9)$$

$$y = \left(\frac{1}{2} + r \right) + \frac{b[tb^2 + 4(3b + 4c)(b + c)w_2]}{4t(b + c)\Delta} \quad (10)$$

This leads to the following propositions.

Proposition 2 *If the downstream leader merges upstream, the Nash equilibrium locations of the two downstream firms, engaged in spatial competition in a vertically related industry, are $\left[\left(\frac{1}{2} - r \right) + \frac{b[bt(3b+4c)+(4b+4c)^2 w_2]}{4t(b+c)\Delta} \right]$ for the integrated firm and $\left[\left(\frac{1}{2} + r \right) + \frac{b[tb^2+4(3b+4c)(b+c)w_2]}{4t(b+c)\Delta} \right]$ for the un-integrated firm.*

Proposition 3 *A merger between the downstream leader and the upstream monopolist induces both downstream firms to move away from the social optimal location $(x^*, y^*) = \left(\frac{1}{2} - r, \frac{1}{2} + r \right)$, to the direction of the un-integrated firm decreasing their in-between distance provided $b \neq 0$.*

The results in Propositions 2 and 3 are different than (and serve to correct) the results

in Beladi et al. (2010) for the case of a merger between the upstream monopolist and the downstream leader.

In case of a merger between M and R_2 , M sets $(w_2, F_2) = (0, 0)$ and the profit functions of R_1 and R_2 are respectively given by

$$\begin{aligned} \Pi_1(x, y) = & \left(c(k - w_1) - \frac{ct}{2} [x^2 + (1 - x)^2] \right) \\ & + \left(\int_0^x b[t(y - x) - w_1] dz \right. \\ & \left. + \int_x^{\left(\frac{x+y}{2} - \frac{w_1}{2t}\right)} b[t(x + y - 2z) - w_1] dz \right) - F_1 \end{aligned} \quad (11)$$

$$\begin{aligned} \Pi_2(x, y) = & \left(ck - \frac{ct}{2} [y^2 + (1 - y)^2] \right) \\ & + \left(\int_{\left(\frac{x+y}{2} - \frac{w_1}{2t}\right)}^y b[t(2z - x - y) + w_1] dz \right. \\ & \left. + \int_y^1 b[t(y - x) + w_1] dz \right) \end{aligned} \quad (12)$$

Either solving the first order condition for profit maximization or letting $w_2 = 0$ in (2) we obtain R_2 's reaction function:

$$y(x) = \frac{4t(b + c) + 2tbx - 2bw_1}{2t(4c + 3b)} \quad (13)$$

Substituting (13) into (11) and solving the first order condition for R_1 's profit-maximization, or equivalently, letting $w_2 = 0$ in (4) and (5), we get the following Nash equilibrium locations

$$x = \left(\frac{1}{2} - r \right) + \frac{b[bt(3b + 4c) - (4b + 4c)^2 w_1]}{4t(b + c)\Delta} \quad (14)$$

$$y = \left(\frac{1}{2} + r \right) + \frac{b[tb^2 - 4(3b + 4c)(b + c)w_1]}{4t(b + c)\Delta} \quad (15)$$

This leads to the following two propositions.

Proposition 4 *If the downstream follower merges upstream, the Nash equilibrium locations of the two downstream firms, engaged in spatial competition in a vertically related industry,*

are $\left[\left(\frac{1}{2} - r\right) + \frac{b[bt(3b+4c)-(4b+4c)^2w_1]}{4t(b+c)\Delta}\right]$ for the un-integrated firm and $\left[\left(\frac{1}{2} + r\right) + \frac{b[tb^2-4(3b+4c)(b+c)w_1]}{4t(b+c)\Delta}\right]$ for the integrated firm.

Proposition 5 *A merger between the downstream follower and the upstream monopolist induces both downstream firms to move away from the social optimal location $(x^*, y^*) = \left(\frac{1}{2} - r, \frac{1}{2} + r\right)$, towards a direction depending on the wholesale price the follower through the upstream monopolist is charging the un-integrated leader.*

The results in Propositions 4 and 5 are different than (and serve to correct) the results in Beladi et al. (2010) for the case of a merger between the upstream monopolist and the downstream follower.

The intuition behind the result in Propositions 3 is the same as in Eleftheriou and Michelacakis (2016). More specifically, the un-integrated firm having lost its competitive edge is forced to give away part of its market share. Moreover, Proposition 5 implies that by controlling the wholesale price as a result of the vertical merger, the downstream follower takes the advantage over the downstream leader. It is precisely for this reason that a market leader is unlikely to accept a situation where his upstream provider has identical interests with his competition follower.

3 Conclusion

We have shown that in contrast to Beladi et al. (2010): (i) in the pre-merger case, downstream firms locate away from the social optimum and (ii) in the case of a vertical merger, downstream firms locate away from the social optimum to either the direction of the un-integrated follower or to the direction determined by the wholesale price charged to the un-integrated leader. The authors intend to further extend the results of the current study in a forthcoming paper.

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