License or entry in oligopoly

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6 September 2016

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Abstract

We consider an incentive of a choice of options for an outside innovating firm to license its new cost reducing technology to incumbent firms, or to enter into the market with or without license in an oligopoly with three firms. We will show that under linear demand and cost functions the results depend on the size of the market. When the market size is large, license to two incumbent firms without entry strategy is the optimum strategy for the innovating firm. However, when the market size is not large, license to one incumbent firm with or without entry strategy may be optimum.

Keywords: license, entry, oligopoly, innovating firm
JEL Classification code: D43, L13.

1 Introduction

When an outside innovating firm has a superior cost reducing technology with which a good can be produced at lower production cost, it may sell licenses to use its technology to incumbent firms, or it may enters into the market and at the same time sell licenses to incumbent firms, or enters into the market without license. We examine an incentive of a choice of options for an outside innovating firm to license its new cost reducing

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technology to incumbent firms without entry, or to enter into the market with or without license to incumbent firms in an oligopoly with three firms. Two incumbent firms and one outside innovating firm. The innovating firm can sell the license to one of two incumbent firms, or both of two incumbent firms.

Assuming linear demand and cost functions, we will show that when the market size is large, license to two incumbent firms with entry strategy is the optimum strategy for the innovating firm. However, when the market size is not so large, license to one incumbent firm with or without entry strategy may be optimum. It seems to be natural that the larger the market size is, the larger the number of active firms and the number of new technology adopting firms.

In the next section we review some related results. In Section 3 we present a model of this paper. In Section 4 we overview our main results. In Section 5 we consider entry with or without license strategy. In Section 6 we consider license only strategy. In Section 7 we calculate license fees and the profits of the firms. We describe grounds of license fees in some details in Section 4. In Section 8 we analyze the optimum strategy for the innovating firm.

2 Some related studies

There are many references about technology adoption or R&D investment in duopoly or oligopoly. Lots of researches focus on the relation between technology licensor and licensee. The difference of means of contracts which are royalties, up-front fees, the combinations of these two and auction are well discussed (Katz and Shapiro (1985) and Kamien and Tauman (1986)). Kamien and Tauman (1986) shows that if the licensor does not have production capacity, fixed fee is better than royalty and it is also better for consumers. This topic under Stackelberg oligopoly is discussed in Kabiraj (2004) when the licensor does not have production capacity, and discussed in Wang and Yang (2004), Kabiraj (2005) and Filippini (2005) when the licensor has production capacity. A Cournot oligopoly with fixed fee under cost asymmetry is analyzed in La Manna (1993). He shows that if technologies can be replicated perfectly, a lower-cost firm always has the incentive to transfer its technology and hence a Cournot-Nash equilibrium cannot be fully asymmetric, but there exists no non-cooperative Nash equilibrium in pure strategies.

On the other hand, using cooperative game theory, Watanabe and Muto (2008) analyses bargaining between licensor with no production capacity and oligopolistic firms. In resent research, market structure and technology improvement is analyzed. Boone (2001) and Matsumura et. al. (2013), respectively, find a non-monotonic relation between intensity of competition and innovation. Also, Pal (2010) shows that technology adoption may change the market outcome. The social welfare is larger in Bertrand competition than in Cournot competition. However, if we consider technology adoption, Cournot competition may make more social welfare than Bertrand competition under differentiated goods market.

Some other studies are worthy of mention. Elberfeld and Nti (2004) examines the adoption of a new technology in oligopoly, where there is ex-ante uncertainty about variable costs of the new technology, and shows that if in equilibrium both old and new technologies are employed, more uncertainty about the new technology increases (decreases) the number of innovating firms and decreases (increases) the product price if the up-front investment is large (small). Zhang et. al. (2014) analyzes the effect of information spillovers when the outcome of R&D is uncertain in a two-stage Cournot oligopoly model where a subset of firms first make a choice between two alternative production technologies independently and then all firms compete in quantity. In Hattori and Tanaka (2014) and Hattori and Tanaka (2015) adoption of new technology in Cournot duopoly and Stackelberg duopoly is analyzed. Rebolledo and Sandonís (2012) presented analyses about the effectiveness of R&D subsidies in an oligopolistic model in the cases of international R&D competition and cooperation.

Sen and Tauman (2007) compared the license system when the licensor is an outsider and that when the licensor is an incumbent firm. Duchene, Sen and Serfes (2015) argued that low license fee can be used to deter entry of potential entrants. In this paper we consider process innovation, that is, cost reducing innovation. Hattori and Tanaka (2016) analyzed similar problems about product innovation, that is, introduction of higher quality good in a duopoly with vertical product differentiation.

3 The model

There are three firms, Firm A, B and C. At present two of them, Firm B and C, produce a homogeneous good. Firm A has a superior cost reducing technology and can produce the good at lower cost than Firm B and C. We call Firm A the innovating firm, and Firm B and C the incumbent firms. Firm A have the following fives options.

(1) To enter into the market without license to incumbent firms.

(2) To enter into the market and license its technology to one incumbent firm.

(3) To enter into the market and license its technology to two incumbent firms.

(4) To license its technology to one incumbent firm, but not enter into the market.

(5) To license its technology to two incumbent firms, but not enter into the market.

Let p be the price, x_A, x_B and x_C be the outputs of Firm A, B and C. Then, the inverse demand function of the good is written as follows.

$$p = p(x_A + x_B + x_C), \text{ when Firm A enters,}$$

$$p = p(x_B + x_C), \text{ when Firm A does not enter.}$$

The cost functions of Firm A, B and C are denoted by $c_A(x_A)$, $c_B(x_B)$ and $c_C(x_C)$. $c_B(\cdot)$ and $c_C(\cdot)$ are the same functions without license. If Firm A licenses its technology to two incumbent firms, all cost functions are the same, and if Firm A licenses its technology to one incumbent firm (for example Firm C), then the cost functions of Firm A and C are the same.

4 Overview of the results

We assume that the market is not so small and the innovating firm does not become a monopolist when it enters into the market without license. We consider the following two cases.

(1) Case 1: The market is large and the output of an incumbent firm which does not buy the license to use cost reducing technology is positive even when only one incumbent firm buys the license.

(2) Case 2: The market is a bit small and the output of an incumbent firm which does not buy the license to use cost reducing technology is zero when only one incumbent firm buys the license.

According to the model of Sen and Tauman (2007) we assume that the innovating firm auctions off one or two licenses to use its superior technology to the incumbent firms conditional on that it enters into the market or not enter[1].

The license fees in various cases are determined based on willingness to pay of the incumbent firms as follows.

(1) If the innovating firm sells the license to one incumbent firm and enter into the market, the license fee is equal to

> "the profit of an incumbent firm when only this firm buys the license and the innovating firm enters" minus "its profit when only the rival firm buys the license and the innovating firm enters".

This is because each incumbent firm knows that irrespective of whether it becomes a licensee or not, there will always be one licensee, and the innovating firm enters into the market.

(2) If the innovating firm sells the license to one incumbent firm and does not enter into the market, the license fee is equal to

> "the profit of an incumbent firm when only this firm buys the license and the innovating firm does not enter" minus "its profit when only the rival firm buys the license and the innovating firm does not enter".

Again this is because each incumbent firm knows that irrespective of whether it becomes a licensee or not, there will always be one licensee, and the innovating firm does not enter.

(3) If the innovating firm sells the licenses to two incumbent firms and enter into the market, the license fee is equal to

[1] In Sen and Tauman (2007) they consider two cases, innovation by an outside innovator and innovation by an incumbent firm. However, they do not consider possibility of entry of an outside innovator into the market.

"the profit of an incumbent firm when two firms buy the licenses and the innovating firm enters" minus "its profit when only the rival firm buys the license and the innovating firm enters".

This is because each incumbent firm knows that if it does not become a licensee, there will be only one licensee, and the innovating firm enters into the market.

(4) If the innovating firm sells the license to one incumbent firm and does not enter into the market, the license fee is equal to

"the profit of an incumbent firm when two firms buy the licenses and the innovating firm does not enter" minus "its profit when only the rival firm buys the license and the innovating firm does not enter".

Again this is because each incumbent firm knows that if it does not become a licensee, there will be only one licensee, and the innovating firm does not enter.

The results depend on the size of the market, or the volume of demand. We will show the following results.

(1) When the size of the market is very large, license to two incumbent firms with entry strategy is the optimum strategy. (Proposition 1 (1))

(2) When the size of the market is a bit large, license to one incumbent firm with entry strategy is the optimum strategy. (Proposition 1 (2))

(3) When the size of the market is a bit small, license to one incumbent firm with entry strategy is the optimum strategy. (Proposition 2 (1))

(4) When the size of the market is very small, license to one incumbent firm without entry strategy is the optimum strategy. (Proposition 2 (2))

It seems to be natural that the larger the market size is, the larger the number of active firms and the number of new technology adopting firms.

Our results are somewhat similar to the results in Sen and Tauman (2007). However, they did not considered entry by an outside innovator.

5 Entry with or without license strategy

First we consider cases where Firm A enters into the market with or without license to incumbent firms.

5.1 The basic model

If Firm A enters into the market, the market becomes a tripoly. The profits of Firm A, B and C are written as

$$\pi_A = p(x_A + x_B + x_C)x_A - c_A(x_A),$$

$$\pi_B = p(x_A + x_B + x_C)x_B - c_B(x_B),$$
$$\pi_C = p(x_A + x_B + x_C)x_C - c_C(x_C).$$

We assume Cournot type behavior of the firms. The conditions for profit maximization are

$$p + p'x_A - c'_A = 0,\ p + p'x_B - c'_B = 0,\ p + p'x_C - c'_C = 0.$$

The second order conditions are

$$2p' + p''x_A - c''_A < 0,\ 2p' + p''x_B - c''_B < 0,\ 2p' + p''x_C - c''_C < 0.$$

Hereafter we assume that the second order conditions in each case are satisfied.

5.2 Linear demand and cost functions

Specifically we assume that the inverse demand function and the cost functions are linear. The inverse demand function is written as follows.

$$p = a - (x_A + x_B + x_C),$$

where a is a positive constant. The marginal cost of Firm A is c_A, and the marginal costs of Firm B and C are c_B, where $0 < c_A < c_B$. However, if Firm B or C buys a license to use cost reducing technology from Firm A, its marginal cost is c_A. There is no fixed cost. We assume $a > 2c_B - c_A$ so that the innovating firm should not be a monopolist when it enters into the market without license. We consider two cases.

(1) Case 1: $a > 3c_B - 2c_A$. Then, the output of an incumbent firm which does not buy the license to use cost reducing technology is positive when one firm buys the license.

(2) Case 2: $a \leq 3c_B - 2c_A$. Then, the output of an incumbent firm which does not buy the license to use cost reducing technology is zero when one firm buys the license.

5.2.1 License to no incumbent firm

When Firm A does not sell a license to use cost reducing technology, the profit maximizing conditions are

$$a - 2x_A - x_B - x_C - c_A = 0,$$
$$a - x_A - 2x_B - x_C - c_B = 0,$$
$$a - x_A - x_B - 2x_C - c_B = 0.$$

Denote the equilibrium values of the outputs and profits of the firms and the price by $x_A^{e0}, x_B^{e0}, x_C^{e0}, \pi_A^{e0}, \pi_B^{e0}, \pi_C^{e0}$ and p^{e0}. Then,

$$x_A^{e0} = \frac{a - 3c_A + 2c_B}{4},\ x_B^{e0} = x_C^{e0} = \frac{a + c_A - 2c_B}{4},\ p^{e0} = \frac{a + c_A + 2c_B}{4},$$

$$\pi_A^{e0} = \frac{(a - 3c_A + 2c_B)^2}{16},\ \pi_B^{e0} = \pi_C^{e0} = \frac{(a + c_A - 2c_B)^2}{16}.$$

5.2.2 License to one incumbent firm

When Firm A sells a license to one incumbent firm, we assume that this firm is Firm C. Then, the profit maximizing conditions are

$$a - 2x_A - x_B - x_C - c_A = 0,$$
$$a - x_A - 2x_B - x_C - c_B = 0,$$
$$a - x_A - x_B - 2x_C - c_A = 0.$$

There are two cases.

(1) Case 1: $a > 3c_B - 2c_A$.

Denote the equilibrium values of the outputs and profits of the firms and the price by x_A^{el1}, x_B^{el1}, x_C^{el1}, π_A^{el1}, π_B^{el1}, π_C^{el1} and p^{el1}. Denote the license fee in this case by L^{el1}. Then,

$$x_A^{el1} = x_C^{el1} = \frac{a - 2c_A + c_B}{4},\ x_B^{el1} = \frac{a + 2c_A - 3c_B}{4},$$
$$p^{el1} = \frac{a + 2c_A + c_B}{4},\ \pi_A^{el1} = \frac{(a - 2c_A + 2c_B)^2}{16},$$
$$\pi_B^{el1} = \frac{(a + 2c_A - 3c_B)^2}{16},\ \pi_C^{el1} - L^{el1} = \frac{(a - 2c_A + c_B)^2}{16} - L^{el1}.$$

(2) Case 2: $a \le 3c_B - 2c_A$.

Then, the equilibrium values of the outputs and profits of the firms are as follows.

$$x_A^{el1} = x_C^{el1} = \frac{a - c_A}{3},\ x_B^{el1} = 0,$$
$$p^{el1} = \frac{a + 2c_A}{3},\ \pi_A^{el1} = \frac{(a - c_A)^2}{9},$$
$$\pi_B^{el1} = 0,\ \pi_C^{el1} - L^{el1} = \frac{(a - c_A)^2}{9} - L^{el1}.$$

5.2.3 Licenses to two incumbent firms

When Firm A sells licenses to both incumbent firms, the profit maximizing conditions are

$$a - 2x_A - x_B - x_C - c_A = 0,$$
$$a - x_A - 2x_B - x_C - c_A = 0,$$
$$a - x_A - x_B - 2x_C - c_A = 0.$$

Denote the equilibrium values of the outputs and profits of the firms and the price by x_A^{el2}, x_B^{el2}, x_C^{el2}, π_A^{el2}, π_B^{el2}, π_C^{el2} and p^{el2}. Denote the license fee in this case by L^{el2}. Then,

$$x_A^{el2} = x_B^{el2} = x_C^{el2} = \frac{a - c_A}{4},\ p^{el2} = \frac{a + 3c_A}{4},$$
$$\pi_A^{el2} = \frac{(a - c_A)^2}{16},\ \pi_B^{el2} - L^{el2} = \pi_C^{el2} - L^{el2} = \frac{(a - c_A)^2}{16} - L^{el2}.$$

6 License only strategy

In this section we assume that Firm A licenses its technology to one or two incumbent firms, but it does not enter into the market.

6.1 The basic model

If Firm A does not enter into the market, it is a duopoly. The profits of the firms are written as

$$\pi_B = p(x_B + x_C)x_B - c_B(x_B),$$
$$\pi_C = p(x_B + x_C)x_C - c_C(x_C).$$

We assume Cournot type behavior of the firms. The conditions for profit maximization are

$$p + p'x_B - c'_B = 0,$$
$$p + p'x_C - c'_C = 0.$$

6.2 Linear demand and cost functions

We assume that the demand function and the cost functions are linear. Similarly to the previous section, the demand function is written as follows.

$$p = a - (x_B + x_C),$$

where a is a positive constant. The marginal costs of Firm B and C are c_B. However, if Firm B or C buys a license to use cost reducing technology from Firm A, its marginal cost is c_A, where $0 < c_A < c_B$. There is no fixed cost.

6.2.1 License to one incumbent firm

When Firm A sells a license to one incumbent firm, we assume that this firm is Firm C. Then, the profit maximizing conditions are

$$a - 2x_B - x_C - c_B = 0,$$

$$a - x_B - 2x_C - c_A = 0.$$

Denote the equilibrium values of the outputs and profits of the firms and the price by $x_B^{l1}, x_C^{l1}, \pi_B^{l1}, \pi_C^{l1}$ and p^{l1}. Denote the license fee in this case by L^{l1}. Then,

$$x_B^{l1} = \frac{a + c_A - 2c_B}{3},\ x_C^{l1} = \frac{a - 2c_A + c_B}{3},$$

$$p^{l1} = \frac{a + c_A + c_B}{3},$$

$$\pi_B^{l1} = \frac{(a + c_A - 2c_B)^2}{9},\ \pi_C^{l1} - L^{l1} = \frac{(a - 2c_A + c_B)^2}{9} - L^{l1}.$$

6.2.2 License to two incumbent firms

When Firm A sells licenses to both incumbent firms, the profit maximizing conditions are

$$a - 2x_B - x_C - c_A = 0,$$
$$a - x_B - 2x_C - c_A = 0.$$

Denote the equilibrium values of the outputs and profits of the firms and the price by $x_B^{l2}, x_C^{l2}, \pi_B^{l2}, \pi_C^{l2}$ and p^{l2}. Denote the license fee in this case by L^{l2}. Then,

$$x_B^{l2} = x_C^{l2} = \frac{a - c_A}{3},$$
$$p^{l2} = \frac{a + 2c_A}{3},$$
$$\pi_B^{l2} - L^{l2} = \pi_C^{l2} - L^{l2} = \frac{(a - c_A)^2}{9} - L^{l2}.$$

7 License fees and the profits of the firms

As stated in Section 4 according to the model of Sen and Tauman (2007) we assume that the innovating firm auctions off one or two licenses to use its superior technology to the incumbent firms conditional on that it enters into the market or not enter. The license fees in various cases are determined based on willingness to pay of the incumbent firms as follows.

(1) If the innovating firm sells the license to one incumbent firm and enter into the market, the license fee is equal to

> "the profit of an incumbent firm when only this firm buys the license and the innovating firm enters" minus "its profit when only the rival firm buys the license and the innovating firm enters".

Denote the license fee in this case by L^{el1}.

(2) If the innovating firm sells the license to one incumbent firm and does not enter into the market, the license fee is equal to

> "the profit of an incumbent firm when only this firm buys the license and the innovating firm does not enter" minus "its profit when only the rival firm buys the license and the innovating firm does not enter".

Denote the license fee in this case by L^{l1}.

(3) If the innovating firm sells the licenses to two incumbent firms and enter into the market, the license fee is equal to

> "the profit of an incumbent firm when two firms buy the licenses and the innovating firm enters" minus "its profit when only the rival firm buys the license and the innovating firm enters".

Denote the license fee in this case by L^{el2}.

(4) If the innovating firm sells the license to one incumbent firm and does not enter into the market, the license fee is equal to

> "the profit of an incumbent firm when two firms buy the licenses and the innovating firm does not enter" minus "its profit when only the rival firm buys the license and the innovating firm does not enter".

Denote the license fee in this case by L^{l2}.

Therefore, we see

$$\begin{aligned} L^{el1} &= \pi_C^{el1} - \pi_B^{el1}, \\ L^{l1} &= \pi_C^{l1} - \pi_B^{l1}, \\ L^{el2} &= \pi_C^{el2} - \pi_B^{el1}, \\ L^{l2} &= \pi_C^{l2} - \pi_B^{l1}. \end{aligned}$$

7.1 Case 1

In this case $a > 3c_B - 2c_A$. Thus, there exists t such that $t > 1$ and

$$a = t(c_B - c_A) + 2c_B - c_A.$$

When t is large (or small), the market size or the volume of demand is large (or small). The license fees and the total profits of the innovating firm in various cases are as follows.

(1) When the innovating firm enters into the market and sells the license to one incumbent firm, the license fee is

$$L^{el1} = \pi_C^{el1} - \pi_B^{el1} = \frac{(t+1)(c_B - c_A)^2}{2},$$

and the total profit of the innovating firm is

$$L^{el1} + \pi_A^{el1} = \frac{(t^2 + 14t + 17)(c_B - c_A)^2}{16}.$$

(2) When the innovating firm enters into the market and sells the licenses to two incumbent firms, the license fee for each incumbent firm is

$$L^{el2} = \pi_C^{el2} - \pi_B^{el1} = \frac{3(2t+1)(c_B - c_A)^2}{16},$$

and the total profit of the innovating firm is

$$2L^{el2} + \pi_A^{el2} = \frac{(t^2 + 16t + 10)(c_B - c_A)^2}{16}.$$

(3) When the innovating firm does not enter into the market and sells the license to one incumbent firm, the license fee is

$$L^{l1} = \pi_C^{l1} - \pi_B^{l1} = \frac{(2t + 3)(c_B - c_A)^2}{3}.$$

(4) When the innovating firm does not enter into the market and sells the licenses to two incumbent firms, the license fee for each incumbent firm is

$$L^{l2} = \pi_C^{l2} - \pi_B^{l1} = \frac{4(t + 1)(c_B - c_A)^2}{9}.$$

The total profit of the innovating firm is

$$2L^{l2} = \frac{8(c_B - c_A)^2(t + 1)}{9}.$$

Also we have

$$\pi_A^{e0} = \frac{(t + 4)^2(c_B - c_A)^2}{16}.$$

7.2 Case 2

Since $2c_B - c_A < a \leq 3c_B - 2c_A$, there exists t such that $0 < t \leq 1$ and

$$a = t(c_B - c_A) + 2c_B - c_A.$$

Then, the license fees and the total profits of the innovating firm in various cases are as follows.

(1) When the innovating firm enters into the market and sells the license to one incumbent firm, the license fee is

$$L^{el1} = \pi_C^{el1} - \pi_B^{el1} = \frac{(t + 2)^2(c_B - c_A)^2}{9},$$

and the total profit of the innovating firm is

$$L^{el1} + \pi_A^{el1} = \frac{2(t + 2)^2(c_B - c_A)^2}{9}.$$

(2) When the innovating firm enters into the market and sells the licenses to two incumbent firms, the license fee for each incumbent firm is

$$L^{el2} = \pi_C^{el2} - \pi_B^{el1} = \frac{(t + 2)^2(c_B - c_A)^2}{16},$$

and the total profit of the innovating firm is

$$2L^{el2} + \pi_A^{el2} = \frac{3(t + 2)^2(c_B - c_A)^2}{16}.$$

(3) When the innovating firm does not enter into the market and sells the license to one incumbent firm, the license fee is

$$L^{l1} = \pi_C^{l1} - \pi_B^{l1} = \frac{(2t+3)(c_B - c_A)^2}{3}.$$

(4) When the innovating firm does not enter into the market and sells the licenses to two incumbent firms, the license fee for each incumbent firm is

$$L^{l2} = \pi_C^{l2} - \pi_B^{l1} = \frac{4(t+1)(c_B - c_A)^2}{9}.$$

The total profit of the innovating firm is

$$2L^{l2} = \frac{8(t+1)(c_B - c_A)^2}{9}.$$

Also, we have

$$\pi_A^{e0} = \frac{(t+4)^2(c_B - c_A)^2}{16}.$$

8 The optimum strategy for the innovating firm

We must compare the profits of the innovator gained by various strategies to investigate its optimum strategy.

8.1 Case 1

Comparing $\pi_A^{el1} + L^{el1}$, $2L^{l2}$, L^{l1} and π_A^{e0}, we get

$$\pi_A^{el1} + L^{el1} - 2L^{l2} = \frac{(9t^2 - 2t + 25)(c_B - c_A)^2}{144},$$

$$\pi_A^{el1} + L^{el1} - L^{l1} = \frac{(t+3)(3t+1)(c_B - c_A)^2}{48},$$

$$\pi_A^{el1} + L^{el1} - \pi_A^{e0} = \frac{(6t+1)(c_B - c_A)^2}{16}.$$

Since $t > 1$, all of them are positive. Thus, license to one incumbent firm with entry strategy is more beneficial than license to one or two incumbent firms without entry strategies and entry only strategy.

Comparing $\pi_A^{el1} + L^{el1}$ and $\pi_A^{el2} + 2L^{el2}$ yields

$$\pi_A^{el1} + L^{el1} - (\pi_A^{el2} + 2L^{el2}) = -\frac{(2t-7)(c_B - c_A)^2}{16}.$$

This is positive when $t < \frac{7}{2}$ and negative when $t > \frac{7}{2}$. Thus, we obtain the following results.

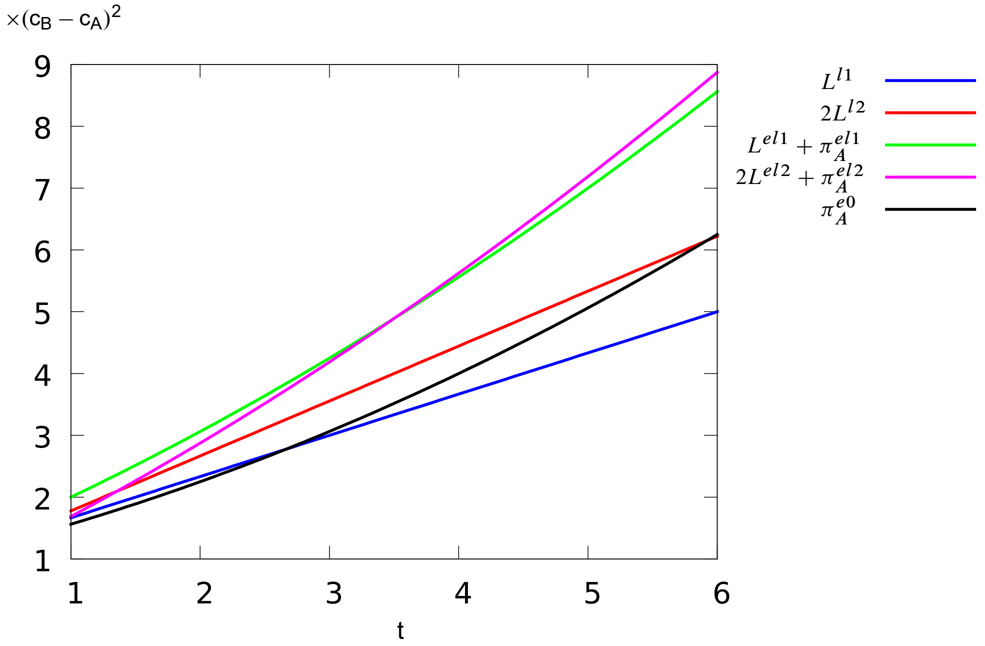

Figure 1: Case 1

Proposition 1. *In Case 1 where* $t > 1$*;*

(1) *when* $t > \frac{7}{2}$*, license to two incumbent firms with entry strategy is the optimum strategy;*

(2) *when* $1 < t < \frac{7}{2}$*, license to one incumbent firm with entry strategy is the optimum strategy.*

When $t = \frac{7}{2}$*, both strategies are optimum.*

Figure 1 illustrates the results of this case.

8.2 Case 2

Comparing L^{l1} and $\pi_A^{el1} + L^{el1}$, we get

$$L^{l1} - \pi_A^{el1} + L^{el1} = -\frac{(2t^2 + 2t - 1)(c_B - c_A)^2}{9}.$$

When $0 < t < \frac{\sqrt{3}-1}{2}$, this is positive, and when $t > \frac{\sqrt{3}-1}{2}$, this is negative. Comparing $L^{l1}, 2L^{l2}, \pi_A^{el2} + 2L^{el2}$ and π_A^{e0}, we get

$$L^{l1} - 2L^{l2} = -\frac{(2t - 1)(c_B - c_A)^2}{9},$$

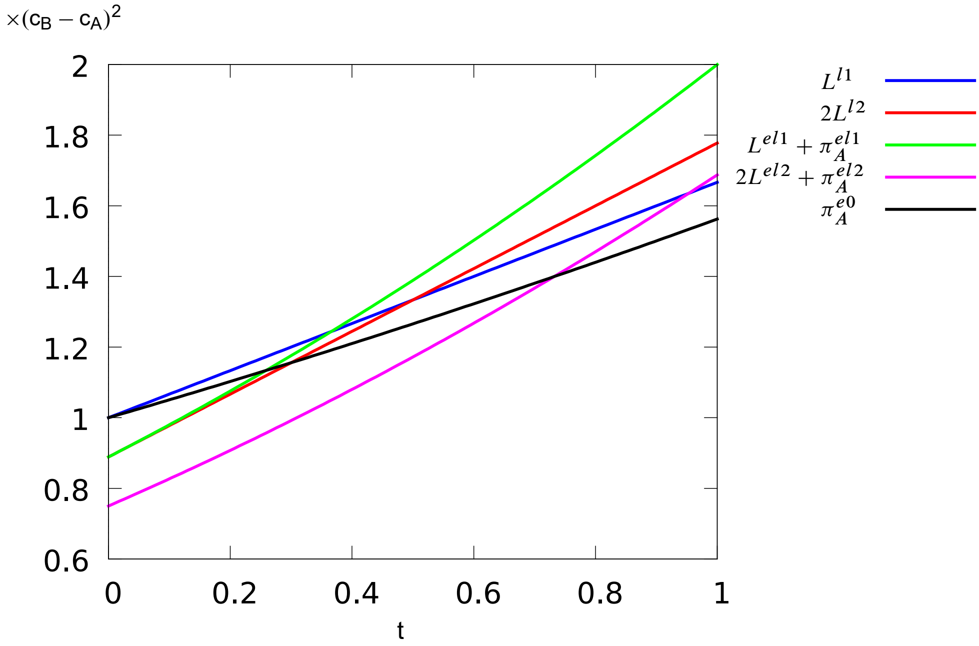

Figure 2: Case 2

$$L^{l1} - (\pi_A^{el2} + 2L^{el2}) = -\frac{(c_B - c_A)^2(9t^2 + 4t - 12)}{48},$$

$$L^{l1} - \pi_A^{e0} = -\frac{t(3t - 8)(c_B - c_A)^2}{48}.$$

If $0 < t < \frac{\sqrt{3}-1}{2}$, they are all positive. Then, license to one incumbent firm without entry strategy is more beneficial than all other strategies.

Comparing $\pi_A^{el1} + L^{el1}$, $2L^{l2}$, $\pi_A^{el2} + 2L^{el2}$ and π_A^{e0}, we get

$$\pi_A^{el1} + L^{el1} - 2L^{l2} = \frac{2(c_B - c_A)^2 t^2}{9},$$

$$\pi_A^{el1} + L^{el1} - (\pi_A^{el2} + 2L^{el2}) = \frac{5(t + 2)^2(c_B - c_A)^2}{144},$$

$$\pi_A^{el1} + L^{el1} - \pi_A^{e0} = \frac{(23t^2 + 56t - 16)(c_B - c_A)^2}{144}.$$

If $t > \frac{\sqrt{3}-1}{2}$, they are all positive. Then, license to one incumbent firm with entry strategy is more beneficial than all other strategies.

Summarizing the results;

Proposition 2. *In Case 2;*

(1) *when* $1 < t < \frac{\sqrt{3}-1}{2}$, *license to one incumbent firm with entry strategy is the optimum strategy.*

(2) *when* $0 < t < \frac{\sqrt{3}-1}{2}$*, license to one incumbent firm without entry strategy is the optimum strategy;*

When $t = \frac{\sqrt{3}-1}{2}$*, both strategies are optimum.*

Figure 2 illustrates the results of this case.

9 Concluding Remarks

We have analyzed a choice of an innovating firm to license its technology to one or two incumbent firms or to enter into the market, which is duopolistic, with or without license in an oligopoly with three firms (tripoly). We have shown that the relative benefit of license and entry depends on the size of the market.

In the future research we want to study the public policy by the government to promote or prevent license or entry by an innovating firm in domestic and international settings, and extend the analysis in this paper to a case of vertical product differentiation.

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