Two consecutive steps in transformation: from Values to Prices and from Sectoral Rates to a Weighted Common Rate

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In Chapter 9. *Formation of a General Rate of Profit (Average Rate of Profit) and Transformation of the Values of Commodities into Prices of Production*, of Capital, volume III, Karl Marx states:

> *"The aggregate price of the commodities I to V would therefore equal their aggregate value, i. e., the sum of the cost-prices I to V plus the sum of the surplus-values, or profits, produced in I to V. It would hence actually be the money-expression of the total quantity of past and newly applied labour incorporated in commodities I to V. And in the same way the sum of the prices of production of all commodities produced in society — the totality of all branches of production — is equal to the sum of their values.*
> *This statement seems to conflict with the fact that under capitalist production the elements of productive capital are, as a rule, bought on the market, and that for this reason their prices include profit which has already been realised, hence, include the price of production of the respective branch of industry together with the profit contained in it, so that the profit of one branch of industry goes into the cost-price of another. But if we place the sum of the cost-prices of the commodities of an entire country on one side, and the sum of its surplus-values, or profits, on the other, the calculation must evidently be right. For instance, take a certain commodity A. Its cost-price may contain the profits of B, C, D, etc., just as the cost-prices of B, C, D, etc., may contain the profits of A. Now, as we make our calculation the profit of A will not be included in its cost-price, nor will the profits of B, C, D, etc., be included in theirs. Nobody ever includes his own profit in his cost-price. <u>If there are, therefore, n spheres of production, and if each makes a profit amounting to p, then their aggregate cost-price = k - np.</u> Considering the calculation as a whole we see that since the profits of one sphere of production pass into the cost-price of another, they are therefore included in the calculation as constituents of the total price of the end-product, and so <u>cannot appear a second time</u> on the profit side. If any do appear on this side, however, then only because the commodity in question is itself an ultimate product, whose price of production does not pass into the cost-price of some other commodity. "[Emphasis added]"*

According to this paragraph, Karl Marx eliminates the profits that appear in the inputs in order not to count profits twice and consequently have overvalued prices of production. Additionally, this way, the surplus value would be exactly the difference between the total value and the value of the cost-price.

It is clear that Karl Marx envisages this operation in a moment previous to the redistribution of the surplus value according to the capital set in motion by labour (invested) and by means of the calculation of a common rate of profit.

If instead of doing so, we do not remove the profits from inputs, we obtain higher sectoral “production prices” that, nevertheless, are proportional to previous values and cost-prices (costs of inputs in value) in the following proportion, based in the rate of surplus value:

$$P'_w=\Lambda'.(1+(1-\Lambda'.B)/\Lambda'.B)$$

Where the symbols are: prices of production measured in wage units P'_w, labour values Λ', non-paid labour time $(1-\Lambda'.B)$ and wage $\Lambda'.B$, or paid labour time, per unit of product, respectively. We must keep in mind that prices and costs are being calculated with <u>different</u> sectoral rates of profit[1].The wage is obtained adding the values of the commodities that form the consumption of workers. The meaning of the formula is that prices are equal to values multiplied by 1 plus the rate of surplus value.

Not removing profits from inputs, apart from maintaining the above referred proportionality, does not affect the weighted rate of profit (1.03859) as can be seen in Tables 1 and 3 in the example below.

1

$$P' = P' * \begin{pmatrix} a_{11} * (1 + r_1) & a_{12} * (1 + r_2) & a_{13} * (1 + r_3) \\ a_{21} * (1 + r_1) & a_{22} * (1 + r_2) & a_{23} * (1 + r_3) \\ a_{31} * (1 + r_1) & a_{32} * (1 + r_2) & a_{33} * (1 + r_3) \end{pmatrix} + (1 + r)' * L$$

P’ row vector of production prices in wage units

$(1+r)'$ row vector of rates of profit

L column vector of labour force employed per unit of product

Example

Table 1

Marxian representation of a capitalist economy (substituting surplus value by a rate of profit calculated using it) Production of a unit of product in every sector				
	C Constant capital	**V Variable capital**	**(1+r_i)** rates of profit based on surplus value	**Value of the unit of product**
Sector 1	0.94528	0.933475	**1.035367**	**1.9452**
Sector 2	1.00225	0.093348	**1.006118**	**1.1023**
Sector 3	0.514785	1.86695	**1.055911**	**2.5149**
Total	2.462315	2.893773		**5.5624**
1+Weighted rate of profit: Total value/(C+V)			**1.038519**	

Table 2

Prices of production (in wage units) calculated with the previous sectoral rates of profit in value (see footnote 1)	
p_{w1}	**2.0840**
P_{w2}	**1.1819**
P_{w3}	**2.6926**
Sum of new prices vector	**5.9585**
Sum of cost-prices	**5.7376**

Table 3

Calculation of aggregates with the above prices of production
C=2.6376
$[p_1 \cdot a_{11}(=2.0840 \cdot 0.3)+p_2 \cdot a_{21}(=1.1819 \cdot 0.10)+p_3 \cdot a_{31}(=2.6926 \cdot 0.10)] \cdot [(1+r_1)(=1.0354)]$
$[p_1 \cdot a_{12}(=2.0840 \cdot 0.2)+p_2 \cdot a_{22}(=1.1819 \cdot 0.10)+p_3 \cdot a_{32}(=2.6926 \cdot 0.2)] \cdot [(1+r_2)(=1.006)]$
$[p_1 \cdot a_{13}(=2.0839 \cdot 0.2)+p_2 \cdot a_{23}(=1.1819 \cdot 0)+p_3 \cdot a_{33}(=2.6926 \cdot 0.05)] \cdot [(1+r_3)(=1.0559)]$
V=3.1
$[p_1 \cdot a_{11}(=2.0840 \cdot 0.1)+p_2 \cdot a_{21}(=1.1819 \cdot 0.1)+p_3 \cdot a_{31}(=2.6926 \cdot 0.25)] \cdot [(1+r_1)(=1.0354)]$
$[p_1 \cdot a_{12}(=2.0840 \cdot 0.01)+p_2 \cdot a_{22}(=1.1819 \cdot 0.01)+p_3 \cdot a_{32}(=2.6926 \cdot 0.025)] \cdot [(1+r_2)(=1.006)]$
$[p_1 \cdot a_{13}(=2.0840 \cdot 0.2)+p_2 \cdot a_{23}(=1.1819 \cdot 0.2)+p_3 \cdot a_{33}(=2.6926 \cdot 0.5)] \cdot [(1+r_3)(=1.0559)]$
Leontief and wage matrices, respectively
$C_1+V_1=2.0126$
$C_2+V_2=1.1735$
$C_3+V_3=2.5514$
C+V=5.7376
(C+V)·1.0385=5.9585=Total price

The proportionality of prices and values can be seen for instance, comparing the following aggregates in Table 1 and Table 3:

Total Price/Total value (5.9589/5.5624)=1.071=1+(1-0.9335)/0.9335

Constant capital (prices)/ Constant capital (values) (2.6378/2.4623)=1.071

Hence, the transformation of values into prices does not represent a problem to be solved.

We can now proceed to the equalization of rates of profit. This can only be done in the sphere of the market: i.e., in prices, and not in the ideal situation which Karl Marx presents as previous to the market, without specifying how the surplus value is to be transferred from one sector to other.

The transformation problem should be limited only to the process of equalizing the sectoral rates of profit, once in a price system. If we intend to have a common rate of profit, even if the rate of profit is the weighted original one in value, then the prices of production have to be slightly different –change its proportionality with values - to accommodate such common rate in every sector. This only affects to the monetary expression of labour time, i.e., the relation between the sum of prices and the sum of values would be different, but not the Marxian rate of profit (weighted rate) which is maintained (i.e. the proportion between the employed inputs and the obtained outputs)

Nevertheless, as we cannot maintain the same common rate of profit in values (the Marxian rate) without changing the sectoral prices, the newly calculated prices then do no keep the double equality (proportionality) with values anymore as we can see in Table 4 where the prices of production have changed in relation with Table 2.

Table 4

New prices of production (in wage units) calculated with the Marxian (weighted) rate (0,0385)	
p_{w1}	**2.0907**
P_{w2}	**1.2137**
P_{w3}	**2.6482**
Sum of new prices vector	**5.9539**
Sum of price cost	**5.7331**

It is worth noting that, consequently, the change of prices over values is a distorting artefact because it artificially changes the value in different sectors so as to achieve an equal remuneration for capitals, despite value is by definition equal in all sector. This also evidences that the system of production prices is, at least initially, independent of the value system.

Therefore, the only way to redistribute value with a common rate of profit is changing the initial relation 1 to 1 between sectoral labour values. It is independent of values or prices.

In our opinion, this derivation of the Marxian text gives a better understanding of the Marxian transformation complexities, and at the same time respects and repairs the Marxian basic insight. It particularly keeps the Marxian calculation of the rate of profits in value terms and the production prices as two separate steps.

Bibliography

Marx, Karl "***Ch. 9: Formation of a General Rate of Profit (Average Rate of Profit) and Transformation of the Values of Commodities into Prices of Production".*** **Capital.** Volume III. https://www.marxists.org/archive/marx/works/1894-c3/ch09.htm

Annex
Data

Initial data			
Leontief I/O matrix, A			
	0.30	0.20	0.20
	0.10	0.10	0.00
	0.10	0.20	0.05
L Labour employed in each industry (*per unit of product)*	1.00	0.10	2.00
B Wage goods *per unit of labour*	0.10	0.10	0.25
B·L matrix, wage matrix			
	0.10	0.01	0.20
	0.10	0.01	0.20
	0.25	0.025	0.25
Sum of inputs in value	5.3561		
Variable capital	2.8938		
Constant capital	2.4624		
Sum of outputs in value	5.5624		
Individual values Λ'			
Λ_1	1.9452		
Λ_2	1.1023		
Λ_3	2.5149		
Rate of profit in value (Marxian)	0.0385		
Wage *(in value, per unit of labour)* $\Lambda'.B$	0.9335		
Rate of surplus value $(1-\Lambda'.B)$	(0.0665=1-0.9335)		
Individual rates of profit in value			
r_1	0.0354		
r_2	0.0061		
r_3	0.0559		
Prices of production (in wage units) calculated with the previous sectoral rates:			
p_{w1}	2.0840		
P_{w2}	1.1819		
P_{w3}	2.6926		
Sum of new prices vector	5.9585		
Sum of price cost	5.7376		
Prices of production (in wage units) calculated with the Marxian rate (0,0385)			
p_{w1}	2.0907		
P_{w2}	1.2137		
P_{w3}	2.6482		
Sum of new prices vector	5.9539		
Sum of price cost	5.7331		
Rate of profit (eigenvalue)	0.043		
Sum of production prices vector (in wage units) (2.11+1.23+2.66)	6		

(All data taken from Vicenç Meléndez-Plumed.
"Some Questions on the Rate of Profits." https://ideas.repec.org/p/pra/mprapa/53664.html)