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Abstract: The rising of Global Value Chains (GVCs) during the last two decades has changed the nature of international trade and investment significantly with many new implications including the generation of value-added. To well understand the international trade in the GVCs context, it is crucial to have a completely decomposition of value-added in the most explicit way. We propose an improved accounting framework based on Koopman, Wang and Wei (2014)'s method by incorporating the Ghosh insight in the value-added demonstration. The new mathematical framework can easily decompose the exports in the form of final demand (including both the intermediate and final good exports). It contributes to the current value-added decomposition GVCs literature because the new method could successfully 1) trace the exports goods propagation route to all the way to the ultimate destination country; 2) trace the value-added terms to include both domestic and foreign double counting terms; 3) show where the value-added double counting terms come from and give explicit expression of domestic and foreign double counting terms. The new framework can also be used for bilateral exports (the most common trade pattern) decomposition directly. At last, we present the dis-aggregated decomposition results for sectoral and bilateral sectoral level trade between China and Brazil based on the WIOD database.

Key words: Input-Output table, Value-added decomposition, Global value chains

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Introduction

The rise of Global Value Chains (GVCs) during the last two decades has changed the nature of international trade and investment significantly with many new implications on the generation of value-added, job opportunities, income, measures of competitiveness, and trade policymaking. "The rise of GVCs is the most important factor shaping contemporary international trade, it has transformed not only international trade but also the international trade relationships. The high level of import intensity in export production has create an unprecedented level of interdependency among countries engaged in supply chains." (Director-General Pascal Lamy, 2012). Moreover, from the perspective of GVCs, more complicated issues emerged, including problems of world trade statistics and measurement (be more specific if necessary). For example, trade in intermediate goods has reached almost two thirds of international trade. Many countries like China or Mexico etc. participate in the GVCs by importing or exporting amounts of intermediate goods. The increasing of intermediate goods trade causes bias in from two prospects in trade statistics. Firstly, because of importing the foreign intermediate goods, the foreign value added will be embodied in the export for one country. The currency official trade statistic cannot measure exactly the domestic value added in the export. The value added bias emerged here are called as 'double counting' issue. Secondly, the exported intermediate goods will be re-exported to a third country or returned back home, the trade statistics bias that also includes the double counted term emerges between the official statistic trade and the fact trade as well, no matter in the gross export or bilateral export situation. The 'double counting' and multi-country production chains together imply that there is a hidden structure of trade in value added under the gross trade flows (Johnson and Noguera, 2012).

Based on that, international and national agencies and academic institutions started to focus on the issues of gross value that distorting the picture of bilateral trade balance and double counting trade flows. Among them, Japan is a pioneer in this field. The Institute of Developing Economies-Japan External Trade Organization (IDE-

JETRO) is one of the earliest to develop international input-output matrices that reflect inter-industrial trade linkages. IDE-JETRO subsequently teamed up with the World Trade Organization (WTO) to develop this work further to measure value-added trade. The WTO launched its “Made-in-the-World” initiative aiming at raising public awareness and deepening analysis of the GVC implications. In addition, more initiatives have also been made to get a more comprehensive picture of the GVCs. The OECD, in co-operation with the WTO, has estimated trade flows in value-added terms of trade flow. Inter-country input-output tables and a full matrix of bilateral trade flows are used to derive data on the value added by each country in the value chain, thus giving a better picture of trade flows related to the activities of firms in GVCs. This cooperation led to the TiVA (Trade in Value added) initiative. Other works contributing to international value-added measurement efforts have been undertaken by the United States International Trade Commission, the World Bank and the International Monetary Fund working with the Global Trade Analysis Project (GTAP) database. More recently, the United Nations Conference on Trade and Development (UNCTAD) derived trade in value-added indicators from EORA database (another academic database initiative).

In order to get a better understanding on the evolution of GVCs, various approaches have been applied in measuring GVCs. In this field, several researchers have examined the issue of vertical specialization on a systematic base to describe the increasing complexity of the production chains using examples of iPods, airplanes, andcars. For example, there are the sociological approaches adopted by Gary Gereffi, Timothy Sturgeon, John Humphrey (1990-2008); case studies by using firm-level data done by Xing and Detert (2010). In addition, the input-output based measures were proposed including the pioneering efforts of Hummels, Ishii, and Yi (2001) (HIY below). They suggested that a country can participate in vertical specialization in two ways: (a) using imported intermediate inputs to produce exports; (b) exporting intermediate goods that are used as inputs by other countries to produce goods for their exports.

HIY(2001) proposed a measure of vertical specialization from the import side(which they call the VS), which is the imported content share in a country's exports. HIY (2001) also proposed a second measure of the vertical specialization from the export side (which they call the VS1). It measures the value of intermediate exports sent indirectly through the third countries to the final destinations. Daudin et al (2011) singled out a particular subset of the VS1, the value of a country's exported goods that are used as imported inputs by the rest of the world to produce final goods and shipped back home. They call it the VS1*. Johnson and Noguera (2012) defined the value-added exports as the value-added produced in the source countries and absorbed in destination countries and proposed using value-added to gross export ratio, the 'VAX ratio' as a summary measurement of the value-added content of trade. Notably, Koopman, Wang, and Wei (referred as KWW below) (2014) relaxed the assumption in HIY and proposed a disaggregated accounting framework to decompose domestic and foreign contents that take the differences in using imported inputs by processing exporters versus normal exporters into consideration. However, KWW method just can decompose the exports flow in the gross country level, so some other scholars tried to extend this method into bilateral level exports flow. For example, Arne J. Nagenast and Robert Stehrer(2014), Alessandro Borin and Michele Mancini(2015).

Our approach of measurement is also related to KWW method which provides an accounting formula that quantifies types of double counted terms for the first time in export statistics. In the KWW mathematics framework, the term 6 and term 9 were called as "pure double counted terms". They are expressed by a coefficient multiplying a specific country's export, so there are two double counted parts in the domestic and foreign value-added parts respectively. However, there are no explicit interpretation about the 'pure double counting terms' in the KWW method, these terms were described as 'they are already captured by other terms in the gross export' in their paper. However where are the 'pure double counting' terms from? Is there another expression form to represent them for a better understanding? Meanwhile, the decomposition form should be symmetric and coherent, it means that the foreign value-added returned home terms should exit in the expression and the same decomposition form can be extended into bilateral and sectoral export level not only in the gross export. So it just was mentioned as in comment of KWW: KWW propose a further decomposition of the residual into what they call "foreign value added" and "pure double counted terms". With some modification foreign value added can be define in this approach as well, but only property in the context of a complete decomposition of global GDP. This needs a more extensive treatment and is left for further research. (Bart Los, Marcel P. Timmer and Gaaitzen J.de Vires, 2016)

If we consider export goods in which the value-added embodied continuous propagation rounds, the process means the value-added returned and being re-exported, after the initial round of exports. This part of value-added will be absorbed by a specific country and part of this will be returned back again and being re-exported again. Meanwhile, the value-added double counted term statistically will accumulate in the subsequent export propagation. This process will be an infinite loop until all the export goods are absorbed by the final demand completely at the end. According to the propagation process, if we consider the infinite rounds of export goods propagation, all the export goods will be finally absorbed by a specific country as the final goods, so the export decomposition will be expressed in a new form, all the decomposition terms should be final demand expression firstly. Concerning to the value-added double counting term, this term will emerge in the subsequent round of every export propagation rather than only in the initial round export. Ultimately, the double counted term should be appeared as an accumulated expression form. So the main purpose of this paper is to seek a new form of decomposition of trade expression in the infinite export goods propagation rounds which is different from KWW method.

According to the motivation above, this paper intends to completely decompose the export (and bilateral export) in the form of final demand, including the intermediate and final goods exports. In term of the decomposition form, we start from the ‘export input-output table’ to construct the export decomposition expression, and then provide a unified and clear mathematical framework to decompose the export goods into various components including terms of intermediate goods and final goods returning home or through the third country etc.in the infinite export goods propagation rounds from the Leontief insight. At the same time we give the value-added decomposition expression which provides another framework to decompose the export value-added into various components including domestic or foreign parts and double counting parts in the value-added infinite rounds propagation from the Ghosh insight. So in this mathematical framework, we can get an explicit exports goods and value-added flow route. The purpose of this paper include three aspects, 1) tracing the exports goods propagation route including the export goods propagation route and ultimate destination country; 2) tracing the value-added terms including domestic or foreign and double counting expression in this mathematical framework; 3) show where the value-added double counting terms come from and give explicit expression of domestic and foreign double counting terms.

This paper is organized as follows, Section 2 presents the general accounting mathematical framework, defines the export goods and value-added decomposition method started from the two country case, discusses the interpretation of decomposition terms. Based on the decomposition, we trace the export goods propagation route and the value-added in export expression in the condition for infinite round of export propagation. Section 3 describes the data sources and assumptions we use to implement the accounting exercise and presents our empirical analysis, and Section 4 concludes.

Methodology

2.1 Trace the export goods route: the Leontief insight

Our methodologies are rooted in Leontief (1936) whose work demonstrated that the amount and type of intermediate inputs needed in the production of one unit of output can be estimated based on the input-output (IO) structure across the industries. Using the linkages across industries, gross output in all stages of production that is needed to produce one unit of final goods can be traced. When the gross output flows associated with a particular level of final demand are known, value added production and trade can be simply derived by multiplying these flows with the value added to gross output ratio in each industry.

In IO table, all gross output must be used either an intermediate good or a final good,

$$X = AX + Y \tag{1}$$

Where, X is the $N \times 1$ gross output vector, Y is the $N \times 1$ final demand vector, and A is the $N \times N$ IO coefficient matrix.

The accounting relationship between gross exports of E_i and final demand in destination *j* in an Inter-Country Input-Output (ICIO) model can be express as:

$$E = \tilde{A}E + \tilde{Y} \tag{2}$$

Here, $\tilde{Y} = Y^F + \tilde{A}Y^D$, Y^D denote the domestic final demand consumed and Y^F is the foreign countries consume the final demand, so $Y = Y^D + Y^F$. In addition, $\tilde{A} = A^F(I - A^D)^{-1}$, A^D is the domestic coefficient in the global ICIO table (The block-diagonal matrix of the A matrix in the ICIO table). A^F is the export matrix of A matrix for use of intermediate input from one country to another country. So we have $A = A^D + A^F$. According to the implication of these matrixes, we can know $\tilde{Y}$ is the vector of export goods' final demand including intermediate goods and final goods demand, for example, the element $\tilde{Y}_i$ in the vector $\tilde{Y}$ means, for country i's export goods, other countries' final demand is $\tilde{Y}_i$ which include two part: the final goods demand Y_i^F and the intermediate goods final demand $\tilde{A}Y_i^D$ which is used to product final goods directly consumed in the destination countries; Similar with the concept matrix A in IO table, the elements in matrix $\tilde{A}$ describe units of intermediate goods produced in domestic and exported used in the production of one unit export goods in destination country, for example, the element $\tilde{A}_{ij}$, means to product country j's one unit export goods, country i need to product $\tilde{A}_{ij}$ units intermediate goods and transport to country j. $\tilde{A}_{ij}E_j$ means country i needs to input $\tilde{A}_{ij}E_j$ as intermediate input in the country j's export E_j, so we can call $\tilde{A}$ as the 'direct export requirement coefficients matrix'. Re-arrange the equation (2) above, we can obtain the equation $E = \tilde{B}\tilde{Y}$, and $\tilde{B} = (I - \tilde{A})^{-1}$, similar with the concept $B = (I - A)^{-1}$ in IO table, we can denote the matrix $\tilde{B}$ as the 'total export requirement coefficients matrix'.

2.1.1 Value-added multiplier coefficient and GDP in the exports

According to the concept of $\tilde{A}$, we can obtain for the country i's export E_i, all of intermediate inputs are $\sum_{j \neq i}^{G} \tilde{A}_{ji}E_i$, and the country i's value-added in the export is $VaE(i)^T = E_i - \sum_{j \neq i}^{G} \tilde{A}_{ji}E_i$. We also can prove for country i's export, the domestic value-added multiplier coefficient is $u(I - \sum_{j \neq i}^{G} \tilde{A}_{ji}) = V_i(I - A_{ii})^{-1}$. Here, define V_i as a 1×N direct value-added coefficient vector. Each element of V_i gives the share of direct domestic value added in total output. This is equal to one minus the intermediate input share from all countries (including domestically produced intermediates): $V_i = u[I - A_{ii} - \sum_{j \neq i}^{G} A_{ji}]$, where *u* is a 1×N unity vector. So the domestic value-added in country i can be expressed as: $VaE(i)^T = V_i(I - A_{ii})^{-1}E_i$.

Also, we can deduce the consistent expression for the domestic value-added (or GDP) from the initial ICIO model. In the ICIO model, the country *i*'s gross output can be written as:

$$X_i = A_{ii}X_i + Y_{ii} + \sum_{j\neq i}^{G} A_{ij}X_j + \sum_{j\neq i}^{G} Y_{ij} = A_{ii}X_i + Y_{ii} + E_i \quad (3)$$

Rearranging the equation (3), we can get

$$X_i = (I - A_{ii})^{-1}Y_{ii} + (I - A_{ii})^{-1}E_i \quad (4)$$

In some literature, the matrix $(I - A_{ii})^{-1}$ is defined as local Leontief inverse. So in the ICIO model, country *i*'s value-added (or GDP) can be calculated as follow:

$$VA_i(GDP_i) = V_iX_i = V_i(I - A_{ii})^{-1}Y_{ii} + V_i(I - A_{ii})^{-1}E_i \quad (5)$$

According to the equation (5), country *i*'s value-added (or GDP) is divided into two parts, one part is the value-added (or GDP) in the final demand and the other part $V_i(I - A_{ii})^{-1}E_i$ is the value-added (or GDP) in the export for country *i*. In this deduction processing, we can also get the expression about the domestic value-added in the export which is consistent with the discussion before, and consider the coefficient $V_i(I - A_{ii})^{-1}$ as the value-added multiplier coefficient for a country's export simultaneously.

In the analysis above, we already known that $\tilde{A}_{ji}E_i$ is the intermediate input which is in the export flow from country *j* to country *i* for the country *i*'s export, so country *j*'s value-added in country *i*'s export can be expressed merged with the value-added multiplier coefficient as $V_j(I - A_{jj})^{-1}\tilde{A}_{ji}E_i$. Also, we can deduce the same expression from the initial ICIO model.

Similarly, we can obtain the expression of country *j*'s GDP as $VA_j(GDP_j) = V_jX_j = V_j(I - A_{jj})^{-1}Y_{jj} + V_j(I - A_{jj})^{-1}E_j$. Meanwhile, we have country *j*'s export expression as $E_j = E_{ji} + \sum_{s\neq i,j}^{G} E_{js}$. So the country *j*'s value-added (or GDP) exported into country *i* is $V_j(I - A_{jj})^{-1}E_{ji}$. If we expand the export expression from *j* to *i* as follow:

$$E_{ji} = A_{ji}X_i + Y_{ji} = A_{ji}(I + (I - A_{ii})^{-1} A_{ii})E_i + A_{ji}(I - A_{ii})^{-1} Y_{ii} + Y_{ji}$$
$$= \tilde{A}_{ji}E_i + \tilde{A}_{ji}Y_{ii} + Y_{ji} \tag{6}$$

In this expansion expression, the part (the country *j*'s value-added or GDP exported into country *i*) $V_j(I - A_{jj})^{-1} E_{ji}$ can be divided into 3 parts respectively: $V_j(I - A_{jj})^{-1} \tilde{A}_{ji}E_i$, $V_j(I - A_{jj})^{-1} \tilde{A}_{ji}Y_{ii}$ and $V_j(I - A_{jj})^{-1} Y_{ji}$. It is easily recognized that the term $V_j(I - A_{jj})^{-1} \tilde{A}_{ji}E_i$ is the country *j*'s value-added (or GDP) in country *i*'s export and the remaining of the terms are absorbed by country *i* at the beginning. If we sum up all countries' (except country *i*) value-added in the country *i*'s export, can obtain the foreign value-added part in the country *i*'s export expressed by $\sum_{j \neq i}^{G} V_j(I - A_{jj})^{-1} \tilde{A}_{ji}E_i$.

Besides, more value-added decomposition analysis for exports will be seen in section 2.2 below.

2.1.2 Export goods decomposition: two countries and one sector case

In this section, we start from a two country and one sector case. Each case is to explore what is the export flow route and how it is absorbed finally by the final demand including the double counted term as well. Assuming in a two country world, in which both countries produce only one kind of goods. Goods in each country can be consumed directly or used as intermediate inputs, and each country exports both intermediate and final goods to the other. The two countries production and trade systems can be written as an ICIO model expression as following:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} y_{11} + y_{12} \\ y_{21} + y_{22} \end{bmatrix} \tag{7}$$

So, the gross output produced in country i can be written as:

$$x_i = a_{ii}x_i + a_{ij}x_j + y_{ii} + y_{ij}, i, j = 1, 2 \tag{8}$$

where, y_{ij} is the final demand in country j for the final good produced in Country i, and a_{ij} is the input-output (IO) coefficient, describing units of intermediate goods produced in i used in the production of one unit gross output in Country j.

Here, we take country 1 as an example for interpretation. Also in the ICIO model, country 1's export can be written as $e_1 = a_{12}x_2 + y_{12}$. In this equation, the term $a_{12}x_2$ is the exported intermediate goods and the term y_{12} is the exported final goods which is absorbed by country 2 finally. According to the export expression, we can rewrite country 1's gross output as $x_1 = a_{11}x_1 + e_1 + y_{11}$. Correspondingly, we also rewrite country 2's gross output as $x_2 = a_{22}x_2 + e_2 + y_{22}$. Re-arranging the equation, we can obtain $x_2 = (1-a_{22})^{-1}e_2 + (1-a_{22})^{-1}y_{22}$. Combining the country 1's export expression, we will have $e_1 = a_{12}(1-a_{22})^{-1}e_2 + a_{12}(1-a_{22})^{-1}y_{22} + y_{12}$.

Then country 1's export expression becomes into 3 terms, every term has its economical interpretation: term 1 $a_{12}(1-a_{22})^{-1}e_2$ means the intermediate goods exported from country 1 to country 2 and being processed into intermediate or final goods in country 2 and being re-exported again to country 1. Term 2 $a_{12}(1-a_{22})^{-1}y_{22}$ means the intermediate goods exported from country 1 to country 2 and being processed into final goods and absorbed by country 2. The last term is the final goods exported from country 1 to country 2 and being absorbed by country 2.

The process above is the country 1's export in round 1. In the round 1 export, the part of country 1's goods $a_{12}(1-a_{22})^{-1}e_2$ which could be intermediate or final goods will be re-exported again. In the two countries case, this part will be exported into country 1 and processed or absorbed there. Here we start to trace this part in the export. For the convenience of writing, we denote $a_{ij}(1-a_{jj})^{-1}$ as $\tilde{a}_{ij}$ here.

According to the export expression above, we have $e_2 = \tilde{a}_{21}e_1 + \tilde{a}_{21}y_{11} + y_{21}$, then we can get $\tilde{a}_{12}e_2 = \tilde{a}_{12}\tilde{a}_{21}e_1 + \tilde{a}_{12}\tilde{a}_{21}y_{11} + \tilde{a}_{12}y_{21}$. Combining country 1's export expression, we can re-write the country 1's export as fallowing:

$$e_1 = \tilde{a}_{12}\tilde{a}_{21}e_1 + \tilde{a}_{12}\tilde{a}_{21}y_{11} + \tilde{a}_{12}y_{21} + \tilde{a}_{12}y_{22} + y_{12} \quad (9)$$

There are three new terms emerged in this new expression: the term $\tilde{a}_{12}\tilde{a}_{21}e_1$ means the goods exported from country 1 to country 2 then processed in country 2 into intermediate goods then exported from country 2 back to country 1 for processing into intermediate or final goods to export again; the term $\tilde{a}_{12}\tilde{a}_{21}y_{11}$ means the goods exported from country 1 to country 2 then being processed in country 2 into intermediate goods then exported back to country 1 for processing into final goods and absorbed by country 1; the term $\tilde{a}_{12}y_{21}$ means the goods exported from country 1 to country 2 then being processed in country 2 into final goods then exported back and absorbed by country 1.

The equation above constitutes an explicit expression which is a nesting relationship in round 1 exports of country 1. Similarly, we can expand the equation combining the export expression in round 1 and write the country 1's export expression in round 2 as:

$$\begin{aligned} e_1 &= \tilde{a}_{12}\tilde{a}_{21}(\tilde{a}_{12}\tilde{a}_{21}e_1 + \tilde{a}_{12}\tilde{a}_{21}y_{11} + \tilde{a}_{12}y_{21} + \tilde{a}_{12}y_{22} + y_{12}) + \tilde{a}_{12}\tilde{a}_{21}y_{11} + \tilde{a}_{12}y_{21} + \tilde{a}_{12}y_{22} + y_{12} \\ &= \tilde{a}_{12}\tilde{a}_{21}\tilde{a}_{12}\tilde{a}_{21}e_1 + (\tilde{a}_{12}\tilde{a}_{21}\tilde{a}_{12}\tilde{a}_{21} + \tilde{a}_{12}\tilde{a}_{21})y_{11} + (\tilde{a}_{12}\tilde{a}_{21}\tilde{a}_{12} + \tilde{a}_{12})y_{21} + (\tilde{a}_{12}\tilde{a}_{21}\tilde{a}_{12} + \tilde{a}_{12})y_{22} + (\tilde{a}_{12}\tilde{a}_{21} + 1)y_{12} \end{aligned}$$

and round n expression as :

$$\begin{aligned} e_1 &= (\tilde{a}_{12}\tilde{a}_{21})^n e_1 + ((\tilde{a}_{12}\tilde{a}_{21})^n \cdots + \tilde{a}_{12}\tilde{a}_{21} + 1)\tilde{a}_{12}\tilde{a}_{21}y_{11} + ((\tilde{a}_{12}\tilde{a}_{21})^n \cdots + \tilde{a}_{12}\tilde{a}_{21} + 1)\tilde{a}_{12}y_{21} \\ &+ ((\tilde{a}_{12}\tilde{a}_{21})^n \cdots + \tilde{a}_{12}\tilde{a}_{21} + 1)\tilde{a}_{12}y_{22} + ((\tilde{a}_{12}\tilde{a}_{21})^n \cdots + \tilde{a}_{12}\tilde{a}_{21} + 1)y_{12} \end{aligned}$$

For the convenience of deduction, we denote the matrix $A^D = \begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix}$ and $A^F = \begin{bmatrix} 0 & a_{12} \\ a_{21} & 0 \end{bmatrix}$, so we will have direct export requirement coefficients matrix as:

$$\tilde{A} = A^F(I - A^D)^{-1} = \begin{bmatrix} 0 & a_{12} \\ a_{21} & 0 \end{bmatrix}\begin{bmatrix} 1-a_{11} & 0 \\ 0 & 1-a_{22} \end{bmatrix} = \begin{bmatrix} 0 & a_{12}(1-a_{22}) \\ a_{21}(1-a_{11}) & 0 \end{bmatrix} = \begin{bmatrix} 0 & \tilde{a}_{12} \\ \tilde{a}_{21} & 0 \end{bmatrix}$$

Also we can obtain the total export requirement coefficients matrix:

$$\tilde{B} = (I - \tilde{A})^{-1} = I + \tilde{A} + \tilde{A}^2 + \cdots \quad (6)$$

In the two countries case, we can write every element expression in the matrix $\tilde{B}$, for example: $\tilde{b}_{11} = \lim_{n\to\infty}[(\tilde{a}_{12}\tilde{a}_{21})^n \cdots + \tilde{a}_{12}\tilde{a}_{21} + 1]$ and $\tilde{b}_{12} = \lim_{n\to\infty}[(\tilde{a}_{12}\tilde{a}_{21})^n \cdots + \tilde{a}_{12}\tilde{a}_{21} + 1]\tilde{a}_{12}$. At the same time, we have $\lim_{n\to\infty}(\tilde{a}_{12}\tilde{a}_{21})^n = 0$. Combining the export expression in the round n, we can re-write the export expression when n is infinite as:

$$\begin{aligned} e_1 &= \tilde{b}_{12}\tilde{a}_{21}y_{11} + \tilde{b}_{12}y_{21} + \tilde{b}_{11}\tilde{a}_{12}y_{22} + \tilde{b}_{11}y_{12} \\ &= (\tilde{b}_{11} - 1)y_{11} + \tilde{b}_{12}y_{21} + \tilde{b}_{12}y_{22} + \tilde{b}_{11}y_{12} \end{aligned} \quad (10)$$

In the equation above, the export is decomposed into the form of final demand completely. Every term in the equation can identify the export goods final destination. The first term $(\tilde{b}_{11} - 1)y_{11}$ means the export goods return to country 1 as intermediate goods, being processed into final goods there and absorbed by country 1 eventually. The second term $\tilde{b}_{12}y_{21}$ means the export goods return to country 1 as final goods and being absorbed immediately there. The term $\tilde{b}_{12}y_{22}$ means the export goods enter country 2 as intermediate goods, being processed into final goods there and absorbed by country 2 finally. The last term $\tilde{b}_{11}y_{12}$ means the export goods enter country 2 as final goods and absorbed immediately by country 2.

With the total export requirement coefficients matrix, we extend the export decomposition expression to the G sectors situation.

2.1.3 G countries and N sectors model:

In this section, we firstly discuss three countries and N sectors case with the total export requirement coefficient matrix and then extend to arbitrary numbers of countries and sectors. In the three countries ICIO table, the relationship between the export and final demand can be given by the block matrix notation as follows.

First of all, we denote the export vector in the three countries ICIO table as

$E=\begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$, and the direct requirement coefficient matrix $A=\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$.

According the concept in section 2.1, we have the domestic coefficient matrix $A^D=\begin{bmatrix} A_{11} & 0 & 0 \\ 0 & A_{22} & 0 \\ 0 & 0 & A_{33} \end{bmatrix}$ and foreign coefficient matrix $A^F=\begin{bmatrix} 0 & A_{12} & A_{13} \\ A_{21} & 0 & A_{23} \\ A_{31} & A_{32} & 0 \end{bmatrix}$, also we denote the final demand vector as $Y=\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix}$, the domestic demand vector as $Y^D=\begin{bmatrix} Y_{11} \\ Y_{22} \\ Y_{33} \end{bmatrix}$ and the foreign demand vector $Y^F=\begin{bmatrix} Y_{12}+Y_{13} \\ Y_{21}+Y_{23} \\ Y_{31}+Y_{32} \end{bmatrix}$, so we have $Y=Y^D+Y^F$. Then we can obtain the direct export requirement matrix $\tilde{A}$ as

$$\tilde{A}=A^F(I-A^D)^{-1}=\begin{bmatrix} 0 & A_{12} & A_{13} \\ A_{21} & 0 & A_{23} \\ A_{31} & A_{32} & 0 \end{bmatrix}\begin{bmatrix} (I-A_{11})^{-1} & 0 & 0 \\ 0 & (I-A_{22})^{-1} & 0 \\ 0 & 0 & (I-A_{33})^{-1} \end{bmatrix}$$

$$=\begin{bmatrix} 0 & A_{12}(I-A_{22})^{-1} & A_{13}(I-A_{33})^{-1} \\ A_{21}(I-A_{11})^{-1} & 0 & A_{23}(I-A_{33})^{-1} \\ A_{31}(I-A_{11})^{-1} & A_{32}(I-A_{22})^{-1} & 0 \end{bmatrix}$$

And $\tilde{Y}=Y^F+\tilde{A}Y^D=\begin{bmatrix} Y_{12}+Y_{13}+A_{12}(I-A_{22})^{-1}Y_{22}+A_{13}(I-A_{33})^{-1}Y_{33} \\ Y_{21}+Y_{23}+A_{21}(I-A_{11})^{-1}Y_{11}+A_{23}(I-A_{33})^{-1}Y_{33} \\ Y_{31}+Y_{32}+A_{31}(I-A_{11})^{-1}Y_{11}+A_{32}(I-A_{22})^{-1}Y_{22} \end{bmatrix}$ we can also denote the total export requirement matrix as $\tilde{B}=(I-\tilde{A})^{-1}=\begin{bmatrix} \tilde{B}_{11} & \tilde{B}_{12} & \tilde{B}_{13} \\ \tilde{B}_{21} & \tilde{B}_{22} & \tilde{B}_{23} \\ \tilde{B}_{31} & \tilde{B}_{32} & \tilde{B}_{33} \end{bmatrix}$.

According to the equation $E=(I-\tilde{A})^{-1}\tilde{Y}=\tilde{B}\tilde{Y}$, we then decompose exports as following. For example, Country 1's exports can be decomposed into:

$$
\begin{aligned}
E_1 &= \tilde{B}_{11}(Y_{12}+Y_{13}) + \tilde{B}_{11}A_{12}(I-A_{22})^{-1}Y_{22} + \tilde{B}_{11}A_{13}(I-A_{33})^{-1}Y_{33} \\
&\quad + \tilde{B}_{12}(Y_{21}+Y_{23}) + \tilde{B}_{12}A_{21}(I-A_{11})^{-1}Y_{11} + \tilde{B}_{12}A_{23}(I-A_{33})Y_{33} \\
&\quad + \tilde{B}_{13}(Y_{31}+Y_{32}) + \tilde{B}_{13}A_{31}(I-A_{11})^{-1}Y_{11} + \tilde{B}_{13}A_{32}(I-A_{22})^{-1}Y_{22} \\
&= (\tilde{B}_{12}Y_{21} + \tilde{B}_{13}Y_{31}) + (\tilde{B}_{11}Y_{12} + \tilde{B}_{13}Y_{32}) + (\tilde{B}_{11}Y_{13} + \tilde{B}_{12}Y_{23}) \\
&\quad + [\tilde{B}_{12}A_{21}(I-A_{11})^{-1} + \tilde{B}_{13}A_{31}(I-A_{11})^{-1}]Y_{11} \\
&\quad + [\tilde{B}_{11}A_{12}(I-A_{22})^{-1} + \tilde{B}_{13}A_{32}(I-A_{22})^{-1}]Y_{22} \\
&\quad + [\tilde{B}_{11}A_{13}(I-A_{33})^{-1} + \tilde{B}_{12}A_{23}(I-A_{33})^{-1}]Y_{33} \\
&= (\tilde{B}_{12}Y_{21} + \tilde{B}_{13}Y_{31}) + (\tilde{B}_{11}Y_{12} + \tilde{B}_{13}Y_{32}) + (\tilde{B}_{11}Y_{13} + \tilde{B}_{12}Y_{23}) \\
&\quad + (\tilde{B}_{11} - I)Y_{11} + \tilde{B}_{12}Y_{22} + \tilde{B}_{13}Y_{33}
\end{aligned} \tag{11}
$$

Similar to the two countries case above, the exports are also decomposed into the form of final demand completely and every term in the equation can identify the final destination of the export goods. Finally there are six terms in the brackets in the above equation. We can interpret each term's economical implication explicitly: term1 and term 4 are country 1's exports and return back to country 1. Term 1 is the return part as final goods returned which comes from country 2 and 3 respectively then absorbed directly by country 1; term 4 is the return part as intermediate goods returned then processed into final goods in country 1 then absorbed by country 1.Term 2 and term 5 mean country 1's export finally absorbed by country 2 directly or indirectly. Term 2 is the final goods entered country 2 which is come from country 1 and 3 respectively and absorbed by country 2 directly; term 5 is the intermediate goods entered country 2 and then processed into final goods there absorbed by country 2 finally. Similarly, we can get the same interpretations about term 3 and term 6.

From the analysis above, we can find the export decomposition expression has following characteristics in the condition of infinite round: 1) every term in the expression clearly identify origin and final destination of the export goods after infinite rounds of exports and re-exports; 2) From the decomposition expression, we can clearly identify the part of exports accounts for the intermediate goods or final goods for the export country.

For example, the term $\tilde{B}_{11}Y_{12}$ in the above equation means, after infinite rounds of exports, return back and re-export again, the ultimate transport route and processing step for this part of country 1's export goods is processed into final goods in country 1 and then exported from country 1 to country 2 consumed in country 2 finally, so this part accounts as country 1's final goods export; but for the term $\tilde{B}_{12}Y_{23}$, it means the ultimate processing step of this part of country 1's export goods is processed into final goods in country 2 and then exported from country 2 to country 3 and finally consumed in country 3. This part of country 3 is final goods import, but pertains to country 1's intermediate goods export and its ultimate processing location is in the country 2; The term $\tilde{B}_{13}Y_{33}$ in the above equation also pertains to the intermediate goods part in country 1's export. , Because the coefficient $\tilde{B}_{13}$ after infinite round of export,re-export, processed nd re-processed, country 1's goods finally reach country 3 to be processed into final goods and consumed in country 3..

According to the export expression in the three country case, we can extend the export decomposition to the G countries and N sectors case as following:

$$\begin{aligned}
E_i &= \sum_{j\neq t}^{G}\sum_{t}^{G}\tilde{B}_{it}Y_{tj} + \sum_{j\neq t}^{G}\sum_{t}^{G}\tilde{B}_{it}A_{tj}(I-A_{jj})^{-1}Y_{jj} \\
&= \sum_{t\neq i}^{G}\tilde{B}_{it}Y_{ti} + \sum_{t\neq j}^{G}\tilde{B}_{it}Y_{tj} + \sum_{t}^{G}\sum_{s\neq t,i,j}^{G}\tilde{B}_{it}Y_{ts} + (\tilde{B}_{ii}-I)Y_{ii} + \tilde{B}_{ij}Y_{jj} + \sum_{s\neq i,j}^{G}\tilde{B}_{is}Y_{ss} \\
&= \sum_{t\neq i}^{G}\tilde{B}_{it}Y_{ti} + \sum_{t}^{G}\sum_{j\neq t,i}^{G}\tilde{B}_{it}Y_{tj} + (\tilde{B}_{ii}-I)Y_{ii} + \sum_{j\neq i}^{G}\tilde{B}_{ij}Y_{jj}
\end{aligned} \tag{12}$$

Three expression forms are given in the equation above for different research purpose.

Expression 1： $\sum_{j\neq t}^{G}\sum_{t}^{G}\tilde{B}_{it}Y_{tj} + \sum_{j\neq t}^{G}\sum_{t}^{G}\tilde{B}_{it}A_{tj}(I-A_{jj})^{-1}Y_{jj}$

In this general expression, we distinguish the goods into intermediate goods and final goods for export destination countries. The first term means the export goods from country i went through country t and finally entered country j **as final goods** and being absorbed by country j directly. The second term means the export goods from country i went through country t and finally entered country j **as intermediate goods** and being processed in country j into final goods then absorbed there.

Also, if we focus on a specific export destination and the export returned back term, we can rewrite the equation in another two forms:

Expression 2： $\sum_{t\neq i}^{G}\tilde{B}_{it}Y_{ti}+\sum_{t\neq j}^{G}\tilde{B}_{it}Y_{tj}+\sum_{t}^{G}\sum_{s\neq t,i,j}^{G}\tilde{B}_{it}Y_{ts}+(\tilde{B}_{ii}-I)Y_{ii}+\tilde{B}_{ij}Y_{jj}+\sum_{s\neq i,j}^{G}\tilde{B}_{is}Y_{ss}$

In this expression form, term 1 and term 4 mean country i's export comes back to country i; term 1 means the export goods from country i and went through country t but finally returned back country i **as final goods** and being absorbed by country i directly. Term 4 means the export goods from country i and finally came back to country i **as intermediate goods** and being processed in country i into final goods then absorbed by country i. Term 2 and term 5 mean country i's export to **specific one country j** and being absorbed by the country j finally. Term 2 is the final goods and term 5 is the intermediate goods. Term 3 and term 6 mean country i's export to country t and being re-exported to country s absorbed by country s finally. Term 3 is the final goods and term 6 is the intermediate goods.

Expression 3: $\sum_{t\neq i}^{G}\tilde{B}_{it}Y_{ti}+\sum_{t}^{G}\sum_{j\neq t,i}^{G}\tilde{B}_{it}Y_{tj}+(\tilde{B}_{ii}-I)Y_{ii}+\sum_{j\neq i}^{G}\tilde{B}_{ij}Y_{jj}$

In this expression form, country j represent all the country i's ultimate export destination. Term 1 and term 3 mean country i's export then return to country i. Term 2 and term 4 mean country i's export to an arbitrary country j and being absorbed by country j finally. Term 2 is the final goods and term 4 is the intermediate goods.

2.1.4 Bilateral trade case

In this section, we start to decompose the bilateral trade in the similar way, from the total export requirement matrix as well. In the bilateral export, we will have the relationship between the bilateral exports. The final demand can be written as following:

$$\begin{aligned} E_{ij} &= A_{ij}X_j+Y_{ij}=A_{ij}(I+(I-A_{jj})^{-1}A_{jj})E_j+A_{ij}(I-A_{jj})^{-1}Y_{jj}+Y_{ij} \\ &=\tilde{A}_{ij}E_j+\tilde{A}_{ij}Y_{jj}+Y_{ij} \end{aligned} \quad (13)$$

In the equation above, the bilateral export was decomposed into three parts in round 1 export firstly, $\tilde{A}_{ij}E_j$ means country i's intermediate goods export to country j and being processed in country j and re-export again there. $\tilde{A}_{ij}Y_{jj}$ means country i's intermediate goods export to country j and being processed to final goods in country j and absorbed by country j finally. Y_{ij} means i's final goods export to country j and then being absorbed by country j directly.

Also we have country j's export ultimate decomposition expression as:

$$E_j = \sum_{t \neq i}^{G} \tilde{B}_{jt}Y_{ti} + \sum_{t \neq j}^{G} \tilde{B}_{jt}Y_{tj} + \sum_{t}^{G} \sum_{s \neq t,i,j}^{G} \tilde{B}_{jt}Y_{ts} + \tilde{B}_{ji}Y_{ii} + (\tilde{B}_{jj} - I)Y_{jj} + \sum_{s \neq i,j}^{G} \tilde{B}_{js}Y_{ss} \tag{14}$$

Combining equation (13) and (14), we can obtain the bilateral export in the infinite round decomposition expression:

$$\begin{aligned} E_{ij} &= \tilde{A}_{ij}Y_{jj} + Y_{ij} + \tilde{A}_{ij}[\sum_{t \neq i}^{G} \tilde{B}_{jt}Y_{ti} + \sum_{t \neq j}^{G} \tilde{B}_{jt}Y_{tj} + \sum_{t}^{G} \sum_{s \neq t,i,j}^{G} \tilde{B}_{jt}Y_{ts} + \tilde{B}_{ji}Y_{ii} + (\tilde{B}_{jj} - I)Y_{jj} + \sum_{s \neq i,j}^{G} \tilde{B}_{js}Y_{ss}] \\ &= \tilde{A}_{ij}\tilde{B}_{jj}Y_{jj} + Y_{ij} + \tilde{A}_{ij}\sum_{t \neq j}^{G} \tilde{B}_{jt}Y_{tj} + \tilde{A}_{ij}[\sum_{t \neq i}^{G} \tilde{B}_{jt}Y_{ti} + \sum_{t}^{G} \sum_{s \neq t,i,j}^{G} \tilde{B}_{jt}Y_{ts} + \tilde{B}_{ji}Y_{ii} + \sum_{s \neq i,j}^{G} \tilde{B}_{js}Y_{ss}] \end{aligned} \tag{15}$$

The bilateral export decomposition form is similar with the gross export. In the equation above, term 1 $\tilde{A}_{ij}\tilde{B}_{jj}Y_{jj}$ means the country i's intermediate goods export to country j and being processed into intermediate or final goods in country j then absorbed by country j finally. This term includes two parts, part 1 is $\tilde{A}_{ij}Y_{jj}$ which is the intermediate goods from country i to country j and directly being processed into final goods and absorbed by country j. Part 2 is $\tilde{A}_{ij}(\tilde{B}_{jj} - I)Y_{jj}$ which is the intermediate goods from country i to country j and being processed into intermediate goods then re-exported again, then finally go back to country j and being processed into final goods in country j and absorbed there. It is an indirect process. Term 2 Y_{ij} is the directly absorbed part in country i's round 1 export. Term 3, $\tilde{A}_{ij}\sum_{t \neq j}^{G} \tilde{B}_{jt}Y_{tj}$ it's the intermediate goods from country i to country j and being processed into intermediate goods then re-export to other countries again, then processed in other countries into final goods at last being absorbed by country j.

The terms $\tilde{A}_{ij}\sum_{t\neq i}^{G}\tilde{B}_{jt}Y_{ti}$ and $\tilde{A}_{ij}\tilde{B}_{ji}Y_{ii}$ is the return back terms which will return country i. The former is final goods returned back and the latter is the intermediate goods returned back thatboth are absorbed by country i finally.

The terms $\tilde{A}_{ij}\sum_{t}^{G}\sum_{s\neq t,i,j}^{G}\tilde{B}_{jt}Y_{ts}$ and $\tilde{A}_{ij}\sum_{s\neq i,j}^{G}\tilde{B}_{js}Y_{ss}$ are the outflow terms which will enter other countries. The former is the export goods entered country j firstly and as final goods entered country s being absorbed there finally and the latter is the intermediate goods entered country j firstly and entered finally entered country s then being processed in country s into final goods and absorbed by country s.

2.2 Trace the value-added in export: the Ghosh insight

The Ghosh model (Ghosh 1958), in turn, is also known as the 'supply–driven' input-output model, since the value-added is the exogenously specified driving force of the model.

Here in an IO table, the output coefficient is defined as $l_{ij} = x_{ij} / x_i$. The output coefficients give the output percentage of industry i that is sold to industry j. The accounting equation above also can be rewritten as:

$$X^T = VA^T + X^T L = VA \cdot G \tag{16}$$

Where $G = (I - L)^{-1}$ denotes the Ghosh inverse; Meanwhile in $G = \hat{X}^{-1} B \hat{X}$, $\hat{X}$ is a $N \times N$ diagonal matrix with output on the diagonal.

Similarly, in the export input-output table, the export can be written as $E^T = VaE^T + E^T \tilde{L} = VaE^T \cdot \tilde{G}$. Here $\tilde{G} = \hat{E}^{-1} \tilde{B} \hat{E}$, $\tilde{L} = \hat{E}^{-1} \tilde{A} \hat{E}$ and $\tilde{L}_{ij} = \hat{E}_i^{-1} \tilde{A}_{ij} \hat{E}_j$. $\tilde{L}_{ij}$

gives the share of country i's goods in country j's export.

To illustrate the relationship between exports and value-added, we can refer to the Taylor expansion.

$$E^T = VaE^T(I + \tilde{L} + \tilde{L}^2 + \tilde{L}^3 + \cdots) \tag{17}$$

In the value-added input VaE^T, the export value is E^T, which is decomposed into three value-added terms: an initial input VaE^T, a direct input $VaE^T \cdot \tilde{L}$ in the first round and indirect input in subsequent rounds amounting to $VaE^T(\tilde{L}^2 + \tilde{L}^3 + \cdots)$. In the export input-output table, the initial input is the domestic value-added input; the direct input is the foreign value-added input; and the indirect input is the value-added double counting terms including both domestic and foreign parts.

In the Ghosh insight, we can give the full expression for specific country i's export as following:

$$\begin{aligned} E_i^T &= VaE(i)^T + VaE(i)^T \tilde{L}_{ii} + \sum_{j \neq i}^{G} VaE(j)^T \tilde{L}_{ji} \\ &+ VaE(i)^T [\tilde{L}]^2{}_{ii} + VaE(i)^T [\tilde{L}]^3{}_{ii} + \cdots \\ &+ \sum_{j \neq i}^{G} VaE(j)^T [\tilde{L}]^2{}_{ji} + \sum_{j \neq i}^{G} VaE(j)^T [\tilde{L}]^3{}_{ji} + \cdots \end{aligned} \tag{18}$$

The expression above gives an explicit interpretation of exports decomposition in the Ghosh insight. Every sub-term in the expression has some economical implication respectively.

The initial effect is the value-added in country i's export which equals to $VaE(i)^T = V_i(I - A_{ii})^{-1} E_i$.

In the first round, the direct effect can be divided into two parts, the effect from the domestic country i and from foreign country j. Because the element in matrix $\tilde{L}_{ii}$ equal to 0, so $VaE(i)^T \tilde{L}_{ii} = 0$; Also the effect from country j can be obtained. Since the foreign value-added in the intermediate goods is imported from country j, these terms equal to

$$\sum_{j \neq i}^{G} VaE(j)^T \cdot \tilde{L}_{ji} = \sum_{j \neq i}^{G} V_j (I - A_{jj})^{-1} \hat{E}_j \cdot \hat{E}_j^{-1} \tilde{A}_{ji} E_i = \sum_{j \neq i}^{G} V_j (I - A_{jj})^{-1} \tilde{A}_{ji} E_i \qquad (19)$$

In the second round, these additional value-added can also be divided into domestic part and foreign parts. It constitutes the value-added double counting terms passed over from country i's export to the third country and returned back home. This implies that for the domestic part, the country i's value-added part is $uVaE(i)^T \sum_{k}^{G} \tilde{L}_{ik} \tilde{L}_{ki}$, reflecting the country i's value-added $uVaE(i)^T \tilde{L}_{ik}$ propagated to country k. $\tilde{L}_{ki}$ part in country k propagated back home. This part has already being counted in the initial round value-added input, so it should be counted as domestic value-added double counting term. We have

$$\begin{aligned} & VaE(i)^T [\tilde{L}]^2{}_{ii} = VaE(i)^T \sum_{k}^{G} \tilde{L}_{ik} \tilde{L}_{ki} \\ & = V_i (I - A_{ii})^{-1} \hat{E}_i \sum_{k}^{G} \hat{E}_i^{-1} \tilde{A}_{ik} \hat{E}_k \cdot \hat{E}_k^{-1} \tilde{A}_{ki} E_i = V_i (I - A_{ii})^{-1} \sum_{k}^{G} \tilde{A}_{ik} \tilde{A}_{ki} E_i \end{aligned} \qquad (20)$$

For the country i's foreign part, the country j's value-added part is $VaE(j)^T \sum_{k}^{G} \tilde{L}_{jk} \tilde{L}_{ki}$, reflecting the country j's value-added $uVaE(j)^T \tilde{L}_{jk}$ propagated to country k. $\tilde{L}_{ki}$ part in country k propagated back to country i. This part also has already being counted in the first round value-added input, so it should be counted as foreign value-added double counting term. Also, we have

$$\begin{aligned} & \sum_{j \neq i}^{G} VaE(j)^T [\tilde{L}]^2{}_{ji} = VaE(j)^T \sum_{k}^{G} \tilde{L}_{jk} \tilde{L}_{ki} \\ & = V_j (I - A_{jj})^{-1} \hat{E}_j \sum_{k}^{G} \hat{E}_j^{-1} \tilde{A}_{jk} \hat{E}_k \cdot \hat{E}_k^{-1} \tilde{A}_{ki} E_i = V_j (I - A_{jj})^{-1} \sum_{k}^{G} \tilde{A}_{jk} \tilde{A}_{ki} E_i \end{aligned} \qquad (21)$$

So in the round 2, the whole foreign double counting value-added part is

$$\sum_{j\neq i}^{G} V_j (I - A_{jj})^{-1} \sum_{k}^{G} \tilde{A}_{jk} \tilde{A}_{ki} E_i \ .$$

Therefore, we can obtain the domestic value-added in export double counting term as

$$\begin{aligned} & VaE(i)^T [\tilde{L}]^2{}_{ii} + VaE(i)^T [\tilde{L}]^3{}_{ii} + \cdots = \\ & V_i (I - A_{ii})^{-1} (\sum_{j}^{G} \tilde{A}_{ij} \tilde{A}_{ji} + \sum_{k}^{G} \sum_{j}^{G} \tilde{A}_{ij} \tilde{A}_{jk} \tilde{A}_{ki} + \cdots) E_i = V_i (I - A_{ii})^{-1} (\tilde{B}_{ii} - I) E_i \end{aligned} \tag{22}$$

And the foreign value-added in export double counting is

$$\begin{aligned} & \sum_{j\neq i}^{G} VaE(j)^T [\tilde{L}]^2{}_{ji} + \sum_{j\neq i}^{G} VaE(j)^T [\tilde{L}]^3{}_{ji} + \cdots \\ & \sum_{j\neq i}^{G} V_j (I - A_{jj})^{-1} (\sum_{k}^{G} \tilde{A}_{jk} \tilde{A}_{ki} + \sum_{t}^{G} \sum_{k}^{G} \tilde{A}_{jk} \tilde{A}_{kt} \tilde{A}_{ti} + \cdots) E_i = \sum_{j\neq i}^{G} V_j (I - A_{jj})^{-1} (\tilde{B}_{ji} - \tilde{A}_{ji}) E_i \end{aligned} \tag{23}$$

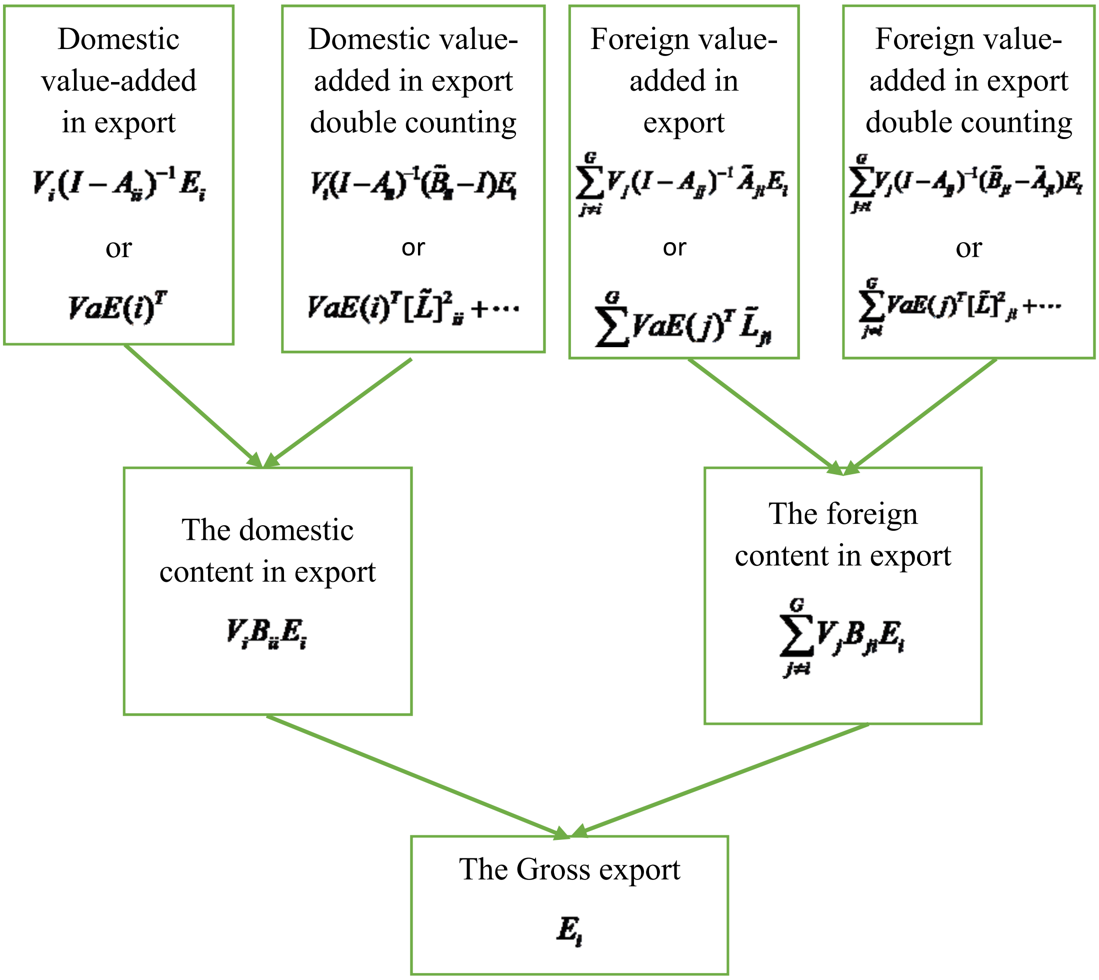
In the value-added decomposition of export, sum of the domestic value-added in export and the double counting export equal to the domestic content in export.

$$V_i (I - A_{ii})^{-1} E_i + V_i (I - A_{ii})^{-1} (\tilde{B}_{ii} - I) E_i = V_i B_{ii} E_i \tag{24}$$

Also, sum of the foreign value-added in export and the double counting export equal to the foreign content in export.

$$\sum_{j\neq i}^{G} V_j (I - A_{jj})^{-1} \tilde{A}_{ji} E_i + \sum_{j\neq i}^{G} V_j (I - A_{jj})^{-1} (\tilde{B}_{ji} - \tilde{A}_{ji}) E_i = \sum_{j\neq i}^{G} V_j B_{ji} E_i \tag{25}$$

The Ghosh insight demonstrates a bottom-up, hierarchical decomposition method of gross exports. The complete gross exports accounting framework made above is also diagrammed in the figure below:



2.3 The value-added decomposition as the exports destination

In the KWW method, the export was divided into domestic and foreign value-added parts. In the infinite round method of this paper, these terms in the export also can be divided into similar parts according to the bilateral trade decomposition framework and value-added decomposition in the Ghosh insight, details are listed in the table below:

Table 1: The decomposition terms compared with KWW method

	Terms
Domestic value-added absorbed by country j as final goods(T1)	$V_i(I-A_{ii})^{-1}\sum_{j\neq i}^{G}(Y_{ij}+\tilde{A}_{ij}\sum_{t\neq j}^{G}\tilde{B}_{jt}Y_{tj})$
Domestic value-added absorbed by country j as intermediate goods(T2)	$V_i(I-A_{ii})^{-1}\sum_{j\neq i}^{G}\tilde{A}_{ij}\tilde{B}_{jj}Y_{jj}$

Domestic value-added absorbed by the third countries through country j(T3)	$V_i(I-A_{ii})^{-1}\sum_{j\neq i}^{G}\tilde{A}_{ij}(\sum_{t}^{G}\sum_{s\neq t,i,j}^{G}\tilde{B}_{jt}Y_{ts}+\sum_{s\neq i,j}^{G}\tilde{B}_{js}Y_{ss})$
Domestic value-added returned home as final goods(T4)	$V_i(I-A_{ii})^{-1}\sum_{j\neq i}^{G}\tilde{A}_{ij}\sum_{t\neq i}^{G}\tilde{B}_{jt}Y_{ti}$
Domestic value-added returned home as intermediate goods(T5)	$V_i(I-A_{ii})^{-1}\sum_{j\neq i}^{G}\tilde{A}_{ij}\tilde{B}_{ji}Y_{ii}$
Domestic value-added double counted term(T6)	$V_i(I-A_{ii})^{-1}(\tilde{B}_{ii}-I)E_i$
Foreign value-added absorbed by country j as final goods(T7)	$\sum_{s\neq i}^{G}\sum_{j\neq i}^{G}V_s(I-A_{ss})^{-1}\tilde{A}_{si}(Y_{ij}+\tilde{A}_{ij}\sum_{t\neq j}^{G}\tilde{B}_{jt}Y_{tj})$
Foreign value-added absorbed by country j as intermediate goods(T8)	$\sum_{s\neq i}^{G}\sum_{j\neq i}^{G}V_s(I-A_{ss})^{-1}\tilde{A}_{si}\tilde{A}_{ij}\tilde{B}_{jj}Y_{jj}$
Foreign value-added double counted term(T12)	$\sum_{j\neq i}^{G}V_j(I-A_{jj})^{-1}(\tilde{B}_{ji}-\tilde{A}_{ji})E_i$
Foreign value-added absorbed by the third countries through country j(T9)	$\sum_{s\neq i}^{G}\sum_{j\neq i}^{G}V_s(I-A_{ss})^{-1}\tilde{A}_{si}\tilde{A}_{ij}(\sum_{t}^{G}\sum_{s\neq t,i,j}^{G}\tilde{B}_{jt}Y_{ts}+\sum_{s\neq i,j}^{G}\tilde{B}_{js}Y_{ss})$
Foreign value-added returned home as final goods(T10)	$\sum_{s\neq i}^{G}\sum_{j\neq i}^{G}V_s(I-A_{ss})^{-1}\tilde{A}_{si}\tilde{A}_{ij}\sum_{t\neq i}^{G}\tilde{B}_{jt}Y_{ti}$
Foreign value-added returned home as intermediate goods(T11)	$\sum_{s\neq i}^{G}\sum_{j\neq i}^{G}V_s(I-A_{ss})^{-1}\tilde{A}_{si}\tilde{A}_{ij}\tilde{B}_{ji}Y_{ii}$

Comparing with KWW method, three new terms emerge here. They are the foreign value-added being absorbed by other countries and returned back to country i finally.

In the terms above, some terms are equal to the terms in KWW method, including:

1) Domestic value-added returned home:

$$V_i\sum_{j\neq i}^{G}B_{ij}Y_{ji}=V_i(I-A_{ii})^{-1}\sum_{j\neq i}^{G}\tilde{A}_{ij}\sum_{t\neq i}^{G}\tilde{B}_{jt}Y_{ti}$$

$$V_i\sum_{j\neq i}^{G}B_{ij}A_{ji}(I-A_{ii})^{-1}Y_{ii}=V_i(I-A_{ii})^{-1}\sum_{j\neq i}^{G}\tilde{A}_{ij}\tilde{B}_{ji}Y_{ii}$$

2) Domestic value-added double counting term:

$$V_i\sum_{j\neq i}^{G}B_{ij}A_{ji}(I-A_{ii})^{-1}E_i=V_i(I-A_{ii})^{-1}(\tilde{B}_{ii}-I)E_i$$

3) Domestic value-added absorbed by other countries:

$$V_i\sum_{j\neq i}^{G}B_{ii}Y_{ij}+V_i\sum_{j\neq i}^{G}B_{ij}Y_{jj}+V_i\sum_{j\neq i}^{G}\sum_{s\neq i,j}^{G}B_{ij}Y_{js}$$
$$=V_i(I-A_{ii})^{-1}[\sum_{j\neq i}^{G}(Y_{ij}+\tilde{A}_{ij}\sum_{t\neq j}^{G}\tilde{B}_{jt}Y_{tj})+\sum_{j\neq i}^{G}\tilde{A}_{ij}\tilde{B}_{jj}Y_{jj}+\sum_{j\neq i}^{G}\tilde{A}_{ij}(\sum_{t}^{G}\sum_{s\neq t,i,j}^{G}\tilde{B}_{jt}Y_{ts}+\sum_{s\neq i,j}^{G}\tilde{B}_{js}Y_{ss})]$$

According to the expression in the KWW method, the decomposition motivation is based on the how often value-added cross the international trade border. For detailed, the term $V_i\sum_{j\neq i}^{G}B_{ii}Y_{ij}$ is value-added in the country's final goods exports; the term $V_i\sum_{j\neq i}^{G}B_{ij}Y_{jj}$ is value-added in the country's intermediate exports used by the direct importer to produce final goods consumed by the direct importer; and the term $V_i\sum_{j\neq i}^{G}\sum_{s\neq i,j}^{G}B_{ij}Y_{js}$ is value-added in the country's intermediate exports used by the direct importing country to produce final goods for third countries. However, the decomposition in this paper is based on the destination and finals or intermediate of exports flows. For example, the first term in the framework represents the value-added was absorbed finally by the exported target country *j* as the final goods (for country j). The sub-term $V_i(I-A_{ii})^{-1}\sum_{j\neq i}^{G}Y_{ij}$ represents the value-added flow in the round 1 and other sub-term represents the value-added flow in the subsequent rounds. We can find the similar interpreting for other value-added decomposition terms. Meanwhile, because the export target country is confirmed in this framework, this decomposition method can be easily extended to the bilateral and sectoral level exports decomposition.

Decomposition result: comparing the gross exports decomposition at the sectoral level between China and Brazil

In this section, we apply our disaggregated accounting framework to the

World Input-output Database (WIOD). The WIOD, developed by a consortium of eleven European research institutions funded by the European Commission, provides a time series of inter-country input-output (ICIO) tables from 1995 to 2011, covering 40 economies including all major industrialized countries and major emerging trading nations. Timmer et al. (2012) provide a detailed description of this database.

3.1 Gross exports

We first demonstrated the decomposition for the gross exports of Chinese and Brazilian electronic sector (sector 14) in the WIOD database because this sector is the most important sector for Chinese export. The decomposition is presented in table 2 as follow. The gross exports column records the country's export in the unit of millions of dollars. The DC column means the domestic content in export and the FC column means the foreign content in export. Other columns mean the decomposition terms interpreted in the table 1.

Although Chinese amount of gross export is much higher than Brazilian, we more focus the structure of value-added in the export in this section. According to the table 2, we can find that Brazil always has higher domestic content in the sector 14 export. Comparing with China's domestic content, Brazilian export always have higher intermediate goods share and China has higher final goods share. Concerning to the foreign part, China has higher foreign double counting term, which means more foreign value-added participates the Chinese production processing for more than 1 time, or we can identify that more foreign intermediate goods was embodied in the Chinese production repeat. For sector 14, it's obviously that China participate the GVCs deeper than Brazil because of not only export amount but also the value-added structure in the exports. However, more foreign value-added input and more final goods exports shows that China still in the bottom of this Chain.

Table 2： The exports decomposition in sector 14

			Domestic Part(%)

		Gross exports	DC	T1	T2	T3	T4	T5	T6
CHN	1995	34032.2	77.9	44.4	21.5	11.3	0.2	0.3	0.1
	2011	721416.6	71.1	35.2	19.5	12.6	0.8	1.6	1.5
BRA	1995	2141.7	86.9	34.9	33.5	18.3	0.1	0.1	0.0
	2011	9916.4	77.9	29.2	28.2	19.9	0.3	0.2	0.1
			Foreign Part(%)						
			FC	T7	T8	T9	T10	T11	T12
CHN	1995		22.1	9.8	4.8	2.5	0.0	0.1	4.9
	2011		28.9	10.7	5.9	3.8	0.3	0.5	7.8
BRA	1995		13.1	4.2	4.0	2.2	0.0	0.0	2.7
	2011		22.1	5.8	5.6	3.9	0.1	0.0	6.7

In the next, we focus another sector which is the most important for Brazil, we chose the food sector, sector 3 in the WIOD database. The decomposition of sector 3 for Chinese and Brazilian exports is presented in table 3 as follow.

Table 3: The exports decomposition in sector 3

			Domestic Parts(%)						
		Gross exports	DC	T1	T2	T3	T4	T5	T6
CHN	1995	8999.5	91.7	82.8	7.4	1.3	0.0	0.0	0.0
	2011	50539.7	88.9	76.7	9.5	2.3	0.1	0.2	0.2
BRA	1995	9671.6	94.0	39.3	44.6	9.9	0.1	0.1	0.0
	2011	52430.8	90.9	49.1	32.8	8.7	0.1	0.1	0.0
			Foreign Parts(%)						
			FC	T7	T8	T9	T10	T11	T12
CHN	1995		8.3	6.3	0.6	0.1	0.0	0.0	1.4
	2011		11.1	8.0	1.0	0.2	0.0	0.0	1.9
BRA	1995		6.0	2.1	2.4	0.5	0.0	0.0	1.0
	2011		9.1	3.9	2.6	0.7	0.0	0.0	1.8

The amount of gross export for two countries is approximate. However if we focus the structure of value-added in the export in this section, we will find more detailed about the difference between China and Brazil. According to the table 3, similar with sector 14 exports, Brazil always has also higher domestic content in this sector export. Comparing with China's domestic content, Brazilian export also have significantly higher intermediate goods share and China has higher final goods share. For sector 3 export value-added structure, it's uncertain that which country participate the GVCs deeper but we can conclude that Brazilian export's position in sector 3 in the GVCs is higher than Chinese.

3.2 Bilateral exports

According to the framework of exports decomposition, we can easily extent the methodology to the bilateral exports. In this subsection, we will demonstrate the bilateral exports decomposition between China and Brazil in sector 3 and 14. The decomposition detailed were showed in the table 4 and 5 as follow:

Table 4: The bilateral exports decomposition in sector 14

			Domestic Parts(%)						
		Exports	DC	T1	T2	T3	T4	T5	T6
CHNtoBRA	1995	259.0	77.9	42.4	32.1	3.2	0.0	0.0	0.1
	2011	17277.5	71.1	44.7	20.8	3.7	0.1	0.3	1.5
BRAtoCHN	1995	11.4	86.9	16.8	50.3	19.6	0.1	0.1	0.0
	2011	525.2	77.9	29.2	28.7	19.5	0.3	0.2	0.1
			Foreign Parts(%)						
			FC	T7	T8	T9	T10	T11	T12
CHNtoBRA	1995		22.1	9.4	7.1	0.7	0.0	0.0	4.9
	2011		28.9	13.5	6.3	1.1	0.0	0.1	7.8
BRAtoCHN	1995		13.1	2.0	6.0	2.3	0.0	0.0	2.7
	2011		22.1	5.8	5.7	3.8	0.1	0.0	6.7

Table 5: The bilateral exports decomposition in sector 3

			Domestic Parts(%)						
		Exports	DC	T1	T2	T3	T4	T5	T6
CHNtoBRA	1995	7.7	91.7	74.8	15.1	1.7	0.0	0.0	0.0
	2011	550.1	88.9	70.9	14.4	3.2	0.1	0.2	0.2
BRAtoCHN	1995	293.2	94.0	19.2	61.6	13.2	0.0	0.0	0.0
	2011	3365.7	90.9	55.0	29.3	6.4	0.1	0.0	0.0
			Foreign Parts(%)						
			FC	T7	T8	T9	T10	T11	T12
CHNtoBRA	1995		8.3	5.7	1.1	0.1	0.0	0.0	1.4
	2011		11.1	7.4	1.5	0.3	0.0	0.0	1.9
BRAtoCHN	1995		6.0	1.0	3.3	0.7	0.0	0.0	1.0
	2011		9.1	4.4	2.3	0.5	0.0	0.0	1.8

Similar with the gross exports structure, we also can find it both has the similar structure with the gross exports in the bilateral level. For sector 14 value-added structure, Brazil always has higher domestic content and Brazilian export always have higher intermediate goods share and China has higher final goods share. Simultaneously, China has higher foreign content term and foreign double counting term. For the sector 3 bilateral exports, although the amount of gross export for two countries is approximate, the amount of export from Brazil to China is much higher than the one from China to Brazil, we can see sector 3 is a significant sector for export from Brazil to China. In this sector bilateral export value-added structure which is similar with the gross exports value-added structure, Brazil always has also higher domestic content in this sector export and has significantly higher intermediate goods share and China has higher final goods share.

Conclusion Remarks

The increasing complexity and sophistication of GVCs brings an urgent challenge to policy makers since "you can't manage what you can't measure". One of the most important starting points for better understanding GVCs is to first develop good measures that can clearly show the value-added distribution and position and the degree of participation of countries and industries in GVCs. In this paper, we presented the difference value-added structure distribution and evolution between China and Brazil in the sectoral and bilateral sectoral exports and analyzed the different position in the GVCs using the WIOD database from 1995 to 2011. We can find that even though the amount of exports is approximate for same sector export between China and Brazil, the distribution of value-added is so different and the role is various as well. So this result can help us better understand different countries' role in the GVCs and global trade.

The methodology proposed in this paper is an extension of KWW method, the contribute of this paper include aspects as follow, 1) tracing the exports goods propagation route including the export goods propagation route and ultimate destination country; 2) tracing the value-added terms including domestic or foreign and double counting expression in this mathematical framework 3) explaining where the value-added double counting terms come from and giving the explicit expression of domestic and foreign value-added double counting terms. The decomposition method of exports can be considered a touchstone for better understanding on the position and participation of countries and industries in various GVCs. The relevant terms in the framework can provide a useful tool in analyzing the determinants of country's role in GVCs as well as doing policy-oriented analyses concerning how to help countries to be involved in and make upgrading in GVCs.

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