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Abstract

This paper examines the direct and indirect effects of a government's wage policy in the public sector on the overall income distribution in the economy. By direct effects we mean the wage differentials in the public sector. Indirect effects refer to the secondary effects of the government's policy through changes in the occupational structure. This analysis is based on a simple model suggested by Tinbergen (1951) and Roy (1951), followed by Houthakker (1976). In the numerical calculations, the model yields realistic conclusions which underline the importance of government wage policy.

Key Word: income distribution, wage policy, public and private sector, Lorentz curve.

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Introduction

Government intervention in the process of wage determination takes a variety of forms. Discussions of this question most commonly refer to statutory regulation of wages (primarily concerned with establishing minimum rates and the protection of low-paid workers), arbitration procedures, and intervention in collective bargaining. The recent growth in Western economies of the public sector relative to the private sector has shifted the focus of this discussion to the direct and indirect effects of wage policies in the public sector on the overall labor market conditions and on income distribution. By *direct effects* I mean the level of pay by the government for different occupations. Indirect effects refer to the "secondary" effects, generated through changes in the occupational structure, as induced by wage policies in the public sector.

This paper focuses on the effects of government's wage policy *within* the public sector on the distribution of income in the private sector, as well as on the overall income distribution. In particuiar, by means of a very simple model originally suggested by Tinbergen. (1951) and followed by Roy (1951) and Houthakker (1976), it examines the effects of the public sector's wage schedule on the distribution of the labor force between sectors and on the distribution of income.

Let us first look at some facts concerning income differentials between the private and public sectors in some Western economies. Smith (1976) has studied pay differences between these sectors for the US. economy. Although no systematic difference was found for the *average* pay across diffenent comparable occupations, there seems to be a clear negative correlation between the wage ratio (public to private) and the level of income. From her findings one can compute Table 1 .

Table 1:
Average Earnings of Men in Different Occupational Groups U.S.A 1958-1970

	Percentage of average for all groups	
	Private sector	Public Sector
Higher Professional	298	180
Lower Professional	124	154
Administrators and managers	271	160
Clerks	100	110
Foremen	149	130
Skilled manual	117	100
Semiskilled manual	85	89
Unskilled manual	79	85

It is quite clear from the table that the spread of wages is much smaller in the public sector than in the private sector. The obvious question that this observation brings to mind is whether the public sector is getting lower abiiity individuals *within* each occupation. If so, then there is presumably an additional effect, namely, if individualis with high ability within the high-paying occupations tend to work in the private sector and if low-ability individuals work in the low-paying occupations, then the income distribution in the private sector tends to be less egalitarian when the wage policy in the public sector is more egalitarian. Some evidence for such tendency in the Israeli economy can be deduced from the studies of Hanoch (1963) and Levy (1975).

We now turn to a simple model of the choice of work among sectors, showing the dependence of income distribution within and between sectors on the public sector's wage policy.

The assumptions of the model are no doubt too simple to provide a realistic description. Nevertheless, they have fairly realistic implications for the distribution of income and the labor force.

A Basic Model of Labor Force Distribution

Each individual is assumed to maximize his earnings by choosing among occupations according to his aptitude for each occupation. We focus on the choice between public sector and private sector employment. Let the individual's aptitudes be summarized by the pair (a_1, a_2), where a_1 is his marginal product in the public sector, and a_2 is his marginal product in the private sector. For simplicity, these productivities are assumed to be independent of the labor force allocation between the sectors. The individual's working time is assumed to be fixed at unity. The time devoted to the ith occupation is $x_i, \quad 0 \leq x_i \leq 1$. The wages paid. by each sector, w_i, are assumed to depend on his aptitude: $w_i = w_i\,(a_i)$. It is thus assumed that the individuai's aptitude can be identified.

The individual is supposed to not favor any occupation and hence to altocate his working time so as to maximize his income. Thus,

$$\max_{x_1, x_2} (w_1 x_1 + w_2 x_2) \tag{1}$$

subject to

$$x_1 + x_2 = 1 \tag{2}$$

The maximum is reached by working all the time in the sector for which his wage is greatest. Normally there is only one such occupation. If there is more than one, then the allocation of time is indeterminate.

This simple micromodel lends itself readily to aggregation over individuals, provided suitable continuity assumptions are made. For this purpose we assume that the pair (a_1, a_2) varies randomly over the individuals with a continuous density function $f\,(a_1, a_2)$. The distribution function $F\,(a_1, a_2)$ is then also continuous. To eliminate the indeterminacy mentioned earlier, only density functions and wage schedules $w_i\,(a_i)$ where ties (i.e. $w_1 = w_2$) have a zero probability are considered. Otherwise, no restrictions are imposed on the joint density function. In particuiar, aptitudes may or may not be independent of each other.

The (cumulative) distribution function for incomes in sector $i = 1, 2$, is denoted $G_i(z)$, and the corresponding density is denoted $g_i(z)$. The overall distribution function is $G(z) = G_1(z) + G_2(z)$ and the density is $g(z) = g_1(z) + g_2(z)$.

If w_i are strictly monotone increasing functions of a_i, then the basic implication of the model for the distribution of income is

$$G(z) = F\left[w_1^{-1}(z_1), \, w_2^{-1}(z_2)\right] \tag{3}$$

where $z = w_1(a_1) = w_2(a_2)$ and w_i^{-1} are the corresponding inverse functions.

The distribution of income in any occupation can be derived similarly. For example,

$$G_1(z) = \int_0^z \int_0^{z_1} f\left[w_1^{-1}(z_1), \, w_2^{-1}(z_2)\right] \frac{1}{\Delta} dz_1 dz_2 \tag{4}$$

where

$$\Delta = w_1^{'}\left[w_1^{-1}(z_1)\right] w_2^{'}\left[w_2^{-1}(z_2)\right], \quad w_i^{'} = {}^{dv_i(a_i)}/_{da_i}$$

It is assumed throughout that the private sector pays individuals their marginal product, that is,

$$w_2(a_2) = a_2 \tag{5}$$

The analysis focuses on the impact of the wage policy in the public sector on income distribution, labor force allocation, and output levels. To simplify, it is confined to linear wage schedules, and to bivariate exponential distributions, selected for computational convenience, rather than realism. Then, let

$$w_1 = \alpha + \beta a_1 \tag{6}$$

where $\alpha, \quad \beta\,(\beta > 0)$ are constants. The density function of aptitudes is assumed to have the form:[1]

$$f(a_1, \, a_2) = \hat{\theta}_1 \hat{\theta}_2 \exp\left(-\hat{\theta}_1 a_1 - \hat{\theta}_2 a_2\right) \tag{7}$$

where $\hat{\theta}_1, \quad \hat{\theta}_2$ ($\hat{\theta}_1 > 0$, $\hat{\theta}_2 > 0$) are constants. Bt Eqs. (4)-(7),

$$\begin{aligned} G_1(z) =& \theta_1\theta_2 \exp(\theta_1\alpha) \int_\alpha^z \int_0^{w_1} \exp(-\theta_1 w_1 - \theta_2 w_2)\, dw_1 dw_2 \\ &= [\theta_1 / (\theta_1 + \theta_2)] \{\exp(-\theta_2\alpha) \exp[-(\theta_1 + \theta_2)(z - \alpha)] - 1\} \\ &\quad - \exp[-\theta_1(z - \alpha)] + 1 \end{aligned} \tag{8}$$

[1] This distribution is the product of two univariate exponentials and therefore does not allow for dependence. Bivariate exponentials with dependence have been introduced by Marshall and Olkin (1967), but they violate this assumption that the probability of ties is zero.

where $\theta_1 = \left({}^{\hat{\theta}_1}/_{\beta}\right)$, $\theta_2 = \hat{\theta}_2$. Similarly,

$$\begin{aligned} G_2(z) =& \theta_1\theta_2 \exp(\theta_1\alpha) \int_\alpha^z \int_\alpha^{w_2} \exp(-\theta_1 w_1 - \theta_2 w_2)\, dw_1 dw_2 \\ =& [\theta_2/(\theta_1+\theta_2)]\{\exp(-\theta_2\alpha)\exp[-(\theta_1+\theta_2)(z-\alpha)] - 1\} \\ &- \exp(-\theta_2 z) + \exp(-\theta_2\alpha) \end{aligned} \tag{9}$$

Adding up, we find

$$\begin{aligned} G(z) =& \exp(-\theta_2\alpha)\exp[-(\theta_1+\theta_2)(z+\alpha)] - \exp[-\theta_1(z-\alpha)] \\ &- \exp(-\theta_2 z) + 1 \\ =& \{1 - \exp[-\theta_1(z-\alpha)]\}\{1 - \exp\{-\theta_2 z\}\} \end{aligned} \tag{10}$$

The income density within the public sector is given by

$$g_1(z) = \theta_1 \exp[-\theta_1(z-\alpha)][e - \exp(-\theta_2 z)] \tag{11}$$

and in the private sector by

$$g_2(z) = \theta_2 \exp(-\theta_2 z)\{1 - \exp[-\theta_1(z-\alpha)]\} \tag{12}$$

Hence, the overall income density is

$$\begin{aligned} g(z) =& \theta_1 \exp[-\theta_1(z-\alpha)] + \theta_2 \exp(-\theta_2 z) \\ &- (\theta_1+\theta_2)\exp(-\theta_2\alpha)\exp[-(\theta_1+\theta_2)(z-a)] \end{aligned} \tag{13}$$

Notice that although. the density furiction of abilities is J-shaped for each sector, the density of income in each sector looks much more like the income distributions encountered in reality (see Figure 1 and Lydall 1968).

Suppose now that the government undertakes a progressive wage policy, that is, $\alpha > 0$, $\beta < 1$. It is immediately seen from Eqs. (8)-(12) that the effect of such a policy on the income distribution *within* each sector and one the overall income distribution is ambiguous. Such comparisons depend, of course, on the constraints imposed on α and β as well as on the parameters $\hat{\theta}_1$ and $\hat{\theta}_2$. This point will be clarified shortly.

In order to calculate inequality measures directly, one is also interested in the cumulative income distribution function for each sector. Let

$$\begin{aligned} I_1(z) = \int_\alpha^z x g_1(x)\, dx =& \alpha + \frac{1}{\theta_1} - \{\exp[-\theta_1(z-\alpha)]\}\left(z + \frac{1}{\theta_1}\right) \\ &- \frac{\theta_1}{\theta_1+\theta_2}[\exp(-\theta_2\alpha)] \\ &\left[\alpha + \frac{1}{\theta_1+\theta_2} - \{\exp[-(\theta_1+\theta_2)(z-\alpha)]\}\left(z + \frac{1}{\theta_1+\theta_2}\right)\right] \end{aligned} \tag{14}$$

$$
\begin{aligned}
I_2(z) = \int_{\alpha}^{z} x g_2(x)\,dx = {} & [\exp(-\theta_2\alpha)]\left(\alpha + \frac{1}{\theta_2}\right) - [\exp(-\theta_2 z)]\left(z + \frac{1}{\theta_2}\right) \\
& - \frac{\theta_1}{\theta_1+\theta_2}[\exp(\theta_2\alpha)] \\
& \left[\exp[-(\theta_1+\theta_2)\alpha]\left(\alpha + \frac{1}{\theta_1+\theta_2}\right) - \{\exp[-(\theta_1+\theta_2)z]\}\left(z + \frac{1}{\theta_1+\theta_2}\right)\right]
\end{aligned} \tag{15}
$$

Denote by $I_i = \lim_{z\to\infty} I_i(z)$, the total income in sector i. By Eqs. (14) and (15)

$$
I_1 = \alpha + \frac{1}{\theta_1} - \frac{\theta_1}{\theta_1+\theta_2}[\exp(-\theta_2\alpha)]\left(\alpha + \frac{1}{\theta_1+\theta_2}\right) \tag{16}
$$

and

$$
I_2 = [\exp(-\theta_2\alpha)]\left[\frac{\theta_1}{\theta_1+\theta_2}\alpha + \frac{1}{\theta_2} - \frac{\theta_2}{(\theta_1+\theta_2)^2}\right] \tag{17}
$$

The Lorenz curve is the cerived relation between $I_i(z)/I_i$ and $G_i(z)/G_i(\infty)$.

The effect of a given "wage policy," (α, β), on outputs in each sector, Y_i, is calculated as follows. In the private sector incomes are equal to the marginal products; hence $Y_2 = I_2$. In the public sector, the relation between output, a_1, and income is given by Eq. (6). Hence, private total output is

$$
\begin{aligned}
Y_1 &= \int_{\alpha}^{\infty} \left(\frac{z-\alpha}{\beta}\right) g_1(z)\,dz \\
&= \frac{1}{\beta}\left[\frac{1}{\theta_1} - \frac{\theta_1}{(\theta_1+\theta_2)^2}\exp(-\theta_2\alpha)\right]
\end{aligned} \tag{18}
$$

Aggregate output Y is

$$
\begin{aligned}
Y = {} & Y_1 + Y_2 = \frac{1}{\beta\theta_1} + \exp(-\theta_2\alpha) \\
& \left[\frac{\theta_1}{\theta_1+\theta_2}\alpha + \frac{1}{\theta_2} - \frac{\theta_2}{(\theta_1+\theta_2)^2} - \frac{\theta_1}{\beta(\theta_1+\theta_2)^2}\right]
\end{aligned} \tag{19}
$$

One can readily calculate that $\alpha = 0$, $\beta = 1$ is a global maximum of Y.[2]

Some Numerical Examples

For numerical illustration, we shall take arbitrary values, $\hat{\theta}_1 = .02$ and $\hat{\theta}_2 = .01$. These values imply that the distribution of abilities is more equal in public

[2]Simply calculate that at $\alpha = 0$, $\beta = 1$, $\partial Y/\partial\alpha = \partial Y/\partial\beta = 0$, and the second-order conditions $\partial^2 Y/\partial\alpha^2 < 0$, $\partial^2 Y/\partial\beta^2 < 0$, $\left(\partial^2 Y/\partial\alpha^2\right)\left(\partial^2 Y/\partial^2\beta\right) - \left(\partial^2 Y/\partial\alpha\partial\beta\right)^2 > 0$, hold.

sector employment, relative to private sector employment. This corresponds to observed facts about distributions of skill and education level in the two sectors. As already noted, although the density function of abilities is J-shaped for each sector, the density of income in each looks like the income distributions encountered in reality (Figure. 1).[3] The comparison of alternative wage policies can be made under alternative assumptions about the restrictions imposed on the parameters (a, B). The calculations have been carried out under the assumption that. the size of empioyment in the private sector (and hence in the public sector as well) is constant.

When $\alpha = 0$, $\beta = 1$, using the chosen values for $\hat{\theta}_1$ and $\hat{\theta}_2$, we have from (9)

$$\lim_{z\to\infty} G_2(z) = \frac{\hat{\theta}_1}{\hat{\theta}_1 + \hat{\theta}_2} = \frac{2}{3} \tag{20}$$

In general, for any (α, β),

$$\lim_{z\to\infty} G_2(z) = \frac{\theta_1}{\theta_1 + \theta_2} = \exp(-\theta_2 \alpha)$$

We therefore restrict the parameters (α, β) by the relation

$$\frac{\theta_1}{\theta_1 + \theta_2} \exp(-\theta_2 \alpha) = \frac{2}{3} \tag{21}$$

which implies that two-thirds of the labor force is employed in the private sector.

From Table 2 and Figure 1 one observes the remarkable effect of changes in β on income distribution within each sector and on the economy as a whole. Imposition of a progressive wage schedule, with $\beta = .7$ ($\alpha = 10.5$), reduces significantly (by a factor of three) the upper tail of the income density in the public sector, and increases, though less dramatically, the inequality in the private sector. It also seems useful to draw the Lorenz curves for the two sectors. In both sectors, the curves pertaining to the $\beta = 1$ and $\beta = .7$ cases intersect, which implies that an evaluation of alternative policies depends on the social welfare function one adopts. It is interesting that the overall inequality as measured by the G_{ini} coefficient increases (from .31 to .32) as β decreases from 1 to .7.

[3] For clarity, the vertical axis in Figure 1 is blown up to a factor of 100.

Table 2:

Income Densities within Each Sector and the Overall Income Density[a]

	$\beta = 1$			$\beta = .7$		
z	g_1	g_2	g	g_1	g_2	g
20	.243	.270	.513	.395	.194	.589
40	.296	.369	.665	.406	.381	.787
60	.272	383	.665	.314	.415	.729
80	.222	.358	.581	.216	.387	.604
100	.171	.318	.489	.141	.339	.479
120	.127	.273	.401	.081	.288	.375
140	.092	.231	.323	.053	.240	.294
160	.065	.193	.259	.032	.199	.231
180	.045	.160	.206	.019	.164	.183
200	.032	.132	.164	.011	.135	.146

[a] One-third of the labor force is in the public sector.

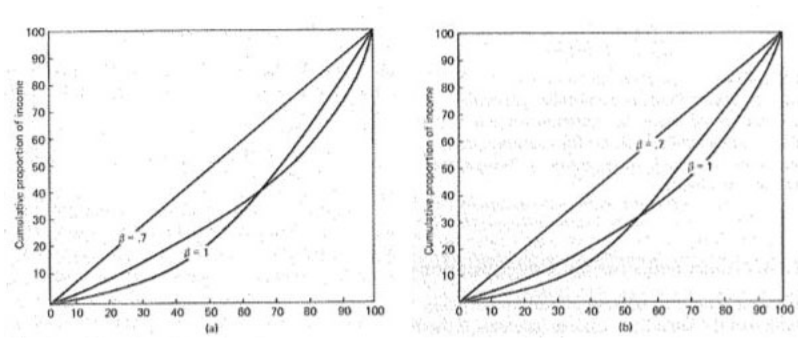

Figure 1: Two graphs show the cumulative proportion of income recipients: (a) is the public sector, (b) is the private sector

The effects of wage policies in the public sector on the size of real outputs are presented in Table 3 . As β decreases from unity to .5, the output of the public sector decreases by about 18%, whereas the output of the private sector increases by about 3.5%. Total output decreases, however, by merely 1%.

The above results clearly depend on the constraint that the allocation of the labor force between the sectors remains constant. An alternative constreint would be to fix the level of output in one of the sectors, or the ratio between the outputs of the two sectors. The results pertaining to these alternative assumptions are as expected. When the size of the output of the public sector is fixed, then a progressive ($\beta < 1$) wage policy in the public sector has a remarkable *negative* effect on the output of the private sector, reflecting the increased proportion of the labor force allocated to the public sector, needed to maintain its output. In all cases, total output decreases (depending on the type of constraint imposed, in some cases very significantly), income inequality decreases in the public sector and increases in the private sector, whereas the overall effect on income distribution is ambiguous.

The next natural step in the analysis is to find the socially optimum wage policy, that is, a wage policy that maximizes, say, a utilitarian social welfare function W,

$$\max_{\alpha,b} W = \int_0^\infty u(z)\, g(z)\, dz \tag{22}$$

However, the constraints of the problem are not obvious. The government may choose wage schedules that satisfy the constraint that the vaiue of the public sector's output is euqal to the total incomes ofthose employed in this sector (assuming that the government's output is sold),

$$\int_0^\infty zg(z) = \int_0^\infty \left(\frac{z-\alpha}{\beta}\right) g(z)\, dz$$

or

$$\alpha = (1-\beta)\, {}^{I_1}/_{G_1} \tag{23}$$

where $G_1 = \lim_{z\to\infty} G_1(z)$.

If the government has available tax-subsidy instruments then the anaiysis becomes more complicated since the optimum wage policy will depend on the nature of these instruments (such as, for example, income taxation) and the issue of the *optimum mix of wage and tax policies* arises. These issues are not anayzed in this paper.

Table 3:
Output in the Public and Private Sectors and Total Output

β	Y_1	Y_2	Y
1.0	27.8	88.9	116.7
.9	27.0	89.6	116.6
.8	26.2	90.3	116.5
.7	25.3	91.0	116.3
.6	24.4	91.6	115.9
.5	23.3	92.1	115.5

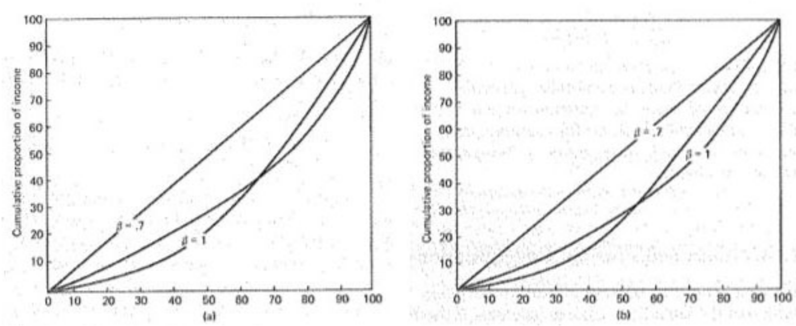

Figure 2: Two graphs show the cumulative proportion of income recipients: (a) is the piblic sector, (b) is the private sector

Reservation and Possible Generalizations

We have examined the effects of wage policies in the public sector on the sectoral distribution of the labor force and on incomes. Although the underlying model has been extremely simple, it seems to yield quite realistic conclusions. However, the assumptions are too simple to provide a framework for certain important issues, three of which are discussed here.

First, one expects a positive correlation between individual aptitudes in different occupations. We have assumed in our examples that aptitudes are distributed independently. It would he interesting to study the effects of wage policies for varying degrees of correlation between aptitudes.

Second, the underlying production functions have been overly simplified. In particular, marginal products a_1 and a_2 are independent of the allocation of the labor force. More generally, one should explore the *interaction* of employment in the public sector on marginal productivity in the private sector, end vice versa. More specifically, suppose that total output Y is a function of two types of employments: public L_1 and private L_2: $Y = F(L_1, L_2)$. Now, suppose

$$L_i = \int_0^\infty a_i X_i(a_i)\, da_i$$

where $X_i(a_i)$ is the number of laborers in section i with aptitude a_i. The function $X_i(a_i)$ is determined by individual maximization as in the basic model. Competition in the private sector implies that $w_1(a_1) = a_1 F_1(L_1, L_2)$, whereas $w_2 = w_2(a_2)$ is a choice function. Clearly, in analyzing this model, terms such as $F_{12} = {}^{\partial^2 Y}/_{\partial L_1 \partial L_2}$, that is, complementarity or substitution between public and private employment, will be crucial.

Finally, it has been assumed throughout that the government (planner), as well as private employers, can identify the true aptitude of each individual. This is a rather extreme assumption, particularly when it concerns the public sector. One could, however, modify the model to include imperfect information. Houthakker (1974), for example, has postulated that the *probability* that an individual with aptitudes (a_1, a_2) will work in. sector i, P_i , is given by

$$P_i = \frac{\exp(ka_i)}{\exp(ka_1) + \exp(ka_2)}$$

Clearly, when $k = 0$, all sectors have an equal probability of being chosen, whereas $k \to \infty$ is the case just analyzed.

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