# Reassessing price adjustment costs in DSGE models

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**Abstract**

Indexation theories have become standard for inflation persistence in DSGE models (Smets and Wouters (2003, 2007)). However, these theories overlook an important stylized fact of U.S. business cycles: high fluctuations in the first difference of inflation. I find that this pattern can be captured by adjustment costs precisely from the first difference of inflation (Pesaran (1991) labels this difference as a "speed change"). I estimate four DSGE models differing in their rigidity assumption and find that a framework with inflation-based adjustment costs has the highest probability to fit U.S. data.

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# 1 Introduction

Dynamic Stochastic General Equilibrium (DSGE) models with nominal rigidities have become useful tools in the analysis of monetary policy (Schorfheide (2000); Smets and Wouters (2003, 2007)). However, the rigidity mechanism is only a consensus to the extent that persistence in inflation is needed to fit the empirical data. An inertial behavior of inflation usually requires the assumption of price indexation to the last period's inflation rate. Prominent examples of indexation with Calvo and Rotemberg pricing (Calvo (1983); Rotemberg (1982)) are Smets and Wouters (2003, 2007) and Ireland (2007). However, less interest has been given to the persistence from a costly first difference of inflation (Pesaran (1991) refers to this first difference as a "speed change"). According to this view, adjustment costs arise whenever the inflation rate differs from its last period's level. The importance of adjustment costs from inflation differences has been studied by Ireland (2001). By estimating a DSGE framework with maximum likelihood methods, Ireland (2001) concludes that these costs are not relevant.

In this paper, I make one main contribution. By using Bayesian estimation techniques and U.S. data, I establish that the costly inflation difference of Pesaran (1991) should in fact be considered at the firm level. However, I depart in several ways from Ireland (2001). First, my measure is the squared absolute first difference of inflation (following the speed-change idea of Pesaran (1991)), as opposed to the squared relative change of inflation formulated by Ireland (2001). Second, I use a more recent dataset for the U.S. economy that includes another time period as in Ireland (2001). Finally, I use Bayesian methods instead of a maximum likelihood estimation.

As a starting point, I draw attention to the costs from inflation first differences by looking at U.S. inflation during the period 1983Q1–2008Q2. Figure 1 shows the squared first difference of the inflation rate based on the GDP deflator. I normalized the series with its corresponding variance to account for the fluctuation range over the whole sample. The crux of figure 1 is that the series reflects a distinct view of the prime cause of adjustment costs, namely first-difference adjustment costs in the spirit of Pesaran (1991). The observation that substantiates this paper is the occurrence of highly pronounced costs during extended periods of time. Since the costs induced by "speed changes" (in the terminology of Pesaran (1991)) have been sizable, I ask whether they must be incorporated into the microeconomic problem of the firm. By answering this question, I clarify whether the aggregate costs from inflation first differences can be explained at the level of the representative monopolistic firm.

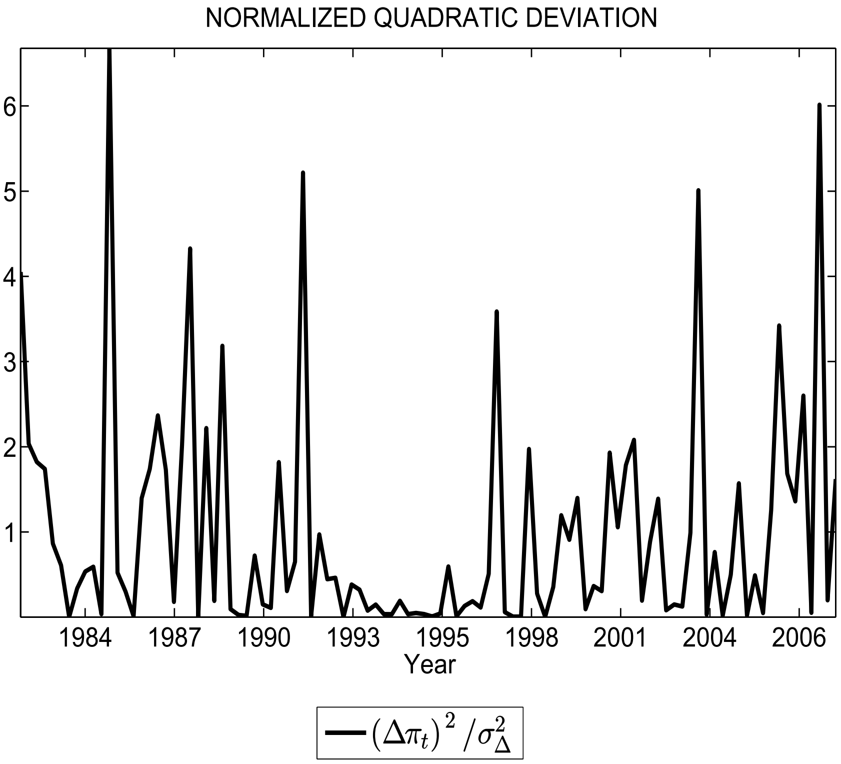


**Figure 1:** Costs from the first difference of inflation in the United States during the time period 1983Q1–2008Q2. *Notes: The inflation rate $\pi_t$ is computed as the first difference of the GDP deflator in natural logarithms. The costs from the first difference of inflation are calculated as the squared first difference of the inflation rate normalized by its sample variance.*

I seek to answer the above-mentioned question using quarterly U.S. data and a framework that has become widespread in the analysis of monetary policy: a standard cashless New Keynesian model (Woodford (2003)). As a centerpiece of the model, I derive the New Keynesian Phillips curve assuming costs from inflation first differences at the firm level. Subsequent Bayesian estimations are tied up with two main objectives. One objective is to determine the size of the parameter that governs these costs. The second objective is to compare the explanatory power to alternative model versions. To that end, I estimate another three competing DSGE models, each with a distinct friction and therefore a distinct New Keynesian Phillips curve. The Bayesian methodology then puts me in the position to evaluate the models in accordance to their marginal likelihoods. The alternative rigidity views considered here are common in the DSGE literature. I consider the standard adjustment cost structure of Rotemberg (1982), its extension with indexation by Ireland (2007), and the indexation extension under Calvo pricing by Christiano, Eichenbaum, and Evans (2005).

The estimation results let me conclude that aggregate first-difference inflation costs can be best described by a corresponding specification at the firm level. I first confirm that the parameter governing the costs from inflation first differences is empirically relevant. Moreover, I find that the consideration of first-difference inflation costs increases the relative marginal likelihood of a DSGE model to its greatest value.

Bayesian estimation techniques have become popular when it comes to evaluating a DSGE model with empirical data. Important contributions are, for instance, An and Schorfheide (2007) and Otrok (2001). Further applications can be found in Krause, López-Salido, and Lubik (2008), Schorfheide (2000), and Smets and Wouters (2003, 2007). I pursue a strategy that is closest to Rabanal and Rubio-Ramírez (2005). Using marginal likelihoods, Rabanal and Rubio-Ramírez (2005) rank New Keynesian models that differ in their New Keynesian Phillips curves. A common weakness of such rankings is the dependence of marginal likelihoods on assigned priors. Therefore, I adjust marginal likelihoods using the training-sample method by Sims (2003). Applications of this procedure can be found in Kriwoluzky and Stoltenberg (2014, 2015).

The rest of the paper is organized as follows. Section 2 outlines an adjustment cost structure that considers the first difference of inflation at the firm level. Section 3 presents a set of DSGE models that differ in their New Keynesian Phillips curves. I present and discuss estimation results in section 4. Section 5 summarizes and concludes.

# 2 Adjustment Costs

A basic DSGE model represents an economy with a continuum of monopolistic firms. Apart from its own demand function, a profit-maximizing firm $i \in [0,1]$ takes into account that adjusting its price level $P_t(i)$ is costly.[1] I summarize these costs in $\Omega_t(i)$. The literature usually assumes that $\Omega_t(i)$ is scaled up by aggregate real output $Y_t$ (see for an example Ireland (2004)). This aims to reflect a rise in adjustment costs due to increased aggregate activity. Thus, the overall adjustment costs the firm has to cope with are given by $\Omega_t(i)Y_t$. The problem of intertemporal profit maximization is

$$\max_{P_t(i)} \quad \mathbb{E}_t \sum_{j=0}^{\infty} \Delta_{t,t+j} \left( \tilde{P}_{t+j}(i) Y_{t+j}(i) - MC_{t+j}(i) Y_{t+j}(i) - \Omega_{t+j}(i) Y_{t+j} \right), \tag{1}$$

subject to $Y_t(i) = \tilde{P}_t^{-\epsilon} Y_t$. The expression $\tilde{P}_t = P_t(i)/P_t$ stands for the relative price, $\epsilon$ denotes the constant elasticity of substitution, $MC_t$ is real marginal cost, and $\Delta_{t,t+j} = \beta^j u_c(t+j)/u_c(t)$ is the stochastic discount factor. Adjustment costs from price changes are commonly formulated according to Rotem-

[1] The typical interpretation for the emergence of adjustment costs is that customer relationships worsen as the price is changed (Rotemberg (1982)).

berg (1982). The corresponding structure is

$$\Omega_t^{\mathcal{R}}(i) = \frac{\psi}{2}\left[\pi_t(i)\right]^2, \qquad (2)$$

where $\pi_t(i) = (P_t(i) - P_{t-1}(i))/P_{t-1}(i)$ represents the (net) inflation rate computed with the price of firm $i$ and $\psi$ denotes a scaling parameter. I argue in this paper that costs from the first difference of firm-level inflation are appropriate because aggregate fluctuations of these costs have been important (see figure (1)). The corresponding adjustment cost structure is modeled as in Pesaran (1991):[2]

$$\Omega_t^{\mathcal{P}}(i) = \frac{\upsilon}{2}\left[\pi_t(i) - \pi_{t-1}(i)\right]^2, \qquad (3)$$

where $\upsilon$ is a scaling parameter. Therefore, (adjustment) costs arise whenever changes in the first difference of inflation $\pi_t(i) - \pi_{t-1}(i)$ occur. This means that the price-setting problem of the firm is subject to a more elaborate structure than in the standard formulation by Rotemberg (1982). To make the implications of this sophistication clear, note that the structure (3) can be rewritten as

$$\Omega_t^{\mathcal{P}}(i) = \frac{\upsilon}{2}\left[\pi_t(i)\right]^2 + \frac{\upsilon}{2}\left[\pi_{t-1}(i)\right]^2 - \upsilon\pi_t(i)\,\pi_{t-1}(i). \qquad (4)$$

The first term corresponds to price adjustment costs (see equation (2)) and discrepancies to Rotemberg (1982) are due to the second and third terms. A price adjustment by the firm gives rise to costs from relative price changes (first term). These costs translate to the next period (second term), which generates dynamically persistent adjustment costs.[3] The third term enters the expression with a negative sign because the deviation $\left[\pi_t(i) - \pi_{t-1}(i)\right]^2$

[2]Similar formulations can be found in Ireland (2001), Price (1992), Fuhrer and Moore (1995), Brayton et al. (1997), Nelson (1998), and Tinsley (2002).

[3]A firm not adjusting its price ($\pi_t(i) = 0$) still encounters price adjustment costs $\frac{\upsilon}{2}\left[\pi_{t-1}(i)\right]^2$ from the last period.

decreases after a movement of $\pi_t(i)$ in the same direction as $\pi_{t-1}(i)$. The Bayesian estimation results I present further below indicate that structure (3) is empirically more relevant than structure (2). This suggests that persistent adjustment costs (second term in (4)) and intertemporal offsetting effects (third term in (4)) at the firm level are important for the description of aggregate data.

I emphasize the empirical benefit of assuming the structure (3) within a fully specified DSGE model. As with any other price-related friction, the consideration of structure (3) leads to a very specific New Keynesian Phillips curve. The first-order condition of problem (1) leads to

$$\hat{\pi}_t = \gamma_{-1}\hat{\pi}_{t-1} + \gamma_{+1}\mathbb{E}_t\left[\hat{\pi}_{t+1}\right] - \gamma_{+2}\mathbb{E}_t\left[\hat{\pi}_{t+2}\right] + \gamma_y\left(\sigma+\eta\right)\left(\hat{y}_t - \hat{y}_t^n\right). \tag{5}$$

Here, $\hat{k}_t$ defines the percentage deviation of a generic variable $k_t$ from its steady state value $k$. Moreover, the natural output level $\hat{y}_t^n$ depends on shocks that I declare in later sections. The $\gamma$-weights depend on deep structural parameters specified below, and the household preference parameters $\sigma = -u_{cc}y/u_c > 0$, $\eta = v_{ll}l/v_l > 0$ represent the coefficient of relative risk aversion and the inverse of the Frisch elasticity of labor supply. In this paper, the comparison of alternative nominal frictions involves estimating alternative versions of the $\gamma$-weights in an otherwise common DSGE model.

The next section presents the set of common equations and the alternative versions of the New Keynesian Phillips curve.

# 3 Economic Models

I put together a collection of representative agent models that can be compared to each other in terms of explanatory power. Each framework includes a distinctive New Keynesian Phillips curve and a set of equations that is shared by all models (as in Rabanal and Rubio-Ramírez (2005)). The DSGE model that is key to this paper contains the New Keynesian Phillips curve calculated with costs from the inflation first difference (equation (3)). I denote this model as $\mathcal{M}_1$. The empirical fit of $\mathcal{M}_1$ is compared to models containing well-known pricing frictions. For example, $\mathcal{M}_2$ includes Rotemberg price adjustment costs with indexation to aggregate inflation (as in Ireland (2007)). This model is nested in the sense that the forward-looking Rotemberg setting is the core underlying inflation model. A model with Calvo pricing and non-nested indexation (following Christiano et al. (2005)) is also considered. I label this model as $\mathcal{M}_3$. Finally, $\mathcal{M}_4$ contains the purely forward-looking New Keynesian Phillips curve that results from standard adjustment costs (Rotemberg (1982)).

The following subsection introduces the equations common to all models.

## 3.1 Common Equations

The equations shared by the models $\mathcal{M}_1$–$\mathcal{M}_4$ are taken from a textbook cashless New Keynesian economy with a representative household (Woodford (2003)). The Euler equation is given by

$$\sigma \left( \mathbb{E}_t \hat{y}_{t+1} - \mathbb{E}_t \hat{y}^n_{t+1} \right) = \sigma \left( \hat{y}_t - \hat{y}^n_t \right) + \hat{R}_t - \mathbb{E}_t \hat{\pi}_{t+1} - \hat{R}^n_t. \tag{6}$$

The natural rates of output $\hat{y}^n_t$ and interest $\hat{R}^n_t$ are given in the flexible-price environment as

$$\hat{y}^n_t = \frac{1}{\sigma + \eta} \left( (1 + \eta) \hat{a}_t + \sigma \tilde{g}_t - \hat{\mu}_t \right) \tag{7}$$

and

$$\hat{R}^n_t = \sigma \left[ \left( \tilde{g}_t - \hat{y}^n_t \right) - \mathbb{E}_t \left( \tilde{g}_{t+1} - \hat{y}^n_{t+1} \right) \right], \tag{8}$$

where $\hat{a}_t$ is a productivity shock to labor as the only production factor, $\tilde{g}_t = (g_t - g)/y$ is a government expenditure shock, and $\hat{\mu}_t$ is a wage-markup shock.

I assume the following simple interest rate rule:

$$\hat{R}_t = \phi_\pi \hat{\pi}_t + \phi_y \left(\hat{y}_t - \hat{y}_t^n\right) + \hat{r}_t, \tag{9}$$

where $\hat{r}_t$ is a monetary policy shock.[4] The shocks $\hat{a}_t$, $\tilde{g}_t$, and $\hat{r}_t$ are supposed to follow stationary AR(1) processes, whereas the shock $\hat{\mu}_t$ is *i.i.d.* The wage-markup shock calls for real wages as an observable variable. I include the following equation for real wages $\hat{w}_t$:

$$\hat{w}_t = \hat{\mu}_t + \eta \left(\hat{y}_t - \hat{a}_t\right) + \sigma \left(\hat{y}_t - \tilde{g}_t\right). \tag{10}$$

A specific DSGE model is obtained by closing the common system (6)–(10) with an equation that specifies the driving forces of inflation. The following subsection presents the set of New Keynesian Phillips curves defining the models $\mathcal{M}_1$–$\mathcal{M}_4$.

## 3.2 New Keynesian Phillips Curves

All versions of the New Keynesian Phillips curve are summarized in equation (5). Table 1 presents the weights $\gamma_{-1}$, $\gamma_{+1}$, $\gamma_{+2}$, $\gamma_y$ depending on structural parameters. Here, $\theta$ and $\psi$ are the standard price rigidity parameters of Calvo (1983) and Rotemberg (1982). Similarly, $\alpha$ is the indexation parameter of Ireland (2007). Note that apart from $\mathcal{M}_4$, the models $\mathcal{M}_1$–$\mathcal{M}_3$ include a New Keynesian Phillips curve that contains a backward inflation term.

Two properties of $\mathcal{M}_1$ have to be emphasized. First, the adjustment cost scaling parameter $\upsilon$ decreases the output gap elasticity $\gamma_y$. The same (qualitative) effect is present in the other models with respect to the price rigidity parameters $\psi$, $\alpha$, $\theta$, and $\omega$. Second, the forward-looking New Keyne-

[4] Therefore, the shocks driving $\hat{y}_t^n$ are observed by the policy maker. Replacing the output gap $\hat{y}_t - \hat{y}_t^n$ with real output $\hat{y}_t$ or an output gap with efficiency output does not alter my conclusions.

sian Phillips curve is not nested by $\upsilon$, which mirrors the Calvo setting with non-nested indexation by Christiano et al. (2005).[5]

**Table 1:** Inflation weights in the New Keynesian Phillips curves.

| Weight | $\mathcal{M}_1$ Pesaran (1991) | $\mathcal{M}_2$ Ireland (2007) | $\mathcal{M}_3$ Christiano et al. (2005) | $\mathcal{M}_4$ Rotemberg (1982) |
|---|---|---|---|---|
| $\gamma_{-1}$ | $\frac{1}{1+2\beta}$ | $\frac{\alpha}{1+\alpha\beta}$ | $\frac{1}{1+\beta}$ | – |
| $\gamma_{+1}$ | $\frac{(2+\beta)\beta}{1+2\beta}$ | $\frac{\beta}{1+\alpha\beta}$ | $\frac{\beta}{1+\beta}$ | $\beta$ |
| $\gamma_{+2}$ | $\frac{\beta^2}{1+2\beta}$ | – | – | – |
| $\gamma_y$ | $\frac{\epsilon-1}{\upsilon(1+2\beta)}$ | $\frac{\epsilon-1}{\psi(1+\alpha\beta)}$ | $\frac{(1-\beta\theta)(1-\theta)}{\theta(1+\beta)}$ | $\frac{\epsilon-1}{\psi}$ |

*Notes: $\mathcal{M}_1$ is the model with adjustment costs from the first difference of inflation (Pesaran (1991)), $\mathcal{M}_2$ is the Rotemberg model with indexation (Ireland (2007)), $\mathcal{M}_3$ is the Calvo model with non-nested indexation (Christiano et al. (2005)), and $\mathcal{M}_4$ is the standard Rotemberg model (Rotemberg (1982)).*

This leads to inflation weights $\gamma_{-1}$, $\gamma_{+1}$ (and $\gamma_{+2}$) that are solely determined by the discount factor $\beta$. I relax this assumption in later sections and nest the forward-looking New Keynesian Phillips curve into $\mathcal{M}_1$ by adding (2) and (3). In this case, $\upsilon$ becomes a nesting parameter also appearing in $\gamma_{-1}$, $\gamma_{+1}$, and $\gamma_{+2}$. Setting $\upsilon = 0$ then eliminates (3), and all inflation weights with the exception of $\gamma_{+1}$ vanish. For the time being, I treat the non-nested version of $\mathcal{M}_1$ as my core specification: $\mathcal{M}_1$ does not contain any (nested) uncertainty on the forward- and backward-looking nature of inflation.

In the following section, I apply U.S. data to the models $\mathcal{M}_1$–$\mathcal{M}_4$.

[5] The analysis of Christiano et al. (2005) considers the price-setting scheme $P_t(i) = \pi_{t-1}^{\varphi} P_{t-1}(i)$ with perfect indexation ($\varphi = 1$). However, nested indexation requires $\varphi \in [0,1]$.

# 4 Estimation

The following subsections present the time series and the equations relating model variables to observables. Bayesian estimations require parameter priors and fixed parameter values, which I also present in further detail. After estimating the models $\mathcal{M}_1$–$\mathcal{M}_4$, I summarize and compare posterior parameter estimates. Moreover, I rank the explanatory power of the models using adjusted marginal data densities. Marginal densities are adjusted with the training sample method by Sims (2003). I briefly review this method, which I employ in order to rule out any influence of specified priors.

## 4.1 Data and Priors

The set of observable variables is determined by the shocks appearing in the common equations. Therefore, I choose the same set of observables as Rabanal and Rubio-Ramírez (2005) and Kriwoluzky and Stoltenberg (2014, 2015): real output $\hat{y}_t^{obs.}$, inflation $\hat{\pi}_t^{obs.}$, the federal funds rate $\hat{R}_t^{obs.}$, and the real wage rate $\hat{w}_t^{obs.}$. The corresponding dataset $Y_T = \left[\hat{y}_t^{obs.}\ \hat{\pi}_t^{obs.}\ \hat{R}_t^{obs.}\ \hat{w}_t^{obs.}\right]'$ consists of quarterly U.S. values for the time period 1983Q1–2008Q2.[6] Before estimation, I detrend each time series by removing a linear quadratic time trend. The observation equation that links the detrended variables

[6] Real GDP is taken from the U.S. Bureau of Economic Analysis (BEA), series *BEA NIPA table 1.1.6, line 1.* Nominal GDP is taken from the U.S. Bureau of Economic Analysis (BEA), series *BEA NIPA table 1.1.5, line 1.* The implicit GDP deflator is calculated as the ratio of nominal to real GDP. The civilian non-institutional population is taken from the Federal Reserve Economic Database (FRED), *http://research.stlouisfed.org/fred2/series/CNP16OV?cid=104.* The interest rate series is also taken from FRED, *http://research.stlouisfed.org/fred2/series/FEDFUNDS.* Nominal hourly wages for the total private industry is taken from *http://research.stlouisfed.org/fred2/series/ahetpi/10.*

$Y_T^{detr.} = \left[\hat{y}_t^{detr.}\ \hat{\pi}_t^{detr.}\ \hat{R}_t^{detr.}\ \hat{w}_t^{detr.}\right]'$ to the variables in the model is given by

$$\begin{bmatrix} \hat{y}_t^{detr.} \\ \hat{\pi}_t^{detr.} \\ \hat{R}_t^{detr.} \\ \hat{w}_t^{detr.} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{y}_t \\ \hat{\pi}_t \\ \hat{R}_t \\ \hat{w}_t \end{bmatrix}. \tag{11}$$

I proceed to describe the priors for the estimated parameters. An overview is given in table 2. I set the prior distributions of most common parameters as in Smets and Wouters (2007). The existing literature also enables me to impose well-known priors on most price rigidity parameters. However, there are no obvious hints for $\upsilon$. In order to make this prior as uninformative as possible, I choose the prior that would be chosen for $\psi$ in the standard Rotemberg model. Equalizing the output gap coefficient of the forward-looking New Keynesian Phillips curve $\mathcal{M}_4$ to the output gap coefficient under standard Calvo pricing $(\sigma + \eta)(1 - \beta\theta)(1 - \theta)/\theta$ leads to $\psi = \theta(\epsilon - 1)(1 - \theta)^{-1}(1 - \theta\beta)^{-1}$. If the assumed prior mean of the Calvo parameter $\theta$ is 0.5 (a price duration of half a year), then $\psi \approx 10$. Therefore, I assume for $\psi$ and $\upsilon$ a normal distribution with mean 10 and a standard deviation (henceforth in brackets) of (4). Moreover, $\theta$ is assumed to follow a beta distribution with standard deviation (0.2). I assume for the inflation indexation parameter $\alpha$ a beta-distribution with 0.5 (0.2). Having described the model-specific priors (for $\upsilon$, $\psi$, $\theta$, and $\alpha$), I turn to the common-model parameters. The persistence of the AR(1) shock processes is determined by parameters $\rho_a$, $\rho_g$, $\rho_r$, for which I employ a relatively uninformative beta-distribution with mean 0.5 and standard deviation (0.2). Prior shocks $\sigma_a = \sigma_g = \sigma_r = \sigma_\mu = 0.02$ are identical across the models $\mathcal{M}_1$–$\mathcal{M}_4$. Relative risk aversion $\sigma$ is assumed to follow a gamma distribution around 1 (logarithmic consumption utility). The same prior is chosen for the inverse of the Frisch elasticity $\eta$. The interest rate rule coefficients are assumed to have prior means $\phi_\pi = 1.7$, $\phi_y = 0.125$, and I impose a normal and gamma distribution with standard deviations (0.5) and (0.05), respectively. The employed dataset does not allow to identify the elasticity of substitution $\epsilon$ and the discount factor $\beta$. I specify them as $\epsilon = 6$ and $\beta = 0.99$. These values are assumed to match a steady state monopolistic markup of 20% and a quarterly real interest rate of 1.01% (see Woodford (2003)).

**Table 2:** Prior distribution of the structural parameters.

| Parameter | Distribution | Mean | Std |
|---|---|---|---|
| $\upsilon$ | normal | 10 | 2 |
| $\psi$ | normal | 10 | 2 |
| $\theta$ | beta | 0.5 | 0.2 |
| $\alpha$ | beta | 0.5 | 0.2 |
| $\sigma$ | gamma | 1 | 0.25 |
| $\eta$ | gamma | 1 | 0.25 |
| $\phi_\pi$ | normal | 1.7 | 0.5 |
| $\phi_y$ | gamma | 0.125 | 0.05 |
| $\rho_a$ | beta | 0.5 | 0.2 |
| $\rho_g$ | beta | 0.5 | 0.2 |
| $\rho_r$ | beta | 0.5 | 0.2 |
| $\sigma_a$ | inv-gamma | 0.02 | inf |
| $\sigma_g$ | inv-gamma | 0.02 | inf |
| $\sigma_r$ | inv-gamma | 0.02 | inf |
| $\sigma_\mu$ | inv-gamma | 0.02 | inf |

Each model is subject to the same procedure. First, I obtain estimates by approximating the posterior mode. Second, I execute a random walk Metropolis-Hastings algorithm to evaluate the posterior distribution. In this exercise, I run two independent chains of 500,000 draws. I discard the first 400,000 draws of each chain and calculate statistics with the remaining ones. For example, the log marginal data density (log marginal likelihood) is the modified harmonic mean estimator (Geweke (1999)) based on 100,000 draws.

I take into account that a simple likelihood comparison is not a satisfactory approach to rank the explanatory power of DSGE models because the estimated marginal likelihood of a model depends on its joint prior distribution (Kriwoluzky and Stoltenberg (2014, 2015)). This implies that any likelihood-based ranking can be arbitrarily affected by the choice of priors. In this paper, each model $\mathcal{M}_i, i = 1, \ldots, 4$ has a singular collection of parameters due to a distinctive New Keynesian Phillips curve. Therefore, the selected price rigidity priors affect the model comparison pursued here. I circumvent this problem by re-estimating the models with a training sample (Sims (2003)). This training sample $Y_{T,0}^{detr.}$ consists of the first 50 observations of the full sample $Y_T^{detr.}$. After re-estimating a model, the log marginal likelihood estimated with $Y_{T,0}^{detr.}$ is subtracted from the number estimated with $Y_T^{detr.}$. To clarify the method, note that each model $\mathcal{M}_i, i = 1, \ldots, 4$ contains a random vector of deep parameters $\Xi_i$. Moreover, I denote a particular realization from the joint posterior distribution as $\xi_i$. The measurement of explanatory power is done with the adjusted log marginal data density

$$\ln p\left(Y_{T,1}^{detr.}|\mathcal{M}_i\right) = \ln p\left(Y_T^{detr.}|\mathcal{M}_i\right) - \ln p\left(Y_{T,0}^{detr.}|\mathcal{M}_i\right) \quad , \quad i = 1 \ldots 4, \tag{12}$$

where

$$p\left(Y_{T,0}^{detr.}|\mathcal{M}_i\right) = \int_{\xi_i \in \Xi_i} p\left(Y_{T,0}^{detr.}|\xi_i, \mathcal{M}_i\right) p\left(\xi_i\right) \mathrm{d}\xi_i \quad , \quad i = 1 \ldots 4, \tag{13}$$

denotes the marginal data density of the training sample. I compute the empirical fit of the models $\mathcal{M}_2$–$\mathcal{M}_4$ relative to $\mathcal{M}_1$ using the posterior odds ratio:

$$PO_{\mathcal{M}_j,\mathcal{M}_1} = \frac{p\left(Y_{T,1}^{detr.}|\mathcal{M}_j\right)}{p\left(Y_{T,1}^{detr.}|\mathcal{M}_1\right)} \quad , \quad j = 2 \ldots 4. \tag{14}$$

A ratio smaller than one indicates the inferiority of model $\mathcal{M}_j$ against model $\mathcal{M}_1$.

## 4.2 Posterior Results

Table 3 summarizes the posterior moments and gives two main insights. First, the posterior mean and the posterior uncertainty values are in line with related studies (see, for example, Smets and Wouters (2007)). Second, monetary policy had an aggressive stance towards deviations of inflation from its target. This is consistent with the common perception in the literature. The mean of the Calvo parameter indicates that prices are reset roughly every second quarter. Moreover, the implicit slope of the New Keynesian Phillips curve $(\sigma + \eta)\,\gamma_y$ is well below unity in all models.

**Table 3:** Posterior estimates of the structural parameters in each model.

| | $\mathcal{M}_1$ | | $\mathcal{M}_2$ | | $\mathcal{M}_3$ | | $\mathcal{M}_4$ | |
|---|---|---|---|---|---|---|---|---|
| Parameter | Mean | Std | Mean | Std | Mean | Std | Mean | Std |
| $\upsilon$ | 9.55 | 1.55 | - | - | - | - | - | - |
| $\psi$ | - | - | 10.62 | 1.66 | - | - | 10.08 | 1.89 |
| $\theta$ | - | - | - | - | 0.48 | 0.04 | - | - |
| $\alpha$ | - | - | 0.72 | 0.14 | - | - | - | - |
| $\sigma$ | 1.77 | 0.25 | 1.80 | 0.26 | 1.86 | 0.26 | 1.56 | 0.24 |
| $\eta$ | 0.98 | 0.16 | 0.96 | 0.16 | 0.92 | 0.15 | 1.14 | 0.20 |
| $\phi_\pi$ | 4.23 | 0.31 | 4.22 | 0.32 | 4.15 | 0.32 | 4.48 | 0.31 |
| $\phi_y$ | 0.15 | 0.05 | 0.13 | 0.05 | 0.14 | 0.05 | 0.13 | 0.05 |
| $\rho_a$ | 0.98 | 0.006 | 0.98 | 0.006 | 0.98 | 0.006 | 0.98 | 0.005 |
| $\rho_g$ | 0.88 | 0.02 | 0.88 | 0.02 | 0.88 | 0.02 | 0.90 | 0.02 |
| $\rho_r$ | 0.81 | 0.04 | 0.81 | 0.04 | 0.80 | 0.04 | 0.83 | 0.04 |
| $\sigma_a$ | 0.003 | 0.003 | 0.003 | 0.003 | 0.003 | 0.003 | 0.003 | 0.003 |
| $\sigma_g$ | 0.008 | 0.008 | 0.008 | 0.008 | 0.009 | 0.009 | 0.009 | 0.009 |
| $\sigma_r$ | 0.003 | 0.003 | 0.003 | 0.003 | 0.003 | 0.003 | 0.003 | 0.003 |
| $\sigma_\mu$ | 0.008 | 0.008 | 0.007 | 0.007 | 0.007 | 0.007 | 0.008 | 0.008 |

*Notes: see the notes for table 1.*

The explanatory power of the models $\mathcal{M}_1$–$\mathcal{M}_4$ is summarized in table 4. A widely accepted observation is that a history dependence of inflation (contained in $\mathcal{M}_1$–$\mathcal{M}_3$) improves the fit of an otherwise forward-looking model ($\mathcal{M}_4$). The crucial finding in this paper is that model $\mathcal{M}_1$ exhibits the highest adjusted marginal likelihood. The posterior odds ratios indicate that the persistence and autocovariation mechanisms contained in $\mathcal{M}_1$ are more adequate to fit the data compared to the indexation schemes in $\mathcal{M}_2$ and $\mathcal{M}_3$. The adjustment costs from the first difference of inflation lead to the strongest improvement of model fit in an otherwise forward-looking setting.

**Table 4:** Log marginal data densities and model probabilities.

| | $\mathcal{M}_1$ Pesaran (1991) | $\mathcal{M}_2$ Ireland (2007) | $\mathcal{M}_3$ Christiano et al. (2005) | $\mathcal{M}_4$ Rotemberg (1982) |
|---|---|---|---|---|
| $\ln p\left(Y_T^{detr.} \vert \mathcal{M}_i\right)$ | 1539.4262 | 1538.4185 | 1538.8316 | 1528.9993 |
| $\ln p\left(Y_{T,0}^{detr.} \vert \mathcal{M}_i\right)$ | 730.5219 | 730.3664 | 730.4182 | 725.8862 |
| $\ln p\left(Y_{T,1}^{detr.} \vert \mathcal{M}_i\right)$ | 808.9043 | 808.0521 | 808.4134 | 803.1131 |
| $PO_{\mathcal{M}_j,\mathcal{M}_1}$ | – | 0.4265 | 0.6121 | 0.0031 |

*Notes: $Y_T^{detr.}$ is the full sample of detrended series (1983Q1–2008Q2), $Y_{T,0}^{detr.}$ is the training sample (first 50 observations), and $Y_{T,1}^{detr.}$ is the comparison sample (last 53 observations). $PO_{\mathcal{M}_j,\mathcal{M}_1}; j = 2,\ldots,4$ is the posterior odds ratio. See also the notes for table 1.*

Note that $\mathcal{M}_1$ does not nest the forward-looking model $\mathcal{M}_4$. In contrast, $\mathcal{M}_4$ is contained in $\mathcal{M}_2$: setting $\alpha = 0$ reproduces $\mathcal{M}_4$. The results in table 4 suggest that nesting inflation persistence via an additional parameter may unnecessarily worsen the explanatory power of a model: $\mathcal{M}_1$ and $\mathcal{M}_3$ are able to outperform model $\mathcal{M}_2$ without an inflation persistence nesting parameter. For the sake of clarity, I first explore whether nesting the standard Rotemberg structure (2) into the first-difference adjustment cost structure (3) changes my estimation results. A simple way to accomplish this is by adding (2) and (3):

$$\Omega_t^{\mathcal{R}}(i) + \Omega_t^{\mathcal{P}}(i). \tag{15}$$

This equation now represents the adjustment cost structure that firm $i$ has to take into account whenever choosing $P_t(i)$. By construction, $\upsilon$ now represents a nesting parameter: setting $\upsilon = 0$ gives the Rotemberg structure $\Omega_t^{\mathcal{R}}(i)$. This is mirrored in the New Keynesian Phillips curve that results from (15), which I label as $\mathcal{M}_{1,4}$. The parameter $\upsilon$ now appears in all weights $(\gamma_{-1}, \gamma_{+1}, \gamma_{+2}, \gamma_y)$, and setting $\upsilon = 0$ leads to model $\mathcal{M}_4$. I repeat the estimation exercise with this New Keynesian Phillips curve and find a posterior mean of the nesting parameter $\upsilon$ which is positive, with a low posterior uncertainty (table 5).[7]

**Table 5:** Selected estimates and marginal likelihoods of the nested model $\mathcal{M}_{1,5}$.

| $\upsilon$ | $\psi$ | $\ln p\left(Y_T^{detr.} \vert \mathcal{M}_{1,4}\right)$ | $\ln p\left(Y_{T,0}^{detr.} \vert \mathcal{M}_{1,4}\right)$ | $\ln p\left(Y_{T,1}^{detr.} \vert \mathcal{M}_{1,4}\right)$ |
|---|---|---|---|---|
| 9.02 (2.11) | 4.60 (3.83) | 1536.6165 | 728.3781 | 808.2384 |

*Notes: see the notes for table 4.*

This indicates that the inflation-difference frictions entering $\Omega_t^{\mathcal{P}}$ remain important for the description of the data. Moreover, the posterior mean of the standard Rotemberg parameter $\psi$ is positive, but statistically insignificant. These results suggest that only inflation frictions (nested by $\upsilon$), but not price rigidities (nested by $\psi$) are necessary for the model to be consistent with the data. However, I showed in equation (4) that inflation-difference adjustment costs can be disentangled into a term à la Rotemberg (1982), a persistence term, and an autocorrelation term. Since adjustment costs from inflation differences already allow for price frictions à la Rotemberg (1982), the additional parameter $\psi$ is estimated to be insignificant. Consequently, I find that the nesting adjustment cost structure (15) leads to a lower marginal likelihood than the non-nested model (3): the explanatory power of the DSGE model worsens due to nested Rotemberg adjustment costs that are insignificant (see tables 4 and 5). A worsening of model fit also appears when estimating model $\mathcal{M}_3$ under the assumption of nested indexation. I allow in the corresponding price-setting scheme $P_t(i) = \pi_{t-1}^{\varphi} P_{t-1}(i)$ for $\varphi \in [0,1]$. The resulting model is labeled as $\mathcal{M}_{3,4}$. Setting the corresponding nesting parameter $\varphi$ equal to zero

[7] I find that increasing the role of the data for the estimates of $\upsilon$ and $\psi$ gives clearer posterior significance results. Therefore, I assign in this exercise a higher prior uncertainty to the parameters $\upsilon$ and $\psi$, namely a standard deviation of 4. All remaining posterior parameter estimates are roughly the same.

then leads to model $\mathcal{M}_4$. In contrast, imposing $\varphi = 1$ gives the non-nested model $\mathcal{M}_3$. Posterior results for model $\mathcal{M}_{3,4}$ are presented in table 6.[8] The indexation parameter $\varphi$ is estimated to be significant. This is due to the inability of other model parameters to capture the inflation persistence present in the data. However, the additional nesting parameter adds uncertainty to the estimation, such that the marginal likelihood worsens.

To conclude the analysis, notice that the model with adjustment costs from the first difference of inflation $\mathcal{M}_1$ has the highest marginal likelihood within the class of non-nested models. Model $\mathcal{M}_{1,4}$ has the highest marginal likelihood within the class of nested models.[9] Therefore, adjustment costs from inflation differences (or "speed changes", according to Pesaran (1991)) are important when formulating a DSGE model.

**Table 6:** Selected estimates and marginal likelihoods of the nested model $\mathcal{M}_{4,5}$.

| $\varphi$ | $\theta$ | $\ln p\left(Y_T^{detr.} \vert \mathcal{M}_{3,4}\right)$ | $\ln p\left(Y_{T,0}^{detr.} \vert \mathcal{M}_{3,4}\right)$ | $\ln p\left(Y_{T,1}^{detr.} \vert \mathcal{M}_{3,4}\right)$ |
|---|---|---|---|---|
| 0.70 (0.16) | 0.52 (0.04) | 1537.1597 | 729.3551 | 807.8046 |

*Notes: see the notes for table 4.*

# 5 Conclusions

In this paper, I documented that adjustment costs from the first difference of inflation should be an important microeconomic element in DSGE models. In a first step, I observed that these costs have been substantial for the aggregate U.S. economy. In a second step, I employed Bayesian estimations to confirm that these costs should be considered at the level of the representative monopolistic firm. The estimations included five DSGE models differing in their

[8] The prior for $\varphi$ is the same as for $\alpha$ in model $\mathcal{M}_2$.

[9] The nested models in this paper are $\mathcal{M}_2$, $\mathcal{M}_{1,4}$, and $\mathcal{M}_{3,4}$. The non-nested models are $\mathcal{M}_1$ and $\mathcal{M}_3$.

price rigidity assumption. I found that a DSGE framework with adjustment costs from inflation differences (in the terminology of Pesaran (1991) denoted as “speed changes”) has the highest probability to fit U.S. data. The inflation difference specification therefore captures data properties that other nominal rigidity settings (Rotemberg (1982); Ireland (2007); Christiano, Eichenbaum, and Evans (2005)) cannot.

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