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Magni, Carlo Alberto

University of Modena and Reggio Emilia

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Economic profit, NPV, and CAPM:  
Biases and violations of Modigliani and Miller's Proposition I

**Carlo Alberto Magni**

Università di Modena e Reggio Emilia, Dipartimento di Economia Politica  
viale Berengario 51, 41100 Modena, Italy  
tel. +39-059-2056777, fax +39-059-2056997, Email: magni@unimo.it

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**Abstract**

The notion of Net Present Value (NPV) is thought to formally translate the notion of economic profit, where the discount rate is the cost of capital. The latter is the expected rate of return of an equivalent-risk alternative that the investor might undertake and is often found by making recourse to the Capital Asset Pricing Model. This paper shows that the notions of *disequilibrium* NPV and economic profit are not equivalent: NPV-minded agents are open to framing effects and to arbitrage losses, which imply violations of Modigliani and Miller's Proposition I. The notion of *disequilibrium* (present) value, deductively derived from the CAPM by several authors and widely used in applied corporate finance, should therefore be dismissed.

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**Abstract.** The notion of Net Present Value (NPV) is thought to formally translate the notion of economic profit, where the discount rate is the cost of capital. The latter is the expected rate of return of an equivalent-risk alternative that the investor might undertake and is often found by making recourse to the Capital Asset Pricing Model. This paper shows that the notions of *disequilibrium* NPV and economic profit are not equivalent: NPV-minded agents are open to framing effects and to arbitrage losses, which imply violations of Modigliani and Miller's Proposition I. The notion of *disequilibrium* (present) value, deductively derived from the CAPM by several authors and widely used in applied corporate finance, should therefore be dismissed.

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## 1 Introduction

Economic profit on one side, (net) present value on the other side. The former is one of the building blocks of economic theory, the latter is a cornerstone in financial economics.

Economic profit is a fundamental notion in economic theory since Marshall (1890). It represents the “excess profit that is gained from an investment over and above the profit that could be obtained from the best alternative foregone” (Rao, 1992, p. 87). That is, economic profit from an investment is the difference between profit from that investment and profit from the best alternative foregone. In other terms, the alternative foregone's profit acts as an opportunity cost (see Buchanan, 1969). As known, many synonyms have been coined to mean ‘economic profit’: ‘excess profit’ (Preinreich, 1938), ‘excess realizable profit’ (Edwards and Bell, 1961), ‘excess income’ (Kay, 1976), ‘abnormal earnings’ (Peasnell, 1981), ‘supernormal profit’ (see Begg, Fischer, and Dornbusch, 1984, p. 121), ‘residual income’ (Biddle, Bowen, and Wallace, 1999). The concept of ‘Goodwill’ (e.g., Preinreich, 1936) is also strictly related to that of excess profit. Other names are Economic Value Added, Cash Value Added, Shareholders Value Creation (see Fernández, 2002).<sup>1</sup>

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<sup>1</sup>See Magni, 2000a, 2000b, 2003, for a nonclassical way of formalizing economic profit.

Net Present Value (NPV) is a fundamental notion in finance since Fisher (1930), although “the technology of discounting is not an invention of twentieth century” (Miller and Napier, 1993, p. 640): Discounted-cash-flow analysis was known and (sometimes) employed since eighteenth century (Brackenborough, McLean and Oldroyd, 2001. See also Parker, 1968; Edwards and Warman, 1981). As known, the NPV is a function of the discount rate, and the latter is often found by making use of the classical Capital Asset Pricing Model (CAPM) (Sharpe, 1964; Lintner, 1965; Mossin, 1966), which puts into effect the NPV methodology.

The notions of economic profit and NPV are often viewed as two sides of the same medal: The NPV is just economic profit disguised in present terms. The common idea of economic profit maximization is then equivalent to the idea of net present value maximization: “The firm attempts to maximize the present value of its net cash flow over an infinite horizon” (Abel, 1990, p. 755) and “the net present value rule is also the basis for the neoclassical theory of investment” (Dixit and Pindyck, 1994, p. 5).

Decision making is straightforward with such equivalent notions. Rubinstein (1973) shows that if the CAPM assumptions are met, a project is worth undertaking if its expected rate of return is greater than the *disequilibrium* required rate of return. His maximization rule is as follows:

The firm should accept the project with the highest excess expected internal rate of return weighted by its cost (p. 174)

This result ... is equivalent to accepting the project with the highest net present value (*ibidem*, footnote 14).

The first quotation just focuses on maximization of economic profit, the second one suggests to maximize net present value. Magni (2007a) shows that the use of disequilibrium values is standard in corporate finance and is widespread in academic papers as well as in textbooks. This paper shows that, contrary to what Rubinstein seems to imply, the alleged equivalence of (disequilibrium) NPV and economic profit does not hold. Such a NPV does not represent economic profit and, in addition, it is a biased measure because it is nonadditive; the same holds for the notion of disequilibrium value.<sup>2</sup> In particular, decision makers abiding by the standard NPV+CAPM methodology give inconsistent answers to the same problem differently framed. In other terms, they are trapped in a sort of mental accounting (Thaler, 1985, 1999) so that their evaluations differ depending on whether outcomes are seen as aggregate or disaggregate

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<sup>2</sup>Problems in the *equilibrium* NPV may be found by Magni (2007a, 2007b as well).

quantities. This amounts to saying that their valuations and choice behaviors do not comply with the principle of description invariance, which prescribes that valuations and decisions must be invariant under changes in description of the same asset. Violations of this principle are known as framing effects (Tversky and Kahneman, 1981; Kahneman and Tversky, 1984; Soman, 2004). This bias bears significant relations to the violation of the principle of arbitrage, which is a well-established principle of economic rationality implying that rational decision makers do not incur arbitrage losses (see Nau and McCardle, 1991; Nau, 1999). In the field of corporate valuation this violation reduces to an infringement of the classical Modigliani and Miller's Proposition I.

The paper is structured as follows. In section 2 it is shown that NPV and economic profit bear a strong formal relation in that the former is the present value of the latter. Section 3 shows an example highlighting the fact that NPV does not represent economic profit, is not additive and does not fulfill the principle of description invariance (i.e. implies framing effects). In contrast, economic profit is additive and frame-independent. Section 4 shows the same results in more formal terms. In section 5 it is shown, on the basis of the previous results, that value itself is nonadditive. Section 6 shows that NPV-minded decision makers incur arbitrage losses. Section 7 shows that the association of CAPM and NPV does not comply with Modigliani and Miller's Proposition I. In particular, the choice behavior of a potential NPV-minded buyer is not invariant under changes in the firm debt-equity ratio.

## 2 Economic profit and NPV as companions

Let  $W^0$  be an investment cost and denote with  $W^1$  the final payoff at time 1. Consider the profit  $W^1 - W^0$ , which we can reformulate as  $rW^0$ , with  $r = \frac{W^1 - W^0}{W^0}$  being the rate of return. Consider also an alternative business for the investor and let  $i$  be the relative rate of return. The corresponding profit is  $W^0(1 + i) - W^0 = iW^0$  and represents an opportunity cost, a foregone return. The economic (excess) profit is given by the difference between the *factual* profit the entrepreneur receives and the *counterfactual* profit she would receive if she invested in the alternative business. Denoting economic profit with  $\pi$  we have:

$$\pi = rW^0 - iW^0. \quad (1)$$

Note that the above equation may also be stated as a difference between two future values:

$$\pi = W^1 - W^0(1 + i). \quad (2)$$

From a financial perspective,  $\pi$  is the Net Future Value. In finance, it is common to work with present values so the notion of Net Present Value (NPV) is introduced, which is given by the

discounted algebraic sum of all cash flows involved in the business. In our simplified one-period case, we have

$$\text{NPV} = -W^0 + \frac{W^1}{1+i}. \quad (3)$$

Economic profit and NPV bear a strong formal relation: NPV is the present value of (1) (or, equivalently, the present value of (2)):

$$\text{NPV} = \frac{\pi}{1+i} = \frac{1}{1+i}(rW^0 - iW^0). \quad (4)$$

In other terms, economic profit and net future value are different names for the same notion, whereas net present value is the present value of economic profit. It is worthwhile noting that eqs. (3)-(4) preserve the sign of eqs. (1)-(2) (as long as  $i > -1$ , as will be assumed here). Decision-making implications of this formal equivalence are straightforward: A business is worth undertaking if and only if the economic profit (the NPV) is positive.

Under uncertainty, the rates  $r$  and  $i$  are expected values and the two rates refer to alternatives equivalent in risk, so that eqs. (1) and (3) are measures of expected excess profit (in final and present terms respectively). What ‘equivalent in risk’ means depends on the model selected. The classical and sophisticated CAPM is the most common tool for measuring an asset’s risk, which is given by its beta:

$$\beta = \frac{\text{cov}(\tilde{r}, \tilde{r}_m)}{\sigma_m^2} = \frac{\text{cov}(\tilde{W}^1, r_m)}{W^0 \sigma_m^2} \quad (5)$$

where  $\tilde{r}_m$  and  $\sigma_m^2$  denotes the market rate of return and its variance (a tilde on a symbol will henceforth highlight randomness).

To calculate excess profit (and NPV) under uncertainty one just has to use the fundamental equation of the CAPM, known as the Security Market Line (SML). Under suitable assumptions, the latter individuates the required rate of return of the business under examination; such a rate is the (opportunity) cost of capital, i.e. the expected rate of return of the counterfactual alternative available to the entrepreneur. We have

$$i = r_f + \beta(r_m - r_f) \quad (6)$$

where  $r_f$  is the risk-free rate and  $r_m$  is the expected market rate of return. Applying this security valuation relation to capital budgeting we have a simple rule: A project should be undertaken if and only if

$$r > r_f + \beta(r_m - r_f) \quad (7)$$

i.e. if and only if its expected rate of return exceeds the cost of capital (see Rubinstein, 1973, p. 171) or, in terms of NPV, if and only if its risk-adjusted NPV is positive:

$$-W^0 + \frac{W^1}{1 + r_f + \beta(r_m - r_f)} > 0 \quad (8)$$

where  $W^1$  is the expected value of  $\widetilde{W}^1$ .

*Remark 1.* It is worth noting that the beta in (5) is a disequilibrium beta, so that the NPV in (8) is a disequilibrium NPV. Beside Rubinstein (1973), the legitimacy of the disequilibrium beta has been deduced from the CAPM by several authors (see Senbet and Thompson, 1978; Magni, 2007c, for a review).

### 3 Nonadditivity and framing effects: An example

Consider the security market described in Table 1, where a risky asset and a risk-free asset are traded and two possible states may occur, conventionally labeled ‘good’ and ‘bad’, with probability 0.8 and 0.2 respectively. The market is complete, is assumed to be in equilibrium (all marketed assets lie on the SML) and arbitrage is not possible.<sup>3</sup> Let us imagine an economic agent comes across the opportunity of investing in a business  $A$  composed of two sub-projects. The first one, say  $A_1$ , consists of an outlay of 15500 euros and generates an outcome of  $\widetilde{W}_{A_1}^1$ , equal to 58000 in good state and 3000 in bad state. The second one, say  $A_2$ , consists in an outflow of 70000 euros and a final risk-free inflow of 72000 at time 1. Suppose also that this two-project business is to be *fully* accepted or *fully* rejected (no sub-project may be undertaken alone). To decide, the investor computes the NPV of the business. The rates of return of  $A_1$  are  $58000/15500 - 1 = 2.7419$  and  $3000/15500 - 1 = -0.8064$  in good and bad state respectively; the expected rate of return is  $r_{A_1} = (2.7419)(0.8) + (-0.8064)(0.2) = 2.0322$ . The covariance of  $\widetilde{r}_{A_1}$  with  $\widetilde{r}_m$  is  $\text{cov}(\widetilde{r}_{A_1}, \widetilde{r}_m) = 0.5677$  and the risk is therefore  $\beta_{A_1} = 0.5677/0.16 = 3.5484$ . The cost of capital is  $i_{A_1} = r_f + \beta_{A_1}(r_m - r_f) = 0.15 + 3.5484(0.45 - 0.15) = 1.2145$ . The economic profit is then

$$W_{A_1}^0(r_{A_1} - i_{A_1}) = 15500(2.0322 - 1.2145) = 12675 \quad (9)$$

while the NPV is

$$\text{NPV}_{A_1} = \frac{12675}{1 + i_{A_1}} = \frac{12675}{1 + 1.2145} = 5723. \quad (10)$$

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<sup>3</sup>As Dybvig and Ingersoll (1982) show, if (i) the CAPM pricing relation holds for all securities in the market, (ii) the market is complete, (iii) the probability that  $\widetilde{r}_m > r_m + \frac{\sigma_m^2}{r_m - r_f}$  is positive, then arbitrage opportunities arise. But in our market of Table 1 condition (iii) is not satisfied.

As for  $A_2$ , its rate of return is  $0.02857=72/70-1$  in both states. As the project is riskless, the cost of capital is  $r_f = 0.15$ , so the excess profit is

$$W_{A_2}^0(r_{A_2} - r_f) = 70000(0.0285 - 0.15) = -8500 \quad (11)$$

and the NPV is

$$\text{NPV}_{A_2} = \frac{-8500}{1 + r_f} = \frac{-8500}{1 + 0.15} = -7391. \quad (12)$$

Consider now a business  $B$  that can be undertaken with an expenditure of 85500 euros whereby the investor will obtain  $\widetilde{W}_B^1$ , equal to 130000 or 75000 in good and bad state respectively. The rate of return of  $B$  is  $130000/85500-1=0.5204$  and  $75000/85500-1=-0.1228$  in good and bad state respectively so that the expected rate of return is  $r_B=(0.8)0.5204+(0.2)(-0.1228)=0.39181$ . It is easy to see that the risk of  $B$  is  $\beta_B=0.6432$  and the cost of capital is therefore  $i_B = 0.15 + 0.6432(0.45 - 0.15)=0.34298$ . The excess profit is

$$W_B^0(r_B - i_B) = 85500(0.39181 - 0.34298) = 4175 \quad (13)$$

and the NPV is

$$\text{NPV}_B = \frac{4175}{1 + i_B} = \frac{4175}{1 + 0.34298} = 3108. \quad (14)$$

It is worthwhile noting that the NPV of business  $B$  differs from the NPV of business  $A$ , which is  $5723 - 7391 = -1668$ . Yet, the two businesses represent the same course of action described in two different ways, because both share the same total investment outlay ( $15500+75000=85500$ ) and the same final outcomes in good and bad state ( $58000+72000=130000$  and  $3000+72000=75000$ ). We have then  $A_1+A_2=B$ . This is a significant result. From a financial perspectives, it means that the NPV is nonadditive (because  $\text{NPV}_{A_1}+\text{NPV}_{A_2}\neq\text{NPV}_{A_1+A_2}$ ); from a cognitive and behavioral outlook, it means that an NPV-minded economic agent incurs framing effects in decision making, because the alternative  $A_1 + A_2$  is rejected (its NPV is negative) and the logically equivalent alternative  $B$  is accepted (its NPV is positive). By contrast, note that the economic profit as translated in (1) gives univocal results: Economic profit from  $B$  is 4175, which coincides with economic profit from the two-project business  $A$  ( $=12675 - 8500$ ).

## 4 Nonadditivity and framing effects: A simple formalization

In general, consider an investment whose initial outlay is  $W^0$  and whose final payoff is the random sum  $\widetilde{W}^1$ , available at time 1. This investment may always be seen as a portfolio of two



investments, one risky and one risk-free, whose outlays are  $W^0 - h$  and  $h$  respectively and whose outcomes are  $\widetilde{W}^1 - k$  and  $k$  respectively, with  $h, k \in \mathbb{R}$ . The economic profit of the investment may be formalized as the sum of these two investments' excess profits. In order to avoid framing effects, description invariance must be guaranteed, which means that economic profit must be invariant under changes in  $h$  and  $k$ . Indeed, considering  $\pi$  and  $i$  as functions of  $h$  and  $k$ , we have

$$\pi(h, k) = \overbrace{[(W^1 - k) - (W^0 - h) - i(h, k)(W^0 - h)]}^{\text{risky excess profit}} + \overbrace{(k - h - r_f h)}^{\text{risk-free excess profit}} \quad (15)$$

with

$$i(h, k) = r_f + \frac{r_m - r_f}{\sigma_m^2} \text{cov}\left(\frac{\widetilde{W}^1 - k}{W^0 - h} - 1, \widetilde{r}_m\right).$$

Substituting the latter in (15) we obtain

$$\begin{aligned} \pi(h, k) &= W^1 - W^0 - \left[r_f + \frac{r_m - r_f}{\sigma_m^2} \text{cov}\left(\frac{\widetilde{W}^1 - k}{W^0 - h} - 1, \widetilde{r}_m\right)\right](W^0 - h) - r_f h \\ &= W^1 - W^0 - \left[r_f + \frac{r_m - r_f}{(W^0 - h)\sigma_m^2} \text{cov}(\widetilde{W}^1, \widetilde{r}_m)\right](W^0 - h) - r_f h \\ &= W^1 - W^0(1 + r_f) - \frac{r_m - r_f}{\sigma_m^2} \text{cov}(\widetilde{W}^1, r_m). \end{aligned}$$

It is then evident that  $\frac{\partial \pi(h, k)}{\partial h} = \frac{\partial \pi(h, k)}{\partial k} = 0$  for all  $h$  and  $k$ , which means that economic profit does not change whatever the way the investment is partitioned (i.e., regardless of aggregation or disaggregation of cash flows).

As for the NPV, seen as a function of  $h$  and  $k$ , things are different:

$$\text{NPV}(h, k) = -(W^0 - h) + \frac{W^1 - k}{1 + r_f + \frac{r_m - r_f}{\sigma_m^2} \text{cov}\left(\frac{\widetilde{W}^1 - k}{W^0 - h} - 1, \widetilde{r}_m\right)} + \left[-h + \frac{k}{1 + r_f}\right] \quad (16)$$

whence

$$\text{NPV}(h, k) = -W^0 + \frac{W^1 - k}{1 + r_f + \frac{r_m - r_f}{\sigma_m^2 (W^0 - h)} \text{cov}(\widetilde{W}^1, \widetilde{r}_m)} + \frac{k}{1 + r_f}. \quad (17)$$

It is evident that, in general,  $\frac{\partial \text{NPV}(h, k)}{\partial h} \neq 0$  as well as  $\frac{\partial \text{NPV}(h, k)}{\partial k} \neq 0$ . Therefore NPV changes as  $h$  and/or  $k$  change, and it is not true that  $\text{NPV}(h_1, k_1) = \text{NPV}(h_2, k_2)$  for all  $h_1, h_2, k_1, k_2$ , as the principle of description invariance requires. As a particular case, the example above described has shown that

$$\pi(70000, 72000) = \pi(0, 0) = 4175$$

whereas

$$\text{NPV}(70000, 72000) = -1668 \neq \text{NPV}(0, 0) = 3108;$$

in the latter case choice behavior depends on the choice of the pair  $(h, k)$ , in the former case it is irrelevant.

## 5 Value is nonadditive

As a consequence, the notion of value in this context is severely undermined. The value  $V$  of an asset is given by  $V = \text{NPV} + W^0$  (where  $W^0$  is the cost to be paid by investors for undertaking it). Referring to the numerical example above where  $B = A_1 + A_2$  and bearing in mind the previous results about NPV, we have

$$\begin{aligned} V_{A_1} + V_{A_2} &= \text{NPV}_{A_1} + W_{A_1}^0 + \text{NPV}_{A_2} + W_{A_2}^0 \\ &= \text{NPV}_{A_1} + \text{NPV}_{A_2} + W_B^0 \\ &\neq \text{NPV}_B + W_B^0 = V_B = V_{A_1+A_2}. \end{aligned}$$

with obvious meaning of  $W_{A_1}^0$ ,  $W_{A_2}^0$ ,  $W_B^0$ . Putting it differently, value is a function of  $h$  and  $k$ :

$$V(h, k) = \text{NPV}(h, k) + W^0 \tag{18}$$

whose partial derivatives are not identically zero (see eq. (17)), and thus value is not invariant under changes in the description of valuation process.

## 6 Arbitrage Losses

The nonadditivity of value and net present value is full of implications. In addition to the framing effect above mentioned we have that our NPV-minded investor is subject to arbitrage losses. To see why, let us refer to the example in section 3. Suppose an economic agent (whom we can call the arbitrageur) asks the investor for a borrowing of 89000 euros whereby he will repay the amount  $\widetilde{W}_B^1$  after one period (the investor accepts to lend money, given that the NPV is easily found to be 112.326). At the same time, the arbitrageur offers our investor two financings: A loan of 15500 whereby the investor will repay  $\widetilde{W}_{A_1}^1$  and a loan of 70000 whereby the investor will reimbursed 72000 euros at the end of the period. These financings are to be accepted or rejected conjunctly. Our investor evidently accepts, given that  $-(\text{NPV}_{A_1} + \text{NPV}_{A_2})$  is positive (=1668). As a result of this choice behavior, our NPV-minded investor receives a

sure loss of 3500 euros, whereas the arbitrageur receives a sure gain of 4000. (Table 2 shows the NPV-minded investor's payoffs. Those for the arbitrageur are the same with opposite sign).

## 7 Violation of Modigliani and Miller's Proposition I

Let us now focus on a world *à la* Modigliani and Miller (1958) where Proposition I holds, so that firm value is not affected by the mix equity-debt. Consider an example of two firms. Firm U is unlevered and all the stocks are owned by entrepreneur U; firm L is levered and all the stocks and bonds are owned by agent L.<sup>4</sup> Let P be a potential buyer and suppose that:

- the two firms will generate the same total cash flow  $\widetilde{W}^1$
- agent U is ready to sell his stocks in exchange of  $W^0$  euros
- agent L is ready to sell his entire endowment in firm L selling the stocks in exchange of  $W^0$  euros but giving free his bonds to the buyer of the firm
- the debt of firm L is risk-free
- agent P is a CAPM enthusiast and selects alternatives via NPV rule.

As a result of the above assumptions, investor P computes the value of both firms as follows.

The value of firm U is<sup>5</sup>

$$V_U = \frac{W^1}{k_U} \quad (19)$$

where  $k_U = r_f + \beta_U(r_f - r_m)$  is the (unlevered) cost of capital. Denoting with  $\widetilde{r}_U$  the rate of return for firm U's stockholders, the unlevered beta is given by

$$\beta_U = \frac{\text{cov}(\widetilde{r}_U, \widetilde{r}_m)}{\sigma_m^2}. \quad (20)$$

As firm U is sold at  $W_0$  and will generate payoff  $\widetilde{W}_1$ , the rate of return for the buyer is  $\widetilde{r}_U = \frac{\widetilde{W}^1}{W^0} - 1$ , (20) becomes

$$\beta_U = \frac{\text{cov}(\frac{\widetilde{W}^1}{W^0} - 1, \widetilde{r}_m)}{\sigma_m^2} = \frac{\text{cov}(\widetilde{W}^1, \widetilde{r}_m)}{W^0 \sigma_m^2}. \quad (21)$$

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<sup>4</sup>Agents U and L are therefore representative agents (for sake of simplicity) but one may equivalently consider agents holding only *some* shares and bonds in a convenient ratio.

<sup>5</sup>The relations presented in this section may be interpreted in two ways: Perpetuity of constant cash flows may be assumed, as usual, or (for coherence with the above sections) one may think of a one-period firm so that  $\widetilde{W}^1$  is the final free cash flow, the rates  $r_f$ ,  $r_m$ ,  $k_U$ ,  $k_e$  are capitalization factors (i.e. 1 plus rate), and  $I$  represents interest+principal repayment.

The value of firm L is easily found. Denoting with  $I$  the cash flow to debt, the equity cash flow is  $\widetilde{W}^1 - I$ . Bearing in mind that the cost of debt equals the risk-free rate we have

$$V_L = \frac{W^1 - I}{k_e} + \frac{I}{r_f} \quad (22)$$

where  $k_e = r_f + \beta_e(r_f - r_m)$ . Denoting with  $\widetilde{r}_e$  the rate of return for firm L's stockholders, the beta of equity is given by

$$\beta_e = \frac{\text{cov}(\widetilde{r}_e, \widetilde{r}_m)}{\sigma_m^2}. \quad (23)$$

As equity is sold at  $W_0$ , the rate of return is  $\widetilde{r}_e = \frac{\widetilde{W}^1 - I}{W^0} - 1$ , so that (23) becomes

$$\beta_e = \frac{\text{cov}(\frac{\widetilde{W}^1 - I}{W^0} - 1, \widetilde{r}_m)}{\sigma_m^2} = \frac{\text{cov}(\widetilde{W}^1 - I, \widetilde{r}_m)}{W^0 \sigma_m^2}. \quad (24)$$

But  $\text{cov}(\widetilde{W}^1 - I, \widetilde{r}_m) = \text{cov}(\widetilde{W}^1, \widetilde{r}_m)$  for  $I$  is a real number. Consequently we have

$$\beta_e = \frac{\text{cov}(\widetilde{W}^1, \widetilde{r}_m)}{W^0 \sigma_m^2} = \beta_U \quad (25)$$

which implies

$$k_e = r_f + \beta_e(r_f - r_m) = r_f + \beta_U(r_f - r_m) = k_U \quad (26)$$

whence

$$V_L = \frac{W^1 - I}{k_U} + \frac{I}{r_f} = \frac{W^1}{k_U} - \frac{I}{k_U} + \frac{I}{r_f} \neq \frac{W^1}{k_U} = V_U.$$

This result contradicts Modigliani and Miller's Proposition I. This just means what we already know: Valuation is not invariant under changes in framing. In this case, we have two financially equivalent firms paying off the same total cash flows. Viewing the latter either as an aggregate quantity or as the sum of two quantities of different nature makes valuation nonequivalent.<sup>6</sup> Analogously, choice behavior may differ. Whenever agent P finds that

$$\text{NPV}_L = V_L - W^0 < 0 < V_U - W^0 = \text{NPV}_U$$

then firm U is purchased and firm L is not. In the opposite case

$$\text{NPV}_U = V_U - W^0 < 0 < V_L - W^0 = \text{NPV}_L$$

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<sup>6</sup>We have assumed that agent L gives free his holdings of bonds. This is not restrictive, as the numerical example in section 3 shows: Assume  $A_1$ 's cash flow is the equity cash flow of a levered firm,  $A_2$ 's cash flow is the cash flow to debt,  $A_1$  and  $A_2$ 's outlays are just the price at which agent L is ready to sell equity and bonds respectively; suppose also  $B$ 's cash flow is the capital cash flow of an unlevered firm and  $B$ 's outlay is the price at which agent U is ready to sell the firm. Then, the values of the two firms differ, as seen.

it is firm L to be purchased.<sup>7</sup> Again, this is a bias in the behavior of our NPV enthusiast.

In contrast, economic profit leads to a correct decision: Economic profit from U is

$$(W^1 - W^0) - k_U W^0,$$

economic profit from L is

$$\left[ \left( (W^1 - I) - W^0 \right) - k_e W^0 \right] + \left[ (I - 0) - (r_f 0) \right]$$

which are equal since  $k_e = k_U$ , as shown in (26).

## 8 Conclusions

The Net Present Value (NPV) of an investment is usually thought to represent economic profit. In applied corporate finance, a widespread way to value an asset (and thus to compute an NPV) is to discount cash flows with a disequilibrium cost of capital calculated via CAPM (for the use of an *equilibrium* NPV see Magni, 2007a, 2007b). The (disequilibrium) NPV of an investment is formally given by the present value of excess profit (value is then computed as the present value of excess profit plus cost). The disequilibrium NPV is validly deducted from the CAPM, as Rubinstein (1973) shows. However this paper, focusing on such a *disequilibrium* NPV shows that:

- while it is true that this NPV is calculated by discounting economic profit (and value is found by adding cost), it does not represent economic profit
- the disequilibrium NPV is nonadditive, which also implies that the disequilibrium value is nonadditive
- NPV-minded decision makers incur framing effects in both valuation (different values and NPVs) and choice behavior (accepting and rejecting the same investment)
- NPV-minded agents are open to arbitrage losses
- the standard disequilibrium NPV+CAPM valuation procedure is not consistent with Modigliani and Miller's Proposition I.

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<sup>7</sup>Obviously, in this case agent P becomes, at the same time, stockholder and bondholder.

Consequently, the association of disequilibrium NPV and CAPM is a flawed methodology and should not be used for project valuation and selection, given that it does not fulfill the principle of description invariance (valuation and judgment must not depend on framing) and the principle of arbitrage (rational decision makers do not incur arbitrage losses).

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**Table 1. The security market**

	Security				
	risky	risk-free	Market	state	probability
Outstanding shares	10		10		
Cash Flow	$\left\{ \begin{array}{l} 165 \\ 65 \end{array} \right.$	$\left\{ \begin{array}{l} 115 \\ 115 \end{array} \right.$	$\left\{ \begin{array}{l} 1,650 \\ 650 \end{array} \right.$	$\left\{ \begin{array}{l} s_1 \\ s_2 \end{array} \right.$	$\left\{ \begin{array}{l} 0.8 \\ 0.2 \end{array} \right.$
Rate of return (%)	$\left\{ \begin{array}{l} 65 \\ -35 \end{array} \right.$	$\left\{ \begin{array}{l} 15 \\ 15 \end{array} \right.$	$\left\{ \begin{array}{l} 65 \\ -35 \end{array} \right.$	$\left\{ \begin{array}{l} s_1 \\ s_2 \end{array} \right.$	$\left\{ \begin{array}{l} 0.8 \\ 0.2 \end{array} \right.$
Expected rate of return (%)	45	15	45		
Covariance with the market rate of return	0.16	0	0.16		
Beta	1	0	1		
Value	100	100	1000		

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**Table 2. Arbitrage loss for an NPV-minded agent**

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	Payoffs	
	Time 0	Time 1
Borrowing ( $=-A_1$ )	15500	$-\widetilde{W}_{A_1}^1$
Borrowing ( $=-A_2$ )	70000	-72000
Lending	-89000	$\widetilde{W}_B^1$
Net Payoffs	-3500	$\widetilde{W}_B^1 - \widetilde{W}_{A_1}^1 - 72000 = 0$

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