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Kawagishi, Taketo and Nakamoto, Yasuhiro

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Taketo Kawagishi*† and Yasuhiro Nakamoto‡

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Abstract

Considering that people can invest in their health-related quality of life, we investigate the effects of public policies (the investment subsidy policy and the direct transfer of investment commodities) on the health-related quality of life in the small open economy. Our main findings are that a temporary increase in these public policies conducted by the domestic government does not have positive impacts on the health-related quality of life in the long run; and alternatively, the foreign aid in the form of direct transfer of investment commodities improves their health.

Keywords: Health-related quality of life, Temporary effects of public policies, Foreign aid.

JEL Classification Code: F41, H21, I12, I18

*Corresponding author

†Faculty of Economics, Tezukayama University, 7-1-1 Tezukayama, Nara 631-8501, Japan; e-mail: tkawagishi@tezukayama-u.ac.jp

‡Faculty of Informatics, Kansai University, 2-1-1 Reizanji-cho,Takatsuki-shi, Osaka, 569-1095, Japan, e-mail: nakamoto@kansai-u.ac.jp

1 Introduction

It has been known that rising longevity has positive effects on the growth rate because expecting a higher chance of surviving leads to more saving for old-age consumption and the investment in education. For instance, Bhargava et al. (2001) show significant effects of adult survival rate on economic growth rates for low income countries. The similar results are given in many empirical researches (e.g., Zhang and Zhang 2005, Ehrlich and Lui 1991, Barro and Sala-i-Martin 1995, Bloom et al. 2003), which means that extending life expectancy is an important factor for developing countries to escape serious poverty. As a result, these findings continue to motivate a lot of researchers to examine how public policies lead to a longer life expectancy in developing countries; however, until now, we have not found the stylized policies.

Some studies have found that the health-related public spending does not yield the desirable improvement for health, especially in developing countries. For instance, Musgrove (1996) and Filmer and Pritchett (1999) conclude that the health-related public spending does not have any impact on health, with a coefficient that is statistically insignificant at conventional levels. Rajkumar and Swaroop (2008) show that the health-related public spending lowers child mortality rates in countries with good governance; however, in poorly governed, developing countries, increasing public spending on health does not lead to better outcomes. Anand and Ravallion (1993) and Bidani and Ravallion (1997) confirm limited impacts of public spending for the poor at the micro level; however, their findings are still consistent with small and insignificant impacts of its spending on aggregate health status. Similarly, World Development Report (World Bank, 2004) mentions that the relationship between public expenditure on health and the state of health is not seen at the statistically significant level. These findings have led to doubts regarding the role of health-related public spending.

There is a health-related paradox between domestic policies and foreign aid. Mishra and Newhouse (2009) give an interesting finding, which is that increased health aid from foreign countries is associated with a statistically significant reduction in infant and child mortality.[1]

[1] To the best of our knowledge, there is no paper which examines the effects of foreign aid on health, except for Mishra and Newhouse (2009). In that regard, Mishra and Newhouse (2009) also mention *despite the vast empirical literature considering the effect of foreign aid on growth, the existing paper that aid improves health outcomes is scare.*

Based on their paper, a possible reason why the expected effect on health is yielded by foreign aid is that foreign aid relaxes government budget constraint. In other words, when foreign aid is conducted, the domestic government does not need to budget for health spending.

In developing countries with serious poverty, the health-related policies have a large impact not only on these countries' aggregate health degrees but also their whole economies. Hence, in this paper, to give a solution for the above paradox, we construct a dynamic general equilibrium model of a small open economy with a health status variable. As for the health status variable, we assume the following. First, the health degrees of agents are incorporated into their utility, which quantifies a health-related quality of life mentioned in Sen (1981), Torrance (1986), Gordon et al. (1993) and Dolan (2000). In other words, we assume that when the level of health-related quality of life increases, their utility level increases as well. Second, the health status variable, represented by the health-related quality of life, is evolved over time. Concretely, increasing the private investment in health derives a higher level of health-related quality of life and increases the level of utility, but increased labor harms the state of health and decreases the level of utility. Under this set-up, the purpose of this paper is to examine the dynamic impacts of the temporary change in health-related public instruments on the health-related quality of life as well as on the domestic economy. The reason why we shed light on the temporary execution is that the government budget of health-related policies will be replaced by other policies during the economic development. This is because as people become richer, the private spending for health increases. Specifically, we introduce two forms of health-related policies: a direct transfer and a subsidy of health investment. Furthermore, to shed light on this finding, we add a direct transfer of health investment in the form of foreign aid. Our main finding is that the temporary enforcement of health-related policies controlled by domestic government does not improve the health-related quality of life; and alternatively, foreign aid improves it in the long run.

Our study is closely related to some of the existing investigations in the dynamic macroeconomic models (e.g., Futagami and Shibata 1998, Corneo and Jeanne 1997, Nakamoto 2009, Momota et al. 2005, Hashimoto and Tabata 2010, Aisa and Pueyo 2013, Chatterjee and Turnovsky 2007). First, the specification of the utility function with a health status variable is similar to the wealth or status seeking preference in Futagami and Shibata (1998), Corneo and Jeanne (1997) and Nakamoto (2009) in the sense that the utility function depends on

a stock variable. The utility function in their papers depends on capital stock as well as consumption, implying that people feel happy as the level of wealth accumulates. On the other hand, our employed utility function depends on the health-related quality of life, not capital stock, meaning that an improvement of health-related quality of life leads to a higher level of utility.

Next, in a two-period overlapping-generation model, Hashimoto and Tabata (2010) and Aisa and Pueyo (2013) introduce health-care goods. That is, when people consume health-care goods, their level of utility increases. In that regard, as mentioned in their papers, the health-care good can be regarded as the consumption good. In addition, they do not introduce any variables which indicate the state of health. In the present paper we suppose that there is an investment commodity which can improve the health-related quality of life.

In what follows, we need to mention the set-up of a two-period overlapping-generation model in Momota et al. (2005). In their paper, the prevalence of the diseases is added to the utility function, which finds that if the competitive equilibrium path is monotonic, the enforcement of one-shot foreign aid improves the welfare of agents.[2] Alternatively, the policy analysis in our paper is more comprehensive in the following three points. First, we employ not only the foreign aid but also two kinds of domestic policies. Second, our policies do not limit one-shot enforcement. In other words, in addition to one-shot policy, we can suppose that each government policy is temporarily changed. Finally, to see the temporary effects more comprehensively, we confirm the permanent effects as well.

Finally, Chatterjee and Turnovsky (2007) and this paper are similarly motivated by the role of foreign aid. In detail, Chatterjee and Turnovsky (2007) incorporate the elastic labor supply into the small open economy, and examine the effects of foreign aid on the economic growth. In our paper, we focus on the effects on the health-related quality of life.

The rest of this paper is organized as follows. In Section 2, we present a baseline model. Section 3 confirms the existence of steady state and stability. Section 4 examines the effects of a temporary/permanent change in the domestic government policies on the health-related quality of life and gives numerical examples. Section 5 analyzes the case where the foreign aid is introduced instead of the domestic government policies, and discusses whether the form

[2]The main finding in their paper is that the external effect of diseases yields a cyclical competitive-equilibrium path; on the other hand, we do not consider any external effects with respect to health.

of utility function affects our main result. Section 6 concludes.

2 Baseline Model

We consider a small open economy where the world interest rate, r, is constant. The population is constant and is normalized to unity. We consider a simple neoclassical production function. Since the aggregate production function is assumed to satisfy constant returns to scale with respect to capital and labor, which is expressed as:

$$y_t = F(\hat{k}_t, l_t) = l_t f(k_t),$$

where y_t is output, $\hat{k}_t$ is capital, l_t is labor and $k_t \equiv \hat{k}_t / l_t$ denotes capital intensity. The production function, $f(k_t)$, is monotonically increasing, strictly concave in k_t and satisfies the Inada conditions. Taking account of competitive factor and final goods markets, the real rent r_t and real wage rate w_t are respectively determined by:

$$r = f'(k_t), \quad w_t = f(k_t) - k_t f'(k_t). \tag{1}$$

Since the real rent is constant in the small open economy, from (1) we can see that the capital intensity is constant and hence, the wage rate is fixed as well ($k_t = k$ and $w_t = w$). As a result, the production function is given by:

$$y_t = l_t f(k). \tag{2}$$

2.1 Households and Health

We express the degree of health-related quality of life as an index $h_t \in R_+$, and a higher level of the variable h_t means that his health-related quality of life is higher. Then, the lifetime utility of the representative agent is given by:

$$U_0 \equiv \int_0^\infty \{u(c_t) + v(h_t)\} e^{-\rho t} dt. \tag{3}$$

where c_t is consumption level at time t and $\rho (> 0)$ is the rate of time preference. In addition, we assume that the instantaneous utility functions $u(\cdot)$ and $v(\cdot)$ are twice continuously differentiable, monotonically increasing and strictly concave in each variable c_t and h_t. In addition, these functions satisfy the Inada conditions. The specification that the health-related

variable is incorporated into the utility function is used in many studies (e.g., Aisa and Pueyo 2013, Hashimoto and Tabata 2010, Momota et al. 2005).

In what follows, denoting by l_t labor supply, we assume that the degree of health-related quality of life h_t is evolved as follows:

$$\dot{h}_t = g(x_t + \eta) - G(l_t), \tag{4}$$

where η the policy instrument concerning health. In addition, we assume that $g'(\cdot) > 0$, $g''(\cdot) < 0$, $G'(\cdot) > 0$, and $G''(\cdot) > 0$. As the variable x_t, we must firstly notice that the health-related quality of life h_t is defined as a measure of well-being in this paper, which is a broader concept beyond a direct measure of health such as longevity. For instance, if more labor deteriorates physical health, the variable x_t may be regarded as health-oriented products such as foods and drinks. When more labor harms mental health, x_t can be thought to be an investment and a consumption goods in improving mental health. In addition, if the health-related quality of life h_t is regarded as a measure of sanitary conditions, working may represent an aspect of deteriorating sanitation (e.g., waste) and the variable x_t represents the hygienic investment of promoting health such as the proper disposal of waste and sewage.[3] Hereafter, following Momota et al. (2005) we call the variable x_t as the (private) health investment. Hence, η can be regarded as a direct transfer of health investment by the government (or, foreign aid).

Turning our interests into the budget constraint, we suppose that using the goods produced by the capital stock, the representative agent has the option of either consumption or health investment. Then, the accumulation of the foreign asset holding, b_t is evolved as:

$$\dot{b}_t = rb_t + l_t f(k) - c_t - (1 - \tau)x_t - z_t, \tag{5}$$

where z_t is the lump-sum tax and τ is a subsidy rate for health investment.[4]

In the accumulation equations for health-related quality of life (4) and foreign asset (5), there are three kinds of policies. First, τ represents a constant rate of investment subsidy,

[3]For instance, since Momota et al. (2005) focused on the prevalence of the diseases, the variable x_t is called as the private health investment to improve water and sanitation systems.

[4]If the health investment x_t is regarded as the health-care goods, the relative price of x_t for consumption goods c_t may be needed. However, the incorporation of relative price does not change the main finding in this paper because the set-up of small open economy makes the relative price fixed through time.

which means that an increase in τ decreases the relative price of investment in the health, and hence, it will further stimulate the investment. Therefore, a higher rate of subsidy for health investment will lead to a higher level of health-related quality of life h_t. Next, η represents a direct transfer of health investment by domestic government. A larger level of direct transfer will lead to a higher level of health-related quality of life. As for these policies, we assume that the domestic government levies lump-sum tax on their budget. Therefore, the government keeps the following balanced budget over time:

$$\tau x_t + \eta = z_t. \tag{6}$$

Finally, we consider the foreign aid of direct transfer of health investment. In this case, the government does not need to impose any tax on the domestic agents and not issue any national bonds in order to receive the foreign aid. Therefore, the optimization choices of agents is not restricted by the balanced budget (6).[5]

2.2 Dynamic system

The representative agent maximizes his lifelong utility (3) subject to the evolution of health-related quality of life (4) and the budget constraint (5). In order to derive the optimal conditions, we set the current-value Hamiltonian:

$$\begin{aligned}\mathcal{H} = u(c_t) + v(h_t) + \lambda_t\big[g(x_t+\eta) - G(l_t)\big] \\ + q_t\big[rb_t + l_t f(k) - c_t - (1-\tau)x_t - z_t\big],\end{aligned}$$

where λ_t and q_t are respectively the costate variables for the equations (4) and (5).

Deriving the optimal conditions, we have:

$$u'(c_t) = q_t, \tag{7a}$$

$$v'(h_t) = -\dot{\lambda}_t + \rho\lambda_t, \tag{7b}$$

$$g'(x_t+\eta)\lambda_t = (1-\tau)q_t, \tag{7c}$$

$$G'(l_t)\lambda_t = q_t f(k), \tag{7d}$$

$$rq_t = -\dot{q}_t + \rho q_t. \tag{7e}$$

[5]Momota et al. (2005) and Chatterjee and Turnovsky (2007) use this form of foreign aid.

The transversality conditions are given by:

$$\lim_{t\to\infty} \lambda_t h_t e^{-\rho t} = 0, \quad \lim_{t\to\infty} q_t b_t e^{-\rho t} = 0.$$

From (7a) and (7e), the well-known Euler equation for consumption is given by:

$$\frac{\dot{c}_t}{c_t} = -\frac{u'(c_t)}{c_t u''(c_t)}(r-\rho). \tag{8}$$

Assuming that $r = \rho$ in the small open economy, we see that $\dot{c}_t = 0$; that is, $c_t = \bar{c}$, which means that $u'(\bar{c}) = q_t = \bar{q}$. Next, using (7c), we obtain:

$$\lambda_t = \frac{(1-\tau)u'(\bar{c})}{g'(x_t+\eta)}. \tag{9}$$

Differentiating (9) with respect to time and then dividing both sides by (9), we have

$$\frac{\dot{\lambda}_t}{\lambda_t} = -\frac{g''(x_t+\eta)}{g'(x_t+\eta)}\dot{x}_t, \tag{10}$$

Substituting (7*b*) and (9) into (10), we can see that:

$$\dot{x}_t = -\frac{g'(x_t+\eta)}{g''(x_t+\eta)}\left\{r - \frac{v'(h_t)g'(x_t+\eta)}{(1-\tau)u'(\bar{c})}\right\}. \tag{11}$$

The equations (7d), (9), and $u'(\bar{c}) = \bar{q}$ yield

$$g'(x_t+\eta)f(k) - (1-\tau)G'(l_t) = 0. \tag{12}$$

Using (12), we can express l_t as follows:

$$l_t = l(x_t, \tau, \eta). \tag{13}$$

Concerning (13), the signs of the partial derivative are

$$l_x \equiv \frac{\partial l_t}{\partial x_t} = \frac{g''(x_t+\eta)f(k)}{(1-\tau)G''(l_t)} < 0, \tag{14a}$$

$$l_\tau \equiv \frac{\partial l_t}{\partial \tau} = \frac{G'(l_t)}{(1-\tau)G''(l_t)} > 0, \tag{14b}$$

$$l_\eta \equiv \frac{\partial l_t}{\partial \eta} = \frac{g''(x_t+\eta)f(k)}{(1-\tau)G''(l_t)} < 0. \tag{14c}$$

We must note that $l_x = l_m$.

Taking (4) and (13) into consideration, we obtain:

$$\dot{h}_t = g(x_t+\eta) - G(l(x_t, \tau, \eta)). \tag{15}$$

From (6) and (13), the accumulation equation of foreign asset (5) can be rewritten as:

$$\dot{b}_t = rb_t + l(x_t, \tau, \eta)f(k) - \bar{c} - x_t - \eta. \tag{16}$$

Therefore, the complete dynamic system is given by (11), (15), and (16) of the small open economy, given a steady-state level of consumption $\bar{c}$, the rate of investment subsidy τ and the direct transfer of health investment η.

3 Steady state and stability

Let us now analyze the existence of uniquely determined steady state and its stability. First, as mentioned in the above, because of the set-up of small open economy, which means that the rate of interest is fixed, we have to require $r = \rho$ for our system to have a finite interior steady-state value for the marginal utility of consumption.

Then, we focus on the subsystem (11) and (15). Denoting by asterisk the steady-state level of each variable, from (11) we show that the $\dot{x}_t = 0$ locus is:

$$g'(x^* + \eta) = \frac{r(1-\tau)u'(\bar{c})}{v'(h^*)}. \tag{17a}$$

where the slope of the $\dot{x}_t = 0$ locus is given by:

$$\left.\frac{dx_t}{dh_t}\right|_{\dot{x}_t=0} = -\frac{g'(x^* + \eta)}{g''(x^* + \eta)}\frac{v''(h^*)}{v'(h^*)} < 0. \tag{17b}$$

Note that x^* approaches to infinity as h^* goes to zero, while x^* approaches to $-\eta$ as h^* goes to infinity. Then, we can depict a lower right curve as seen in Figure 1.

Turning into the equation $\dot{h}_t = 0$, from (15) we can see that

$$g(x^* + \eta) = G(l(x^*, \tau, \eta)),$$

which can be easily confirm that the steady-state level of health investment x^* is uniquely determined where as in Figure 1, $\dot{h}_t = 0$ is given by a horizontal line. As a result, from Figure 1, we can easily see that there exists a unique steady state.

(Insert Figure 1 here.)

To examine the stability of the steady state, from (11) and (15) we obtain the linearly approximated equations:

$$\begin{bmatrix} \dot{x}_t \\ \dot{h}_t \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{bmatrix} x_t - x^* \\ h_t - h^* \end{bmatrix}, \tag{18}$$

where

$$J_{11} \equiv \frac{\partial \dot{x}_t}{\partial x_t} = \frac{v'(h^*)g'(x^*+\eta)}{(1-\tau)u'(\bar{c}^*)} = r > 0 \ , \quad J_{12} \equiv \frac{\partial \dot{x}_t}{\partial h_t} = r\frac{g'(x^*+\eta)}{g''(x^*+\eta)}\frac{v''(h^*)}{v'(h^*)} > 0 \ ,$$
$$J_{21} \equiv \frac{\partial \dot{h}_t}{\partial x_t} = g'(x^*+\eta) - G'(l^*)l_x > 0 \ , \quad J_{22} \equiv \frac{\partial \dot{h}_t}{\partial h_t} = 0 \ .$$

Concerning the coefficient matrix of (18), we can see that

$$\mathrm{Tr}J = r > 0, \quad \mathrm{Det}J = -r\frac{g'(x^*+\eta)}{g''(x^*+\eta)}\frac{v''(h^*)}{v'(h^*)}\left\{g'(x^*+\eta) - G'(l^*)l_x\right\} < 0. \tag{19}$$

From (19), the subsystem has the saddle-path stability because the two eigenvalues of the coefficient matrix have opposite sign. Let us denote an eigenvalue by a_i $(i = 1, 2)$. Solving a_1 and a_2 $(a_1 < a_2)$, we get

$$a_1 = \frac{1}{2}\left(r - \sqrt{r^2 + 4J_{12}J_{21}}\right) < 0, \quad a_2 = \frac{1}{2}\left(r + \sqrt{r^2 + 4J_{12}J_{21}}\right) > 0.$$

Then, we obtain the following proposition.

Proposition 1 *The steady-state equilibrium is uniquely determined and satisfies the saddle-path stability.*

Next, we turn to the linear approximation of (h_t, x_t) given by:

$$h_t = h^* + A_1 e^{a_1 t} + A_2 e^{a_2 t}, \tag{20a}$$

$$x_t = x^* + \Theta_1 A_1 e^{a_1 t} + \Theta_2 A_2 e^{a_2 t}, \tag{20b}$$

where

$$\Theta_1 \equiv \frac{a_1}{J_{21}} < 0, \quad \Theta_2 \equiv \frac{a_2}{J_{21}} > 0.$$

From the transversality conditions, we must notice that $A_2 = 0$. Furthermore, it follows from (20a) that

$$A_1 = h_0 - h^*.$$

Thus, the stable adjustment path is given by:

$$h_t = h^* + (h_0 - h^*)e^{a_1 t}, \tag{21a}$$

$$x_t = x^* + \Theta_1(h_0 - h^*)e^{a_1 t}. \tag{21b}$$

Therefore, the phase diagram of the subsystem is illustrated in Figure 2.[6]

(Insert Figure 2 here.)

We move on to the linearization of (16). The linearized equation is given by

$$\dot{b}_t = \Omega(x_t - x^*) + r(b_t - b^*), \quad \Omega \equiv f(k)l_x - 1 < 0. \tag{22}$$

Solving (22), we have

$$b_t = b^* + \left\{b_0 - b^* - \Pi(h_0 - h^*)\right\}e^{rt} + \Pi(h_0 - h^*)e^{a_1 t}, \quad \Pi \equiv \frac{\Omega\Theta_1}{a_1 - r} < 0. \tag{23}$$

Regarding (23), the following equality must hold due to the intertemporal solvency condition of the economy:

$$b_0 - b^* - \Pi(h_0 - h^*) = 0. \tag{24}$$

In this case, (23) yields

$$b_t = b^* + \Pi(h_0 - h^*)e^{a_1 t}. \tag{25}$$

Taking account of $\dot{x}_t = \dot{h}_t = \dot{b}_t = 0$ with the intertemporal solvency condition, we find

[6] From (11), the dynamics of x_t become

$$\frac{\partial \dot{x}_t}{\partial h_t} = r\frac{g'(x_t + \eta)}{g''(x_t + \eta)}\frac{v''(h_t)}{v'(h_t)} > 0.$$

Thus, $\dot{x}_t > 0$ above the $\dot{x}_t = 0$ locus, while $\dot{x}_t < 0$ below the $\dot{x}_t = 0$ locus. From (15), the dynamics of h_t are given by

$$\frac{\partial \dot{h}_t}{\partial x_t} = g'(x_t + \eta) - G'(l_t)l_x > 0.$$

Thus, $\dot{h}_t > 0$ above the $\dot{h}_t = 0$ locus, while $\dot{h}_t < 0$ below the $\dot{h}_t = 0$ locus. Thus, through this process, we can confirm that the phase diagram of the subsystem is illustrated in Figure 2.

that the steady state is determined by the following expressions:

$$v'(h^*)g'(x^* + \eta) = r(1 - \tau)u'(\bar{c}^*), \tag{26a}$$

$$g(x^* + \eta) - G(l(x^*, \tau, \eta)) = 0, \tag{26b}$$

$$rb^* + l(x^*, \tau, \eta)f(k) - \bar{c}^* - x^* - \eta = 0, \tag{26c}$$

$$b_T - b^* = \Pi(h_T - h^*). \tag{26d}$$

where we have to notice that $\bar{c} = \bar{c}^*$ over time.

4 The effects of public policies by domestic government

In this section, we analyze how a change in public policies affects the health-related quality of life as well as the whole economy. Before proceeding with our analysis, we need to mention the policy result of the Ramsey paradigm of a closed economy, which is that a temporary change influences the economy during the transition; however, after the temporary change is removed, the economy gradually returns to the original steady state. As a result, it can be easily presumed that a temporary change of policy instrument in a closed economy does not have any impact on the long-run level of health-related quality of life. On the other hand, this paper employs the framework of a small open economy. As a result, even if the public policy is temporarily changed or conducted, the position of long-run steady state does not coincide with the original one, implying that the health-related quality of life increases or decreases. Then, we will show that the public policies conducted by domestic government do not have positive effects on the health-related quality of life.

4.1 Temporary effects

As the economy develops, the wealth of residents increases, and hence, they will spend more for their health, which implies that the role of public spending about health will be gradually restricted. Hence, it would be difficult for the health-related policies to keep the same scale in the governmental budget permanently. Then, it is more important to focus on the temporary effects of health-related policies, compared to the permanent effects.

For the small open economy, the rate of interest is exogenously given, so that it must hold $r = \rho$ for the economy to go toward the steady state. Hence, from (8) the consumption growth

rate is zero. That is, the level of consumption is fixed over time except for an unanticipated change in public policies. If public policies unanticipatedly change, the agents in the small economy will change their consumption behaviors. Concretely, let us consider the case in which the government unanticipatedly announces a change of the policy instruments from the original level, τ_0 (or η_0), to a new level τ_1 (or η_1), at time $t = 0$, the policy is conducted during some duration $t = [0, T)$, and hereafter, the policy instruments permanently return to the original level τ_0 (or η_0) at time $t = [T, \infty]$. Then, the consumption behavior of agents is as follows. At the initial period, because of the unanticipated policy change, the initially consumption jumps, and hereafter, the level of consumption becomes fixed. In particular, we must notice that even if the policy instrument returns to the original level at time T, the level of consumption does not go back to its original level but still remains under the assumption of perfect foresight. That is, since the agents can initially anticipate the policy change at time T, so that their levels of consumption do not change at time T. As a result, the newly level of fixed consumption leads to the long-run impact of temporary change in public policies.

Then, we can show the following main finding.

Proposition 2 *A temporary increase in the subsidy rate for the private health investment, τ, decreases the level of health-related quality of life in the long run. Alternatively, a temporary increase in the direct transfer of health investment, η, does not lead to any changes in the health-related quality of life.*

Proof. See Appendix A. ■

The intuitive explanations about each policy are as follows. As for the health-related investment subsidy, the key element in determining the long-run effects is the behavior of consumption. An increase in the subsidy rate for the health investment leads to a negative impact on consumption at the initial period, because the policy decreases the relative price of health-investment commodities and hence, it causes further investment in health. Therefore, the direction of the initial jump in consumption is downward, thereby showing that the growth rate of health investment in (11) is negatively affected over time. Put differently, the current health investment is substituted with the future investment, which causes the delay in the health investment. Finally, considering that only the negative impact on consumption

remains in the long run, we can argue that there exists the delay in health investment in the long run. As a result, the long-run level of health-related quality of life decreases.

Now, we confirm the dynamic movement of the economy in Figure 3. Suppose that the economy is in the original steady state E_0 initially. When the government raises τ from τ_0 to τ_1 at time $t = 0$ and restores τ_1 to τ_0 after $t = T$, the graph of the $\dot{x}_t = 0$ locus first shifts upward (from "$\dot{x}_t = 0$ $(t = 0)$" to "Tem: $\dot{x}_t = 0$ $(t < T)$" in Figure 3 (A)) and then moves downward (from "Tem: $\dot{x}_t = 0$ $(t < T)$" to "Tem: $\dot{x}_t = 0$ $(t > T)$"). Regarding the $\dot{h}_t = 0$ locus, the graph first shifts upward (from "$\dot{h}_t = 0$ $(t = 0)$" to "Tem: $\dot{h}_t = 0$ $(t < T)$") and then moves downward (from "Tem: $\dot{h}_t = 0$ $(t < T)$" to "Tem: $\dot{h}_t = 0$ $(t > T)$"). In this case, the private health investment, x_t, first jumps to "Q'" on an unstable path, because the dynamic path needs to ride on the new saddle path at time $t = T$. Due to this, although health status first improves, it is ultimately deteriorated in comparison with health status in the original steady state. Besides, Figure 3 (B) shows the relationship between h_t and b_t. Under a temporary rise in τ, the steady-state value of b_t ultimately increases.

(Insert Figure 3 here.)

In what follows, we consider the effects of direct transfer of health investment by domestic government. From (26b), we can see that $\frac{\partial x^*}{\partial \eta} = -1$ because $l_x = l_\eta$ in (13), meaning that the relationship between the health investment h_t and the transfer η is completely substituted. Since an increase of one unit of direct transfer of health investment causes a decrease of one unit of private health investment, there is a crowding out effect for health investment. In summary, due to the substitution of public for private spending, additional public provision in domestic government has a negligible net marginal effect, which can be seen in a real economy, as mentioned in Rajkumar and Swaroop (2008) and Mishra and Newhose (2009). Under the complete substitution, even if the direct transfer of health investment temporarily changes, the economic movement does not change, by which we conclude that the level of health-related quality of life does not change over time.

4.2 Permanent effects

To understand the temporary effects in the last subsection more comprehensively, let us examine the permanent effects of public policies. Firstly, we must notice that as confirmed in

the last subsection, the direct transfer of health investment in the form of (4) does not have any impacts on the whole economy due to the crowding out effect. Therefore, this conclusion is not related to the implementation period, and hence, we focus on the permanent effect of subsidy policy for the health investment.

Suppose that the subsidy rate τ increases permanently. Then, we can see that the shift of the $\dot{x}_t = 0$ locus is as follows:

$$\frac{\partial x_t}{\partial \tau} = \underbrace{-\frac{(1-\tau)ru'(\bar{c})}{v'(h_t)g''(x_t)}}_{+} \left\{ \frac{1}{1-\tau} - \frac{u''(\bar{c})}{u'(\bar{c})} C_\tau \right\}. \tag{27}$$

Since $C_\tau < 0$, the sign of $\partial x_t/\partial \tau$ is ambiguous. In the curly brackets of (27), if the effect of first term dominates that of second term, we can see that $\partial x_t/\partial \tau > 0$. In other words, when the subsidy rate τ is large enough, this case can be held. Then, as depicted in Figure 3 (A), the $\dot{x}_t = 0$ locus shifts upward by increasing the subsidy rate τ. Next, we can see the shift of the $\dot{h}_t = 0$ locus as follows:

$$\frac{\partial x_t}{\partial \tau} = \frac{G'(l_t)l_\tau}{g'(x_t) - G'(l_t)l_x} > 0, \tag{28}$$

which leads to an upward movement of the $\dot{h}_t = 0$ locus by increasing the subsidy rate τ.

Now, let us see the impact of a permanent rise in τ in Figure 3 in which our interest is the upward shift of $\dot{x}_t = 0$, because the long-run effect on the health-related quality of life is the opposite with that obtained by the temporary increase. Suppose that the economy is in the original steady state E_0 initially as in Figure 3 (A). Then, if the government raises τ permanently at time $t = 0$, the locus of $\dot{x}_t = 0$ moves upwardly from "$\dot{x}_t = 0$ $(t = 0)$" to "Per: $\dot{x}_t = 0$ $(t > 0)$." Furthermore, since the $\dot{h}_t = 0$ locus also shifts upward under a permanent rise in τ, the graph of the $\dot{h}_t = 0$ locus moves from "$\dot{h}_t = 0$ $(t = 0)$" to "Per: $\dot{h}_t = 0$ $(t > 0)$." In this case, the private health investment, x_t, first jumps to "Q" in order to ride on the new saddle path, and then health status improves along this saddle path. As a result, the economy reaches the new steady state E_1, where health status is better than the original one. Besides, Figure 3 (B) depicts the relationship between h_t and b_t. Under a permanent rise in τ, the steady-state value of b_t ends up decreasing.

As for the health-related quality of life, the results can be summarized as follows:

Proposition 3 *Suppose that the subsidy rate for the health investment is large enough such as the positive sign of $\partial x_t/\partial \tau$ in* (27)*. Then, a permanent rise in the subsidy rate for the*

health investment improves the health-related quality of life in the long run. On the other hand, when the subsidy rate for the health investment is not large, this result can be reversed.

4.3 Numerical examples

In the section 4.1 and 4.2, we have confirmed the main finding of this paper. While the direct transfer of health-related investment commodities does not have any impacts on the whole economy over time, we concluded that a temporary increase in the rate of investment subsidy decreases the health-related quality of life in the long run and a permanent increase may improve it. Then, our concern is to see the quantitative impacts of investment subsidy policy on the health-related quality of life. In addition, we are interested in providing numerical confirmation of our results in Proposition 2 and furthermore, seeing that Figures 2 and 3 are mathematically correct.

First, we specify the utility function as the constant-relative-risk-aversion type: $u(c_t) = c_t^{1-\sigma}/(1-\sigma)$ and $v(h_t) = h_t^{1-\phi}/(1-\phi)$ where σ and ϕ are preference parameters with positive signs. In what follows, the production of the single homogeneous commodity is given by the production process: $f(k_t) = Ak_t^{\xi}$ where A and ξ are the production parameters. While these types of utility and the production functions are frequently used, we assume that the functions $g(\cdot)$ and $G(\cdot)$ about the health-related quality of life are as follows:

$$g(x_t) = \frac{x_t^{1-\alpha}}{1-\alpha}, \quad G(l_t) = \frac{l_t^{1+\beta}}{1+\beta}, \tag{29}$$

where $0 < \alpha < 1$ and $\beta > 0$.

The parameter values used in our simulations are:

- The production parameters: $\xi = 0.3, \quad A = 0.2$.
- Taste parameters: $\sigma = 1.5, \quad \phi = 3.5$.
- Health parameters: $\alpha = 0.5, \quad \beta = 0.75$.
- Interest rate and tax rate: $r = (\rho =)0.04, \quad \tau_0 = 0, \quad \tau_1 = 0.25$.
- Initial values: $b_0 = 1, \quad h_0 = 1.859$.

Some of these parameters are standard ones: $\xi = 0.3$, $\rho = r = 0.04$ and $\sigma = 1.5$. In that case, setting $A = 0.2$ yields the fixed value of capital stock, $k = 1.78$. As for the health parameters, we assume that the elasticity of marginal utility with respect to consumption is lower than with respect to the health-related quality of life, $\phi = 3.5$, which supposes that the investment

commodity about health is a luxury good for the poor in the developing countries. Next, we set the remaining parameters in the functions $g(\cdot)$ and $G(\cdot)$ as follows: $\alpha = 0.5$ and $\beta = 0.75$. Finally, while the original rate of investment subsidy is zero, we assume that its rate is set as $\tau_1 = 0.25$, which means that the relative price of investment commodities decreases by the 75 percent level.

Figure 4(a) and 4(b) show the quantitative effects of the temporary change in the investment subsidy. Alternatively, Figure 5(a) and 5(b) give those of permanent change where we focus on the case where the health-related quality of life increases in the long run. The labels given in each figure corresponds to that in Figure 3. For example, the label E_0 in these figures represents the original steady state. Then, our findings are as follows.

First, the dynamic behaviors of health investment, the health-related quality of life and the foreign asset are qualitatively the same with those in Figure 3. In particular, a temporary rise in the rate of investment subsidy leads to a decrease in the health-related quality of life in the long run. Concretely, the long-run level of health-related quality of life at E_2 is given by $h^* = 1.840$, which implies that the temporary policy leads to about 1 percent decrease in the health-related quality of life. This change seems to be a slight shift. This is because the long-run impact of investment subsidy policy is determined by only the initial jump of private consumption.

Based on this, the current concern is to see the quantitative change in private consumption because this change leads to the decrease in the health-related quality of life. Concretely, we see that $\bar{c} = 0.1980$ at the original steady state E_0, which leads to a downward jump $\bar{c} = 0.1933$, which shows around 2 percent decrease in the consumption. This slight shift causes a small impact on the health-related quality of life. As a result, since the distance between h_0 and h_2^* is very close, the foreign asset does not move dramatically , and furthermore, each level of foreign asset at E_0 and E_2 is very close as well as seen in Figure 4(b).

Finally, we turn to the permanent effects of investment subsidy in Figure 5. Based on our setting, the permanent increase in the rate of investment subsidy improves the health-related quality of life in the long run. In detail, the 25 percent increase in the rate of investment subsidy increases to $h^* = 1.886$ at E_1, which is the 1.5 percent increase in the health-related quality of life.

(Insert Figures 4 and 5 here.)

5 Discussion

5.1 The foreign aid

In the last section, we have confirmed the effects of public policies conducted by domestic government, concluding that these policies negatively affect the health-related quality of life in the long run. Alternatively, taking account of the real economy, a lot of developing countries are assisted by foreign countries in various forms. Therefore, our concern is now to examine the effects of foreign aid on the health-related quality of life. In particular, we focus on the form of direct transfer of health investment. This is because this type of assistance (e.g., transfers of foods and medicines) is broadly observed in foreign aid.[7]

Let us denote the direct transfer of health commodities in the form of foreign aid as the variable m. Since residents in the recipient country do not have any tax burden, the budget constraint can be rewritten as:

$$\dot{b}_t = rb_t + l_t f(k) - c_t - x_t, \tag{30a}$$

where the evolution of health-related quality of life is fundamentally the same except for the notation:

$$\dot{h}_t = g(x_t + m) - G(l_t). \tag{30b}$$

Following the similar analysis in Section 2.2 and Section 3, in the steady state we find that

$$v'(h^*)g'(x^* + m) = ru'(\bar{c}^*), \tag{31a}$$

$$g(x^* + m) = G(l(x^*, m)), \tag{31b}$$

$$rb^* + l(x^*, m)f(k) = \bar{c}^* + x^*, \tag{31c}$$

$$b_T - b^* = \Pi^m(h_T - h^*). \tag{31d}$$

[7] We considered foreign aid which causes an increase in the subsidy rate of health investment; however, to the best of our knowledge, such foreign aid does not exist in the sense that a foreign country determines the relative price of investment commodities for health.

where $\Pi^m(<0)$ is the same as $\Pi(<0)$ in (23) except for the public policies. From (31a)-(31d), we obtain the following proposition.

Proposition 4 *A temporary rise in the foreign aid, m, improves the health-related quality of life in the long run. Additionally, a permanent rise in the foreign aid improves the health-related quality of life in the long run as well.*

Proof. See Appendix B. ■

Comparing this proposition with Proposition 2, we can easily understand the reason why the foreign aid improves the health-related quality of life. First, based on $\tau = 0$, let us compare $(26a)$ and $(26b)$ with $(31a)$ and $(31b)$. Then, these equations are completely the same each other. Thus, from $(26b)$ and $(31b)$, we can see $\partial x^*/\partial m = \partial x^*/\partial \eta = -1$, and furthermore, substituting this result into $(26a)$ or $(31a)$ yields $h^* = h^*(\bar{c}^*)$ where $\partial h^*/\partial \bar{c}^* > 0$. Importantly, because of a crowding-out effect, an increase of one unit of direct transfer of health investment causes a decrease of one unit of private health investment, meaning that since the total amount of investment commodities $(x + m)$ or $(x + \eta)$ does not change, the direct transfer of investment commodities does not have a direct impact on the health-related quality of life. This conclusion is consistent irrespective of whether the policy is conducted by the domestic government.

On the other hand, unlike the domestic policy, the foreign aid does not impose any burden for the domestic government. That is, it can be easily presumed that the domestic residents who are not imposed by any tax have more wealth to spare in the case of foreign aid, which leads to a positive impact on private consumption. Concretely, considering that the growth rate of private consumption is zero, we find that the agent initially increases the level of his consumption. That is, it holds $\bar{c}^* = \bar{c}(m)$ where $\partial \bar{c}^*/m > 0$.[8] Finally, substituting $\bar{c}^* = \bar{c}(m)$ into $h^* = h^*(\bar{c}^*)$ yields $h^* = h^*(\bar{c}^*(m))$ where $\partial h^*/\partial m = \frac{\partial h^*}{\partial \bar{c}^*}\frac{\partial \bar{c}^*}{\partial m} > 0$. Intuitively, in contrast to the investment subsidy in the last section, such a higher level of private consumption causes the negative impact on the growth rate of private health investment in (11), which means that the current investment is further stimulated. As a result, the effect causes a higher level of health-related quality of life in the long run.

[8] In detail, see Appendix B.

5.2 Non-separable utility function

Hashimoto and Tabata (2010) and Aisa and Pueyo (2013) introduce the non-separable utility function with non-health-care and health-care goods in the multiplicative form. In other words, when people consume health-care goods, consuming non-health-care goods leads to a higher level of utility. For instance, suppose that health-care goods are medicine and non-health-care goods are food. When people consume the health-care enough goods, the state of their health will be largely improved, so that consuming the food leads to a higher satisfaction because they can enjoy eating more, which would be reasonable as an extension of our paper. Therefore, in this subsection we modify the utility function as follows:

$$U_0 \equiv \int_0^\infty u(c_t, h_t)e^{-\rho t}dt. \tag{32}$$

Unlike (3) in the baseline model, we assume the non-separable utility function between the consumption and the health-related quality of life where we assume that $u_{ch}(c_t, h_t)$ has a positive sign as in Hashimoto and Tabata (2010) and Aisa and Pueyo (2013).

Then, we can conclude that the main finding in the baseline model does not change. To give the intuitive explanations, let us confirm the dynamic equation of private health investment:[9]

$$(3):\ \dot{x}_t = -\frac{g'(x_t+\eta)}{g''(x_t+\eta)}\left\{r - \frac{v'(h_t)g'(x_t+\eta)}{(1-\tau)\bar{q}}\right\}, \tag{33a}$$

$$(32):\ \dot{x}_t = -\frac{g'(x_t+\eta)}{g''(x_t+\eta)}\left\{r - \frac{u_h(c(h_t,\bar{q}),h_t)g'(x_t+\eta)}{(1-\tau)\bar{q}}\right\}. \tag{33b}$$

where $u_h = \partial u/\partial h$. We must note that $\bar{q}$ is the shadow value for the budget constraint (5), and its value is fixed because $-\dot{q}/q = r - \rho = 0$. For the convenience of intuitive explanation, we make use of the shadow value in (33a) and (33b), not the marginal utility of private consumption. Then, we can easily confirm that except for $v'(h_t)$ and $u_h(c(h_t,\bar{q}),h_t)$, both equations are the same, and furthermore importantly, in both equations it holds that $\partial\dot{x}_t/\partial\bar{q} < 0$. Since the dynamic equation of health-related quality of life in (4) is the same irrespective of whether the utility function is separable or not, we can presume that the main findings of public policies are also the same.

[9]Concretely, we can maximize (32) subject to (4) and (5) as in the baseline model. Then, we can derive (33b) from the similar procedure of derivation for (11)

6 Conclusion

In this paper, we considered the health-related quality of life and the agent's investment behavior for health in the small open economy. In other words, we assumed that people can invest in their health, and hence, their health-related quality of life can be improved. Using the model, we found that the temporary changes of public policies (the investment subsidy policy and the direct transfer of investment commodities) conducted by the domestic government do not have positive impacts on the health-related quality of life in the long run. Furthermore, this result does not change irrespective of whether the utility function is separable between the consumption good and the health-related quality of life. On the other hand, we found that a temporary increase in direct transfer of investment commodities from a foreign country improves the health-related quality of life.

Appendix A.

As for the temporary effects, we follow the procedure in Sen and Turnovsky (1990) and Turnovsky (1997).

The effect of a temporary change in τ

To simplify the exposition, we first focus on the case where $\tau \neq 0$ and $\eta = 0$. From (26a)-(26d), each steady-state value is given as follows:

$$x^* = X(\tau), \quad X_\tau > 0, \tag{A.1a}$$

$$h^* = H(\bar{c}^*, \tau), \quad H_{\bar{c}^*} > 0, \quad H_\tau > 0, \tag{A.1b}$$

$$b^* = B(\bar{c}^*, \tau), \quad B_{\bar{c}^*} > 0, \quad B_\tau > 0, \tag{A.1c}$$

$$\bar{c}^* = C(\tau, b_T, h_T), \quad C_\tau < 0. \tag{A.1d}$$

In what follows, we provide the explanation about the derivations of (A.1a)-(A.1d). Under $\eta = 0$, it follows from (26b) that $x^* = X(\tau)$, where

$$X_\tau \equiv \frac{\partial x^*}{\partial \tau} = \frac{G'(l^*)l_\tau}{g'(x^*) - G'(l^*)l_x} > 0.$$

Furthermore, (26a), (26c), and $x^* = X(\tau)$ yield

$$v'(h^*)g'(X(\tau)) = r(1-\tau)u'(\bar{c}^*),$$

$$rb^* + l(X(\tau), \tau)f(k) - \bar{c}^* - X(\tau) = 0,$$

so that we obtain $h^* = H(\bar{c}^*, \tau)$ and $b^* = B(\bar{c}^*, \tau)$, where[10]

$$H_{\bar{c}^*} \equiv \frac{\partial h^*}{\partial \bar{c}^*} = \frac{r(1-\tau)u''(\bar{c}^*)}{v''(h^*)g'(x^*)} > 0 \ , \quad H_\tau \equiv \frac{\partial h^*}{\partial \tau} = -\frac{v'(h^*)g''(x^*)X_\tau + ru'(\bar{c}^*)}{v''(h^*)g'(x^*)} > 0 \ ,$$

$$B_{\bar{c}^*} \equiv \frac{\partial b^*}{\partial \bar{c}^*} = \frac{1}{r} > 0 \ , \qquad B_\tau \equiv \frac{\partial b^*}{\partial \tau} = -\frac{1}{r}\Big[f(k)\big\{l_x X_\tau + l_\tau\big\} - X_\tau\Big] > 0 \ .$$

It follows from (26d), $h^* = H(\bar{c}^*, \tau)$, and $b^* = B(\bar{c}^*, \tau)$ that

$$B(\bar{c}^*, \tau) - b_T = -\Pi\big\{h_T - H(\bar{c}^*, \tau)\big\}.$$

This leads to $\bar{c}^* = C(\tau, b_T, h_T)$, where[11]

$$C_\tau \equiv \frac{\partial \bar{c}^*}{\partial \tau} = -\frac{B_\tau - \Pi H_\tau}{B_{\bar{c}^*} - \Pi H_{\bar{c}^*}} < 0.$$

[10] The detailed derivations of $H_\tau > 0$ and $B_\tau > 0$ are available upon request.

[11] Employing $H_{\bar{c}^*} > 0$, $B_{\bar{c}^*} > 0$, and $\Pi < 0$ yields $B_{\bar{c}^*} - \Pi H_{\bar{c}^*} > 0$. In addition, $H_\tau > 0$, $B_\tau > 0$, and $\Pi < 0$ lead to $B_\tau - \Pi H_\tau > 0$. Thus, $C_\tau < 0$ holds.

We now examine the effect of a change in τ on the health-related quality of life in the long run. The economy moves along the following unstable path under $0 \leq t < T$:

$$h_t = h_1^* + A_1 e^{a_1 t} + A_2 e^{a_2 t},$$

where $h_1^* = H(\bar{c}_1^*, \tau_1) = H(C(\tau_1, b_0, h_0), \tau_1)$. Under $T \leq t$, on the other hand, the economy moves along the following stable path:

$$h_t = h_2^* + A_1' e^{a_1' t},$$

where $h_2^* = H(\bar{c}_2^*, \tau_0) = H(C(\tau_0, b_T, h_T), \tau_0)$. Calculating $h_2^* - h_1^*$ and $h_1^* - h_0^*$, we obtain[12]

$$h_2^* - h_1^* \equiv H(\bar{c}_2^*, \tau_0) - H(\bar{c}_1^*, \tau_1) = -H_\tau d\tau, \tag{A.2a}$$

$$h_1^* - h_0^* \equiv H(\bar{c}_1^*, \tau_1) - H(\bar{c}_0^*, \tau_0) = H_{\bar{c}^*} C_\tau d\tau + H_\tau d\tau. \tag{A.2b}$$

Taking $H_{\bar{c}^*} > 0$ and $C_\tau < 0$ into account, we find that the following expression holds from (A.2a) and (A.2b):[13]

$$h_2^* - h_0^* = H_{\bar{c}^*} C_\tau d\tau < 0.$$

Therefore, the agent's health status gets worse in the new steady state when the value of τ temporarily rises.

The effect of a temporary change in η

We next move on to the case where $\tau = 0$ and $\eta \neq 0$. From (26a)-(26d), each steady-state value is given as follows:

$$x^* = X(\eta), \quad X_\eta < 0, \tag{A.3a}$$

$$h^* = H(\bar{c}^*, \eta), \quad H_{\bar{c}^*} > 0, \quad H_\eta = 0, \tag{A.3b}$$

$$b^* = B(\bar{c}^*, \eta), \quad B_{\bar{c}^*} > 0, \quad B_\eta = 0, \tag{A.3c}$$

$$\bar{c}^* = C(\eta, b_T, h_T), \quad C_\eta = 0. \tag{A.3d}$$

In what follows, we explain about the derivations of (A.3a)-(A.3d). When $\tau = 0$, (26b) yields $x^* = X(\eta)$, where

$$X_\eta \equiv \frac{\partial x^*}{\partial \eta} = -\frac{g'(x^* + \eta) - G'(l^*) l_\eta}{g'(x^* + \eta) - G'(l^*) l_x} = -1 < 0, \quad (\because l_x = l_\eta).$$

[12] We employ $\bar{c}_1^* = \bar{c}_2^*$.

[13] Note that $d\tau > 0$.

From (26a), (26c), and $x^* = X(\eta)$, we see that

$$v'(h^*)g'(X(\eta)+\eta) = ru'(\bar{c}^*),$$
$$rb^* + l(X(\eta),\eta)f(k) - \bar{c}^* - X(\eta) - \eta = 0.$$

These expressions result in $h^* = H(\bar{c}^*, \eta)$ and $b^* = B(\bar{c}^*, \eta)$, where

$$H_{\bar{c}^*} \equiv \frac{\partial h^*}{\partial \bar{c}^*} = \frac{ru''(\bar{c}^*)}{v''(h^*)g'(x^*+\eta)} > 0\ , \quad H_\eta \equiv \frac{\partial h^*}{\partial \eta} = -\frac{v'(h^*)g''(x^*+\eta)(X_\eta+1)}{v''(h^*)g'(x^*+\eta)} = 0\ ,$$
$$B_{\bar{c}^*} \equiv \frac{\partial b^*}{\partial \bar{c}^*} = \frac{1}{r} > 0\ , \qquad B_\eta \equiv \frac{\partial b^*}{\partial \eta} = -\frac{1}{r}\big(X_\eta+1\big)\big\{l_x f(k) - 1\big\} = 0\ .$$

Note that we employ $X_\eta = -1$ regarding the derivations of $H_\eta = 0$ and $B_\eta = 0$. It follows from (26d), $h^* = H(\bar{c}^*, \eta)$, and $b^* = B(\bar{c}^*, \eta)$ that

$$B(\bar{c}^*, \eta) - b_T = -\Pi\big\{h_T - H(\bar{c}^*, \eta)\big\}.$$

Solving this expression, we obtain $\bar{c}^* = C(\eta, b_T, h_T)$, where

$$C_\eta \equiv \frac{\partial \bar{c}^*}{\partial \eta} = -\frac{B_\eta - \Pi H_\eta}{B_{\bar{c}^*} - \Pi H_{\bar{c}^*}} = 0 \quad (\because H_\eta = 0 \text{ and } B_\eta = 0).$$

We now check the effect of a change in η on the health-related quality of life in the long run. The economy moves along the following unstable path under $0 \le t < T$:

$$h_t = h_1^* + A_1 e^{a_1 t} + A_2 e^{a_2 t},$$

where $h_1^* = H(\bar{c}_1^*, \eta_1) = H(C(\eta_1, b_0, h_0), \eta_1)$. Under $T \le t$, on the other hand, the economy moves along the following stable path:

$$h_t = h_2^* + A_1' e^{a_1' t},$$

where $h_2^* = H(\bar{c}_2^*, \eta_0) = H(C(\eta_0, b_T, h_T), \eta_0)$. Calculating $h_2^* - h_1^*$ and $h_1^* - h_0^*$, we obtain[14]

$$h_2^* - h_1^* \equiv H(\bar{c}_2^*, \eta_0) - H(\bar{c}_1^*, \eta_1) = -H_\eta d\eta = 0,$$
$$h_1^* - h_0^* \equiv H(\bar{c}_1^*, \eta_1) - H(\bar{c}_0^*, \eta_0) = H_{\bar{c}^*} C_\eta d\eta + H_\eta d\eta = 0,$$

because $H_\eta = 0$ and $C_\eta = 0$. Thus, we find that the following expression holds:

$$h_2^* - h_0^* = 0.$$

[14] We employ $\bar{c}_1^* = \bar{c}_2^*$. In addition, note that $d\eta > 0$.

Therefore, the agent's health status does not change in the new steady state when the value of η temporarily rises.

Appendix B.

From (31a)-(31d), each steady-state value is given as follows after some manipulation:

$$x^* = X(m), \quad X_m < 0, \tag{B.1a}$$

$$h^* = H(\bar{c}^*, m), \quad H_{\bar{c}^*} > 0, \quad H_m = 0, \tag{B.1b}$$

$$b^* = B(\bar{c}^*, m), \quad B_{\bar{c}^*} > 0, \quad B_m < 0, \tag{B.1c}$$

$$\bar{c}^* = C(m, b_T, h_T), \quad C_m > 0. \tag{B.1d}$$

Note that the signs of X_m, $H_{\bar{c}^*}$, H_m, and $B_{\bar{c}^*}$ are obtained from the same calculations as in Appendix B (replace η with m in each expression), whereas the signs of B_m and C_m are given by

$$B_m \equiv \frac{\partial b^*}{\partial m} = -\frac{1}{r}\Big[\big(X_m + 1\big)\big\{l_x f(k) - 1\big\} + 1\Big] = -\frac{1}{r} < 0 \quad (\because X_m = -1),$$
$$C_m \equiv \frac{\partial \bar{c}^*}{\partial m} = -\frac{B_m - \Pi H_m}{B_{\bar{c}^*} - \Pi H_{\bar{c}^*}} = -\frac{B_m}{B_{\bar{c}^*} - \Pi H_{\bar{c}^*}} > 0.$$

We now consider the effect of a temporary change in m on h^*. Taking the same analysis as in Appendix A into consideration, we ultimately see that

$$h_2^* - h_1^* \equiv H(\bar{c}_2^*, m_0) - H(\bar{c}_1^*, m_1) = -H_m dm = 0,$$
$$h_1^* - h_0^* \equiv H(\bar{c}_1^*, m_1) - H(\bar{c}_0^*, m_0) = H_{\bar{c}^*} C_m dm + H_m dm = H_{\bar{c}^*} C_m dm > 0,$$

because $H_m = 0$, $H_{\bar{c}^*} > 0$ and $C_m > 0$ (note that $dm > 0$). Thus, we find that the following expression holds:

$$h_2^* - h_0^* = H_{\bar{c}^*} C_m dm > 0.$$

Therefore, the agent's health status gets better in the new steady state when the value of m temporarily rises.

We now examine the effect of a permanent change in a foreign aid on the health-related quality of life in the long run. The permanent effect of a change in foreign aid on health status is given by

$$\frac{dh^*}{dm} = H_{\bar{c}^*} C_m + H_m = \frac{r u''(\bar{c}^*)}{v''(h^*) g'(x^* + m) - r^2 u''(\bar{c}^*) \Pi^m} > 0.$$

Thus, if a foreign aid increases, then the health-related quality of life improves. However, since $H_m = 0$, an increase in a foreign aid does not lead to improvement of the health-related quality of life directly. The indirect effect of a foreign aid on consumption ultimately raises the health-related quality of life.

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Figures

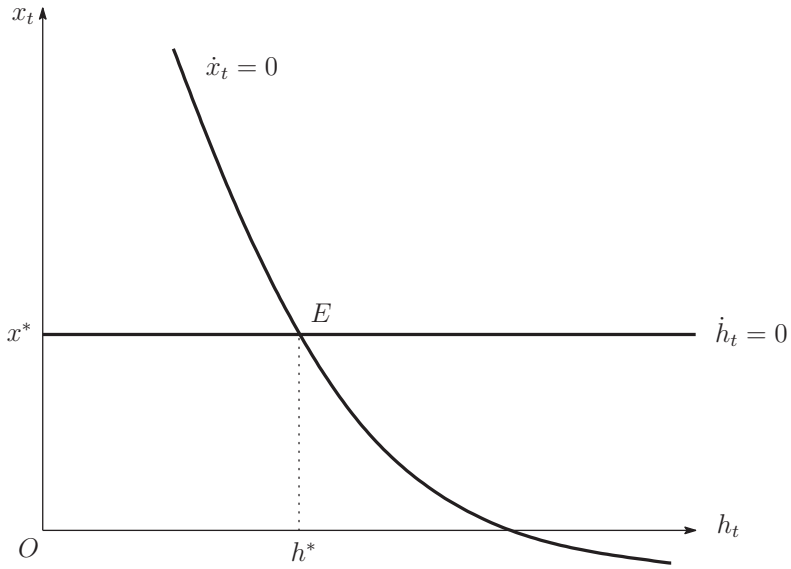

Figure 1: The graph of the $\dot{x}_t = 0$ locus and that of the $\dot{h}_t = 0$ locus

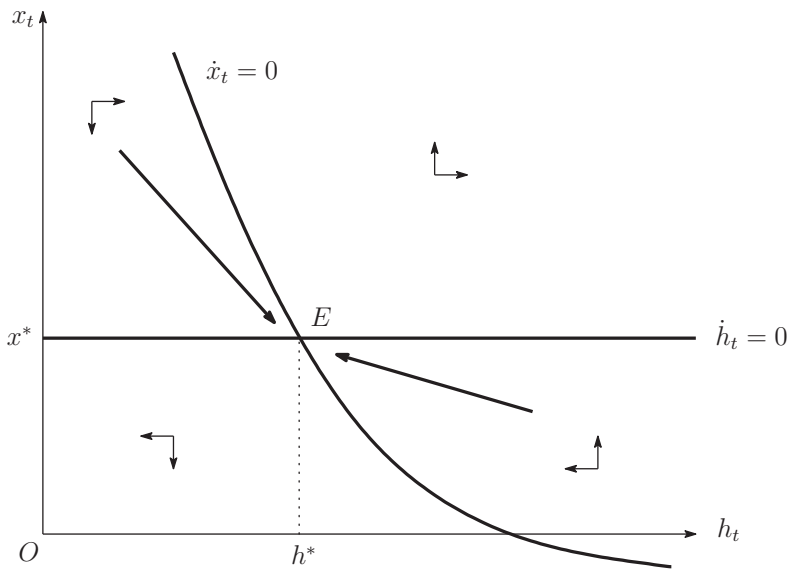

Figure 2: Phase diagram of the subsystem

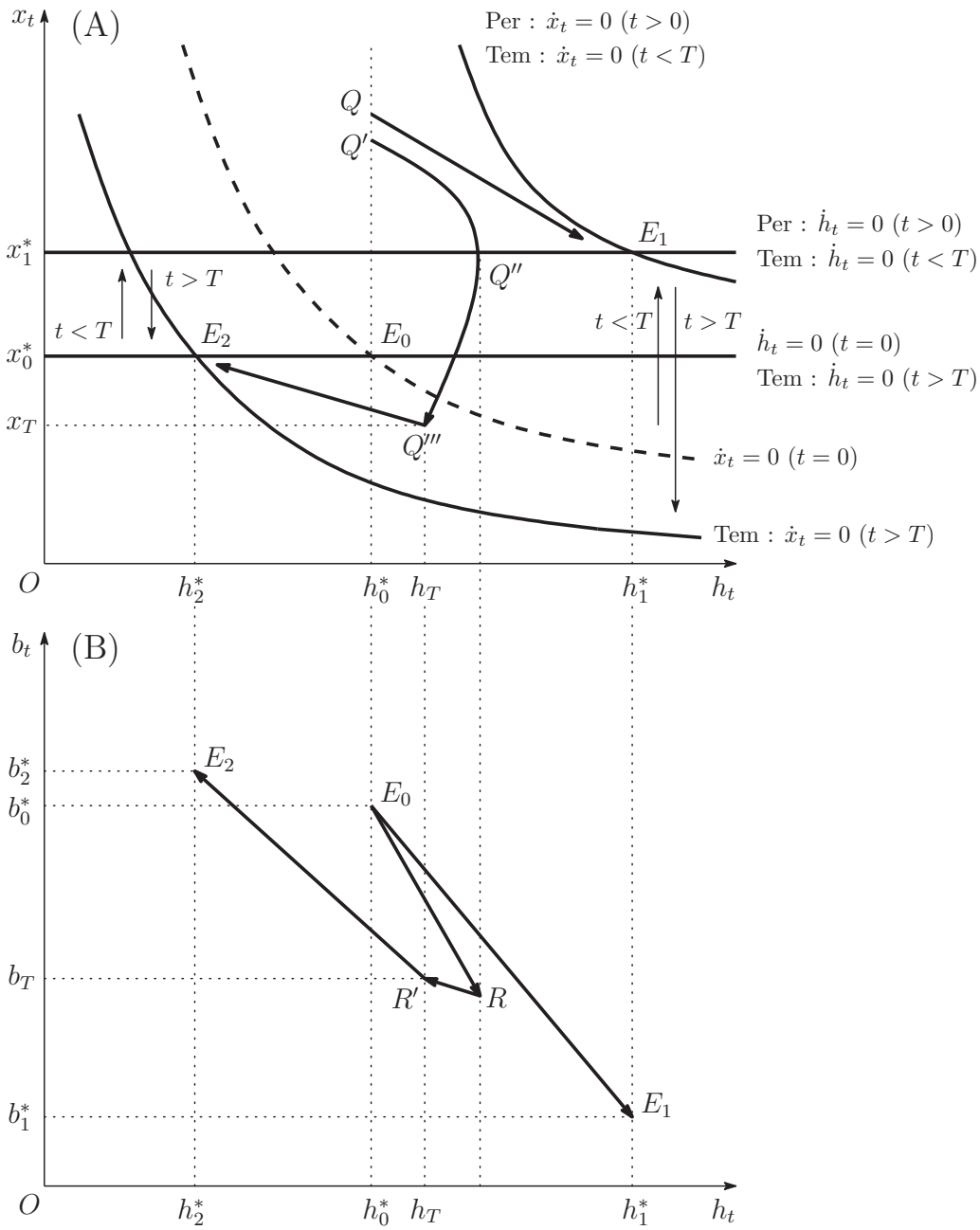

Figure 3: The effect of a rise in τ

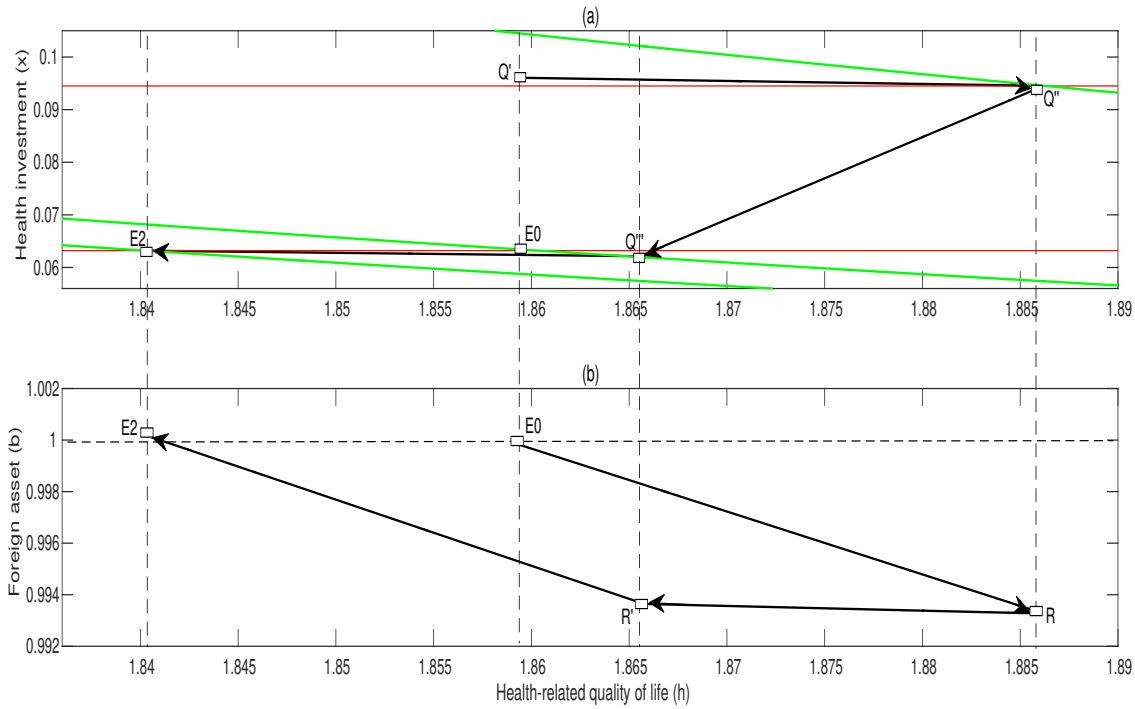

Figure 4: Temporary effect of a rise in τ

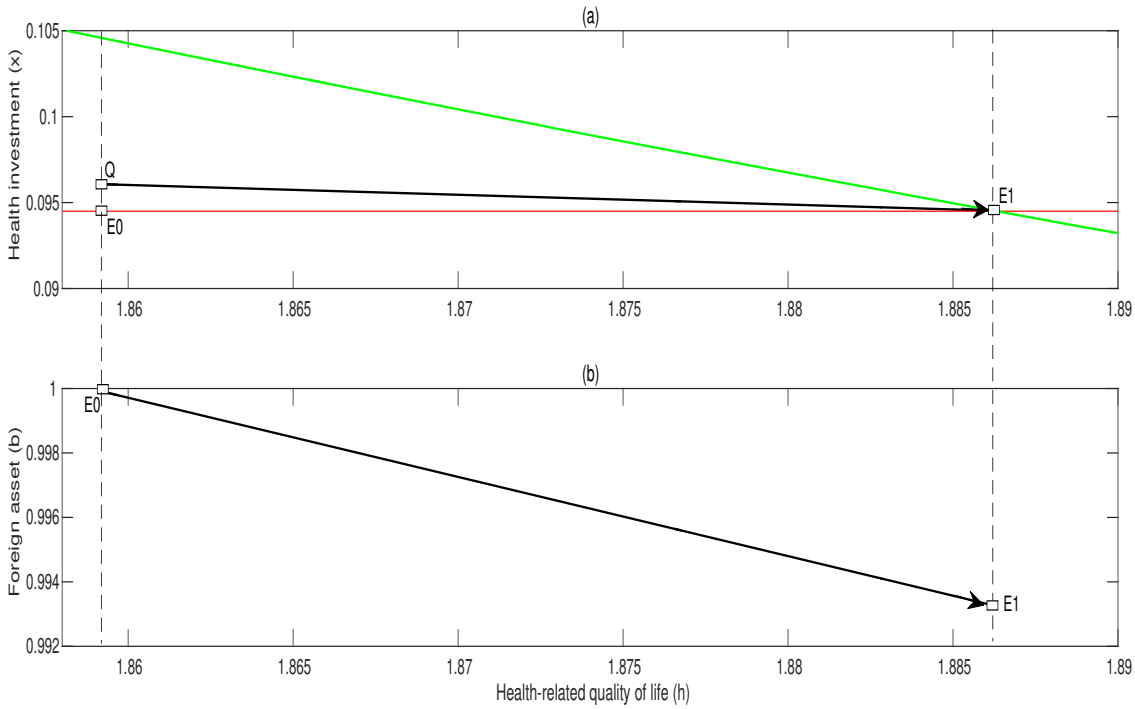

Figure 5: Permanent effect of a rise in τ