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Jan G. Jørgensen*    Philipp J.H. Schröder†

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Abstract

Recent literature on the workhorse model of intra-industry trade has explored heterogeneous cost structures at the firm level. These approaches have proven to add realism and predictive power. This paper presents a new and simple heterogeneous-firms specification. We develop a symmetric two-country intra-industry trade model where firms are of two different marginal costs types and where fixed export costs are heterogeneous across firms. This model traces many of the stylized facts of international trade. However, we find that with heterogeneous fixed export costs there exists a positive bilateral tariff that maximizes national and world welfare.

JEL: F12, F13, F15

Key Words: Intra-industry trade, trade liberalization, monopolistic competition, heterogeneous firms, welfare, protectionism.

*Department of Business and Economics, University of Southern Denmark, Denmark
†Department of Economics, Aarhus School of Business, University of Aarhus, Denmark.

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Corresponding author: Philipp J.H. Schröder, Aarhus School of Business, University of Aarhus, Fuglesangs Allé 4, DK-8210 Aarhus V, Denmark, Tel.: + 45 8948 6392, Fax: + 45 8948 6197, E-mail: psc@asb.dk.

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1 Introduction

Recently, firm-level heterogeneity has been introduced to intra-industry trade models, e.g. by Schmitt and Yu (2001), Montagna (2001), Jean (2002), Melitz (2003), Helpman, Melitz and Yeaple (2004) or Yeaple (2005). These specifications, where firms are heterogeneous with respect to their cost structures, have provided important new insights, frequently reconciling theory with the stylized facts of international trade, see Bernard, Jensen and Schott (2006), Greenaway and Kneller (2007), Helpman, Melitz and Rubinstein (2007). For example Schmitt and Yu (2001) resolve the puzzle of scale economies and the volume of intra-industry trade by introducing firm-level heterogeneous fixed exporting costs. Montagna (2001) examines trade between countries with efficiency asymmetries when firms are heterogeneous with respect to marginal costs. Melitz (2003) features firm-level heterogeneous marginal costs and analyzes intra-industry reallocations, showing that additional gains from trade stem from the induced productivity improvements.

However, thus far the literature has not fully examined the implications of these new – and more realistic – assumptions for the welfare effects of trade policies such as tariffs. Melitz (2003), Falvey, Greenaway and Yu (2004) and Baldwin and Forslid (2006), examine the welfare effects of reducing iceberg and fixed export costs in a Melitz-type (2003) setting with firm-level heterogeneous marginal costs. Bernard, Redding and Schott (2007) model iceberg cost reductions for heterogeneous firms in a neoclassical trade setting. The present paper contributes to this literature. In particular, we examine trade policy by introducing fully redistributed bilateral ad valorem tariffs – instead of the customary iceberg costs – into a simple symmetric two-county Krugman-type (1980) intra-industry trade model with firm-level heterogeneous fixed costs of exporting as in Schmitt and Yu (2001). We find that, even though free trade welfare exceeds autarky welfare, a positive bilateral tariff exists that maximizes national and world welfare. Thus small bilateral tariffs increase welfare. The underlying mechanism is that small tariffs force fairly inefficient (high fixed export costs) producers to cease their trading activity. This saving is paired with a volume reduction occurring for all remaining traded varieties, due to the tariff driven price increase. In sum, these effects compensate consumers for the loss in imported varieties, via domestic entry and larger consumption volumes of home varieties. This effect is at work, even though we employ assumptions that promote free trade as the welfare optimum. For example, the firm-specific fixed costs of exporting, i.e. creating variety via imports, are always lower than the cost of creating a new domestic variety.

Modelling tariffs explicitly, including the re-distribution of revenues, our
paper follows an empirically based criticism of the iceberg costs approach, e.g. Hummels and Skiba (2004). In terms of welfare results, iceberg costs specifications may raise additional issues. If one captures trade liberalization as improvements in the transport technology (reductions in iceberg costs), such technological improvements should have a positive impact on welfare irrespective of their trade implications.\footnote{Put differently, if the world loses only 5 instead of 10 containers for every 1000 containers that are shipped, surely welfare must increase.} In contrast, modelling tariffs explicitly and with full redistribution of revenues, helps to disentangle actual tariff liberalization effects from transport technology effects, see e.g. Schröder (2004). Moreover, with marginal cost heterogeneity iceberg cost specifications (e.g. Melitz, 2003) imply that the more productive firms not only are more productive in terms of producing output, but also in terms of shipping their output; thus creating a trade bias for these firms.

A second contribution of the present paper is to extend the literature by combining a simple model of firm-level heterogeneous fixed export costs (i.e. Schmitt and Yu, 2001; Jørgensen and Schröder, 2006) with an element of marginal cost heterogeneity, where firms can be of two types, and most importantly by introducing an entry mechanism in the manner of Hopenhayn (1992) and Melitz (2003). In particular, in our model firms make their entry decisions subject to sunk costs and based on expected profits, knowing only the distribution of firm heterogeneity in the population but not their own realization. Arguably, the entry mechanism employed in previous heterogeneous fixed export costs models, i.e. Schmitt and Yu (2001) and Jørgensen and Schröder (2006), is problematic, because here firms’ entry decisions are based on reaching breakeven on their home market operation alone. Accordingly, the examined situation does not depict an equilibrium, since export profits exist that fail to trigger industry entry.\footnote{Despite these shortcomings we find in Jørgensen and Schröder (2006), inter alia, that the possibility of a welfare increase from bilateral tariffs exists. However, only when modelled under an Melitz (2003) entry mechanism, as in the present paper, it is possible to establish that such effect is not simply the result of the un-realized profit opportunities.} Despite being somewhat simpler than the well-known Melitz (2003) model with marginal costs heterogeneity, the fixed costs heterogeneity model of the present paper captures the central stylized facts of international trade well. For example, within the model we have partitioning and the export-active firms turn out to be larger and more productive (lower average costs) than their non-exporting counterparts. Also, by allowing for two levels of marginal costs the model can, in line with Melitz (2003), generate a class of firms that exit immediately after entry.

The role of fixed export costs – which are at the center of the present
paper – is generally emphasized in the firm-level heterogeneity literature, e.g. Melitz (2003), Greenaway and Kneller (2007). Fixed export costs or export market entry costs are associated with items such as administrative burdens, the adjustments of product designs to local tastes or regulations, information requirements or the costs of maintaining a distribution network abroad. The literature on the internationalization of firms provides ample evidence of such fixed costs of exporting and how such costs vary substantially across firms, see e.g. Leonidou (1995, 2004), Leonidou and Katsikeas (1996), Roberts and Tybout (1997), Morgan and Katsikeas (1998), Das, Roberts and Tybout (2001) and Bugamelli and Infante (2003). For example, Das, Roberts and Tybout (2001) estimate an empirical model with marginal and fixed export costs heterogeneity based on panel data for Colombian chemical producers and conclude that “... sunk [export market entry] costs vary considerably across plants” (Das, Roberts and Tybout, 2001, p. 23). Bugamelli and Infante (2003) provide econometric evidence based on a large panel dataset of Italian manufacturing firms and highlight the costs of collecting information about foreign markets and consumer tastes as a main entry barrier. They find substantial differences among firms’ abilities to collect and operationalize such information.

Finally, the present paper’s emphasis on fixed costs heterogeneity rather than marginal costs heterogeneity can also be motivated from recent trends in the organisational theory literature. In the past decades industry structures for numerous sectors have moved away from traditional integrated firms – covering the full range of activities in the value chain – to production networks, i.e. Fine (1998), Gereffi (1999), Sturgeon (2000, 2002). In such production networks an original brand name manufacturer (OBM), also referred to as “manufacturer without a factory”, specializes exclusively on design and marketing activities (which are fixed cost activities) while turn-key suppliers, also referred to as full-package suppliers, cover the actual manufacturing process (marginal cost activities). Empirical studies identify such patterns for example in the apparel, consumer electronics, or footwear industry; e.g. Frenkel (2001), Hess and Coe (2006). Furthermore, turn-key suppliers rely essentially on identical production methods and individual turn-key suppliers may service several competing OBMs, e.g. Tokatli and Kizilgun (2004). For such production networks it then makes sense to think about OBM firms to differ predominantly in their fixed costs, say the power of their brand name, their ability to penetrate new foreign markets, etc., rather than their marginal costs. In line with this reasoning marginal cost heterogeneity modelling as in Melitz (2003) would then reflect sectors dominated by traditional integrated firms, while heterogeneous fixed cost models – as presented in the present paper – would relate to situations prevailing in industries featuring
production networks.

The next section presents the model. In Section 3, we derive the welfare effect of imposing bilateral ad valorem tariffs. In section 4 we discuss the results and relate our findings to existing literature. Section 5 concludes.

2 The Model

The starting point is a standard Krugman-type (1980) model of intra-industry trade, yet with the feature of firm-level heterogeneous fixed costs of exporting as in Schmitt and Yu (2001). Consumers in two identical countries, home and foreign, love variety and have identical preferences, in which all consumption goods, $c$, enter symmetrically. Utility is given by

$$U = \sum v(c_i)$$

$$= \sum c_i^\theta, \quad \theta \in (0, 1).$$

More specifically we can write (1) as

$$U = \sum_{i_d=1}^{N_d} c_{d,i_d}^\theta + \sum_{i_t=1}^{N_t} c_{t,i_t}^\theta + \sum_{i_f=1}^{N_f} c_{f,i_f}^\theta,$$

where $c_{d,i_d}$ is consumption of variant $i_d$ of non-exported domestic products, $c_{t,i_t}$ is consumption of variant $i_t$ of the exported domestic products and $c_{f,i_f}$ is consumption of variant $i_f$ of imported products. The number of variants actually produced ($n_d$, $n_t$, and $n_f$) is assumed to be large, although smaller than $N_d$, $N_t$ and $N_f$. Furthermore, denoting foreign variables by $^*$, the symmetry of the setup implies $n_t = n_f^* = n_f = n_t^*$ and that trade is balanced.

Firms

Firms can produce their specific variant for the home market alone or for both the home and foreign market. The decision to start production and subsequently start the export activity is firm-endogenous, where some firms may decide not to start production at all and where not all producing firms will export. We avoid several of the complexities of modelling the probability of firm ‘death’ as presented in Melitz (2003), and instead apply an alternative version, simply envisaging two separate rounds. In particular, production and sales for the home market (and the fixed production costs $\alpha$) are sunk in the sense that they are assumed to occur prior to an exporting round, in which the individual fixed export costs are revealed and export production
– if the firm chooses so – and sales take place. Afterwards all firms die with probability 1.3

Prior to entry firms are uncertain about their marginal cost type and their firm-specific fixed export costs, yet homogenous with respect to fixed costs of production and tariffs. Upon entry the marginal cost type \((\beta_h, \beta_l)\) is revealed while the firm specific export costs \((a_i)\) is disclosed after one round of production for the domestic market has occurred; i.e. this resembles a situation where firms learn something when servicing their home market that they can put to use when accessing foreign markets. The fixed cost \(\alpha\), marginal costs \(\beta_h, \beta_l\), and fixed export costs \(a_i\) are all expressed in terms of labor, \(L\), which is the only factor of production and is remunerated at the economy-wide wage rate \(w\), normalized to 1. Upon entry, but prior to production firms are revealed their marginal cost type, where \(\beta_l = \phi \beta_h\) and \(\phi > 1\). With probability \(\gamma\) firms are of the high productivity type. For a large enough \(\phi\) low productivity firms may choose not to start producing. The additional firm-specific fixed export cost, \(a_i\), is heterogeneous across firms and, for simplicity, assumed to be uniformly distributed on the interval \([0, \alpha]\), with \(F(.)\) denoting the distribution function which is public knowledge. The fixed costs of exporting represent, for example, the cost of building up a distribution network abroad, the cost of collecting information or additional costs of adapting a product to foreign specifications or tastes.

To enter, firms face initial sunk fixed entry costs; loosely speaking these are the costs for participating in the lottery for firm specific marginal cost and fixed export costs.4 Such sunk entry costs capture costs that are distinct from the fixed cost of production, \(\alpha\). We operate here with a distinction into monetary and labour costs of these sunk costs.5 The costs \(e_k > 0\) represent e.g. some entry fee or the threshold return (premium) demanded by entrepreneurs in order to cover the risk they take when establishing a firm, and are thus expressed in monetary units, and redistributed. The costs \(e_w > 0\) represent actual labour costs and are expressed in terms of labour.6 To simplify notation we let \(e_w = \delta f\) and \(e_k = (1 - \delta)f\) such that from the entrepreneurs point of view the sunk cost of entry is simply \(f\).

Trade is costly. Apart from firm specific fixed export costs \((a_i)\), both

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3We are grateful to Marc Melitz for pointing out this short-cut.
4The participation constraint sets an upper limit on the entry costs so that entrepreneurs accept to enter the lottery.
5In a general equilibrium setting such a distinction matters. Monetary costs must be redistributed – as are the tariffs in our model – while labour lottery entry costs do ‘vanish’, more in the fashion of an iceberg cost. We hold that entrepreneurial activity when launching a firm may contain both types of costs.
6In Melitz (2003) all entry costs are of the \(e_w\) type.
countries charge the same ad valorem tariff \( \tau \in (0, 1) \) on imports, i.e., a bilateral tariff. The presence of fixed export costs and the tariff creates an asymmetry between trading and non-trading firms, and hence, the profit functions of a pure domestic firm only servicing the home market, and an exporting home firm servicing both markets, are

\[ \pi_d = p_d x_d - (\alpha + \beta_d x_d)w \]

\[ \pi_z = p_t x_t + (1 - \tau)p_z x_z - (\alpha + a_i + \beta_j(x_t + x_z))w , \]

where \( j = h, l \), \( x_d \) is the production of a pure domestic firm, and \( x_t \) and \( x_z \) are the output of an exporting firm to the home and the foreign market respectively. Finally, various market-clearing relations complete the model: goods market clearing \( Lc_{d,j,i} = x_{d,j,i} \), \( Lc_{t,j,i} = x_{t,j,i} \), and \( L^*_c f_{j,i} = x_{t,j,i} \), where the foreign index \( i_f \) and the home index \( i_t \) denote one and the same variant; income expenditure clearing \( Lw + R = p_d x_d n_d + p_t x_t n_t + p_f x_f n_f + p_d x_d n_d + p_t x_t n_t + p_f x_f n_f \), where \( R \) denotes the net profits of all home firms (which sum to zero in the free entry-exit equilibrium), the sum of entry costs \( e_k \) (including firms failing to start production), and all tariff revenues assumed to be lump-sum redistributed to consumers; and similar relations for the foreign country. In equilibrium some firms make profits and others make losses whereby free entry and exit ensures zero total profit.

**Prices and quantities**

Maximization of (2) leads to the familiar inverse demand functions of the form \( p_d = \frac{\theta c_{d,i}^{\theta - 1}}{\lambda} \) for a non-traded home good \( i_d \), and similar for traded products and different marginal cost types. Then, profit maximization of (3) with respect to \( x_d \) and maximization of (4) with respect to \( x_t \) and \( x_z \) results in the prices

\[ p_{dh} = p_{th} = \frac{\beta_h}{\theta} \]

\[ p_{dt} = p_{ti} = \frac{\phi \beta_h}{\theta} \]

\[ p_{zh} = \frac{p_{dh}}{1 - \tau} \]

\[ p_{zi} = \frac{p_{di}}{1 - \tau} \]

for products of low and high marginal cost firms on the home and the foreign market respectively. Since \( p_{t,i} = p_{d,i} \), consumers do not distinguish between
non-traded home products and traded home products within a marginal cost
category; and hence, sales quantities of trading firms on their home market
must be identical to that of non-trading firms, i.e. \( x_{dj} = x_{ij} \). Yet, high
marginal cost goods are more expensive than low marginal cost goods and
exported goods are more expensive than domestically produced goods within
each marginal costs category. By symmetry \( p_{zj} = p_{zj}^* \), i.e. the price that
a home firm charges abroad is the same as the price charged by foreign
exporters on our home market. In equilibrium, maximization of utility (2)
requires that the ratio of the marginal utility of an extra consumption unit
equals the price ratio, e.g. \( \frac{\theta c}{\theta_t - 1} \frac{d}{\theta c} = \frac{p_d}{p_t} = 1 - \tau \). Utilizing the goods market
clearing conditions, this implies within a given marginal cost category
\[
x_{zj} = x_{zj}^* = x_{dj}(1 - \tau) \frac{1}{1 - \tau} .
\] (9)
and across marginal cost categories e.g.
\[
x_{dl} = \left( \frac{1}{\phi} \right) \frac{1}{1 - \tau} x_{dh} .
\] (10)
Thus within the same marginal cost category exporting firms charge the same
price on their home market and have the same sales volume as non-trading
firms, but charge higher prices with lower sales of their variety on the foreign
market. By the same token, domestic consumers pay more and consume less
of imported product varieties compared to domestically produced varieties.
Similarly across marginal cost categories low marginal cost firms charge lower
prices and have larger sales volumes.

With these relations in place, production scale can be determined as
driven by free entry/exit. Firms know the distribution of \( a_i \)'s, the values of
\( \gamma, \alpha, \beta_h, \beta_l \) and the relations given in (9) and (10). Furthermore, there must
exist some cut-off levels, \( \bar{a}_h \) and \( \bar{a}_l \), of the firm specific fixed export costs
denoting the firm that is exactly indifferent between engaging in exports and
being a non-trading firm. Firms determine their entry subject to expected
profits and sunk cost, accordingly in equilibrium some firms will make profits
(those that do export) and some losses (those that only service the home
market).\(^7\) Entry of firms occurs until expected profits equal entry cost \( e_w +
\(^7\)Here, we departs significantly from Schmitt and Yu (2001) and Jørgensen and Schröder
(2006). In these previous models of fixed cost heterogeneity firms determine entry subject
to reaching breakeven on their home market operation. This is problematic, because
positive profits from exporting exists in equilibrium that fail to trigger entry. The novelty
of the present fixed cost heterogeneity model is that we overcome this problem, by following
Melitz (2003).
Marginal cost heterogeneity can here, as in Melitz (2003), generate the feature that not all firms that enter the industry will actually launch production. In particular, we set $\phi$ larger than $\bar{\phi}$ such that high marginal cost firms will exit immediately. Given this assumption and inserting (3) and (4) in (11) while realizing that the expected fixed costs of exporting must be $\bar{a}_h^2$, the equation reads:

$$\gamma [\bar{a}_h \alpha (p_{th} x_{th} + (1 - \tau) p_{zh} x_{zh} - \left( \alpha + \frac{\bar{a}_h}{2} + \beta_h (x_{th} + x_{zh}) \right)) + \left( 1 - \frac{\bar{a}_h}{\alpha} \right) (p_{dh} x_{dh} - (\alpha + \beta_h x_{dh}))] = f .$$

(12)

Inserting the quantity relations (9) and (10) from above (12) can be solved for $x_{dh}$ to yield:

$$x_{dh} = \theta \frac{\bar{a}_h^2 + 2 \frac{\tau}{\gamma} \alpha + 2 \alpha^2}{2 (\alpha + \bar{a}_h (1 - \tau))} ,$$

(13)

which is also the home market production scale of exporting firms ($x_{th}$) and can be plugged into (9) to determine $x_{zh}$. Note that in autarky (where the tariff is the prohibitive $\tau = 1$ and accordingly $\bar{a}_h = 0$), the production scale reaches the textbook case, namely $x_{dh} \mid_{\text{autarky}} = \frac{\theta}{(1 - \theta) \bar{a}_h} (\alpha + \frac{\tau}{\gamma})$. Furthermore, we are now able to investigate the participation constraint of the model. An upper limit of the participation constraint is given by a hypothetical situation where all firms break even on the home market and all firms are export active. This results in expected profits, when participating in the lottery, of $\alpha \gamma$ and thus $f < \frac{\alpha \gamma}{2}$. The actual participation constraint, however, will be lower as additional entry triggered by expected positive profits competes domestic production scale below breakeven.\(^8\)

**The indifferent firm**

With the prices and quantities derived above, it is straightforward to identify the firm which is indifferent as to becoming an exporting firm or becoming a

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\(^8\)Given the $x_{dh}$ which we derive below, one can calculate the precise requirement $\bar{\phi} = \left( \frac{(2/\gamma + \gamma (\alpha + \alpha^2)/(\alpha + \alpha(1 - \tau)/\gamma))^{1/2}}{\gamma (\alpha + \alpha^2)/(\alpha + \alpha(1 - \tau)/\gamma)} \right)^{1/2} \), which is greater than 1.

\(^9\)The actual participation constraint is cumbersome since it depends on various other parameters, however, since all our subsequent results apply for all $f \in [0, \frac{\alpha \gamma}{2}]$, they must also apply for the parameter range defined by the actual participation constraint.
pure domestic firm. This firm is characterized by a fixed cost of exporting $\bar{a}_h$ such that \( \pi_{z_h, \bar{a}_h} = \pi_{d_h} \) must hold resulting in the condition \((1-\tau)p_{z_h} x_{z_h} - (\bar{a}_h + \beta_h x_{z_h}) = 0, \) i.e. the indifferent firm makes zero profits from the exporting activity. After setting in \( p_{z_h}, \) and \( x_{z_h} \) from above, one can solve:

\[
\bar{a}_h = \left( \sqrt{\alpha^2 + 2\alpha \frac{f}{\gamma} + \alpha(1-\tau)} \frac{(1-\tau)^{\frac{1}{1-\theta}}}{(1-\tau)^{\frac{1}{\theta}}} \right) (1-\tau). \tag{14}
\]

All firms \( i \) such that \( a_i \in [0, \bar{a}_h] \) make non-negative profits from exporting, while all firms \( i \) such that \( a_i \in (\bar{a}_h, \alpha] \) are non-trading firms. Notice that by (14) we have \( \bar{a}_h > 0 \) and that in the free trade situation \( (\tau = 0) \) we have \( \bar{a}_h|_{\tau=0} < \alpha. \) The reason for \( \bar{a}_h|_{\tau=0} < \alpha \) is as follows. With zero tariffs, the sales scale on the home and the foreign market respectively are identical \((x_{d_h} = x_{z_h}).\) Since exporting promises expected profits, this scale is competed so small (via entry) that the home sales will not breakeven, accordingly the exporting activity of a firm with fixed export costs \( a_i = \alpha \) must also result in negative profits, hence also with free trade \( (\tau = 0) \) some firms are non-trading firms choosing to minimize their losses by refraining from exporting. Furthermore, \( \bar{a}_h \) decreases in the tariff rate (see appendix A.1), implying that the least efficient (high \( a_i \)) firms will cease their trading activity in response to a tariff increase. Finally, given the above equations, it is easy to verify that export-active firms are larger and more productive than their non-exporting counter parts.\(^{10}\)

**The number of firms**

The total number of firms participating in the industry is denoted by \( m. \) However, given that \( \phi > \bar{\phi} \) only \( \gamma m \) will actually start production while \( (1-\gamma)m \) exit immediately. A fraction \( F(\bar{a}_h) \) of all producing firms will furthermore be export active, such that \( n_{t_h} = \frac{\alpha}{\alpha} \gamma m \) while \( n_{d_h} = (1 - \frac{\alpha}{\alpha}) \gamma m \) are pure domestic firms. The number of firms is derived from either the income expenditure clearing condition or from labour market clearing. In the former case \( L + R = p_{d_h}x_{d_h}n_{d_h} + p_{t_h}x_{t_h}n_{t_h} + p_{f_h}x_{f_h}n_{f_h} \) where \( R = \tau p_{z_h}x_{z_h}n_{t_h} + e_k m; \) i.e. tariff revenues and the part of the initial entry cost expressed in monetary units. In the latter case \( n_{d_h}(\alpha + \beta_h x_{d_h}) + n_{t_h}(\alpha + \frac{\alpha}{2} + \beta_h x_{d_h} +

\[\text{In particular, comparing the average costs of non-trading firms, } AC_d = \frac{\alpha + \frac{f}{\gamma}}{x_{d_h}} + \beta_h, \text{ to that of the export-indifferent firm, } AC_z|_{\bar{a}_h} = \frac{\alpha + \alpha_h + \frac{f}{x_{d_h}x_{z_h}} + \beta_h, \text{ one can simplify } AC_z|_{\bar{a}_h} < AC_d \text{ to } (\alpha + \frac{f}{\gamma})^2(1-\tau)^{\frac{1}{1-\theta}} > 0.\]
\[ \beta_h x_{zh} + e_w m = L. \] Solving either conditions for \( m \) yields:

\[ m = \frac{L(1 - \theta)}{\alpha \gamma + f(\delta + \theta(1 - \delta)) + \frac{a^2 \gamma}{2\alpha}}. \] (15)

Because of trade, consumers also have access to foreign varieties, in particular due to symmetry \( n_{zh} = n_{zh}^* = n_{fh} \) and accordingly the number of varieties available on the home market are given by \( \hat{n} = n_{zh} + 2n_{th}. \)

### 3 Welfare Results

Consumer utility is our measure of welfare. Given goods market clearing and (2), we can write

\[ U = n_{zh}(\frac{x_{zh}}{L})^\theta + n_{th}(\frac{x_{th}}{L})^\theta + n_{fh}(\frac{x_{fh}}{L})^\theta, \]

and setting in values from above and simplifying gives:

\[ U = \frac{L(1 - \theta)}{\alpha} \left( \sqrt{g} (1 - \tau) \frac{1}{\tau^\frac{1}{\theta}} + \alpha (1 - \tau) \frac{1}{\tau^\frac{1}{\theta}} - \alpha (1 - \tau) \frac{1}{\tau^\frac{1}{\theta}} \right) k^\theta, \] (16)

where \( g = \alpha \left( \alpha + 2 \left( \frac{f}{\gamma} + \alpha \right) (1 - \tau) \frac{1}{\pi^2} \right), \)

and \( k = \frac{\theta(\sqrt{g} - \alpha)}{L\beta_h (1 - \theta)(1 - \tau) \frac{1}{\pi^2}}. \)

The following results can be stated.

**Proposition 1.** Consumer utility under free trade exceeds that under autarky, yet, there exists a strictly positive bilateral tariff, \( \hat{\tau} \), that maximizes total national (and world) consumer utility. In particular, \( U_{\tau=0} > U_{\text{autarky}} \) and \( \frac{\partial U}{\partial \tau}_{\tau=0} > 0. \)

**For proof, see appendix A.2.** To illustrate proposition 1, consider figure 1 which plots utility (16) normalized by autarky utility \( U_{\text{autarky}} \) as a function of \( \tau \) for various values of \( \theta \), i.e. thus representing the welfare gains from trade.\(^{11}\) To the right, for \( \tau \) close to 1, we are in the autarky situation and accordingly \( U/(U_{\text{autarky}}) = 1. \) To the left, for \( \tau = 0 \), we are in the free trade situation, and welfare in both countries is clearly above the autarky level. However, imposing a small bilateral tariff increases welfare until we reach the welfare maximizing bilateral tariff, \( \hat{\tau} \), beyond which welfare starts to decrease towards the autarky level.

\(^{11}\)The expression for \( U_{\text{autarky}} \) is given in appendix A.2 equation (A.3). The parameter values for the plot are \( \alpha = 3, \beta_h = 0.5, f = 1, L = 1000, \gamma = 0.9, \delta = 0.3 \)
4 Discussion

What proposition 1 implies is in fact that there is too much trade in the free trade situation. National and world welfare increases when imposing small bilateral tariffs. The welfare maximizing bilateral ad valorem tariff is strictly positive, less than 1 and increases in the degree of product differentiation, \( \theta \), (love of variety). Accordingly, trade liberalization, in particular the bilateral reduction of tariffs smaller than \( \hat{\tau} \), will be welfare-reducing.

It is important to note that the present model employs several assumptions that promote free trade as a welfare optimum, i.e. we have stacked the deck against our finding: there are no wasteful transport costs and the firm-specific fixed costs of exporting are always less than the cost of creating a new variety. Still, we find that in this model there is too much trade in the free trade equilibrium. More resources are used on the exporting/importing activity than is welfare-optimal, measured as total consumer utility.

To provide some intuition for the above result, it is useful to break down the contributing factors. First, examine the number of firms given in (15) and in particular the number of varieties available on the home market given by \( \hat{n} = n + n_f \). It turns out that with the imposition of a small bilateral tariff, the exit of trading firms and therewith the loss of \( n_t \) and \( n_f \) is compensated by the entry of additional pure domestic firms \( n_d \), in fact slightly increasing \( \hat{n} \).
at first before it falls for larger tariffs. Accordingly, within the consumption basket foreign products have been replaced with home products. The second contribution to a utility increase stems from the changes in the output volumes, \(x_d\) and \(x_z\) that can be consumed. As can be seen from (13) and (9), a tariff increases the output volume of domestic varieties available to domestic consumers and reduces the output volume directed at the foreign market (and hence, the consumption volume of each imported variety). Thus even if the number of available varieties was just constant (and not increasing), then the pure shift from foreign varieties to home varieties paired with an increase in the amounts consumed of each home variety would constitute a utility increase.

To see the logic of these changes in the number of available varieties and the consumption volumes, consider the following reasoning. A small bilateral tariff reduces the number of imported varieties and – via the imposed price increase of foreign products – the import volume of all remaining varieties. However, overall a small tariff still increases welfare because the least efficient exporters (high \(a_i\)) are the first to cease their trading activity, exactly as a result of fixed export costs heterogeneity. Paired with the additional resources saved by reducing the trading activity of all remaining exporting firms, enough resources are freed for the production of more home varieties in larger quantities. That is, the tariff reduces the volume of each remaining importer/exporter but converts it into additional domestic entry and consumption. However, beyond the welfare maximizing bilateral tariff, \(\hat{\tau}\), an additional increase in the tariff further cuts imported volumes, and more importantly, it forces fairly efficient exporters out of the trading activity. Thus, additional variants produced relatively cheaply (i.e. by foreign exporters who have fairly low fixed export costs, \(a_i\)) are replaced with variants produced relatively expensively (i.e. by new home producers incurring the fixed production costs, \(\alpha\)).

In line with this reasoning, it turns out that the total fixed costs per available variety that occur to a country \(((\gamma \alpha + n_t \bar{a})/\bar{n})\) as a function of \(\tau\) are U-shaped. Thus a small bilateral tariff, by forcing expensive (high \(a_i\)) exporters/importers out reduces the amount of fixed costs that society has to tie up in order to generate variety.

The finding of welfare-reducing tariff liberalization contradicts much of the existing literature, see e.g. Markusen and Venables (1988), Fukushima and Kim (1989), Lockwood and Wong (2000). Also in intra-industry trade

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12Formal proof of \(\frac{\partial n_i}{\partial \tau} \big|_{\tau=0} > 0\) is given in appendix A.3. Appendix A.4 shows a plot of the number of firms and \(\bar{n}\).

13Appendix A.5 shows a plot of the output volumes \(x_d\) and \(x_z\).
models, bilateral tariffs are usually welfare-reducing, e.g. Gros (1987), Jørgensen and Schröder (2005). The main difference between the above models and the present model is that the earlier work assumes firms to be homogeneous in their cost structure. However, Melitz (2003), Falvey, Greenaway and Yu (2004) and Baldwin and Forslid (2004), all using a Melitz-type (2003) framework with firm-level heterogeneous marginal costs, examine, inter alia, iceberg trade cost reductions, which are often interpreted to represent trade liberalization, and find, in line with earlier literature, an overall welfare gain. Furthermore, Melitz (2003) and Baldwin and Forslid (2004) note the possibility for an anti-variety effect. In contrast to the anti-variety effect in the present model, this situation only emerges once the fixed costs of exporting are larger than the fixed costs of pure domestic production, and thus the export activity of a firm ties up more resources than would be required for an additional domestic variety. This case is explicitly ruled out in the present model.

The possibility of welfare-reducing trade liberalization is, however, found in Montagna (2001), in a framework where firms have heterogeneous marginal costs and when comparing the no-trade to free-trade equilibrium, and in Jørgensen and Schröder (2006) for fixed cost heterogeneity. However, both these papers feature un-used profit opportunities, and thus unrealized entry in equilibrium, i.e. do not employ the Hopenhayn (1992)-type entry mechanism introduced in the present paper. The welfare-reducing effect found by Montagna (2001) is different from the present effect, since the reduction in welfare in Montagna (2001) occurs for the more efficient of two asymmetric countries when moving from no-trade to free-trade and is contingent on parameter constellations where trade allows relatively inefficient firms to enter and when consumers’ taste for variety is low. Furthermore, Jørgensen and Schröder (2006) assume identical marginal costs across firms and full redistribution of sunk entry costs, while the present paper provides analytical results even though both these assumptions are relaxed. Earlier, Schmitt (1990) has established welfare-reducing effects of small reciprocal tariff reductions in a characteristics space model in the tradition of Hotelling. Moreover, in his model the move from no-trade to free-trade is welfare-improving. Both results are in line with our present findings. However, apart from being placed in a different framework, the underlying mechanism is very different. In Schmitt (1990) the zero tariff situation features excessive product diversity, a distortion that is counter balanced by the small tariffs.

On the other hand, small unilateral tariffs may increase welfare (Gros, 1987), and unilateral tariffs can induce a home market effect in the presence of transportation costs (Helpman and Krugman, 1989). Furthermore, similar situations occur in Brander-Spencer-type settings.
Tariffs versus icebergs

The above finding of welfare increasing small bilateral tariffs, has been derived under an explicit model of tariffs including the re-distribution of tariff revenues. In this respect our approach departs from the customary iceberg costs specifications, but is more suited to examine actual tariff liberalization effects instead of effects from improved transport technologies (reduced iceberg costs). Ceteris paribus improved transport technology will be welfare improving irrespective of the impact on trade patterns. The question than remains is, if the established welfare effect of the present paper is solely driven by this modelling difference of redistributed revenues. The answer is no. A simple way to introduce an iceberg type setting is to let tariff revenues be wasted. In particular, we assume that the state uses all tariff earnings and entry fees $e_m$ to purchase goods following the consumption pattern of consumers, but subsequently throws out these goods. Accordingly, the $m, n_t, n_d, x_d$ and $x_z$ derived above still apply, and consumers – in line with iceberg cost settings – still have access to the full range of variety. The only revision in our welfare expression then becomes that consumers only consume a share (proportional to the share of wages in total earnings) of each good. In particular the consumers’ share, scaling down the $c_d, c_t$ and $c_f$ in (2), is $I = \frac{L}{L + rP_t n_t x_z + (1-\delta) f m}$. Estimating the revised utility expression $U_I$, and finding the derivative with respect to $\tau$ at $\tau = 0$ in the limit of $\theta \to 0$ gives

$$\frac{\partial U_I}{\partial \tau} |_{\tau=0, \theta \to 0} = \frac{f L \gamma (\sqrt{\alpha \gamma (2f + 3\alpha \gamma)} - \alpha \gamma) (1 - \delta)}{\sqrt{\alpha \gamma (2f + 3\alpha \gamma)} (f + 3\alpha \gamma - \sqrt{\alpha \gamma (2f + 3\alpha \gamma) + f \delta})^2} > 0,$$

and in the limit of $\theta \to 1$ it becomes

$$\frac{\partial U_I}{\partial \tau} |_{\tau=0, \theta \to 1} = -\frac{2(f + 2\alpha \gamma - \sqrt{\alpha \gamma (2f + 3\alpha \gamma)})}{\beta h (2f + 3\alpha \gamma - \sqrt{\alpha \gamma (2f + 3\alpha \gamma)})} < 0.$$  \hspace{1cm} (18)

Thus, for sufficiently strong consumer preferences for variety, small bilateral “wasted” tariffs can increase welfare by expelling high fixed export cost firms from the market, i.e. replicating the result established in Proposition 1. However, with sufficiently low love of variety the “waste” aspect in iceberg tariffs dominates and welfare is maximized at $\tau = 0$.$^{15}$

$^{15}$A related observation occurs for the initial sunk entry costs. In the iceberg-like case, where the entry lottery fee would consists exclusively of labour costs (the here excluded case of $\delta = 1$), welfare would be maximized at $\tau = 0$. 

5 Conclusion

This paper examines the welfare impact of trade policy in an intra-industry trade model with firm-level heterogeneity. This new type of specifications, where firms are heterogeneous with respect to their cost structures, has generated important new insights, frequently reconciling theory with the stylized facts of international trade, e.g. Schmitt and Yu (2001), Montagna (2001), Melitz (2003), but has not yet been used to examine trade policies systematically.

Our model examines bilateral ad valorem tariffs in a symmetric two-country intra-industry trade model, with firm-level heterogeneous fixed costs of exporting. The focus on fixed cost heterogeneity – instead of marginal cost heterogeneity – is justified both on empirical grounds and by the simplicity of the resulting model. In particular, since all firms charge the same price, straightforward analytical solutions can be provided throughout the paper. Still, fixed costs heterogeneity has previously received less attention. Our model traces many of the stylized facts of international trade well. However, we find that in this model there is in fact too much trade in the free trade equilibrium. More resources are used on the exporting/importing activity than is welfare-optimal, measured as total consumer utility. There exists a strictly positive bilateral tariff that maximizes national and world welfare. Accordingly, trade liberalization, in particular the reciprocal reduction of small tariffs, is welfare-reducing. The underlying mechanism for our result is that even though small bilateral tariffs reduce the number of traded varieties, the number of available varieties in both countries is maintained and consumption volumes of home products increase. This mechanism is at work even though the fixed costs of creating a new domestic variety are always larger than the firm-specific fixed costs of exporting and even though there are no wasteful transport costs. In particular our finding is in contrast to Melitz (2003), and subsequent work, who examines iceberg trade cost reductions in a setting where firms are heterogeneous in their marginal costs. Yet, also when iceberg like costs are introduced into the present framework, small bilateral “wasted” tariffs can be welfare improving.

Going beyond the specific model presented here, the present paper hopes to have shown that the application of more realistic and powerful specifications for the workhorse model of intra-industry trade does not only answer many of the conflicts between stylized facts and theory, it also raises important new issues. Future research should address the welfare effects of trade policies for different forms of firm-level heterogeneity and for more types of trade barriers.
A Appendix

A.1 Derivative of $\frac{\partial \bar{a}_h}{\partial \tau}$

From (14) we have $\bar{a}_h = \left( \sqrt{\alpha^2 + 2\alpha(\frac{f}{\gamma} + \alpha)(1 - \tau)^{\frac{2}{1-\gamma}} - \alpha} \right) (1 - \tau)^{\frac{1}{1-\gamma}}$. It follows immediately that:

$$\frac{\partial \bar{a}_h}{\partial \tau} = -\frac{\alpha \left( \sqrt{\alpha(\alpha + 2(\frac{f}{\gamma} + \alpha)(1 - \tau)^{\frac{2}{1-\gamma}}) - \alpha} \right) (1 - \tau)^{\frac{2}{1-\gamma}}}{(1 - \theta) \sqrt{\alpha \left( \alpha + 2(\frac{f}{\gamma} + \alpha)(1 - \tau)^{\frac{2}{1-\gamma}} \right)}} < 0. \quad (A.1)$$

A.2 Proof of Proposition 1.

Proof. Total consumer utility under free trade exceeds that under autarky; in particular, $U|_{\tau=0} > U|_{\text{autarky}}$.

Evaluating (16) at $\tau = 0$ gives:

$$U|_{\tau=0} = \frac{L(1 - \theta)}{\alpha} \frac{\sqrt{\alpha(2\frac{f}{\gamma} + 3\alpha)}}{3\alpha - \sqrt{\alpha(2\frac{f}{\gamma} + 3\alpha) + \frac{f}{\gamma}(\delta + \theta(1 - \delta) + 1)}} \theta \quad (A.2)$$

Under autarky (where the tariff is prohibitive and accordingly $\bar{a}_h = 0$) we have $x_d|_{\text{autarky}} = \frac{\theta(\alpha + \frac{f}{\gamma})}{(1 - \theta)}$ and accordingly $m|_{\text{autarky}} = \frac{L(1 - \theta)}{\alpha\gamma + f(\delta + \theta(1 - \delta))}$. Setting in these values, total utility under autarky is given by:

$$U|_{\text{autarky}} = \frac{L(1 - \theta) \left( \frac{\frac{f}{\gamma} + \alpha}{L(1 - \theta)} \right)^{\theta}}{\alpha + \frac{f}{\gamma}(\delta + \theta(1 - \delta))} \quad (A.3)$$

Hence, we want to show that for all $\alpha > 0, 0 < f < \frac{\alpha^2}{2}, \theta \in [0, 1], \delta \in [0, 1]$:

$$\frac{\sqrt{\alpha(2\frac{f}{\gamma} + 3\alpha)} \left( \sqrt{\alpha(2\frac{f}{\gamma} + 3\alpha) - \alpha} \right)^{\theta}}{\alpha \left( 3\alpha - \sqrt{\alpha(2\frac{f}{\gamma} + 3\alpha) + \frac{f}{\gamma}(\delta + \theta(1 - \delta) + 1)} \right)} > \frac{\left( \frac{f}{\gamma} + \alpha \right)^{\theta}}{\alpha + \frac{f}{\gamma}(\delta + \theta(1 - \delta))} \quad (A.4)$$

Step 1: Define $s = \frac{f}{\gamma} \iff f = \gamma \alpha s$ and insert in (A.4), which leads to:

$$\frac{\sqrt{2s + 3} \left( \sqrt{2s + 3} - 1 \right)^{\theta}}{s + 3 - \sqrt{2s + 3} + s(\delta + \theta(1 - \delta))} > \frac{(s + 1)^{\theta}}{1 + s(\delta + \theta(1 - \delta))} \quad (A.5)$$
Step 2: Define \( v = \sqrt{2s+3} \) \( \iff \) \( s = \frac{v^2-3}{2} \). As \( 0 < f < \frac{\alpha}{2} \) and \( s = \frac{f}{\gamma_0} \) we have that \( \sqrt{3} < v < \sqrt{4} \). By substituting for \( s \) in (A.5) we get:

\[
\frac{v^2(\theta - 1)^\theta}{v^2 - 2v + (v^2 - 3)(\delta + \theta(1 - \delta))} > \frac{(\theta - 1)^\theta}{(v^2 - 3)(\delta + \theta(1 - \delta)) + 2} \tag{A.6}
\]

Note, that \((\theta - 1)^\theta = (v + 1)^\theta (\theta - 1)^\theta\). Since \( v > \sqrt{3} > 1 \), we have that \((v - 1)^\theta > 0 \) and (A.6) leads to:

\[
\frac{v^2\theta}{v^2 + 2v + (v^2 - 3)(\delta + \theta(1 - \delta))} > \frac{(v + 1)^\theta}{(v^2 - 3)(\delta + \theta(1 - \delta)) + 2} \tag{A.7}
\]

\[
\Downarrow
\]

\[
v(v^2 - 3)(\delta + \theta(1 - \delta)) + 2v > \left(\frac{v + 1}{2}\right)^\theta (v^2 - 3)(\delta + \theta(1 - \delta)) + \left(\frac{v + 1}{2}\right)^\theta (v^2 - 3)(1 - \delta) = \left(\frac{v + 1}{2}\right)^\theta \tag{A.8}
\]

Step 3: Define \( LHS(\theta) = v(v^2 - 3)(\delta + \theta(1 - \delta)) + 2v = v(2 + (v^2 - 3)\delta) + v(v^2 - 3)(1 - \delta)\theta \) and \( RHS(\theta) = \left(\frac{v + 1}{2}\right)^\theta (v^2 - 3)(\delta + \theta(1 - \delta)) + \left(\frac{v + 1}{2}\right)^\theta (v^2 - 3)(1 - \delta)\theta \). \( LHS(\theta) \) is linear in \( \theta \) with a slope and an intercept that depend on \( v \). Furthermore, \( LHS(0) = 2v + v\delta(v^2 - 3) \) and \( LHS(1) = v^3 - v \). \( RHS(\theta) \) looks linear in \( \theta \) but it is multiplied with the factor \( \left(\frac{v + 1}{2}\right)^\theta \). It is evident that \( RHS(0) = v^2 - 2v + 3 + (v^2 - 3)\delta \) and that \( RHS(1) = v^3 - v \). Hence, \( LHS(1) = RHS(1) \).

Now we want to show that \( LHS(0) > RHS(0) \) for all relevant \( v \). It is true since, \( LHS(0) > RHS(0) \) \( \iff \) \( 2v + v(v^2 - 3)\delta > (v^2 - 3)\delta + v^2 - 2v + 3 \) \( \iff \) \( (1 - v)(v - 3) + (v - 1)(v^2 - 3)\delta > 0 \) and \( \sqrt{3} < v < \sqrt{4} \).

Step 4: We want to show that \( RHS(\theta) \) is convex in \( \theta \) where \( \theta \in [0, 1] \) and \( \sqrt{3} < v < \sqrt{4} \). Differentiating \( RHS(\theta) \) with respect to \( \theta \) we get:

\[
RHS'(\theta) = (v^2 - 3)(1 - \delta) \left(\theta \left(\frac{v + 1}{2}\right)^\theta \ln \left(\frac{v + 1}{2}\right) + \left(\frac{v + 1}{2}\right)^\theta\right) + ((v^2 - 3)\delta + v^2 - 2v + 3) \left(\frac{v + 1}{2}\right)^\theta \ln \left(\frac{v + 1}{2}\right) \tag{A.9}
\]
From (A.9) it follows that:

\[ R_{H}^{R} \left( \psi \right) = \left( \nu^{2} - 3 \right) \left( 1 - \delta \right) \left( \ln \left( \frac{\nu + 1}{2} \right) \theta \left( \frac{\nu + 1}{2} \right)^{\theta} \ln \left( \frac{\nu + 1}{2} \right) + \left( \frac{\nu + 1}{2} \right)^{\theta} \right) \]

\[ + \left( \nu^{2} - 3 \right) \left( 1 - \delta \right) \left( \left( \frac{\nu + 1}{2} \right)^{\theta} \ln \left( \frac{\nu + 1}{2} \right) \right) \]

\[ + \left( \nu^{2} - 3 \right) \delta \left( \nu^{2} - 2\nu + 3 \right) \ln \left( \frac{\nu + 1}{2} \right) \left( \frac{\nu + 1}{2} \right)^{\theta} \ln \left( \frac{\nu + 1}{2} \right) \]

From (A.10) it follows that \( R_{H}^{R} \left( \psi \right) > 0 \) for all \( \sqrt{3} < \nu < \sqrt{4} \), \( \theta \in [0, 1] \). Hence, \( R_{H} \left( \psi \right) \) is convex, and therefor \( L_{H} \left( \psi \right) > R_{H} \left( \psi \right) \). We have now shown that \( U \mid \tau = 0 > U \mid \text{autarky} \). \( \square \)

**Proof.** There exists a strictly positive bilateral tariff that maximizes total national and world consumer utility; in particular \( \frac{\partial U}{\partial \tau} \mid \tau = 0 > 0 \). By differentiation (16) with respect to \( \tau \) and evaluating the expression in \( \tau = 0 \) we get:

\[ \frac{\partial U}{\partial \tau} \mid \tau = 0 = \frac{2fL\alpha \left( \frac{f}{\gamma} + 2\alpha - \sqrt{\alpha(2\frac{f}{\gamma} + 3\alpha)} \right) \left( 1 - \theta \right) \left( \frac{\sqrt{\alpha(2\frac{f}{\gamma} + 3\alpha)} - \alpha}{\gamma,\psi(1-\theta)} \right)^{\theta}}{\sqrt{\alpha(2\frac{f}{\gamma} + 3\alpha)} \left( \sqrt{\alpha(2\frac{f}{\gamma} + 3\alpha)} - \alpha \right) \left( \frac{f}{\gamma} \left( 1 + \delta + \theta(1 - \delta) \right) + 3\alpha - \sqrt{\alpha(2\frac{f}{\gamma} + 3\alpha)} \right)^{2}} \]

(A.11) is positive as:

\[ \frac{f}{\gamma} + 2\alpha > \sqrt{\alpha(2\frac{f}{\gamma} + 3\alpha)} \]

\[ \Downarrow \]

\[ f^{2} + \alpha^{2}\gamma^{2} + 2f\alpha\gamma > 0 \]

\( \square \)

**A.3 Proof of** \( \frac{\partial \tilde{n}}{\partial \tau} \mid \tau = 0 > 0 \). **Proof.** The number of available varieties increases for a small tariff. The number of varieties available on the home market is given by \( \tilde{n} = n + n_{f} \).
From (15) and using the fact that \( n_t = n_t^* = n_f = F(\bar{a})n \) it follows that

\[
\tilde{n} = \frac{L(1 - \theta)}{\alpha} \frac{\alpha - \alpha(1 - \tau)^{\frac{1}{1-\theta}} + \sqrt{\alpha(\alpha + 2(\frac{L}{\gamma} + \alpha)(1 - \tau)^{\frac{2}{1-\theta}})(1 - \tau)^{\frac{1}{1-\theta}}}}{\frac{L}{\gamma}(1 + \delta + \theta(1 - \delta)) + \alpha(2 + (1 - \tau)^{\frac{2}{1-\theta}}) - \sqrt{\alpha(\alpha + 2(\frac{L}{\gamma} + \alpha)(1 - \tau)^{\frac{2}{1-\theta}})(1 - \tau)^{\frac{2}{1-\theta}}}}
\]  

(A.12)

The derivative of \( \tilde{n} \) in (A.12) with respect to \( \tau \), and evaluated at the free trade situation, \( \tau = 0 \), gives:

\[
\frac{\partial \tilde{n}}{\partial \tau} \bigg|_{\tau=0} = \frac{L(1 - \theta)\frac{L}{\gamma} \left( \sqrt{\alpha(2\frac{L}{\gamma} + 3\alpha)} - \alpha \right)}{\sqrt{\alpha(2\frac{L}{\gamma} + 3\alpha) \left( \frac{L}{\gamma}(1 + \delta + \theta(1 - \delta)) + 3\alpha - \sqrt{\alpha(2\frac{L}{\gamma} + 3\alpha)} \right)^2}} > 0.
\]

(A.13)
A.4 The number of firms and available varieties

Figure A.1 plots the number of firms, $m$, the number of pure domestic producers, $n_{dh}$, and exporting producers, $n_{th}$, and the total number of available varieties, $\tilde{n}_h$, as a function of $\tau$. Other parameter values are $\alpha = 3$, $\beta_h = 0.5$, $\theta = 0.4$, $f = 1$, $L = 1000$, $\gamma = 0.9$, $\delta = 0.3$. 

![Figure A.1: Number of firms and available varieties](image)

Figure A.1: Number of firms and available varieties
A.5 Production scale

Figure A.2 plots the production scale, $x_{dh}$, that is sold by domestic non-exporting and exporting firms on the domestic market, and the production scale, $x_{zh}$, sold by foreign exporters on the domestic market (which is identical to the sales that domestic exporters have on the foreign market). Other parameter values are $\alpha = 3$, $\beta_h = 0.5$, $\theta = 0.4$, $f = 1$, $L = 1000$, $\gamma = 0.9$, $\delta = 0.3$.

Figure A.2: Output (production scale)
References


Melitz, Marc J. (2003), The Impact of Trade on Intra-industry Reallocations


