Licensing under general demand and cost functions

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14 February 2016

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September 23, 2016

Abstract

We consider a Cournot duopoly under general demand and cost functions, where an incumbent patentee has a cost reducing technology that it can license to its rival by using combinations of royalties and upfront fees (two-part tariffs). We show that for drastic technologies: (a) licensing occurs and both firms stay active if the cost function is superadditive and (b) licensing does not occur and the patentee monopolizes the market if the cost function is additive or subadditive. For non drastic technologies, licensing takes place provided the average efficiency gain from the cost reducing technology is higher than the marginal gain computed at the licensee's reservation output. Optimal licensing policies have both royalties and fees for significantly superior technologies if the cost function is superadditive. By contrast, for additive and certain subadditive cost functions, optimal licensing policies have only royalties and no fees.

Keywords: Patent licensing; superadditive function; subadditive function; royalties; two-part tariff

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1 Introduction

A patent grants an innovator monopoly rights over an innovation for a given period of time. It seeks to provide incentives to innovate as well as to diffuse innovations. Licensing is a standard way of diffusion of innovations. Initiated by Arrow (1962), the study of different aspects of patent licensing has constituted an important area of modern industrial economics. Comparing a monopoly with a perfectly competitive industry, Arrow argued that perfect competition provides a higher incentive to innovate. Licensing in oligopolies was first studied by Kamien and Tauman (1984, 1986) and Katz and Shapiro (1985, 1986). Since then, the literature has been extended to address issues such as informational asymmetries (Gallini and Wright 1990; Choi 2001), product differentiation (Muto 1993; Faulí-Oller and Sandonís 2002), incumbent innovators (Marjit 1990; Wang 1998; Kamien and Tauman 2002; Sen and Tauman 2007) and intertemporal aspects (Jensen 1992, Saracho 2004, Avagyan et al. 2014).

In spite of its richness, the existing literature has been restrictive in regard to one important aspect of production: return to scale. The vast majority of the papers in the literature assume that firms operate under linear cost functions.[1] The current paper seeks to expand the literature by studying licensing between rival firms operating under general cost functions. Our motivation is based on theoretical and also on empirical grounds. On the one hand, we are interested in seeing how the general properties of cost functions (e.g., subadditivity, superadditivity) affect licensing policies, in particular the diffusion of new technologies and the optimal combination of fees and royalties. On the other hand, empirical data shows that licensing policies vary from industry to industry. Given that modern industries differ also in the degree of returns to scale in production, our second goal is to bring forward the latter as a empirical factor that potentially interacts with actual licensing policies.[2]

We carry out our analysis in a Cournot duopoly with a general demand function. Initially both competing firms produce under the same cost function. One of the firms obtains a patent on a technological innovation that changes this cost function. The patentee firm can either use the new technology exclusively or license it to its rival. The licensing policies available to the patentee consist of all combinations of linear royalties and upfront fees (two-part tariffs). The main results of the paper can be summarized as follows:

1. A drastic technology[3] is licensed and both firms are active in the market if the cost function resulting from the new technology is superadditive. If the cost function is additive or subadditive, then a drastic technology is not licensed and the patentee becomes a monopolist.

2. A non drastic technology is licensed if at the licensee's reservation output (i.e. its equilibrium output without a license), the average efficiency gain from the technology is (weakly) higher than the marginal gain.[4]

3. Whenever licensing occurs, it is always optimal to set positive royalties. Furthermore if the technology is significantly superior, upfront fees are also positive for superadditive cost

[1]Sen and Stamatopoulos (2009) is an exception. This paper analyzes licensing in a Cournot duopoly with an outside patentee, quadratic cost functions and linear demand.

[2]More on this important issue appears in Section 3.3.

[3]A cost-reducing technological innovation is *drastic* (Arrow, 1962) if it is significant enough to create a monopoly if only one firm has the new technology; otherwise, it is *non drastic*.

[4]At any output the average efficiency gain from the technology is the difference between the pre and post innovation average costs, while the marginal gain is the difference between the corresponding marginal costs.

functions. For additive cost functions, licensing involves only royalties and no fees and the same conclusion holds for subadditive functions under certain additional assumptions.

To see the basic intuition behind these findings, consider first a drastic technology. Under subadditivity (increasing returns to scale), the maximum profit attainable in the industry is the monopoly profit. As the patentee can become a monopolist by exclusively using a drastic technology, it has no incentive to license such a technology to its rival. Under superadditivity (decreasing returns), the monopoly profit might no longer be the maximum attainable profit. In this case higher cost efficiency, and consequently higher industry profit, is achieved when both firms are active. As a result, licensing occurs and the market is not monopolized.

In the case of a non drastic technology, there is a specific rate of royalty that induces a market equilibrium which replicates the equilibrium under no licensing (i.e., market prices are the same under the two regimes). Under this royalty, the market profit of the patentee is the same as its profit under no licensing. Adding the royalty revenues tilts the scale in favor of licensing. The assumption that the average efficiency gain from the new technology is higher than the marginal gain guarantees that the licensee accepts this policy.

Optimal royalties are determined by two factors. First, the patentee intends to create a relatively inefficient rival and second, it has to consider how the rival's efficiency affects its own marginal cost. Under increasing returns to scale, these two factors work in the same direction. A less efficient rival implies larger output and lower marginal cost for the patentee. Hence, the patentee has incentives to restrict the rival's output under increasing returns, which is achieved by setting higher royalties. Under decreasing returns, the two factors do not work in the same direction, as creating a less efficient rival results in larger output and higher marginal cost for the patentee. For this reason the patentee is inclined to set lower royalties (and higher fees) under decreasing returns.

The paper is organized as follows. Section 2 presents the model. The results are derived in section 3. The last section concludes.

2 The model

Consider a Cournot duopoly with firms 1 and 2. For $i = 1, 2$, let $q_i \geq 0$ be the quantity produced by firm i and $Q = q_1 + q_2$ be the industry output. Let $p(Q) : \mathbb{R}_+ \to \mathbb{R}_+$ be the price function or the inverse demand function. We assume

A1 There exists $Q_0 > 0$ such that (i) $p(Q) = 0$ for $Q \geq Q_0$ and (ii) for $Q \in [0, Q_0)$, $p(Q)$ is positive and twice continuously differentiable with $p'(Q) < 0$, that is, $p(Q)$ is decreasing for $Q \in [0, Q_0)$.

A2 $p'(Q) + qp''(Q) < 0$ for all $Q \in [0, Q_0)$ and $q \in [0, Q]$.

The set of all feasible technologies is $S = [0, \bar{\varepsilon}]$. The total cost of producing q units under technology $\varepsilon \in S$ is given by $c_\varepsilon(q) : \mathbb{R}_+ \to \mathbb{R}_+$. A higher ε corresponds to a better technology and has a lower cost of production. Specifically we assume

A3 For every $\varepsilon \in S$: (i) $c_\varepsilon(0) = 0$, (ii) $c_\varepsilon(q)$ is twice continuously differentiable[5] with $c'_\varepsilon(q) > 0$ for $q > 0$, that is, $c_\varepsilon(q)$ is increasing in q, and (iii) $p(0) > c'_\varepsilon(0)$.

A4 For any $q > 0$, both the total cost $c_\varepsilon(q)$ and the marginal cost $c'_\varepsilon(q)$ are decreasing and differentiable in ε.

[5] At $q = 0$, we consider the right derivative for both functions p and c_ε.

A5 For every $\varepsilon \in S$: $p'(Q) - c''_\varepsilon(q) < 0$ for all $Q \in [0, Q_0)$ and $q \in [0, Q]$.

Initially both firms produce under the least efficient technology $0 \in S$ and have cost $c_0(q)$. Firm 1 has a patent for a superior technology $\varepsilon \in (0, \overline{\varepsilon}]$ that results in cost function $c_\varepsilon(q)$. Firm 1 produces with the new technology. It may also license this technology to its rival firm 2. The set of licensing policies we consider is the set of all combinations of royalties and upfront fees (two-part tariffs) of the form (r, α) where $r \geq 0$ is a unit royalty and $\alpha \geq 0$ is an upfront fee.

A1-A5 are standard assumptions which guarantee uniqueness and stability of Cournot equilibrium (see, e.g., Gaudet and Salant 1991; Kamien et al. 1992; Vives 2001) under any licensing configuration.

2.1 Cost functions

In this paper we consider three classes of cost functions: superadditive, subadditive and additive. Superadditivity represents production technologies characterized by decreasing returns to scale in production whereas subadditivity represents increasing returns. Additivity corresponds to constant returns.

Definition 1 For $\varepsilon \in S$, the cost function c_ε is *superadditive* if $c_\varepsilon(q + \widetilde{q}) > c_\varepsilon(q) + c_\varepsilon(\widetilde{q})$ for all $q, \widetilde{q} > 0$ and it is *subadditive* if $c_\varepsilon(q + \widetilde{q}) < c_\varepsilon(q) + c_\varepsilon(\widetilde{q})$ for all $q, \widetilde{q} > 0$.

Remark 1 If $c(q) : \mathbb{R}_+ \to \mathbb{R}_+$ is a convex (concave) function with $c(0) = 0$, then it is superadditive (subadditive) but the converse is not true. Consider the function $c(q)$ defined on $q \geq 0$ as $c(q) = q \exp(-1/q^2)$ for $q > 0$ and $c(0) = 0$. This function is superadditive, but not convex.[6]

Definition 2 For $\varepsilon \in S$, the cost function c_ε is *additive* if it satisfies Cauchy's basic equation $c_\varepsilon(q + \widetilde{q}) = c_\varepsilon(q) + c_\varepsilon(\widetilde{q})$ for all $q, \widetilde{q} \geq 0$.

Remark 2 If an additive function c_ε is continuous at a point, then $\exists\ k_\varepsilon$ such that

$$c_\varepsilon(q) = k_\varepsilon q \text{ for all } q \geq 0 \tag{1}$$

For the proof, see Theorem 1, p.34 of Aczél (1966). By (1) and A3 it follows that if a technology ε has additive cost function, then $c'_\varepsilon(q) = k_\varepsilon > 0$. An additive cost function thus results in constant marginal cost of production. If the initial technology 0 and the new technology ε both have additive cost functions, then the magnitude of the cost reduction from the new technology for every unit of production is $k_0 - k_\varepsilon > 0$. This case has been extensively studied in the literature of patent licensing (see, e.g., Wang 1998; Sen and Tauman 2007).

The following lemma provides a useful characterization of three different classes of cost functions in terms of marginal costs.

Lemma 1 *Let $c(q) : \mathbb{R}_+ \to \mathbb{R}_+$ be a twice continuously differentiable cost function with $c(0) = 0$. For any $q > 0$, $c'(q) \geq c'(0)$ if c is superadditive, $c'(q) \leq c'(0)$ if c is subadditive and $c'(q) = c'(0)$ if c is additive.*

Proof See the Appendix. ■

[6]For more such counter-examples, see Bourin and Hiai (2015), who explore a large class of superadditive functions. For the early literature on superadditive functions, see, e.g., Bruckner (1962, 1964), Beckenbach (1964).

3 The licensing game G

The strategic interaction between the two firms is modeled as an extensive-form game G that has three stages. In the first stage, firm 1 decides whether to license the new technology ε to firm 2 or not. If firm 1 decides to license, it offers firm 2 a licensing policy (r, α). In the second stage, firm 2 decides whether to accept or reject any licensing policy offered. Finally, in the third stage the two firms simultaneously choose quantities q_1, q_2 in the Cournot duopoly.

If firm 2 has a license of technology ε under a policy (r, α), it pays firm 1 the upfront fee α and in addition pays royalty r for every unit it produces. If firm 2 does not have a license, it produces with technology 0. Let λ be the indicator variable with $\lambda = 1$ if firm 2 has a license and $\lambda = 0$ if it does not. The payoff functions of firms 1, 2 (these are functions of $(r, \alpha), \lambda, q_1, q_2$) in the game G are

$$\Pi^1_\varepsilon = p(Q)q_1 - c_\varepsilon(q_1) + \lambda(rq_2 + \alpha), \Pi^2_\varepsilon = p(Q)q_2 - \lambda[c_\varepsilon(q_2) + rq_2 + \alpha] - (1-\lambda)c_0(q_2) \quad (2)$$

We determine Subgame Perfect Nash Equilibrium (SPNE) outcome of G. Working backwards, we begin with the Cournot stage of this game and then move to the initial stages.

3.1 Cournot stage

Consider the third stage of G where firms compete in quantities. For $i = 1, 2$, let π_i be the duopoly profit of firm i in the Counrot stage. From (2), we have

$$\pi_1 = p(Q)q_1 - c_\varepsilon(q_1), \quad \pi_2 = p(Q)q_2 - \lambda[c_\varepsilon(q_2) + rq_2 + \alpha] - (1-\lambda)c_0(q_2) \quad (3)$$

To find SPNE of G, for every $r \geq 0$ and $\lambda \in \{0, 1\}$, we need to determine Nash equilibrium (NE) of the corresponding Cournot duopoly where firms 1, 2 choose q_1, q_2 to obtain profits given by[7] (3). As the cost functions of both firms satisfy A3-A5, existence and uniqueness of NE of the Cournot duopoly is guaranteed under both cases of licensing and no licensing. The notion of *drastic* techn! ology (Arrow 1962) will be useful for our analysis.

Definition 3 The new technology is *drastic* if it is significant enough to create a monopoly when only one firm uses it; otherwise, it is *non drastic*.

Remark 3 To characterize drastic technologies, consider the monopoly problem. For any $\varepsilon \in S$, let $\phi_\varepsilon(q) := p(q)q - c_\varepsilon(q)$ denote the profit function of a monopolist who faces inverse demand $p(Q)$ and produces under cost $c_\varepsilon(Q)$. By A1-A5, $\phi_\varepsilon(q)$ is concave in q and the monopoly problem $\max_{q \geq 0} \phi_\varepsilon(q)$ has a unique solution. Denote this solution by q^m_ε (the monopoly output). Let $p^m_\varepsilon = p(q^m_\varepsilon)$ be the monopoly price and $\pi^m_\varepsilon = \phi_\varepsilon(q^m_\varepsilon)$ be the monopoly profit. A1-A5 also ensure that q^m_ε is increasing (and hence p^m_ε is decreasing) in ε.

Lemma 2 describes the key features of Cournot equilibrium under the two cases of licensing and no licensing.

Lemma 2 *For all $\varepsilon \in S$, $r \geq 0$ and $\lambda \in \{0, 1\}$, the Cournot duopoly has a unique NE. For $i = 1, 2$, let $\widehat{q}^i_\varepsilon, \widehat{\pi}^i_\varepsilon$ denote NE (Cournot) output, profit of firm i when firm 2 does not have a license and let $\widehat{Q}_\varepsilon = \widehat{q}^1_\varepsilon + \widehat{q}^2_\varepsilon$. Let $q^i_\varepsilon(r), \pi^i_\varepsilon(r), Q_\varepsilon(r)$ be the corresponding expressions when firm 2 has a license with royalty r.*

[7] Since α is a lump-sum transfer paid upfront, it has no effect on the outcomes of the Cournot duopoly. As firm 1's choice of q_1 does not affect its royalty revenue λrq_2, this revenue can be left out from the profit of firm 1 at the Cournot stage. However, for firm 2, its choice of q_2 does affect its royalty payments λrq_2, so these payments are part of its cost function in the Cournot duopoly.

(i) *Suppose firm 2 does not have a license. If* $p^m_\varepsilon > c'_0(0)$*, then* $\widehat{q}^1_\varepsilon, \widehat{q}^2_\varepsilon$ *are both positive and if* $p^m_\varepsilon \le c'_0(0)$*, then* $\widehat{q}^1_\varepsilon = q^m_\varepsilon, \widehat{q}^2_\varepsilon = 0$*. Consequently a technology* $\varepsilon \in S$ *is drastic if* $p^m_\varepsilon \le c'_0(0)$ *and it is non drastic otherwise. Moreover,* $\widehat{q}^1_\varepsilon, \widehat{\pi}^1_\varepsilon, \widehat{Q}_\varepsilon$ *are increasing in* ε *and for non drastic technologies* $\widehat{q}^2_\varepsilon, \widehat{\pi}^2_\varepsilon$ *are decreasing in* ε*.*

(ii) *Suppose firm 2 has a license with royalty* $r \ge 0$*. There exists* $\overline{r}_\varepsilon \equiv p^m_\varepsilon - c'_\varepsilon(0) > 0$ *such that if* $r < \overline{r}_\varepsilon$*, then* $q^1_\varepsilon(r), q^2_\varepsilon(r)$ *are both positive and if* $r \ge \overline{r}_\varepsilon$*, then* $q^1_\varepsilon(r) = q^m_\varepsilon, q^2_\varepsilon(r) = 0$*. Moreover,* $q^1_\varepsilon(r), \pi^1_\varepsilon(r)$ *are increasing and* $q^2_\varepsilon(r), \pi^2_\varepsilon(r), Q_\varepsilon(r)$ *are decreasing in* r *for* $r \le \overline{r}_\varepsilon$*.*

(iii) *For any non drastic technology* ε*, there are royalties* $\widehat{r}_\varepsilon, r^*_\varepsilon \in (0, \overline{r}_\varepsilon)$ *such that* $\pi^2_\varepsilon(r) \gtreqless \widehat{\pi}^2_\varepsilon \Leftrightarrow r \lesseqgtr \widehat{r}_\varepsilon$ *and* $q^2_\varepsilon(r) \gtreqless \widehat{q}^2_\varepsilon \Leftrightarrow r \lesseqgtr r^*_\varepsilon$*. Moreover* $q^1_\varepsilon(r^*_\varepsilon) = \widehat{q}^1_\varepsilon$ *and* $Q_\varepsilon(r^*_\varepsilon) = \widehat{Q}_\varepsilon$*.*

Proof See the Appendix. ∎

Lemma 2 characterizes Cournot equilibrium and delivers the standard comparative statics results. When there is no licensing, expectedly the quantity of firm 1 increases in the quality of the new technology ε. Our assumptions guarantee that standard effects will then follow: A2 implies that best-replies are negatively sloped, hence quantity of firm 2 falls in ε. A5 implies that the net effect of ε on industry output is positive.

When there is licensing with royalty r, expectedly $q^2_\varepsilon(r)$ (whenever positive) falls in r. Then A2 implies that $q^1_\varepsilon(r)$ increases in r and A5 implies that the net effect of r on industry output $Q_\varepsilon(r)$ is negative. The last part of the lemma identifies two specific thresholds of royalties: one that equates firm 2's Cournot profits with and without a license and the other one that equates its Cournot outputs. These thresholds will be useful to determine licensing policies that are acceptable to firm 2.

Since $p^m_0 > c'_0(0)$, it follows by Lemma 2 that the initial technology 0 is non drastic. Note that p^m_ε is continuous and decreasing in ε. Henceforth we assume that $\exists\ \varepsilon_D \in (0, \overline{\varepsilon})$ such that $p^m_\varepsilon > c'_0(0)$ if $\varepsilon \in [0, \varepsilon_D)$ and $p^m_\varepsilon \le c'_0(0)$ if $\varepsilon \in [\varepsilon_D, \overline{\varepsilon}]$. This ensures that sets of non drastic and drastic technologies are both non empty. Any technology $\varepsilon < \varepsilon_D$ is non drastic and $\varepsilon \ge \varepsilon_D$ is drastic.

3.2 Technology transfer stages

Given the analysis of the previous section, we next move to the initial stages of G. If firm 2 accepts a licensing policy (r, α), the payoff $\Pi^1_\varepsilon(r, \alpha)$ of firm 1 is the sum of its duopoly profit and licensing revenue. For firm 2, note from (3) that royalty payments are already included as part of cost in its duopoly profit. So firm 2's payoff $\Pi^2_\varepsilon(r, \alpha)$ is its duopoly profit net of upfront fee. Using the equilibrium values of profits and quantities from Lemma 2, we have

$$\Pi^1_\varepsilon(r, \alpha) = \pi^1_\varepsilon(r) + r q^2_\varepsilon(r) + \alpha, \ \ \Pi^2_\varepsilon(r, \alpha) = \pi^2_\varepsilon(r) - \alpha$$

By Lemma 2, if firm 2 rejects the licensing policy, it obtains $\widehat{\pi}^2_\varepsilon$. Hence for any r, it is optimal for firm 1 to set the fee α equal to $\alpha_\varepsilon(r) := \pi^2_\varepsilon(r) - \widehat{\pi}^2_\varepsilon$ making firm 2 just indifferent between accepting and rejecting the licensing offer.[8] Therefore, if firm 1 decides to offer a license, its problem reduces to choose $r \ge 0$ to maximize

$$\Pi^1_\varepsilon(r) = \pi^1_\varepsilon(r) + r q^2_\varepsilon(r) + \pi^2_\varepsilon(r) - \widehat{\pi}^2_\varepsilon = p(Q_\varepsilon(r))Q_\varepsilon(r) - c_\varepsilon(q^1_\varepsilon(r)) - c_\varepsilon(q^2_\varepsilon(r)) - \widehat{\pi}^2_\varepsilon \tag{4}$$

[8] If royalty r is such that $\pi^2_\varepsilon(r) < \widehat{\pi}^2_\varepsilon$, then even with zero fee firm 2's payoff with a license is lower than its payoff without a license. As upfront fees are non-negative, such a royalty will not be accepted by firm 2. By Lemma 2(iii), it follows that any licensing policy with royalty $r > \widehat{r}_\varepsilon$ is not acceptable to firm 2.

On the other hand, if firm 1 does not offer a license, it obtains $\widehat{\pi}^1_\varepsilon$. We are now in a position to determine optimal licensing policies for firm 1.

3.3 Optimal licensing policies

The following general result shows that royalties must be positive whenever licensing occurs.

Proposition 1 *If firm* 1 *offers a license to firm* 2, *it is always optimal to set a positive royalty.*

Proof See the Appendix. ■

Remark 4 Proposition 1 holds under A1-A5 without any further assumptions. However, setting a positive royalty may not necessarily be optimal for a patentee firm when it competes in an oligopoly of size $n \geq 3$. In the duopoly model firm 2's reservation payoff $\widehat{\pi}^2_\varepsilon$ (i.e., its payoff when it does not have a license) is independent of the rate of royalty r. But in an oligopoly of general size, if a specific firm is without a license, some other firms might have a license; so the reservation payoff of a non-licensee does depend on r. This may lead to a conclusion different from Proposition 1 (see, e.g., Sen and Tauman 2007).

Further characterization of optimal licensing policies depends on whether the new technology is drastic or non drastic.

3.3.1 Drastic technologies

Consider a drastic technology ε. For this case, if firm 2 does not have a license it exits the market and firm 1 becomes a monopolist, i.e., $\widehat{\pi}^2_\varepsilon = 0$ and $\widehat{\pi}^1_\varepsilon = \pi^m_\varepsilon$. Proposition 2 shows that firm 1's decision to license or not depends crucially on the nature of new technology. It should be also noted that these results do not require any additional assumption on the initial technology 0 apart from A3-A5.

Proposition 2 *Consider a drastic technology ε, i.e., $\varepsilon \in [\varepsilon_D, \overline{\varepsilon}]$. The following hold.*

(I) *Regardless of whether there is licensing or not, firm* 2 *obtains zero net payoff.*

(II) *If $c_\varepsilon(q)$ is subadditive or additive, licensing does not occur. Firm* 1 *becomes a monopolist and obtains π^m_ε.*

(III) *If $c_\varepsilon(q)$ is superadditive and $c'_\varepsilon(q^m_\varepsilon) > c'_\varepsilon(0)$, then licensing occurs.*[9] *Any optimal licensing policy has positive royalty and upfront fee. Both firms are active and firm* 1 *obtains more than π^m_ε.*

Proof Firm 2's net payoff under any optimal licensing policy is its payoff without a license: $\widehat{\pi}^2_\varepsilon$. Since $\widehat{\pi}^2_\varepsilon = 0$ for a drastic technology ε, (I) follows.

For (II)-(III), note that by not offering a license, firm 1 obtains the monopoly profit π^m_ε. By Lemma 2(ii), offering a license with royalty $r \geq \overline{r}_\varepsilon$ for a drastic technology ε results in the same outcome as not offering a license. So it is sufficient to consider licensing policies with $r \in [0, \overline{r}_\varepsilon)$. In that case both firms produce positive outputs.

(II) Let $c_\varepsilon(q)$ be subadditive or additive. Taking $\widehat{\pi}^2_\varepsilon = 0$ in (4), for $r \in [0, \overline{r}_\varepsilon)$, we have

$$\Pi^1_\varepsilon(r) = p(Q_\varepsilon(r))Q_\varepsilon(r) - c_\varepsilon(q^1_\varepsilon(r)) - c_\varepsilon(q^2_\varepsilon(r)) \leq p(Q_\varepsilon(r))Q_\varepsilon(r) - c_\varepsilon(Q(r)) < \pi^m_\varepsilon \quad (5)$$

[9]Superaddivity and $c_\varepsilon(0) = 0$ already imply $c'_\varepsilon(q) \geq c'_\varepsilon(0)$ for all $q > 0$ (see Lemma 1). The inequality in the proposition requires the marginal cost at the monopoly output q^m_ε to be different from that at 0.

where the first inequality is strict if $c_\varepsilon(q)$ subadditive and holds with equality if $c_\varepsilon(q)$ is additive. The second inequality is due to the fact that the monopolist's profit is maximized at q^m_ε and $Q_\varepsilon(r) < q^m_\varepsilon$ for $r \in [0, \overline{r}_\varepsilon)$ (Lemma 2(ii)).

(III) Let $c_\varepsilon(q)$ be superadditive and $c'_\varepsilon(q^m_\varepsilon) > c'_\varepsilon(0)$. Then $\Pi^1_\varepsilon(r)$ is decreasing at $r = \overline{r}_\varepsilon$ (see Lemma A3 of Appendix). Together with Proposition 1, this implies that it is optimal for firm 1 to license and any optimal policy must have $r \in (0, \overline{r}_\varepsilon)$ and upfront fee $\pi^2_\varepsilon(r) - \widehat{\pi}^2_\varepsilon = \pi^2_\varepsilon(r) > 0$. As $\Pi^1_\varepsilon(r) = \pi^m_\varepsilon$ for $r \geq \overline{r}_\varepsilon$, under any optimal policy firm 1 obtains more than π^m_ε. ■

Under additivity or subadditivity of the new technology (constant or increasing returns to scale), the maximum achievable payoff is the monopoly profit π^m_ε. Since firm 1 can obtain the monopoly profit by using a drastic technology exclusively, licensing does not occur. Under superadditivity (decreasing returns), the monopoly profit is no longer the upper bound of industry profit, as production of output by a single firm creates cost inefficiencies. For this case firm 1 has an incentive to keep firm 2 active in the market. The presence of two active firms increases efficiency and results in higher surplus than the monopoly profit, which firm 1 extracts via a fee.

We note that the result that a firm might sell a drastic technology to its rival has been obtained in a different context by Faulí-Oller and Sandonís (2002) who analyzed licensing in a differentiated goods duopoly under constant returns to scale. In that framework, a patentee transfers a drastic technology to its rival as it does not want to close the rival's profitable market. Proposition 2 brings forward an alternative motive of licensing of a drastic technology, namely, cost efficiency under decreasing returns.

3.3.2 Non drastic technologies

To characterize optimal licensing policies for a non drastic technology ε, let $F(q) := c_0(q) - c_\varepsilon(q)$. Note by A4 that $F(q), F'(q)$ are both positive for any $q > 0$. $F(q)$ represents the *total gain in efficiency* for q units obtained from the superior technology ε. Accordingly, $F(q)/q$ stands for the *average gain* and $F'(q)$ the *marginal gain*. For $q > 0$, define

$$\mathcal{H}(q) := \frac{F'(q)}{F(q)/q} \tag{6}$$

The function $\mathcal{H}(q)$ is the ratio of average and marginal gains in efficiency obtained from using the superior technology ε.

Recall the two threshold levels of royalties $r^*_\varepsilon, \widehat{r}_\varepsilon$ from Lemma 2(iii). At $r = r^*_\varepsilon$, the Cournot outputs of firms coincide with their no-licensing levels and at $r = \widehat{r}_\varepsilon$, the Cournot profit of firm 2 equals its profit without a license. For a licensing policy with royalty r, the maximum upfront fee is $\alpha_\varepsilon(r) = \pi^2_\varepsilon(r) - \widehat{\pi}^2_\varepsilon$. A policy $(r, \alpha_\varepsilon(r))$ is acceptable to firm 2 if and only if $r \in [0, \widehat{r}_\varepsilon]$. Lemma 3 utilizes the function $\mathcal{H}(q)$ together with $r^*_\varepsilon, \widehat{r}_\varepsilon$ to determine values of royalties that make a licensing agreement beneficial for both firms.

Lemma 3 *The following hold for a non drastic technology* ε.

(i) $\Pi^1_\varepsilon(r^*_\varepsilon) > \widehat{\pi}^1_\varepsilon$, *i.e., for firm* 1, *licensing under policy* $(r^*_\varepsilon, \alpha(r^*_\varepsilon))$ *yields a higher payoff than the no licensing.*

(ii) *Suppose* $\mathcal{H}(\widehat{q}^2_\varepsilon) \leq 1$. *Then* $r^*_\varepsilon \leq \widehat{r}_\varepsilon$ *and consequently there exist licensing policies acceptable to firm* 2 *in which firm* 1 *obtains a higher payoff than no licensing.*

Proof See the Appendix. ■

At $r = r_\varepsilon^*$, the Cournot profit of firm 1 is the same as its profit under no licensing, but it obtains additional licensing revenue, yielding a surplus for firm 1. Whether firm 2 also obtains a surplus there depends on the inequality $\mathcal{H}(\widehat{q}_\varepsilon^2) \leq 1$, which says that the marginal efficiency gain obtained from the new technology is lower than the average gain when both are computed at firm 2's Cournot output without a license. As licensing with $r = r_\varepsilon^*$ results in same Cournot outputs and price as no licensing, firm 2's Cournot profit is higher under licensing if and only if its total cost is lower under licensing, i.e.,

$$\pi_\varepsilon^2(r_\varepsilon^*) \geq \widehat{\pi}_\varepsilon^2 \Leftrightarrow c_\varepsilon(\widehat{q}_\varepsilon^2) + r_\varepsilon^* \widehat{q}_\varepsilon^2 \leq c_0(\widehat{q}_\varepsilon^2) \Leftrightarrow r_\varepsilon^* \leq F(\widehat{q}_\varepsilon^2)/\widehat{q}_\varepsilon^2$$

Since $r_\varepsilon^* = F'(\widehat{q}_\varepsilon^2)$ (see the proof of Lemma 3 in the Appendix), it follows that $\pi_2(r_\varepsilon^*) \geq \widehat{\pi}_\varepsilon^2 \Leftrightarrow \mathcal{H}(\widehat{q}_\varepsilon^2) \leq 1$. In particular this inequality implies that the unit royalty does not exceed the average gain in efficiency from the superior technology. This ensures that licensing under royalty r_ε^* leaves a surplus to firm 2.

3.3.3 Additive and superadditive cost functions

Using the condition $\mathcal{H}(\widehat{q}_\varepsilon^2) \leq 1$, we can characterize optimal licensing policies for additive and superadditive cases.

Proposition 3 *Consider a non drastic technology ε, i.e., $\varepsilon \in (0, \varepsilon_D)$. If $\mathcal{H}(\widehat{q}_\varepsilon^2) \leq 1$, then licensing occurs. The optimal licensing policies have the following properties.*

(I) *If $c_\varepsilon(q)$ is additive, then the unique optimal licensing policy for firm 1 has royalty $r = \widehat{r}_\varepsilon$ and zero upfront fee, i.e., it is a pure royalty policy.*

(II) *If $c_{\varepsilon_D}(q)$ is superadditive and $c'_{\varepsilon_D}(q^m_{\varepsilon_D}) > c'_{\varepsilon_D}(0)$, then $\exists\ 0 < \widehat{\varepsilon} < \varepsilon_D$ such that for all $\varepsilon \in (\widehat{\varepsilon}, \varepsilon_D)$, any optimal licensing policy for firm 1 has both positive royalty as well as positive upfront fee.*

Proof See the Appendix. ■

Note from (1) that if $c_\varepsilon(q)$ is additive, then $c_\varepsilon(q) = k_\varepsilon q$. If the initial technology 0 also has an additive cost function, then $c_0(q) = k_0 q$. In that case for any $q > 0$, we have $F'(q) = F(q)/q = k_0 - k_\varepsilon$ and $\mathcal{H}(q) = 1$. Therefore the condition $\mathcal{H}(\widehat{q}_\varepsilon^2) \leq 1$ holds for the standard case studied in the literature where both new and initial technologies have constant returns. The conclusion of part (I) is the same as the result of Sen and Tauman (2007) for a Cournot duopoly with an incumbent patentee.

Observe that ε_D is the threshold that separates non drastic and drastic technologies. Part (II) of the proposition shows that if technology ε_D has a superadditive cost function, then for all sufficiently significant non drastic technologies (i.e., technologies ε that are close enough to ε_D) any optimal policy has both royalty and upfront fee.

3.3.4 Subadditive cost functions

When $c_\varepsilon(q)$ is subadditive, the condition $\mathcal{H}(\widehat{q}_\varepsilon^2) \leq 1$ alone is not enough to characterize optimal licensing policies.[10] More structure on the cost function is needed.

[10] As shown in the proof of Proposition 3, for technologies ε that are close enough to ε_D, in the superadditive case firm 1's payoff is decreasing at $r = \widehat{r}_\varepsilon$ (the maximum acceptable royalty for firm 2). This implies that any optimal policy has royalty $r < \widehat{r}_\varepsilon$ and the upfront fee is positive. Applying similar reasoning for the subadditive case will show that firm 1's payoff is increasing at $r = \widehat{r}_\varepsilon$. But this does not conclusively say whether it is optimal to set $r = \widehat{r}_\varepsilon$ (and zero fee) or $r < \widehat{r}_\varepsilon$ (and positive fee).

Definition Let $x_0 > 0$. A function $f : \mathbb{R}_+ \to \mathbb{R}_+$ is *interval-wise decreasing at* x_0 if $f(x) > f(x_0)$ for $x < x_0$ and $f(x) < f(x_0)$ for $x > x_0$.

Note that interval-wise decreasing property at x_0 implies $f(x) < f(y)$ if $x < x_0 < y$, but it does not specify any order between $f(x), f(y)$ when x, y are both lower (or both higher) than x_0. We shall use this concept to impose further structure on a cost function. Note from Lemma 2 that when there is licensing with zero royalty, both firms produce the same Cournot output. Denote this output by $q_\varepsilon(0)$ and note that $q^2_\varepsilon(r) < q_\varepsilon(0) < q^1_\varepsilon(r)$ for any $r > 0$. Proposition 4 shows that optimal licensing policies can be completely characterized if the marginal cost function $c'_\varepsilon(q)$ is interval-wise decreasing at[11] $q_\varepsilon(0)$. Before stating th! e proposition, it will be useful to see the implications of interval-wise decreasing property of the marginal cost function.

Lemma 4 *The following hold if $c'_\varepsilon(q)$ is interval-wise decreasing at $q_\varepsilon(0)$.*

(i) *$c_\varepsilon(q)$ cannot be additive or superadditive.*

(ii) *For any $r \in (0, \overline{r}_\varepsilon)$, $c_\varepsilon(q^1_\varepsilon(r) + q^2_\varepsilon(r)) < c_\varepsilon(q^1_\varepsilon(r)) + c_\varepsilon(q^2_\varepsilon(r))$.*

Proof (i) Since $c'_\varepsilon(q)$ is interval-wise decreasing at $q_\varepsilon(0) > 0$, we have $c'_\varepsilon(0) > c'_\varepsilon(q_\varepsilon(0))$. Then by Lemma 1 it follows that $c_\varepsilon(q)$ cannot be additive or superadditive.

(ii) We drop subscript ε for brevity. Recall from Lemma 2(ii) that for $r \in (0, \overline{r})$, $0 < q^2(r) < q(0) < q^1(r)$. Since $c'(q)$ is interval-wise decreasing at $q(0)$, we have

$$c'(x) > c'(q(0)) \text{ for } x < q^2(r) \text{ and } c'(x) < c'(q(0)) \text{ for } x > q^1(r)$$

Hence $c(q^1(r) + q^2(r)) - [c(q^1(r)) + c(q^2(r))] = \int_{q^1(r)}^{q^1(r)+q^2(r)} c'(x)\mathrm{d}x - \int_0^{q^2(r)} c'(x)\mathrm{d}x$, which is lower than $\int_{q^1(r)}^{q^1(r)+q^2(r)} c'(q(0))\mathrm{d}x - \int_0^{q^2(r)} c'(q(0))\mathrm{d}x = 0$. This completes the proof. ■

Lemma 4 shows that if $c'_\varepsilon(q)$ is interval-wise decreasing at $q_\varepsilon(0)$, then $c_\varepsilon(q)$ is outside the set of superadditive and additive functions and the "subaddivity inequality" $c_\varepsilon(q + \widetilde{q}) < c_\varepsilon(q) + c_\varepsilon(\widetilde{q})$ holds for all pairs $(q, \widetilde{q})$ that arise as Cournot equilibrium under any licensing policy.

Proposition 4 *Consider a non drastic technology ε. Suppose $\mathcal{H}(\widehat{q}^2_\varepsilon) \leq 1$ and $c'_\varepsilon(q)$ is interval-wise decreasing at $q_\varepsilon(0)$. Then the unique optimal licensing policy for firm 1 has royalty $r = \widehat{r}_\varepsilon$ and zero upfront fee, i.e., it is a pure royalty policy.*

Proof See the Appendix. ■

In particular, if $c_\varepsilon(q)$ is subadditive and satisfies the conditions of Proposition 4, then pure royalty emerges as the unique optimal licensing policy. Putting together the results of Propositions 3 and 4, the general conclusion is that for non drastic technologies a patentee licenses to its rival by setting the maximum possible royalty and zero upfront fee for constant or increasing returns (subject to some additional structure on the cost function). By contrast, for decreasing returns, royalties are set lower and there are positive fees.

The result that royalties tend to be higher under increasing returns seems to be consistent with some real life observations on royalty rates in various industries. A recent survey by Held and Parker (2011) computed the *average* royalty rates in some major industries in USA and Canada for the period 2008-2011. The higher average royalty rates were found in sectors such us aerospace (commercial and military), transportation and information technology & equipment,

[11]Note that if $c_\varepsilon(q)$ is concave, then $c'_\varepsilon(q)$ is decreasing for all $q \geq 0$ and in particular it is interval-wise decreasing at any $q > 0$. Thus, interval-wise decreasing property of the marginal cost at a certain point is a weaker requirement than concavity of the cost function.

i.e., sectors usually associated with increasing returns to scale. Of course, one needs to look at industry-specific data for a better understanding of the observed licensing policies. Still, some sort of association between returns to scale and optimal licensing policies seems to be in place.

4 Concluding remarks

This paper has analyzed optimal licensing policies of a cost-reducing innovation in a market with general cost functions. In a Cournot duopoly with one of the firms as a patentee, we have derived a fairly complete characterization of licensing policies under a general setting. We have shown that if licensing occurs, then royalties must be positive (Proposition 1), licensing of drastic technologies depends on whether the cost function generated by the new technology is superadditive or subadditive (Proposition 2), licensing of non drastic technologies depends on the relation between the marginal and average efficiency gains obtained from the superior technology (Proposition 3), for significantly superior technologies, superadditivity ensures that fees are positive (Proposition 3) and subadditivity or additivity results in maximum possible royalty and zero fees (Propositions 3,4).

In conclusion, we suggest some extensions. The analysis of licensing in a market with $n \geq 3$ firms operating under general cost functions is the most natural direction for future work. Another interesting direction is to consider markets where firms compete in prices under non-constant returns. This case is interesting not only for licensing but also from a market equilibrium viewpoint: it is known that a multiplicity of price equilibria emerges when two firms operate in homogeneous goods markets under decreasing returns (Dastidar 1995). The introduction of licensing changes the objective function of the incumbent patentee in the price stage of the game from $p_1 d_1(p_1, p_2) - c_\varepsilon(d_1(p_1, p_2))$ to $p_1 d_1(p_1, p_2) - c_\varepsilon(d_1(p_1, p_2)) + r d_2(p_1, p_2)$, where $d_i(p_1, p_2)$ denotes the demand function of firm i and p_i denotes the price set by firm i. It will be interesting to see how this modification of the objective function affects the set of price equilibria.

Appendix

Proof of Lemma 1 We prove the result when $c(q)$ is superadditive. The proofs for subadditive and additive cases follow by similar reasoning. Fix any $q > 0$. By superadditivity, for any $\delta > 0$, we have $c(q + \delta) > c(q) + c(\delta)$ so that $[c(q + \delta) - c(q)]/\delta > c(\delta)/\delta$. Hence

$$c'(q) = \lim_{\delta \uparrow 0}[c(q + \delta) - c(q)]/\delta \geq \lim_{\delta \uparrow 0}[c(\delta)/\delta]$$

Since $c(0) = 0$, by L'Hôpital's rule we have $\lim_{\delta \uparrow 0} c(\delta)/\delta = \lim_{\delta \uparrow 0} c'(\delta) = c'(0)$, proving that $c'(q) \geq c'(0)$. ■

Some comparative statics results Consider a Cournot duopoly with two firms $1, 2$ where the demand function satisfies A1-A2. Let $i, j \in \{1, 2\}$, $i \neq j$. Firm i has cost $\tau(q)$ and firm j has cost $\gamma_t(q)$ where $t \geq 0$ is an exogenously given parameter. For any $q \geq 0$, $\gamma_t(q)$ is twice continuously differentiable in t and both $\tau(q), \gamma_t(q)$ satisfy A3-A5 for all $t \geq 0$. The profit functions of firms i, j are $\pi_i = p(Q)q_i - \tau(q_i)$ and $\pi_j = p(Q)q_j - \gamma_t(q_j)$. Also assume that the unique NE of the Cournot duopoly is determined from the first order conditions, where both firms produce positive output. Let $q_1(t), q_2(t)$ and $\pi_1(t), \pi_2(t)$ be the NE outputs and profits of firms $1, 2$ and $Q(t)$ be the industry output.

Lemma A1 lists the comparative statics properties of outputs and profits of firms with respect to the parameter t. We shall apply these results in specific cases of interest. For instance (a) to see how firm 1's profit changes with respect to technology ε in the case when firm 2 does not have the new technology, we take $i = 2, j = 1$ and $t = \varepsilon$ and (b) to see how firm 2's profit as licensee changes with respect to royalty r, we take $i = 1, j = 2$ and $t = r$.

Lemma A1 *Denote $g^q(t) := \partial\gamma_t(q)/\partial t$ and $h^q(t) := \partial\gamma_t'(q)/\partial t$*

(i) *If $h^q(t) > 0$ for all $q > 0$, then $q_i(t), \pi_i(t)$ are increasing and $q_j(t), Q(t)$ are decreasing in t.*

(ii) *If $h^q(t) < 0$ for all $q > 0$, then $q_i(t), \pi_i(t)$ are decreasing and $q_j(t), Q(t)$ are increasing in t.*

(iii) *If $g^q(t) > 0, h^q(t) > 0$ for all $q > 0$, then $\pi_j(t)$ is decreasing in t; if $g^q(t) < 0, h^q(t) < 0$ for all $q > 0$, then $\pi_j(t)$ is increasing in t.*

Proof Without loss of generality, let $i = 1, j = 2$. Note that

$$\partial\pi_1/\partial q_1 = p'(Q)q_1 + p(Q) - \tau'(q_1) \text{ and } \partial\pi_2/\partial q_2 = p'(Q)q_2 + p(Q) - \gamma_t'(q_2) \tag{7}$$

For $i, j = 1, 2$ and $i \neq j$, denote $a_i(q_1, q_2) := \partial\pi_i^2/\partial q_i^2$, $b_i(q_1, q_2) := \partial\pi_i^2/\partial q_i\partial q_j$. The Jacobian of the marginal profits is

$$J = \begin{pmatrix} a_1 & b_1 \\ b_2 & a_2 \end{pmatrix}$$

From (7) and A2, A5, $a_1 = q_1p''(Q) + 2p'(Q) - \tau''(q_1) < 0$, $a_2 = q_2p''(Q) + 2p'(Q) - \gamma_t''(q_2) < 0$ and $b_i = q_ip''(Q) + p'(Q) < 0$. Note that $|J| = a_1a_2 - b_1b_2$. By the uniqueness of the NE, $(-1)^2|J| = |J| > 0$ (see, e.g., Dixit 1986; Dastidar 2000).

As both firms produce positive output in the unique NE $(q_1(t), q_2(t))$, using first order conditions (f.o.c.) in (7), we have

$$p'(Q(t))q_1(t) + p(Q(t)) - \tau'(q_1(t)) = 0 \text{ and } p'(Q(t))q_2(t) + p(Q(t)) - \gamma_t'(q_2(t)) = 0$$

Totally differentiating the above with respect to t and using the definition of h_t^q:

$$a_1q_1'(t) + b_1q_2'(t) = 0 \text{ and } b_2q_1'(t) + a_2q_2'(t) - h_t^{q_2(t)} = 0$$

Solving this system of equations, we have

$$q_1'(t) = -b_1h_t^{q_2(t)}/|J|,\ q_2'(t) = a_1h_t^{q_2(t)}/|J|,\ Q'(t) = q_1'(t) + q_2'(t) = (a_1 - b_1)h_t^{q_2(t)}/|J| \tag{8}$$

As $a_1 < 0, b_1 < 0, a_1 - b_1 = p'(Q(t)) - \tau''(q_1(t)) < 0$ (by A5) and $|J| > 0$, by (8), $q_1'(t) > 0, q_2'(t) < 0$ and $Q'(t) < 0$ if $h_t^q > 0$ and the reverse inequalities holds if $h_t^q < 0$.

Totally differentiating firm 1's NE profit $\pi_1(t) = p(Q(t))q_1(t) - \tau(q_1(t))$ with respect to t and using the f.o.c. of firm 1, we have $\pi_1'(t) = [\partial\pi_1/\partial q_2]q_2'(t) = p'(Q(t))q_1(t)q_2'(t)$. Since $p' < 0$ and $q_1(t) > 0$, it follows that $\text{sign}[\pi_1'(t)] = -\text{sign}[q_2'(t)]$. Hence $\pi_1'(t) > 0$ if $h_t^q > 0$ and $\pi_1'(t) < 0$ if $h_t^q < 0$. This completes the proof of (i)-(ii).

Totally differentiating firm 2's NE profit $\pi_2(t) = p(Q(t))q_2(t) - \gamma_t(q_2(t))$ with respect to t, using the f.o.c. of firm 2 and the definition of g_t^q, we have $\pi_2'(t) = [\partial\pi_2/\partial q_1]q_1'(t) - g_t^{q_2(t)} = p'(Q(t))q_2(t)q_1'(t) - g_t^{q_2(t)}$. As $p' < 0$ and $q_2(t) > 0$, it follows that (a) if $g_t^q > 0$ and $h_t^q > 0$, then $q_1'(t) > 0$ and hence $\pi_2'(t) < 0$ and (b) if $g_t^q < 0$ and $h_t^q < 0$, then $q_1'(t) < 0$ and hence $\pi_2'(t) > 0$. This completes the proof of (iii). ■

Lemma A2 *If firm 2 has a license with zero royalty, its Cournot output as well as profit are higher than their no-licensing levels.*

Proof The result is immediate for a drastic technology. Consider a non drastic technology. Take $i = 1, j = 2$ in Lemma A1. Firm 1's cost is $\tau(q) = c_\varepsilon(q)$. Compare two scenarios: (a) if firm 2 has a license with royalty $r = 0$, its cost is $\gamma_\varepsilon(q) = c_\varepsilon(q)$ and (b) if 2 does not have a license, its cost is $\gamma_0(q) = c_0(q)$. Hence firm 2's cost is $\gamma_t(q) = c_t(q)$ where $t = \varepsilon$ for (a) and $t = 0$ for (b). By Assumption A4 we have $g_t^q < 0$ and $h_t^q < 0$ for all $q > 0$. Then by Lemma A1 ((ii),(iii)), it follows that $q_2(t)$ and $\pi_2(t)$ are both increasing in t, i.e., the NE output and profit of firm 2 in (a) are higher than the output and profit in (b). ■

Proof of Lemma 2 Since for any $\lambda \in \{0, 1\}$ and $r \geq 0$, A3-A5 hold for cost functions of both firms in (3), the existence and uniqueness of NE follow by A1-A5 (for a proof see, e.g., Gaudet and Salant, 1991). Note from A1-A3 that at the NE, the industry output must be positive and lower than Q_0. The following observations will be useful to prove (i)-(ii).

Observation 1 $p_\varepsilon^m > c_\varepsilon'(0)$.

Proof By the first order condition of the monopoly problem: $p_\varepsilon^m = c_\varepsilon'(q_\varepsilon^m) - p'(q_\varepsilon^m)q_\varepsilon^m$. By the mean value theorem, $\exists\ \overline{q} \in (0, q_\varepsilon^m)$ such that $c_\varepsilon'(q_\varepsilon^m) - c_\varepsilon'(0) = q_\varepsilon^m c_\varepsilon''(\overline{q})$. Hence $p_\varepsilon^m - c_\varepsilon'(0) = q_\varepsilon^m[c_\varepsilon''(\overline{q}) - p'(q_\varepsilon^m)]$. By A5, this expression is negative, which proves the result. ■

Observation 2 At the NE, we must have $q_1 > 0$.

Proof By contradiction. Suppose $q_1 = 0$ at the NE. Then (a) $q_2 = \widetilde{q}^m$ (the monopoly output under cost $\widetilde{c}(q) = \lambda[c_\varepsilon(q) + rq] + (1-\lambda)c_0(q)$) and (b) $q_1 = 0$ must be a best response of firm 1 to $q_2 = \widetilde{q}^m$. As π_1 is concave in q_1 (by A2 and A5), for (b) to hold, we must have $\pi_1'(q_1 = 0, q_2 = \widetilde{q}^m) \leq 0$, which holds iff $p(\widetilde{q}^m) \leq c_\varepsilon'(0)$. Since $\widetilde{c}(q) \geq c_\varepsilon(q)$, we have $p(\widetilde{q}^m) \geq p_\varepsilon^m$, implying that $p_\varepsilon^m \leq c_\varepsilon'(0)$, contradicting Observation 1. ■

(i) Suppose firm 2 does not have a license. Then its cost function is $c_0(q)$. By Observation 2, the NE must have either (a) $(q_1 = q_\varepsilon^m, q_2 = 0)$, or (b) $(q_1 > 0, q_2 > 0)$. For (a) to be NE, $q_2 = 0$ must be a best response of firm 2 to $q_1 = q_\varepsilon^m$. As π_2 is concave in q_2 (by A2 and A5), this occurs iff $\pi_2'(q_1 = q_\varepsilon^m, q_2 = 0) \leq 0$ which holds iff $p_\varepsilon^m \leq c_0'(0)$.

To complete the proof of (i), take $i = 2, j = 1$ in Lemma A1. For non drastic technologies both firms produce positive output and the NE is determined from the first order conditions. Firm 2's cost is $\tau(q) = c_0(q)$ and firm 1's cost is $\gamma_t(q) = c_\varepsilon(q)$. Taking $t = \varepsilon$ in Lemma A1, it follows that $g^q(t) < 0, h^q(t) < 0$ for all $q > 0$ (by Assumption A4). The last statement of (i) then follows by Lemma A1 (ii)-(iii).

(ii) Suppose firm 2 has a license with royalty r. Then its cost function is $c_\varepsilon(q) + rq$. By Observation 2, the NE must have either (a) $(q_1 = q_\varepsilon^m, q_2 = 0)$, or (b) $(q_1 > 0, q_2 > 0)$. For (a) to be NE, $q_2 = 0$ must be a best response of firm 2 to $q_1 = q_\varepsilon^m$. As π_2 is concave in q_2 (by A2 and A5), this occurs iff $\pi_2'(q_1 = q_\varepsilon^m, q_2 = 0) \leq 0$ which holds iff $p_\varepsilon^m \leq c_\varepsilon'(0) + r$, i.e., $r \geq \overline{r}(\varepsilon) \equiv p_\varepsilon^m - c_\varepsilon'(0) > 0$.

To complete the proof of (ii), take $i = 1, j = 2$ in Lemma A1. When firm 2 has a license with royalty $r < \overline{r}_\varepsilon$, both firms produce positive output and the NE is determined from the first order conditions. Firm 1's cost is $\tau(q) = c_\varepsilon(q)$ and firm 2's cost is $\gamma_r(q) = c_\varepsilon(q) + rq$. Taking $t = r$ in Lemma A3, we have $g^q(r) = q > 0$ and $h^q(r) = 1 > 0$ for all $q > 0$. The last statement of Lemma 1 then follows from Lemma A1 (i),(iii).

(iii) By (i), for a non drastic technology ε, we have $\pi_\varepsilon^2(\overline{r}_\varepsilon) = 0 < \widehat{\pi}_\varepsilon^2$ and $q_\varepsilon^2(\overline{r}_\varepsilon) = 0 < \widehat{q}_\varepsilon^2$. The first part of (iii) follows by noting that (a) $\pi_\varepsilon^2(r), q_\varepsilon^2(r)$ are both decreasing for $r \in [0, \overline{r}_\varepsilon)$ (part (ii)) and (b) $\pi_\varepsilon^2(0) > \widehat{\pi}_\varepsilon^2$, $q_\varepsilon^2(0) > \widehat{q}_\varepsilon^2$ (i.e. at zero royalty, firm 2's Cournot profit and output are both higher than their no-licensing levels, see Lemma A2).

To prove the second part, let $r = r_\varepsilon^*$. As $q_\varepsilon^2(r_\varepsilon^*) = \widehat{q}_\varepsilon^2$, the first order condition of firm 1 implies $\pi_1'(q_\varepsilon^1(r_\varepsilon^*), q_\varepsilon^2(r_\varepsilon^*)) = \pi_1'(q_\varepsilon^1(r_\varepsilon^*), \widehat{q}_\varepsilon^2) = 0$. Since $\pi_1'(\widehat{q}_\varepsilon^1, \widehat{q}_\varepsilon^2) = 0$ and $\pi_1(q_1, q_2)$ is concave in q_1 for any

q_2, we have $q^1_\varepsilon(r^*_\varepsilon) = \widehat{q}^1_\varepsilon$ and hence $Q_\varepsilon(r^*_\varepsilon) = \widehat{Q}_\varepsilon$. ∎

Lemma A3 *Let $\psi_\varepsilon(r) := \partial\Pi^1_\varepsilon(r)/\partial r$. Denote by $q_\varepsilon(0)$ the Cournot output of each firm when there is licensing with zero royalty. For all $\varepsilon \in S$:*

(i) $\psi_\varepsilon(0) > 0$.

(ii) *If $c_\varepsilon(q)$ is additive, then $\psi_\varepsilon(r) > 0$ for all $r \in [0, \overline{r}_\varepsilon)$.*

(iii) *If $c'_\varepsilon(q)$ is interval-wise decreasing at $q_\varepsilon(0)$, then $\psi_\varepsilon(r) > 0$ for all $r \in [0, \overline{r}_\varepsilon)$.*

(iv) *$\psi_\varepsilon(\overline{r}_\varepsilon) \leq 0$ if $c_\varepsilon(q)$ is superadditive with strict inequality if and only if $c'_\varepsilon(q^m_\varepsilon) > c'_\varepsilon(0)$.*

Proof (i) Differentiating (4) with respect to r, we have (suppressing the subscript ε in quantities and cost functions),

$$\psi_\varepsilon(r) = [p'(Q(r))Q(r) + p(Q(r))]\partial Q(r)/\partial r - \sum_{i=1}^{2} c'(q^i(r))\partial q^i(r)/\partial r$$

As $Q(r) = \sum_{i=1}^{2} q^i(r)$, we have $\partial Q(r)/\partial r = \sum_{i=1}^{2} \partial q^i(r)/\partial r$. Using this above

$$\psi_\varepsilon(r) = [p'(Q(r))Q(r) + p(Q(r)) - c'(Q(r))]\partial Q(r)/\partial r + \sum_{i=1}^{2}[c'(Q(r)) - c'(q^i(r))]\partial q^i(r)/\partial r \quad (9)$$

When there is licensing with no royalty ($r = 0$), firms $1, 2$ have the same Cournot output $q(0) = Q(0)/2$. Evaluating (9) at $r = 0$, we have $\psi_\varepsilon(0) = [p'(Q(0))Q(0)+p(Q(0))-c'(q(0))][\partial Q(r)/\partial r|_{r=0}]$. As $Q(r)$ is decreasing in r for $r \in [0, \overline{r}_\varepsilon)$ (Lemma 2(ii)), we have $\text{sign}[\psi_\varepsilon(0)] = -\text{sign}[p'(Q(0))Q(0)+p(Q(0)) - c'(q(0))]$.

The first order condition of firm 1 in the Cournot duopoly has $p'(Q(r))q^1(r) + p(Q(r)) - c'(q^1(r)) = 0$, so that for $r = 0$, we have $p'(Q(0))q(0)+p(Q(0))-c'(q(0)) = 0$. Hence $\text{sign}[\psi_\varepsilon(0)] = -\text{ sign } [p'(Q(0))q(0)] > 0$ (since $p' < 0$).

(ii) Let $\phi(q) = p(q)q - c(q)$ be the profit function of a monopolist. From (9), we have

$$\psi_\varepsilon(r) = \phi'(Q(r))\partial Q(r)/\partial r + \sum_{i=1}^{2}[c'(Q(r)) - c'(q^i(r))]\partial q^i(r)/\partial r \quad (10)$$

As $Q(r) > q^m_\varepsilon$ for $r \in [0, \overline{r}(\varepsilon))$, we have $\phi'(Q(r)) < 0$. As $Q(r)$ is decreasing in r, the first term of (10) is positive. If $c(q)$ is additive, then $c'(q) = c'(0)$ for all $q > 0$, so the second term of (10) is zero, proving that $\psi_\varepsilon(r) > 0$.

(iii) Suppose $c'_\varepsilon(q)$ is interval-wise decreasing at $q(0)$. Since $q^2(r) \leq q(0) \leq q^1(r)$ for any $r \in [0, \overline{r}(\varepsilon))$ and $\partial q^2(r)/\partial r < 0 < \partial q^1(r)/\partial r$, the second term of (10) is bounded below by $[c'(Q(r)) - c'(q(0))]\partial Q(r)/\partial r$. By Lemma 2(ii), we have $Q(r) > Q(\overline{r}_\varepsilon) = q^m_\varepsilon$ and $q(0) = q^1(0) < q^1(\overline{r}_\varepsilon) = q^m_\varepsilon$. Hence $Q(r) > q(0)$. Interval-wise decreasing property then implies $c'(Q(r)) < c'(q(0))$. As $\partial Q(r)/\partial r < 0$, the second term of (10) is bounded below by zero. Since the first term is positive, we conclude that $\psi_\varepsilon(r) > 0$.

(iv) Evaluating (10) at $r = \overline{r}_\varepsilon$ (taking the derivative $\partial\Pi^1_\varepsilon(r)/\partial r$ from the left) and noting that $Q(\overline{r}_\varepsilon) = q^1(\overline{r}_\varepsilon) = q^m_\varepsilon$ and $q^2(\overline{r}_\varepsilon) = 0$, we have

$$\psi_\varepsilon(\overline{r}_\varepsilon) = \phi'(q^m_\varepsilon)[\partial Q(r)/\partial r|_{r=\overline{r}_\varepsilon}] + [c'(q^m_\varepsilon) - c'(0)][\partial q^2(r)/\partial r|_{r=\overline{r}_\varepsilon}].$$

The first term above is zero. Since $q^2(r)$ is decreasing in r, we have $\text{sign}[\psi_\varepsilon(\overline{r}_\varepsilon)] = -\text{ sign } [c'(q^m_\varepsilon) - c'(0)]$. Then the result follows by Lemma 1. ∎

Proof of Proposition 1 Lemma A3(i) implies that $\Pi^1_\varepsilon(r)$ is increasing at $r = 0$, which proves the result. ■

Proof of Lemma 3 (i) For brevity, we drop the subscript ε from all expressions except c_ε. From (4) and by Lemma 2(iii), firm 1's payoff under the policy $(r^*, \alpha(r^*))$ is $\Pi^1(r^*) = p(\widehat{Q})\widehat{Q} - c_\varepsilon(\widehat{q}^1) - c_\varepsilon(\widehat{q}^2) - [p(\widehat{Q})\widehat{q}^2 - c_0(\widehat{q}^2)] = \widehat{\pi}^1 + [c_0(\widehat{q}^2) - c_\varepsilon(\widehat{q}^2)] > \widehat{\pi}^1$.

(ii) The first order condition of firm 2 under no licensing implies $p'(\widehat{Q})\widehat{q}^2 + p(\widehat{Q}) - c_0'(\widehat{q}^2) = 0$. Since $q^2(r^*) = \widehat{q}^2, Q(r^*) = \widehat{Q}$, when there is licensing with $r = r^*$, the first order condition of firm 2 implies $p'(\widehat{Q})\widehat{q}^2 + p(\widehat{Q}) - c_\varepsilon'(\widehat{q}^2) - r^* = 0$. From these two equations, we have $r^* = c_0'(\widehat{q}^2) - c_\varepsilon'(\widehat{q}^2) = F'(\widehat{q}^2)$. Since $\pi^2(r^*) - \widehat{\pi}^2 = c_0(\widehat{q}^2) - c_\varepsilon(\widehat{q}^2) - r^*\widehat{q}^2 = F(\widehat{q}^2) - r^*\widehat{q}^2$, using (6) and the value of r^* we have $\pi^2(r^*) - \widehat{\pi}^2 = F(\widehat{q}^2)[1 - \mathcal{H}(\widehat{q}^2)]$. Hence, if $\mathcal{H}(\widehat{q}^2) \leq 1$, then we have $\pi^2(r^*) \geq \widehat{\pi}^2 = \pi^2(\widehat{r})$ implying that $r^* \leq \widehat{r}$. The last part of (ii) follows from part (i). ■

Proof of Proposition 3 As $\mathcal{H}(\widehat{q}^2_\varepsilon) \leq 1$, we know from Lemma 3 that licensing occurs. As no $r > \widehat{r}_\varepsilon$ is acceptable to firm 2, consider licensing policies $(r, \alpha_\varepsilon(r))$ where $r \in [0, \widehat{r}_\varepsilon]$.

(I) If $c_\varepsilon(q)$ is additive, then by Lemma A3 it follows that $\Pi^1_\varepsilon(r)$ is increasing for $r \in [0, \widehat{r}_\varepsilon]$, implying that the unique optimal licensing policy is to set $r = \widehat{r}_\varepsilon$. The corresponding upfront fee is $\alpha(\widehat{r}_\varepsilon) = \pi^2_\varepsilon(\widehat{r}_\varepsilon) - \widehat{\pi}^2_\varepsilon = \widehat{\pi}^2_\varepsilon - \widehat{\pi}^2_\varepsilon = 0$.

(II) To prove (II), note that $\pi^2_\varepsilon(r)$ is decreasing for $r \in [0, \overline{r}_\varepsilon]$ and $\pi^2_\varepsilon(\widehat{r}_\varepsilon) = \widehat{\pi}^2_\varepsilon$. If $\varepsilon = \varepsilon_D$, then $\widehat{\pi}^2_\varepsilon = \pi^2_\varepsilon(\overline{r}_{\varepsilon_D}) = 0$ (since $\varepsilon = \varepsilon_D$ corresponds to a drastic technology). As $\pi^2_\varepsilon(\widehat{r}_\varepsilon) = \widehat{\pi}^2_\varepsilon$, we have $\widehat{r}_{\varepsilon_D} = \overline{r}_{\varepsilon_D}$.

Consider the function $\psi_\varepsilon(r) = \partial\Pi^1_\varepsilon(r)/\partial r$. As $\widehat{r}_{\varepsilon_D} = \overline{r}_{\varepsilon_D}$, by the continuity of $\psi_\varepsilon(r)$ with respect to ε we have

$$\lim_{\varepsilon\uparrow\varepsilon_D} \psi_\varepsilon(\widehat{r}_\varepsilon) = \psi_{\varepsilon_D}(\widehat{r}_{\varepsilon_D}) = \psi_{\varepsilon_D}(\overline{r}_{\varepsilon_D}) \tag{11}$$

As Lemma A3 holds for all $\varepsilon \in [0, \overline{\varepsilon}]$, in particular it holds for $\varepsilon = \varepsilon_D$. Under the conditions on $c_{\varepsilon_D}(q)$, we have $\psi_{\varepsilon_D}(\overline{r}_{\varepsilon_D}) < 0$ and by (11), so is $\lim_{\varepsilon\uparrow\varepsilon_D} \psi_\varepsilon(\widehat{r}_\varepsilon)$. Consequently for all values of ε that are sufficiently close to ε_D, we have $\psi_\varepsilon(\widehat{r}_\varepsilon) < 0$, i.e., $\exists\ 0 < \widehat{\varepsilon} < \varepsilon_D$ such that for all $\varepsilon \in (\widehat{\varepsilon}, \varepsilon_D)$, we have $\psi_\varepsilon(\widehat{r}_\varepsilon) < 0$, implying that $\Pi^1_\varepsilon(r)$ is decreasing at $r = \widehat{r}_\varepsilon$. Hence it is optimal for firm 1 to choose $r \in (0, \widehat{r}_\varepsilon)$. In that case, $\pi^2_\varepsilon(r) > \widehat{\pi}^2_\varepsilon$ and the upfront fee is $\pi^2_\varepsilon(r) - \widehat{\pi}^2_\varepsilon > 0$. ■

Proof of Proposition 4 As $\mathcal{H}(\widehat{q}^2_\varepsilon) \leq 1$, we know from Lemma 3 that licensing occurs. As no $r > \widehat{r}_\varepsilon$ is acceptable to firm 2, consider licensing policies $(r, \alpha_\varepsilon(r))$ where $r \in [0, \widehat{r}_\varepsilon]$. If $c_\varepsilon'(q)$ is interval-wise decreasing at $q = q_\varepsilon(0)$, then by Lemma A3 it follows that $\Pi^1_\varepsilon(r)$ is increasing for $r \in [0, \widehat{r}_\varepsilon]$, implying that the unique optimal licensing policy is to set $r = \widehat{r}_\varepsilon$. The corresponding upfront fee is $\alpha(\widehat{r}_\varepsilon) = \pi^2_\varepsilon(\widehat{r}_\varepsilon) - \widehat{\pi}^2_\varepsilon = \widehat{\pi}^2_\varepsilon - \widehat{\pi}^2_\varepsilon = 0$. ■

Acknowledgements

For helpful comments, we are grateful to two anonymous reviewers and Tsogbadral Galaabaatar.

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