Technology Shocks, the Service Sector and Economic Growth

Mitra Thakur, Gogol

Jawaharlal Nehru University, New Delhi, India

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Gogol Mitra Thakur*†

Abstract

Advances in ICTs as well as financial developments have greatly increased the scope for joint utilisation of various industrial goods and services. For example, consumption of many durable goods like telecommunication equipment (e.g. mobile sets), various electronic products, computer hardware and automobiles leads to joint purchases of services such as telecommunications, software services, insurance and other financial services. In this paper, we propose a specification for demand interlinkage between industry and the service sector, indicative of such developments, wherein final demand for service not only depends on industrial output but also on the relative price of service. This specification implies that a labour productivity increase in the service sector, say due to adoption of ICTs, can generate enough demand to increase both the growth rate in the economy and the relative size of the service sector if demand for service per unit industrial output is sufficiently elastic with respect to its relative price.

JEL Classification: O11; O14; O41

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1. Introduction

The world economy today is a predominantly service economy. Services contributed 70 percent of the world GDP in the year 2013. In case of high income economies, the

*Ph.D. Candidate, Centre for Economic Studies and Planning, Jawaharlal Nehru University, New Delhi-110067, India. Email: gogol86@gmail.com

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average share of services in GDP was 74 percent in the same year and the share of services in male and female employment for the period 2009-2012 were 64 percent and 86 percent respectively.[1] Traditionally, however, a rising share of the service sector in any economy has been perceived as a matter of concern. This view, formalized by the two-sector 'unbalanced growth' model of Baumol (1967), posits that the inherent nature of services is such that labour productivity improvements are rare phenomena. Therefore, if output share of the service sector does not decline then resources continuously shift away from more productive sectors to the service sector, causing stagnation in the economy.

Advances in information and communication technologies (ICTs) and their rapid adoption in many services have ensured that this traditional view regarding expansion of services has few takers today.[2] Particularly important in this regard is the fact that ICT using services such software and IT, telecommunications, banking & finance have emerged as important sectors in not only advanced economies but also in developing economies such as India. According to Eichengreen and Gupta (2013), ICT using services are driving the expansion in the output share of the service sector at much lower levels of per capita income after 1990 than before. Most contributions in the literature that attempt to reverse Baumol's negative relationship between the expansion of the service sector and economic growth focus on the importance of various services for endogenous productivity and output growth in the economy.[3] This paper, however, draws attention to the implication of adoption of ICTs in various services for economic growth and structural change. In that, we focus on two aspects. First, new kinds of demand interlinkages between the service sector and industry ushered in by development of ICTs (as well as financial developments). Second, possible increase in labour productivity of the service sector due to adoption of ICTs.

The importance of demand interlinkages between sectors for economic growth

[1]Source: Table 2.3 and Table 4.2 of *World Development Indicators* (2015) for employment shares and GDP share respectively.

[2]Adoption of ICTs in services not just limited to advanced economies. For example, Qiang et al. (2006), using data from Investment Climate Surveys (ICS) conducted by the World Bank between 1999 and 2003 covering 20,000 firms from 26 sectors in 56 low- and middle-income countries, find that 55 and 50 percent of service firms use e-mails and websites, respectively, to interact with clients and in both use of websites and percentage of employees using computers, service sector firms are much ahead of manufacturing firms. Among the various services are telecommunications and IT services, real estate and hotels & restaurants were the heaviest users of e-mails and websites whereas percentage of employees using computers was highest (67 percent) in case of accounting and finance sector.

[3]Section 2 contains a short review of this literature.

was stressed upon by Kaldor (1989, pp. 431-432), who argued, "...industrial growth is dependent on the exogenous component of demand for industry...growth of purchasing power of the primary sector..." A crucial assumption that allows Kaldor to arrive at the above conclusion is availability of an unlimited supply of labour to industry at a subsistence wage rate fixed in terms of food. On the contrary, as pointed out by Dutt (1992), the only factor of production in both sectors, labour, in Baumol's model is always fully employed in the economy and, therefore, expansion of the service sector necessarily shift resources away from the more productive sector. Dutt (1992) considers a two-sector growth model consisting of a productive sector and an unproductive sector, where the latter is meant to represent overlapping sets of service, non-market and unproductive activities.[4] Contrary to Baumol, Dutt assumes that resources are not fully employed and shows that the two sectors can grow in a balanced manner because of demand interlinkages. In this paper, we show that Dutt's framework also implies that an exogenous increase in labour productivity of the industry will increase both the relative size of the sector in the economy and the aggregate growth rate of the economy. On the other hand, a similar increase in the labour productivity of the service sector has the opposite effect. This negative association of exogenous increase in labour productivity of the service sector and the growth rate implied by Dutt's model is slightly disappointing. This is because widespread adoption ICTs in various service activities can be expected to have a positive impact on the labour productivity of the service sector.

However, we also show that in models such as that of Dutt, where resources are not fully utilised and there are demand interlinkages between the two sectors, implications of sector-specific technology shocks for growth and structural change depend upon the specification of demand interlinkages. This leads to the main argument of this paper, that in contemporary times advances in ICTs as well as financial developments have greatly increased the scope for joint utilisation various industrial goods and services. For example, consumption of many durable goods like telecommunication equipment (e.g. mobile sets), various electronic products and computer hardware make sense only if purchased with telecommunication and software services. Similarly, because of financial developments, purchase of durable goods like automobiles give rise to purchase of insurance and other financial services. Even in case of investment demand, firms might employ financial and business consultancy services in order to arrange financing for their investments. We propose a specification for demand interlinkage

[4] In this paper, we narrowly interpret Dutt's two sectors as industry and service.

between industry and the service sector, indicative of such developments, wherein final demand for service not only depends on the industrial output but also on the relative price of service. We show that if demand for service per unit industrial output is sufficiently elastic with respect to its relative price then an exogenous increase in the labour productivity of the service sector, say due to adoption of ICTs, generates enough demand to increase not only the growth rate of the economy but also the relative size of the service sector.

As regard to the structure of the paper, the next section presents the 'unbalanced growth' model of Baumol (1967) and also provides a brief review of recent theoretical contributions that emphasise on the contribution of various services towards endogenous productivity and output growth in order to counter Baumol's argument. Section 3 presents the two-sector demand constrained growth model of Dutt (1992) which emphasises balanced growth between industry and the service sector as a result of demand interlinkages between the two. We examine the effects of sector-specific technology shocks on the balanced growth rate of the model. Further, using two simple variants of this model, we show that effects of sector specific technology shocks on both the growth rate and the structure of the economy are sensitive to specifications of demand interlinkages between the two sectors. In section 4 we present a two sector demand constrained growth model similar to Dutt (1992), where the demand for a service generated per unit of industrial output is negatively related to the relative price of the service. In this model we show improvements in labour productivity of the service sector can increase both the growth rate of the economy as well as the relative size of the service sector. Finally, section 5 concludes the paper.

2. Service Sector and Stagnation

Baumol (1967) argues that labour productivity increase in services is at best sporadic compared to industry, where it rises in a cumulative fashion. As a result, if the ratio of outputs of the service sector and industry is not allowed to decline then resources shift towards service sector away from 'technologically progressive' industry causing stagnation in the economy. In the 'unbalanced growth' model of Baumol (1967) there are two sectors - the industrial sector, which produces a single good, and the service sector, which produces a single service. Production technologies of the two sectors are specified as $X_j = x_j L_j$ where $j \in \{i, s\}$ with i and s denoting the industrial sector and the service sector respectively.[5] X_j and L_j represents output and employment in sector j. Labour is the only factor of production in both the sectors and is fully employed in the economy, i.e. $L_i + L_s = L$, which is the total labour supply in the economy. x_j is labour productivity in sector j. Baumol assumes that x_i grows exponentially at a constant rate, say $\eta > 0$, whereas x_s is a constant. Thus if $\frac{X_i}{X_s}$ does not increase sufficiently then employment share of the service sector must approach one over time as $\frac{L_i}{L_s} = \frac{x_s X_i}{x_i X_s}$ approaches zero because $\frac{x_i}{x_s}$ grows at the constant rate η. Further, the growth rate of aggregate labour productivity approaches zero in this case. Since it is generally agreed that services have greater income elasticity of demand than industrial goods, particularly at higher levels of per capita income, the 'unbalanced growth' model of Baumol (1967) predicts that as economies develop, more and more resources will shift to the provisioning of technologically stagnant services causing stagnation. This association of the service sector with stagnation led Rowthorn and Ramaswamy (1997, p. 22) to argue that "...growth of living standards in the advanced economies is likely to be increasingly influenced by productivity developments in the service sector". However, note that merely allowing labour productivity growth in the service sector does not prevent a decline in the growth rate in Baumol's model. Growth rates of labour productivity in the two sectors have to be exactly equal.

In Baumol's model both sectors produce only final output. Oulton (2001), therefore, argues that the 'unbalanced growth' model of Baumol (1967) is not suitable for explaining implications of expansion of services such as business services, that are primarily required as intermediate inputs, for growth. In a two-sector model where the single service is required just as an intermediate input in industry, Oulton (2001) shows that under the assumption of perfect competition, a slower rate of labour productivity growth in service sector does not necessarily imply increase in the service sector's share of primary input usage. Further, if the elasticity of substitution between the service input and the primary input in industry is greater than one and the growth rate of labour productivity in the service sector is positive then the service sector's share of primary input usage asymptotically increases to approach one and the growth rate of total factor productivity (TFP) increases to approach the sum of the labour productivity growth rates of the two sectors. However, Sasaki (2007) using a CES production function for industry shows that once final demand for service is included in Oulton's model, a slower growth rate of labour productivity in the service

[5] Throughout this paper, notations with subscript i refer to the industry sector and notations with subscript s refer to the service sector.

sector ultimately causes a decline in the growth rate of TFP if the consumption ratio of the service and the industrial good is held constant. This result of Sasaki (2007) suggests that just highlighting the role of services as an intermediate input is not enough to counter the gloomy predictions of Baumol (1967).

Quite a few of the other contributions in the literature, which deal with this issue, resort to endogenous growth theory. For example Pugno (2006) extends Baumol's model by including human capital stock of the economy in the production functions of both sectors. Pugno (2006) argues that consumption of services like health, education and cultural services contributes towards human capital formation. Using a linear human capital production function, Pugno shows that expansion of these services need not necessarily lead to a decline in growth rate of the economy so long as their contribution towards human capital formation is substantial. Similarly, Vincenti (2007), in a model based on two hypotheses - service sector produces a positive externality on industry, via R & D and general human capital improvements, and 'learning by doing' in both sectors - shows that the share of service employment can be positively related to the growth rate of the economy. Sasaki (2012) combines 'learning by doing' in industry along with the hypothesis of Pugno (2006) that consumption of services leads to human capital formation and generates a U-shaped relationship between the growth rate of the economy and the employment share of the service sector. De (2014) argues that services such as finance, insurance, software and various other business services that use ICTs are part of the 'new economy' and contribute towards creation of 'intangible capital'. Extending the Uzawa-Lucas model by including 'intangible capital' as a separate non-rival but excludable factor in the production of the final good and a separate sector for its production, De (2014) shows that accumulation of 'intangible capital' can result in sustained growth in the economy.

Although these contributions highlight the importance of various services for endogenous technological progress and growth, it is important to realise that the negative relation between the expansion of the service sector and economic growth as implied by Baumol (1967) is to a large extent determined by the macroeconomic structure of the model. Particularly consequential is the assumption of full employment of labour because of which any expansion in the service sector necessarily shifts resources away from the more productive industry sector. This point is made by Dutt (1992), who also shows that if resources are not fully employed there can be balanced growth between industrial and service sectors, with each sector generating demand for the other. In the next section we discuss the demand constrained two-sector model of Dutt (1992).

3. Balanced Growth between Industry & Service

Unlike Baumol, Dutt (1992) assumes that production in both the sectors require both capital and labour as inputs. The industrial sector produces a tangible good which is used both as consumption good and as capital good. The service sector produces an intangible service which is required as an overhead input in the industrial sector in a constant proportion, say $\lambda > 0$, to its capital stock. Thus, the total service input required by the industrial sector is

$$N_s = \lambda K_i \tag{1}$$

where K_i is the capital stock of the industrial sector. There is no technological progress. Both the sectors are both assumed to be characterized by the presence of excess capacity and imperfect competition. Price in both the sectors is determined by applying a fixed mark-up on unit prime cost in the following manner.

$$P_j = (1 + z_j)\frac{W}{x_j} \tag{2}$$

where P_j, x_j and z_j are price, labour productivity and price mark-up in sector $j \in \{i, s\}$. x_j and z_j are assumed to be positive constants for all $j \in \{i, s\}$. Nominal wage W is exogenously given and is assumed to be the same in both the sectors. These assumptions imply that the relative price of the service in terms of the industrial good, p, is a constant as shown below.

$$p = \frac{P_s}{P_i} = \frac{(1 + z_s)x_i}{(1 + z_i)x_s} \tag{3}$$

Real wage in terms of the industrial good, $\frac{W}{P_i}$, is a positive constant following assumptions regarding W and P_i. Both sectors can employ as much labour as they require at this real wage.

There is capital accumulation in both the sectors. Dutt (1992) assumes that rates of investment of the two sectors are increasing linear functions of their respective rates of capacity utilization. Let X_j, K_j and I_j be output, capital stock and investment of sector $j \in \{i, s\}$. Then rates of capacity utilization in the industrial service sectors are $\frac{X_i}{K_i}$ and $\frac{X_s}{K_s}$ respectively. The rate of investment of sector $j \in \{i, s\}$ is given by

$$\frac{I_j}{K_j} = \alpha_j + \beta_j \frac{X_j}{K_j} \tag{4}$$

α_j and β_j, for all $j \in \{i, s\}$, are positive constants. There is no depreciation of capital.[6] Savings behaviour in the model is such that all wages and a fraction of profits in the economy are used for consumption. Consumption expenditure incurred on the industrial good then, using (2), is $C_i = P_i X_i/(1 + z_i) + P_s X_s/(1 + z_s) + (1 - s)\{z_i P_i X_i/(1 + z_i) + z_s P_s X_s/(1 + z_s) - P_s X_s\}$ or,

$$C_i = P_i X_i - \frac{s z_i P_i X_i}{1 + z_i} + \frac{s P_s X_s}{1 + z_s} \tag{5}$$

where $s \in (0, 1)$ is a constant.

In the short run of the model, the capital stock of both the sectors -K_i and K_s- are assumed to be given. Since prices are fixed any mis match between demand and supply in the two sectors is corrected via adjustments of output of respective sectors. The short-run dynamics can be represented in the following manner. For $j \in \{i, s\}$,

$$\dot{X}_j = \psi_j [d_j - X_j] \tag{6}$$

where, for all $j \in \{i, s\}$, $\dot{X}_j$ is the time derivative of X_j, ψ_j is a positive constant and d_j is real demand of sector j's output. By definition, $d_i = \frac{C_i}{P_i} + I_i + I_s$ and $d_s = N_s$. Substituting for d_i and d_s, using (1), (4) and (5), in (6) reduces the short run dynamics of the model to

$$\begin{aligned} \dot{X}_i &= \psi_i [-(\frac{s z_i}{1 + z_i} - \beta_i) X_i + (\frac{sp}{1 + z_s} + \beta_s) X_s + \alpha_i K_i + \alpha_s K_s] \\ \dot{X}_s &= \psi_s [-X_s + \lambda K_i] \end{aligned} \tag{7}$$

Short-run equilibrium requires $X_i > 0$ and $X_s > 0$ such that $\dot{X}_i = \dot{X}_s = 0$. Let $X_i^* = [\alpha_i K_i + \alpha_s K_s + \lambda K_i \{sp/(1 + z_s) + \beta_s\}]/\Omega$ and $X_s^* = \lambda K_i$ where $\Omega = \frac{s z_i}{1+z_i} - \beta_i$.

Proposition 1. *If $\Omega > 0$ then (X_i^*, X_s^*) is a unique and asymptotically stable short-run equilibrium of (7).*

[6]Constant rates of depreciation in both the sectors can be easily accommodated in such models without any significant effect on the conclusions. For simplicity of exposition, through out this paper, we are going to assume that there is no depreciation of capital.

Proof. Suppose $\Omega > 0$. Setting the right hand sides of the two equations in (7) equal to zero yields the following system of linear equations.

$$\begin{bmatrix} -(\frac{sz_i}{1+z_i} - \beta_i) & (\frac{sp}{1+z_s} + \beta_s) \\ 0 & -1 \end{bmatrix} \begin{bmatrix} X_i \\ X_s \end{bmatrix} = \begin{bmatrix} -(\alpha_i K_i + \alpha_s K_s) \\ -\lambda K_i \end{bmatrix} \tag{8}$$

Ω is the determinant of 2×2 matrix in (8). Since $\Omega \neq 0$,*Cramer's rule* yields the unique solution of (8) as $X_i = [\alpha_i K_i + \alpha_s K_s + \lambda K_i\{sp/(1+z_s) + \beta_s\}]/\Omega = X_i^*$ and $X_s = \lambda K_i = X_s^*$. $\Omega > 0$ implies $X_i^* > 0$ and $X_s^* > 0$ as α_i, α_s, K_i, K_s, λ, s, p, z_s and β_s are all positive. For stability, notice that the Jacobian matrix for (7) is

$$\begin{bmatrix} -\psi_i(\frac{sz_i}{1+z_i} - \beta_i) & \psi_i(\frac{sp}{1+z_s} + \beta_s) \\ 0 & -\psi_s \end{bmatrix}$$

with determinant $\psi_i\psi_s\Omega > 0$ and trace $-\psi_i\Omega - \psi_s < 0$ when $\Omega > 0$, as ψ_i and ψ_s are both positive. $\square$

In the long run, Dutt considers capital accumulation in the two sectors as a result of investments carried out in the short equilibrium described above. For this analysis it is assumed that $\Omega > 0$ (note that in this case Ω is the reciprocal of the expenditure multiplier for industrial output) and the economy is always in a short-run equilibrium given by X_i^* and X_s^*.[7] In the absence of depreciation, growth rate of capital stock of sector j, say g_j, is equal to its rate of investment $\frac{I_j}{K_j}$ where $j \in \{i, s\}$. Substituting (X_i^*, X_s^*) for (X_i, X_s) in (4) gives the following expressions for g_i and g_s.

$$g_i = \alpha_i + \frac{\beta_i}{\Omega}\{\alpha_i + \frac{\alpha_s}{k} + \lambda(\frac{sp}{1+z_s} + \beta_s)\} \tag{9}$$

$$g_s = \alpha_s + \beta_s\lambda k \tag{10}$$

where $k = \frac{K_i}{K_s}$ is the relative capital stock of industry sector vis-a-vis the service sector, hence forth referred to as the relative capital stock of the industrial sector. The long-run dynamics of the model is captured by changes in the relative capital stock of the industrial sector k because of different rates of growth of capital stocks

[7] Implicitly it is also being assumed that full capacity output-capital ratios of the two sectors- say, $\bar{u}_j$ where $j \in \{i, s\}$- are such that, given K_i and K_s, X_i^* and X_s^* allow for excess capacity in both the sectors. We make this assumption regarding $\bar{u}_i$ and $\bar{u}_s$ throughout the rest of this paper.

of the two sectors. For all $k > 0$, the rate of change in k is

$$\dot{k} = k[g_i - g_s] \tag{11}$$

Substituting for g_i from (9) and g_s from (10) in (11), we obtain, for all $k > 0$,

$$\dot{k} = k[\alpha_i + \frac{\beta_i}{\Omega}\{\alpha_i + \frac{\alpha_s}{k} + \lambda(\frac{sp}{1+z_s} + \beta_s)\} - \alpha_s - \beta_s \lambda k] \tag{12}$$

Existence of steady state in the long run requires $\dot{k} = 0$ in (12) for some $k > 0$. Let $k^* = \frac{-b-\sqrt{b^2-4ac}}{2a}$ where $a = -\beta_s\lambda\Omega$, $b = (\alpha_i - \alpha_s)\Omega + \alpha_i\beta_i + \beta_i\lambda(\frac{sp}{1+z_s} + \beta_s)$ and $c = \alpha_s\beta_i$.

Proposition 2. *Given $\Omega > 0$, k^* is a unique and asymptotically stable steady state of* (12) *in* $\mathbb{R}_{++}$.

Proof. We can rearrange (12) as $\dot{k} = \frac{ak^2+bk+c}{\Omega}$ where $a = -\beta_s\lambda\Omega$, $b = (\alpha_i - \alpha_s)\Omega + \alpha_i\beta_i + \beta_i\lambda(\frac{sp}{1+z_s} + \beta_s)$ and $c = \alpha_s\beta_i$. In the steady state $ak^2 + bk + c = 0$ as $\Omega > 0$. Now $a < 0$ and $c > 0$ as α_s, β_i, β_s, λ and Ω are all positive. $a < 0$ and $c > 0$ imply $b^2 - 4ac > 0$. Therefore $ak^2 + bk + c = 0$ has two distinct real roots, $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$. Since $a < 0$, the steady state value of k is $k^* = \frac{-b-\sqrt{b^2-4ac}}{2a}$. For stability, define a function $V : \mathbb{R}_{++} \mapsto \mathbb{R}$ such that $V(k) = (g_i - g_s)^2$, where g_i and g_s are given by (9) and (10) respectively. Notice that, by definition, $V(k^*) = 0$ and $V(k \neq k^*) > 0$ for all $k \in \mathbb{R}_{++}$. Also, using (9), (10) and (11), $\dot{V} = -2k(g_i - g_s)^2(\frac{\beta_i\alpha_s}{\Omega k^2} + \beta_s\lambda) < 0$ for all $k \in \mathbb{R}_{++}$ and $k \neq k^*$ as Ω, α_s, β_i, β_s and λ are all positive. Thus, V is a *strict Liapunov function* for k^*.[8] □

Figure 1 illustrates the intuition behind Proposition 1. In this figure, we show the relative capital stock of the industrial sector k on the x-axis and the growth rates of capital stocks of the two sectors g_i and g_s on the y-axis. The downward sloping curve g_i represents (9) and the upward sloping line g_s represents (10). These two curves intersect at k^*, which is the long-run steady state. At k^* capital stocks of both the sectors grow at the same rate g^*. g^* is a positive constant, as can be checked by substituting k^* in either (9) or (10). Moreover, since labour productivity is constant in both sectors, it follows that growth rates of output as well as employment of both sectors is g^* at the steady state. In Figure 1, at any $k < k^*$, the industrial sector

[8] On Liapunov stability theorem see, for example, Hirsch et al. (2004, pp. 194-195).

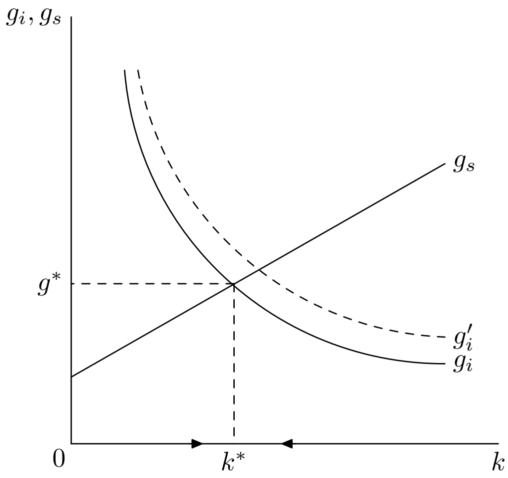

Figure 1: Balanced growth of industry and service in Dutt (1992)

accumulates at a higher rate than the service sector so, from (**??**), k increases and continues to increase so long as it is below k^*. Similarly, at any $k > k^*$ the service sector accumulates at a faster rate than the industrial sector causing a decreases in k, which continues to decrease so long it is above k^*.

Dutt (1992) offers two conclusions from this model. First, an increase in λ increases the growth rate of the economy. This is fairly obvious as, from (9) and (10), an increase in λ will shift both g_i and g_s curves upwards in Figure 1. Second, there is no inverse relation between expansion of the service sector and the growth rate of the economy if the latter is measured by g_i. This conclusion is rather peculiar, because outside the steady state, the rate of accumulation of the industrial sector can not be taken as the growth rate of the economy in the model. Moreover since the steady state is globally stable, it is more natural to consider the rate at which both sectors grow at the steady state as the growth rate of the economy. Changes in the steady state growth rate depend on the nature of shifts in g_i and g_s curves. For example, if for some reason g_i curve shifts downwards in Figure 1 and the g_s curve remains unaffected then there is a relative expansion of the service sector and a decline in the growth rate. Analyzing the implications of an exogenous labour productivity increase in the industrial sector on the steady state appears to provide a more interesting comparison of this demand constrained model of Dutt (1992) with Baumol (1967).

An increase in x_i increases the relative price of service, as p in (3) is an increasing function of x_i. This in turn increases the growth rate of capital stock of the industrial sector for all k but, given k, has no effect on the growth rate of capital stock of the service sector, from (10). In Figure 1 this is shown as an upward shift of the schedule for growth rate of capital stock of the industrial sector to shift upwards from g_i to g_i' in response to an increase in x_i. The new steady state is given by the intersection point of the dashed curve g_i' and the upward sloping line g_s at which both relative capital stock of the industrial sector and the growth rate of economy are greater. Thus, contrary to the conclusions of Baumol (1967), in the demand constrained model of Dutt (1992) an exogenous labour productivity increase in the industrial sector increases both the relative size of the sector (measured in terms of the intersectoral ratio of capital stocks) and the growth rate of the economy. On the other hand, an increase in the labour productivity of the service sector has the opposite effect on the steady state because in this case p decreases as it is a decreasing function of x_s in (3). Proposition 3 proves this formally.

Proposition 3. *Given that* $\Omega > 0$. *Let* $g^* = \alpha_i + \frac{\beta_i}{\Omega}\{\alpha_i + \frac{\alpha_s}{k^*} + \lambda(\frac{sp}{1+z_s} + \beta_s)\} = \alpha_s + \beta_s \lambda k^*$. *Then* $\frac{\partial g^*}{\partial p} > 0$.

Proof. Suppose $\Omega > 0$. By the definition of g^*, $\frac{\partial g^*}{\partial p} = \beta_s \lambda \frac{\partial k^*}{\partial p}$. Thus $\frac{\partial g^*}{\partial p} > 0$ if and only if $\frac{\partial k^*}{\partial p} > 0$ as β_s and λ are positive. Now,

$$\frac{\partial k^*}{\partial p} = -\frac{1}{2a}\{1 + \frac{b}{\sqrt{b^2 - 4ac}}\}\frac{\partial b}{\partial p}$$

Since $b = (\alpha_i - \alpha_s)\Omega + \alpha_i\beta_i + \beta_i\lambda(\frac{sp}{1+z_s} + \beta_s)$, $\frac{\partial b}{\partial p} = \frac{\beta_i \lambda s}{1+z_s} > 0$ as β_i, λ and z_s are positive. Also $a = -\beta_s\lambda\Omega < 0$ and $c = \alpha_s\beta_i > 0$ as Ω, β_s, λ, α_s and β_i are positive. $a < 0$ and $c > 0$ imply $b^2 - 4ac > b^2$. Then, it must be that $-1 < \frac{b}{\sqrt{b^2-4ac}} < 1$ or $\{1 + \frac{b}{\sqrt{b^2-4ac}}\} > 0$. Thus, it follows that $\frac{\partial k^*}{\partial p} > 0$. ☐

3.1 Alternative Specifications for Demand Interlinkages

Implications of sector specific technology shocks in the demand constrained model of Dutt (1992), given by Proposition 3, are not so much driven by the fact that resources are not fully utilised - existence of surplus labour and excess capacity - but by the specification as well as the functional forms of demand interlinkages between the two sectors. To bring this out, let us separately consider two variants of this model. First, instead of assuming that production in the industrial sector requires the service as an overhead input, let us assume that it requires the service as an intermediate input.

Specifically let $N_s = \lambda_1 X_i$ where λ_1 is a positive constant. Substituting for d_i and d_s in (6), using (4), (5) and $N_s = \lambda_1 X_i$, we can represent the short run dynamics in this case as the following system of differential equations.

$$\begin{aligned}\dot{X}_i &= \psi_i[-(\frac{sz_i}{1+z_i} - \beta_i)X_i + (\frac{sp}{1+z_s} + \beta_s)X_s + \alpha_i K_i + \alpha_s K_s] \\ \dot{X}_s &= \psi_s[\lambda_1 X_i - X_s]\end{aligned} \tag{13}$$

Short-run equilibrium now requires that there exists $X_i > 0$ and $X_s > 0$ such that $\dot{X}_i = \dot{X}_s = 0$ in (13). Let $X^*_{i1} = \frac{\alpha_i K_i + \alpha_s K_s}{\Omega_1}$ and $X^*_{s1} = \frac{\lambda(\alpha_i K_i + \alpha_s K_s)}{\Omega_1}$ where $\Omega_1 = \frac{sz_i}{1+z_i} - \beta_i - \lambda_1(\frac{sp}{1+z_s} + \beta_s)$.[9]

Proposition 4. *If $\Omega_1 > 0$ then (X^*_{i1}, X^*_{s1}) is a unique and asymptotically stable short-run equilibrium of (13).*

Proof. Similar to the proof of Proposition 1, see appendix A. □

Assuming $\Omega_1 > 0$ and substituting (X^*_{i1}, X^*_{s1}) for (X_i, X_s) in (4) yields the growth rate of capital stock of the two sectors as

$$g_i = \alpha_i + \frac{\beta_i(\alpha_i + \frac{\alpha_s}{k})}{\Omega_1} \tag{14}$$

$$g_s = \alpha_s + \frac{\beta_s \lambda(\alpha_i k + \alpha_s)}{\Omega_1} \tag{15}$$

The long-run dynamics is now obtained by substituting for g_i and g_s from (14) and (15) respectively in (11):

$$\dot{k} = k[\alpha_i + \frac{\beta(\alpha_i + \frac{\alpha_s}{k})}{\Omega_1} - \alpha_s - \frac{\beta_s \lambda(\alpha_i k + \alpha_s)}{\Omega_1}] \tag{16}$$

for all $k > 0$. Like in the previous model, there exists a stable long run steady state with a constant relative capital stock of industrial sector, $k^*_1 = \frac{-b_1 - \sqrt{b_1^2 - 4a_1 c_1}}{2a_1}$ where $a_1 = -\alpha_i \beta_s \lambda_1$, $b_1 = (\alpha_i - \alpha_s)\Omega_1 + \alpha_i \beta_i - \lambda_1 \alpha_s \beta_s$ and $c_1 = \alpha_s \beta_i$.

Proposition 5. *Given $\Omega_1 > 0$, k^*_1 is a unique and asymptotically stable steady state of (16) in $\mathbb{R}_{++}$.*

[9] Note that once we replace (1) with $N_s = \lambda_1 X_i$ the price equation for industry changes to $P_i = (1 + z_i)(\frac{W}{x_i} + P_s \lambda_1)$. Nonetheless, as can be easily verified, the relative price of service, p, is still strictly increasing in x_i and strictly decreasing in x_s.

Proof. Similar to the proof of Proposition 2, see appendix A. □

Despite these similarities between the two models, implications of sector-specific technology shocks on the steady state are not exactly same. Like the previous model, in this model too, a *ceteris paribus* increase in x_i (x_s) unambiguously increases (decreases) g_1^*, however, unlike the previous model, effect on k_1^* is ambiguous. This is because, in this case, changes in the relative price of service p affect the expenditure multiplier, Ω_1^{-1}. In the instance of an increase in x_i, which increases p, the expenditure multiplier increases as $\frac{\partial \Omega_1}{\partial p} = -\frac{\lambda_1 s}{1+z_s} < 0$. This means that, for any arbitrary combination of capital stocks of the two sectors, short-run equilibrium output levels of the two sectors increase which translate into increase in growth rates of their capital stocks for all values of k resulting in an increase. In terms of Figure 1, in this case an increase in x_i will shift both the g_i and g_s curves upwards. As a consequence, the steady state growth rate in the model will increase whereas the effect on the steady state k will depend on which of the two curves shifts more. We formally prove this in Proposition 6.

Proposition 6. *Let* $g_1^* = \alpha_i + \frac{\beta_i\{\alpha_i + (\alpha_s/k_1^*)\}}{\Omega_1} = \alpha_s + \frac{\beta_s \lambda_1(\alpha_i k_1^* + \alpha_s)}{\Omega_1}$. *Then* $\frac{\partial g_1^*}{\partial p} > 0$.

Proof. From $g_1^* = \alpha_s + \frac{\beta_s \lambda_1(\alpha_i k_1^* + \alpha_s)}{\Omega_1}$,

$$\frac{\partial g_1^*}{\partial p} = \frac{\beta_s \lambda_1}{\Omega_1^2}\{\Omega_1 \alpha_i \frac{\partial k_1^*}{\partial p} - (\alpha_i k_1^* + \alpha_s)\frac{\partial \Omega_1}{\partial p}\} \tag{17}$$

Now from the definition of Ω_1, $\frac{\partial \Omega_1}{\partial p} = -\frac{\lambda_1 s}{1+z_s} < 0$ as λ_1, s and z_s are positive. Next,

$$\frac{\partial k_1^*}{\partial p} = -\frac{1}{2a_1}\frac{\partial b_1}{\partial p}\{1 + \frac{b_1}{\sqrt{b_1^2 - 4a_1c_1}}\}$$

Since $b_1 = (\alpha_i - \alpha_s)\Omega_1 + \alpha_i\beta_i - \lambda_1\alpha_s\beta_s$ we have $\frac{\partial b_1}{\partial p} = (\alpha_i - \alpha_s)\frac{\partial \Omega_1}{\partial p}$. Substituting for $\frac{\partial b_1}{\partial p}$ in above expression yields

$$\frac{\partial k_1^*}{\partial p} = -\frac{(\alpha_i - \alpha_s)}{2a_1}\frac{\partial \Omega_1}{\partial p}\{1 + \frac{b_1}{\sqrt{b_1^2 - 4a_1c_1}}\} \tag{18}$$

Now $a_1 = -\alpha_i\beta_s\lambda_1 < 0$ and $c_1 = \alpha_s\beta_i > 0$ as α_i, α_s, β_i, β_s and λ_1 are all positive. $a_1 < 0$ and $c_1 > 0$ imply $b_1^2 - 4a_1c_1 > b_1^2$. Therefore, $-1 < \frac{b_1}{\sqrt{b_1^2 - 4a_1c_1}} < 1$ or $\{1 + \frac{b_1}{\sqrt{b_1^2 - 4a_1c_1}}\} > 0$. Thus, from (18), it follows that $\frac{\partial k_1^*}{\partial p} < 0$ if and only if $\alpha_i - \alpha_s > 0$

as $\frac{\partial \Omega_1}{\partial p} < 0$ and $a_1 < 0$. And, from (17), if $\frac{\partial k_1^*}{\partial p} \geq 0$ then $\frac{\partial g^*}{\partial p} > 0$ as $\frac{\partial \Omega_1}{\partial p} < 0$ and all other factors in the right hand side of (17) are positive. To complete the proof, we need to show that $\frac{\partial g_1^*}{\partial p} > 0$ when $\frac{\partial k_1^*}{\partial p} < 0$. Suppose at $p_1 > 0$, $\frac{\partial k_1^*}{\partial p} < 0$ and $\frac{\partial g_1^*}{\partial p} \leq 0$. Let k_{11}^* be the steady state of (16) when relative price of service is p_1. Also, let $g_{11}^* = \alpha_i + \frac{\beta_i\{\alpha_i + (\alpha_s/k_{11}^*)\}}{\Omega_1(p_1)} = \alpha_s + \frac{\beta_s\lambda_1(\alpha_i k_{11}^* + \alpha_s)}{\Omega_1(p_1)}$, where $\Omega_1(p_1) = \frac{sz_i}{1+z_i} - \beta_i - \lambda_1(\frac{sp_1}{1+z_s} + \beta_s)$. Since $\frac{\partial k_1^*}{\partial p} < 0$ and $\frac{\partial g_1^*}{\partial p} \leq 0$ at p_1, there exists a $p_2 > p_1$ such that $k_{12}^* < k_{11}^*$ and $g_{12}^* \leq g_{11}^*$, where k_{12}^* is the steady state of (16) when relative price of service is p_2 and $g_{12}^* = \alpha_i + \frac{\beta_i\{\alpha_i + (\alpha_s/k_{12}^*)\}}{\Omega_1(p_2)} = \alpha_s + \frac{\beta_s\lambda_1(\alpha_i k_{12}^* + \alpha_s)}{\Omega_1(p_2)}$ with $\Omega_1(p_2) = \frac{sz_i}{1+z_i} - \beta_i - \lambda_1(\frac{sp_2}{1+z_s} + \beta_s)$. Now $g_{12}^* \leq g_{11}^*$ implies $\alpha_i + \frac{\beta_i\{\alpha_i + (\alpha_s/k_{12}^*)\}}{\Omega_1(p_2)} \leq \alpha_i + \frac{\beta_i\{\alpha_i + (\alpha_s/k_{11}^*)\}}{\Omega_1(p_1)}$. This in turn implies $\Omega_1(p_1) < \Omega_1(p_2)$ since $\alpha_s > 0$ and $k_{11}^* > k_{12}^*$ imply $\frac{\alpha_s}{k_{11}^*} < \frac{\alpha_s}{k_{12}^*}$. However this is a contradiction as it must be that $\Omega_1(p_1) > \Omega_1(p_2)$ since $\frac{\partial \Omega_1}{\partial p} < 0$ for all p and $p_1 < p_2$. $\square$

Next, consider another simple change in the model of Dutt (1992). Instead of the classical savings function, let us assume that consumption expenditure incurred on the industrial good is a constant fraction of the the value added. So consumption expenditure incurred on the industrial good now is $C_i = cP_iX_i$ where $c \in (0,1)$ is a constant. Also we revert back to (1), that is the service input in the industrial sector is an overhead input rather than an intermediate input. Using (1), (4) and $C_i = cP_iX_i$, to substitute for d_i and d_s in (6) we can represent the short run dynamics of this model as the following system of differential equations.

$$\begin{aligned} \dot{X}_i &= \psi_i[-(1-c-\beta_i)X_i + \beta_s X_s + \alpha_i K_i + \alpha_s K_s] \\ \dot{X}_s &= \psi[-X_s + \lambda K_i] \end{aligned} \tag{19}$$

Let $X_{i2}^* = \frac{\alpha_1 K_i + \alpha_s K_s + \beta_s \lambda K_i}{\Omega_2}$ and $X_{i2}^* = \frac{\alpha_1 K_i + \alpha_s K_s + \beta_s \lambda K_i}{\Omega_2}$ where $\Omega_2 = 1 - c - \beta_i > 0$.

Proposition 7. *If $\Omega_2 > 0$ then (X_{i2}^*, X_{s2}^*) is a unique and asymptotically stable short-run equilibrium of (19).*

Proof. Similar to the proof of Proposition 1, see appendix A. $\square$

For the long run, once again assuming that $\Omega_2 > 0$ and the economy is always in a short-run equilibrium, we obtain growth rates of capital stocks of the two sectors by substituting (X_{i2}^*, X_{s2}^*) for (X_i, X_s) in (4),

$$g_i = \alpha_i + \frac{\beta_i}{\Omega_2}(\alpha_i + \frac{\alpha_s}{k} + \beta_s \lambda) \tag{20}$$

$$g_s = \alpha_s + \beta_s \lambda k \tag{21}$$

The long run dynamics of this model is then obtained by substituting for g_i and g_s respectively from (20) and (21) in (11).

$$\dot{k} = k[\alpha_i + \frac{\beta_i}{\Omega_2}(\alpha_i + \frac{\alpha_s}{k} + \beta_s \lambda) - \alpha_s - \beta_s \lambda k] \tag{22}$$

There exists a stable steady state of (22) with a constant relative capital stock of industrial sector, $k_2^* = \frac{-b_2 - \sqrt{b_2^2 - 4a_2c_2}}{2a_2}$ where $a_2 = -\beta_s\lambda$, $b_2 = (\alpha_i - \alpha_s)\Omega_2 + \alpha_i\beta_i + \beta_i\beta_s\lambda$ and $c_2 = \alpha_s\beta_i$.

Proposition 8. *Given $\Omega_2 > 0$, k_2^* is a unique and asymptotically stable steady state of (22) in $\mathbb{R}_{++}$.*

Proof. Similar to the proof of Proposition 2, see appendix A. □

In this case exogenous increase in labour productivity in either of the sectors has no effect on the steady state, as shown in Proposition 9. This is because growth rate of capital stocks of both the sectors are independent of the relative price of the service.

Proposition 9. *Let $g_2^* = \alpha_i + \frac{\beta_i}{\Omega_2}(\alpha_i + \frac{\alpha_s}{k_2^*} + \beta_s\lambda) = \alpha_s + \beta_s\lambda k_2^*$. Then $\frac{\partial g_2^*}{\partial p} = 0$.*

Proof. From the definitions of k_2^* and g_2^*, it follows that $\frac{\partial g_2^*}{\partial p} = 0$. □

4. ICTs and Service-Led Growth

In the models examined in the previous section, there is no growth-boosting effect of labour productivity increases in the service sector. This is somewhat perplexing considering the widespread application of ICTs in various services. In those models sector specific technology shocks affected the growth rate via their effect on the relative price of the service on the demand for the industrial good, d_i. On the other hand, the demand for the service, d_s, was completely determined by either the capital stock or the output of the industrial sector. In this section we argue that if d_s depends on both p and X_i then an increase in labour productivity in the service sector can not only increase the relative size of the service sector but also the growth rate of the economy.

Due to advances in ICTs and in electronics, many services are today required to complement the use of various industrial products. For example purchase of computer hardware without software and Internet services is not very useful. Similar is the case for mobile telephony and other electronic goods in general. This is not only true for consumption of industrial products but can also be true for investment demand. For example it is possible that a firm can raise more funds for investment if it employs the services of a financial firm to underwrite its shares. Moreover there is no reason why the joint utilization of industrial goods and services needs to be a perfectly complementary one. With lower prices of various services, more of services can be purchased along various industrial goods. In a two-sector model with inter-sectoral demand linkages, this aspect can be incorporated by stipulating that industrial output can be utilized for consumption or used as investment good only if it is purchased along with service output. Formally let, $P_s d_s = \theta P_i X_i$ or,

$$d_s = \frac{\theta X_i}{p} \tag{23}$$

where θ is a positive constant. Thus we assume that demand for service d_s is now positively related to output of the industrial sector X_i and negatively related to the relative price of service $p = \frac{P_s}{P_i}$. For the sake of simplicity, we do not consider demand for services as inputs in the industrial sector in this section.[10] Price levels of the industrial good and the service are given by (2) and the relative price of service, p is given by (3).

Like in the models of the previous section, the industrial good is demanded for consumption and as capital good by both the sectors. Therefore demand for industrial output once again is $d_i = \frac{C_i}{P_i} + I_i + I_s$. Investment demands of the two sectors, I_i and I_s, is described by (4). As far as C_i is concerned, in this section we are going to assume that consumption expenditure incurred on the industrial sector is a constant fraction of total value added in the economy. Since we have assumed that the entire demand for service is final demand, total value added in the economy now is equal

[10] We can include a price sensitive term for intermediate input demand for the service, such as $N_s = \frac{\lambda_2 X_i}{p}$ where $\lambda_2 > 0$ is a constant, without significantly effecting any result.

to $P_iX_i + P_sX_s$. Thus,

$$C_i = c(P_iX_i + P_sX_s) \tag{24}$$

where $c \in (0, 1)$.[11] Using (4), (23) and (24), to substitute for d_i and d_s in (6) we obtain the short-run dynamics of this model as the following system of two differential equations.

$$\begin{aligned} \dot{X}_i &= \psi_i[-(1 - c - \beta_i)X_i + (cp + \beta_s)X_s + \alpha_i K_i + \alpha_s K_s] \\ \dot{X}_s &= \psi_s[\frac{\theta}{p}X_i - X_s] \end{aligned} \tag{25}$$

Let $X^*_{i3} = \frac{\alpha_i K_i + \alpha_s K_s}{\Omega_3}$ and $X^*_{s3} = \frac{\theta(\alpha_i K_i + \alpha_s K_s)}{p\Omega_3}$ where $\Omega_3 = 1 - c(1 + \theta) - \beta_i - \frac{\theta \beta_s}{p}$.

Proposition 10. *If $\Omega_3 > 0$ then (X^*_{i3}, X^*_{s3}) is a unique and asymptotically stable short-run equilibrium of (25).*

Proof. Similar to the proof of Proposition 1, see appendix A. □

For the long-run analysis, we once again assume that $\Omega_3 > 0$ and the economy is always in a short run equilibrium and capital stocks of both the sectors grow because of investment carried out in the short run. Substituting (X^*_{i3}, X^*_{s3}) for (X_i, X_s) in yields the following expressions for growth rates of capital stock of the two sectors.

$$g_i = \alpha_i + \frac{\beta_i(\alpha_i + \frac{\alpha_s}{k})}{\Omega_3} \tag{26}$$

$$g_s = \alpha_s + \frac{\beta_s \theta(\alpha_i k + \alpha_s)}{p\Omega_3} \tag{27}$$

And the long-run dynamics of this model is given by the following differential equation derived from (11), (26) and (27). For all $k > 0$,

$$\dot{k} = k[\alpha_i + \frac{\beta_i(\alpha_i + \frac{\alpha_s}{k})}{\Omega_3} - \alpha_s - \frac{\beta_s \theta(\alpha_i k + \alpha_s)}{p\Omega_3}] \tag{28}$$

Proposition 11 shows that there exists a unique and asymptotically stable steady state

[11] It can be verified that, if, instead of (23) and (24), we assume that the service is used only for consumption and derive consumption demands for the two sectors as constant fractions of the total consumption expenditure, obtained using the classical savings function, then implications of sector-specific technology shocks are no different from what is discussed in this section (with the exception of subsection 4.2, where there can be some differences). However, the algebra becomes much more cumbersome.

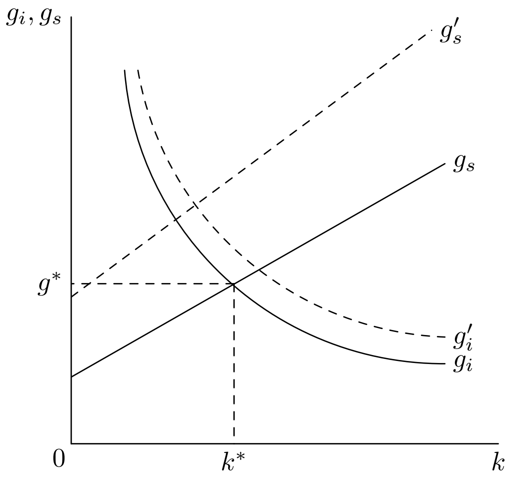

Figure 2: Effect of an increase in x_s when d_s is described by (23)

of (28), $k_3^* = \frac{-b_3 - \sqrt{b_3^2 - 4a_3c_3}}{2a_3}$ where $a_3 = -\alpha_i\beta_s\theta$, $b_3 = (\alpha_i - \alpha_s)p\Omega_3 + \alpha_i\beta_i p - \alpha_s\beta_s\theta$ and $c_3 = \alpha_s\beta_i p$.

Proposition 11. *Given $\Omega_3 > 0$, k_3^* is a unique and asymptotically stable steady state of (28) in $\mathbb{R}_{++}$.*

Proof. Similar to the proof of Proposition 2, see appendix A. $\square$

Now let us examine the implication of an increase in labour productivity of the service sector in this model. We know from (3), that an increase in x_s decreases p. A fall in p, however, has completely different effect on the steady state in this model compared to the models in the previous sections. Here, a lower relative price of service means more service demand per unit industrial output as from (23), $\frac{\partial d_s}{\partial p} = -\frac{\theta X_i}{p^2} < 0$. Further, since greater service demand means greater service output, there is an increase in consumption and investment demand for the industrial good generated by the service sector because of which the industrial output increases. This effect is reflected in an increase in the expenditure multiplier for the industrial output, Ω_3^{-1}, as $\frac{\partial \Omega_3}{\partial p} = \frac{\theta\beta_s}{p^2} > 0$. As a consequence, the short-run equilibrium output of the industrial sector in X_{i3}^* increases, which in turn combines with increase in service demand per unit industrial output to increase the short-run equilibrium output of the service sector X_{s3}^*. Since short-run equilibrium output of both sectors increase because of a rise in x_s irrespective of their capital stocks, growth rates of capital stock of both sectors increase for all $k > 0$. We show this in Figure 2, where schedules for the growth rate of capital stock of both sectors shift upwards from g_i to g_i' and g_s g_s'

because of the increase in x_s. The new steady state is given by the intersection point of the curves labeled g'_i and g'_s in Figure 2. Clearly in this case increase in labour productivity of the service sector increases the steady state growth rate.

Proposition 12. *Let* $g_3^* = \alpha_i + \frac{\beta_i\{\alpha_i+(\alpha_s/k_3^*)\}}{\Omega_3} = \alpha_s + \frac{\beta_s\theta(\alpha_i k_3^*+\alpha_s)}{p\Omega_3}$. *Then* $\frac{\partial g_3^*}{\partial p} < 0$.

Proof. From $g_3^* = \alpha_s + \frac{\beta_s\theta(\alpha_i k_3^*+\alpha_s)}{p\Omega_3}$,

$$\frac{\partial g_3^*}{\partial p} = \frac{\beta_s\theta}{p^2\Omega_3^2}[p\Omega_3\alpha_i\frac{\partial k_3^*}{\partial p} - (\alpha_i k_3^* + \alpha_s)\{\Omega_3 + p\frac{\partial \Omega_3}{\partial p}\}] \tag{29}$$

From the definition of Ω_3, $\Omega_3 + p\frac{\partial\Omega_3}{\partial p} = 1 - c(1+\theta) - \beta_i > 0$ since $\Omega_3 > 0$. Therefore sign of $\frac{\partial g_3^*}{\partial p}$ in (29) depends on the sign of $\frac{\partial k_3^*}{\partial p}$ as p, Ω_3, p, α_s, k_3^*, β_s and θ are all positive. Next,

$$\begin{aligned}\frac{\partial k_3^*}{\partial p} &= -\frac{1}{2a_3}\{\frac{\partial b_3}{\partial p} + \frac{1}{2\sqrt{b_3^2 - 4a_3c_3}}(2b_3\frac{\partial b_3}{\partial p} - 4a_3\frac{\partial c_3}{\partial p})\} \\ &= -\frac{1}{2a_3}\frac{\partial b_3}{\partial p}\{1 + \frac{b_3}{\sqrt{b_3^2 - 4a_3c_3}}\} + \frac{\partial c_3}{\partial p}\end{aligned} \tag{30}$$

Now $a_3 < 0$, $c_3 > 0$ and $\frac{\partial c_3}{\partial p} = \alpha_s\beta_i > 0$ as α_i, α_s, β_i, β_s, p and θ are all positive. Also, $b_3^2 - 4a_3c_3 > b_3^2$ as $a_3 < 0$ and $c_3 > 0$. Therefore $-1 < \frac{b_3}{\sqrt{b_3^2-4a_3c_3}} < 1$ or $\{1 + \frac{b_3}{\sqrt{b_3^2-4a_3c_3}}\} > 0$. However, sign of $\frac{\partial b_3}{\partial p} = \{(\alpha_i - \alpha_s)(\Omega_3 + p\frac{\partial\Omega_3}{\partial p}) + \alpha_i\beta_i\}$ is ambiguous because of which sign of $\frac{\partial k_3^*}{\partial p}$ is also ambiguous. Then it follows from (29) that $\frac{\partial g_3^*}{\partial p} < 0$ if $\frac{\partial k_3^*}{\partial p} \le 0$. To complete the proof, we need to show that $\frac{\partial g_3^*}{\partial p} < 0$ when $\frac{\partial k_3^*}{\partial p} > 0$. Suppose, on the contrary, that at some arbitrary value of $p = p_1$, $\frac{\partial k_3^*}{\partial p} > 0$ and $\frac{\partial g_3^*}{\partial p} \ge 0$. Let k_{31}^* be the steady state of (28) when relative price of service is p_1. Also, let $g_{31}^* = \alpha_i + \frac{\beta_i\{\alpha_i+(\alpha_s/k_{31}^*)\}}{\Omega_3(p_1)} = \alpha_s + \frac{\beta_s\theta(\alpha_i k_{31}^*+\alpha_s)}{p_1\Omega_3(p_1)}$, where $\Omega_3(p_1) = 1 - c(1+\theta) - \beta_i - \frac{\theta\beta_s}{p_1}$. Since $\frac{\partial k_3^*}{\partial p} > 0$ and $\frac{\partial g_3^*}{\partial p} \ge 0$ at p_1, there exists a $p_2 > p_1$ such that $k_{32}^* > k_{31}^*$ and $g_{32}^* \ge g_{31}^*$, where k_{32}^* is the steady state of (28) when relative price of service is p_2 and $g_{32}^* = \alpha_i + \frac{\beta_i\{\alpha_i+(\alpha_s/k_{32}^*)\}}{\Omega_3(p_2)} = \alpha_s + \frac{\beta_s\theta(\alpha_i k_{32}^*+\alpha_s)}{p_2\Omega_3(p_2)}$ with $\Omega_3(p_2) = 1 - c(1+\theta) - \beta_i - \frac{\theta\beta_s}{p_2}$. Now $g_{32}^* \ge g_{31}^*$ implies $\alpha_i + \frac{\beta_i\{\alpha_i+(\alpha_s/k_{32}^*)\}}{\Omega_3(p_2)} \ge \alpha_i + \frac{\beta_i\{\alpha_i+(\alpha_s/k_{31}^*)\}}{\Omega_3(p_1)}$. This, in turn, implies $\Omega_3(p_1) > \Omega_3(p_2)$ since $k_{32}^* > k_{31}^*$ and $\alpha_s > 0$. However, this is a contradiction because $p_1 < p_2$ and $\frac{\partial\Omega_3}{\partial p} = \frac{\beta_s\theta}{p^2} > 0$ for all p imply $\Omega_3(p_1) < \Omega_3(p_2)$. □

Although in Figure 2, we show that steady state relative capital stock of the industry sector decreases, the effect on the steady state relative capital stock of the industrial sector depends on which of the two schedules shifts more and is, therefore, ambiguous. From (30) in the proof Proposition 12, $\frac{\partial k^*}{\partial p} > 0$ if $\frac{\partial b_3}{\partial p} = \{(\alpha_i - \alpha_s)(\Omega_3 + p\frac{\partial \Omega_3}{\partial p}) + \alpha_i\beta_i\} = \{(\alpha_i - \alpha_s)(1 - c(1+\theta) - \beta_i) + \alpha_i\beta_i\} > 0$. Thus this model predicts if application of ICTs causes an increase in labour productivity of the service sector, then both the growth rate of economy and the relative size of the service sector can increase. Rise in labour productivity in the industrial sector, on the other hand, decreases growth rate in this model. However, unlike Baumol (1967), where the decline in growth rate is because of a shift in resources to a stagnant service sector, here it is due to an increase in the relative price of the service. As a result, outputs and growth rates of capital stocks of both sectors decline in the short run and the long run respectively, which causes a decrease in the steady state growth rate of the model. Such an unambiguous conclusion regarding effect of sector specific technology shocks on the growth rate, however, does not follow if we adopt a slightly more general specification for demand for the service.

4.1 A More General Formulation of d_s

Instead of (23), let demand for the service be given by $d_s = \theta(p)X_i$ where θ now is a function $\theta : \mathbb{R}_{++} \mapsto \mathbb{R}_{++}$ with derivative $\theta'(p) < 0$ for all $p \in \mathbb{R}_{++}$. Using (4), (24), $d_s = \theta(p)X_i$ and (6) we obtain the short run dynamics of this model as the following system of two differential equations.

$$\begin{aligned} \dot{X}_i &= \psi_i[-(1 - c - \beta_i)X_i + (cp + \beta_s)X_s + \alpha_i K_i + \alpha_s K_s] \\ \dot{X}_s &= \psi_s[\theta(p)X_i - X_s] \end{aligned} \tag{31}$$

Let $X^*_{i4} = \frac{\alpha_i K_i + \alpha_s K_s}{\Omega_4}$ and $X^*_{s4} = \frac{\theta(p)(\alpha_i K_i + \alpha_s K_s)}{\Omega_4}$ where $\Omega_4 = 1 - c - \beta_i - \theta(p)(cp + \beta_s)$.

Proposition 13. *If $\Omega_4 > 0$ then (X^*_{i4}, X^*_{s4}) is a unique and asymptotically stable short-run equilibrium of (31).*

Proof. Similar to the proof of Proposition 1, see appendix A. □

As usual, substituting (X^*_{i4}, X^*_{s4}) for (X_i, X_s) in (4) we obtain growth rates of capital stocks of the two sectors as the following.

$$g_i = \alpha_i + \frac{\beta_i(\alpha_i + \frac{\alpha_s}{k})}{\Omega_4} \tag{32}$$

$$g_s = \alpha_s + \frac{\beta_s\theta(p)(\alpha_i k + \alpha_s)}{\Omega_4} \tag{33}$$

The long run dynamics is now obtained by substituting for g_i and g_s from (32) and (33) respectively in (11).

$$\dot{k} = k[\alpha_i + \frac{\beta_i(\alpha_i + \frac{\alpha_s}{k})}{\Omega_4} - \alpha_s - \frac{\beta_s\theta(p)(\alpha_i k + \alpha_s)}{\Omega_4}] \tag{34}$$

for all $k > 0$. Let $k_4^* = \frac{-b_4 - \sqrt{b_4^2 - 4a_4c_4}}{2a_4}$ where $a_4 = -\alpha_i\beta_s\theta(p)$, $b_4 = (\alpha_i - \alpha_s)\Omega_4 + \alpha_i\beta_i - \alpha_s\beta_s\theta(p)$ and $c_4 = \alpha_s\beta_i$.

Proposition 14. *Given $\Omega_4 > 0$, k_4^* is a unique and asymptotically stable steady state of (34) in $\mathbb{R}_{++}$.*

Proof. Similar to the proof of Proposition 2, see appendix A. □

Like in the previous model, a decrease in p increases demand for service per unit industrial output $\theta(p)$, which tends to increase the industrial output. However in this case, the total indirect effect on the industrial output per unit increase in the industrial output, $(cp+\beta_s)\theta(p)$, may not be positive. As a result increase in p now has an ambiguous effect on the expenditure multiplier, Ω_4^{-1}, and, therefore, on the steady state growth rate too. Nonetheless in Proposition 15 we show if $\theta(p)$ is sufficiently elastic then a decrease in p increases the steady state growth rate.

Proposition 15. *Let $g_4^* = \alpha_i + \frac{\beta_i\{\alpha_i + (\alpha_s/k_4^*)\}}{\Omega_4} = \alpha_s + \frac{\beta_s\theta(p)(\alpha_i k_4^* + \alpha_s)}{\Omega_4}$. Then $\frac{\partial g_4^*}{\partial p} < 0$ if $\frac{p\theta'(p)}{\theta(p)} < -\frac{cp}{cp+\beta_s}$ for all $p > 0$.*

Proof. Suppose $\frac{p\theta'(p)}{\theta(p)} < -\frac{cp}{cp+\beta_s}$ for all $p > 0$. From the definition Ω_4, $\frac{\partial\Omega_4}{\partial p} = -(cp + \beta_s)\theta'(p) - c\,\theta(p)$. Now $\frac{p\theta'(p)}{\theta(p)} < -\frac{cp}{cp+\beta_s}$ implies $-(cp+\beta_s)\theta'(p) - c\,\theta(p) > 0$ as p, $\theta(p)$, c and β_s are all positive. Thus $\frac{p\theta'(p)}{\theta(p)} < -\frac{cp}{cp+\beta_s}$ for all $p > 0$ implies $\frac{\partial\Omega_4}{\partial p} > 0$ for all p. Next, from $g_4^* = \alpha_s + \frac{\beta_s\theta(p)(\alpha_i k_4^* + \alpha_s)}{\Omega_4}$,

$$\frac{\partial g_4^*}{\partial p} = \frac{\beta_s}{\Omega_4^2}[\Omega_4\{\theta'(p)(\alpha_i k_4^* + \alpha_s) + \theta(p)\alpha_i\frac{\partial k_4^*}{\partial p}\} - \theta(p)(\alpha_i k_4^* + \alpha_s)\frac{\partial\Omega_4}{\partial p}] \tag{35}$$

Since $\theta'(p) < 0$ and $\frac{\partial\Omega_4}{\partial p} > 0$ for all p, it follows from (35) that $\frac{\partial g_4^*}{\partial p} < 0$ if $\frac{\partial k_4^*}{\partial p} \leq 0$ as α_i, α_s, β_s, $\theta(p)$, k_4^* and Ω_4 are all positive. To complete the proof, we need to show that $\frac{\partial g_4^*}{\partial p} < 0$ when $\frac{\partial k_4^*}{\partial p} > 0$. Suppose, on the contrary that at some arbitrary value of $p = p_1 > 0$, $\frac{\partial k_4^*}{\partial p} > 0$ and $\frac{\partial g_4^*}{\partial p} \geq 0$. Let k_{41}^* be the steady state of (34) when relative price of service is p_1. Also, let $g_{41}^* = \alpha_i + \frac{\beta_i\{\alpha_i + (\alpha_s/k_{41}^*)\}}{\Omega_4(p_1)} = \alpha_s + \frac{\beta_s\theta(p)(\alpha_i k_{41}^* + \alpha_s)}{\Omega_4(p_1)}$, where

$\Omega_4(p_1) = 1 - c - \beta_i - \theta(p_1)(cp_1 + \beta_s)$. Since $\frac{\partial k_4^*}{\partial p} > 0$ and $\frac{\partial g_4^*}{\partial p} \geq 0$ at p_1, there exists a $p_2 > p_1$ such that $k_{42}^* > k_{41}^*$ and $g_{42}^* \geq g_{41}^*$, where k_{42}^* is the steady state of (34) when relative price of service is p_2 and $g_{42}^* = \alpha_i + \frac{\beta_i\{\alpha_i + (\alpha_s/k_{42}^*)\}}{\Omega_4(p_2)} = \alpha_s + \frac{\beta_s\theta(p)(\alpha_i k_{42}^* + \alpha_s)}{\Omega_4(p_2)}$ with $\Omega_4(p_2) = 1 - c - \beta_i - \theta(p_2)(cp_2 + \beta_s)$. Now, $g_{42}^* \geq g_{41}^*$ implies $\alpha_i + \frac{\beta_i\{\alpha_i + (\alpha_s/k_{42}^*)\}}{\Omega_4(p_2)} \geq \alpha_i + \frac{\beta_i\{\alpha_i + (\alpha_s/k_{42}^*)\}}{\Omega_4(p_2)}$. This, in turn, implies $\Omega_4(p_1) > \Omega_4(p_2)$ because $k_{41}^* < k_{42}^*$ and $\alpha_s > 0$. However this is a contradiction since $\frac{p\theta'(p)}{\theta(p)} < -\frac{cp}{cp+\beta_s}$ for all $p > 0$ implies $\frac{\partial \Omega_4}{\partial p} > 0$ for all p and, therefore, $p_1 < p_2$ implies $\Omega_4(p_1) < \Omega_4(p_2)$. □

4.2 Sensitiveness of z_s to Changes in x_s

So far we have assumed that labour productivity shocks have no effect on the 'degree of monopoly'. Introduction of new technology, however, can alter the concentration of market power in the economy or in particular sectors. For example if new technology in a sector is introduced by new entrants then the degree of concentration in the sector might decline whereas, in case new technology is introduced by an incumbent then it might increase as the new technology can act as a barrier to entry. Even when new technology is introduced by a new entrant, market power can become more concentrated if either an incumbent acquires or merges with the entrant firm or the new entrant drives out incumbent firms from the market. Now we are going to examine a scenario where adoption of ICTs in various services, in addition to increasing the labour productivity of the service sector, can also change the concentration of market power in the sector.

Let the mark-up in the service sector be a differentiable function of its labour productivity. That is, let $z_s = z_s(x_s)$. The derivative $z_s'(x_s) > 0$ would mean that an increase in labour productivity of the service sector is accompanied by an increase in the 'degree of monopoly' of the sector and $z_s'(x_s) < 0$ would mean that an increase in labour productivity of the service sector is accompanied by a decrease in the 'degree of monopoly' of the sector. In order to consider the implication for either $z_s'(x_s) > 0$ or $z_s'(x_s) < 0$ in our analysis we need to include the rate of profit of the service sector in its investment function. By definition, the rate of profit of the service sector is $r_s = \frac{ph_s X_s}{K_s}$ where h_s is the profit share in the service sector, which, from (2), is positively related to z_s, i.e $h_s = \frac{z_s}{1+z_s}$. Let the investment function of the service sector be

$$\frac{I_s}{K_s} = \alpha_s + (\beta_s + \gamma_s p h_s)\frac{X_s}{K_s} \tag{36}$$

For simplicity, let the demand for service be given by (23). Using (4), (23), (24) and (36) to substitute for d_i and d_s in (6), the short-run dynamics is now given by the following system of differential equations.

$$\begin{aligned} \dot{X}_i &= \psi_i[-(1-c-\beta_i)X_i + (cp+\beta_s+\gamma_s p h_s)X_s + \alpha_i K_i + \alpha_s K_s] \\ \dot{X}_s &= \psi_s[\frac{\theta}{p}X_i - X_s] \end{aligned} \tag{37}$$

Let $X^*_{i5} = \frac{\alpha_i K_i + \alpha_s K_s}{\Omega_5}$ and $X^*_{s5} = \frac{\theta(\alpha_i K_i + \alpha_s K_s)}{p\Omega_5}$ where $\Omega_5 = 1 - c - \beta_i - \theta c - \frac{\theta\beta_s}{p} - \gamma_s h_s$.

Proposition 16. *If $\Omega_5 > 0$ then (X^*_{i5}, X^*_{s5}) is a unique and asymptotically stable short run equilibrium of (37).*

Proof. Similar to the proof of Proposition 1, see appendix A. □

Substituting for X^*_{i5} for X_i in (4) and X^*_{s5} in (36) yields the growth rate of capital stocks of both sectors in the long run as the following.

$$g_i = \alpha_i + \frac{\beta_i(\alpha_i + \frac{\alpha_s}{k})}{\Omega_5} \tag{38}$$

$$g_s = \alpha_s + \frac{(\beta_s + \gamma_s p h_s)\theta(\alpha_i k + \alpha_s)}{p\Omega_5} \tag{39}$$

The long run dynamics is now obtained by substituting for g_i and g_s from (38) and (39) respectively in (11) as the following differential equation.

$$\dot{k} = k[\alpha_i + \frac{\beta_i(\alpha_i + \frac{\alpha_s}{k})}{\Omega_5} - \alpha_s - \frac{(\beta_s + \gamma_s p h_s)\theta(\alpha_i k + \alpha_s)}{p\Omega_5}] \tag{40}$$

for all $k > 0$. Let $k^*_5 = \frac{-b_5 - \sqrt{b_5^2 - 4a_5 c_5}}{2a_5}$ where $a_5 = -\alpha_i\theta(\beta_s + \gamma_s p h_s)$, $b_5 = (\alpha_i - \alpha_s)p\Omega_5 + \alpha_i\beta_i p - \alpha_s\theta(\beta_s + \gamma_s p h_s)$ and $c_5 = \alpha_s\beta_i p$.

Proposition 17. *Given $\Omega_5 > 0$, k^*_5 is a unique and asymptotically stable steady state of (40) in $\mathbb{R}_{++}$.*

Proof. Similar to the proof of Proposition 2, see appendix A. □

A rise in labour productivity of the service sector affects the steady state of this model in a much more complicated manner. First of all, with the price mark-up in service sector z_s being a function of x_s, an increase in x_s no longer necessarily decreases p. Since $z_s = z_s(x_s)$, from (3) we obtain

$$\frac{\partial p}{\partial x_s} = p(\frac{z_s'(x_s)}{1+z_s} - \frac{1}{x_s}) \tag{41}$$

From (41), if a rise in labour productivity of the service sector is associated with a sufficiently large increase in the degree of monopoly of the sector then the relative price of the service increases instead of decreasing. Second, a rise in x_s effects the multiplier for industrial output, Ω_5^{-1}, not only by changing the relative price of service but also through a change in profit share of the service sector. The net effect can be ambiguous. To see this, note that the derivative of Ω_5 with respect to x_s is,

$$\frac{\partial \Omega_5}{\partial x_s} = \frac{\beta_s \theta}{p^2}\frac{\partial p}{\partial x_s} - \gamma_s \frac{\partial h_s}{\partial x_s} \tag{42}$$

And finally, the third source of complication is that increase in x_s now has another potentially ambiguous effect on investment of the service sector through the term ph_s in (36) in addition to its effect on the same through the short-run equilibrium service output. We end this section with Proposition 18 which provides a sufficient condition for an increase in x_s to increase the steady state growth rate when $z_s'(x_s) > 0$.

Proposition 18. *Let* $g_5^* = \alpha_i + \frac{\beta_i\{\alpha_i + (\alpha_s/k_5^*)\}}{\Omega_5} = \alpha_s + \frac{(\beta_s + \gamma_s p h_s)\theta(\alpha_i k_5^* + \alpha_s)}{p\Omega_5}$. *Then* $\frac{\partial g_5^*}{\partial x_s} > 0$ *if* $\frac{z_s}{x_s} < z'(x_s) < \frac{1+z_s}{x_s}$.

Proof. Suppose $\frac{z_s}{x_s} < z'(x_s) < \frac{1+z_s}{x_s}$. From $g_5^* = \alpha_s + \frac{(\beta_s + \gamma_s p h_s)\theta(\alpha_i k_5^* + \alpha_s)}{p\Omega_5}$,

$$\begin{aligned}\frac{\partial g_5^*}{\partial x_s} = [&p\Omega_5\{\gamma_s\theta(\alpha_i k_5^* + \alpha_s)(h_s\frac{\partial p}{\partial x_s} + p\frac{\partial h_s}{\partial x_s}) + (\beta_s + \gamma_s p h_s)\theta\alpha_i\frac{\partial k_5^*}{\partial x_s}\} \\ &- (\beta_s + \gamma_s p h_s)\theta(\alpha_i k_5^* + \alpha_s)(\Omega_5\frac{\partial p}{\partial x_s} + p\frac{\partial \Omega_5}{\partial x_s})] \times \frac{1}{p^2\Omega_5^2}\end{aligned} \tag{43}$$

Now $\frac{z_s}{x_s} < z'(x_s)$ implies $z_s'(x_s) > 0$ as z_s and x_s are positive. Therefore, $\frac{\partial h_s}{\partial x_s} = \frac{z_s'(x_s)}{(1+z_s)} > 0$. Also from (41), $z'(x_s) < \frac{1+z_s}{x_s}$ implies $\frac{\partial p}{\partial x_s} < 0$ as $p > 0$. Further, from (41) and $h_s = \frac{z_s}{1+z_s}$, $(h_s\frac{\partial p}{\partial x_s} + p\frac{\partial h_s}{\partial x_s}) = \frac{p}{1+z_s} \times (z_s'(x_s) - \frac{z_s}{x_s})$. Thus $\frac{z_s}{x_s} < z'(x_s)$ implies $(h_s\frac{\partial p}{\partial x_s} + p\frac{\partial h_s}{\partial x_s}) > 0$. Next, from (42) note that, $\frac{\partial p}{\partial x_s} < 0$ and $\frac{\partial h_s}{\partial x_s} > 0$ imply $\frac{\partial \Omega_5}{\partial x_s} < 0$ as β_s, θ and γ_s are positive. Then, from (43), $(h_s\frac{\partial p}{\partial x_s} + p\frac{\partial h_s}{\partial x_s}) > 0$, $\frac{\partial p}{\partial x_s} < 0$ and $\frac{\partial \Omega_5}{\partial x_s} < 0$ imply $\frac{\partial g_5^*}{\partial x_s} > 0$ if $\frac{\partial k_5^*}{\partial x_s} \geq 0$. Finally, in order to complete the proof, we need to show $\frac{\partial g_5^*}{\partial x_s} > 0$ when $\frac{\partial k_5^*}{\partial x_s} < 0$. Suppose, on the contrary, $\frac{\partial k_5^*}{\partial x_s} < 0$ and $\frac{\partial g_5^*}{\partial x_s} \leq 0$ at some arbitrary value of $x_s = x_{s1}$. Let k_{51}^*, p_1, h_{s1} and $\frac{1}{\Omega_5(x_{s1})}$ be the steady state of (40), the relative price of the service, profit share in service sector and the multiplier respectively when labour productivity of the service is x_{s1}. Also, let $g^*_{51} = \alpha_i + \frac{\beta_i\{\alpha_i+(\alpha_s/k^*_{51})\}}{\Omega_5(x_{s1})} = \alpha_s + \frac{(\beta_s+\gamma_s p_1 h_{s1})\theta(\alpha_i k^*_{51}+\alpha_s)}{p_1\Omega_5(x_{s_1})}$. Since $\frac{\partial k^*_5}{\partial x_s} < 0$ and $\frac{\partial g^*_5}{\partial x_s} \leq 0$ at $x_s = x_{s1}$, there exist a $x_{s2} > x_{s1}$ such that $k^*_{51} > k^*_{52}$ and $g^*_{51} \geq g^*_{52}$, where k^*_{52} is the the steady state of (40) when $x_s = x_{s2}$, $g^*_{52} = \alpha_i + \frac{\beta_i\{\alpha_i+(\alpha_s/k^*_{52})\}}{\Omega_5(x_{s2})} = \alpha_s + \frac{(\beta_s+\gamma_s p_2 h_{s2})\theta(\alpha_i k^*_{52}+\alpha_s)}{p_2\Omega_5(x_{s_2})}$ with p_2 the relative price of the service, h_{s2} profit share in service sector and $\frac{1}{\Omega_5(x_{s2})}$ the multiplier respectively when $x_s = x_{s2}$. Now, $g^*_{51} \geq g^*_{52}$ implies $\alpha_i + \frac{\beta_i\{\alpha_i+(\alpha_s/k^*_{51})\}}{\Omega_5(x_{s1})} \geq \alpha_i + \frac{\beta_i\{\alpha_i+(\alpha_s/k^*_{52})\}}{\Omega_5(x_{s2})}$. This implies $\Omega_5(x_{s1}) < \Omega_5(x_{s2})$ because $k^*_{51} > k^*_{52}$ and $\alpha_s > 0$. However this is a contradiction because $\frac{\partial \Omega_5}{\partial x_s} < 0$ and $x_{s1} < x_{s2}$ imply $\Omega_5(x_{s1}) > \Omega_5(x_{s2})$. □

5. Conclusion

We sum up this paper with the following comments. First, the result of Baumol (1967) that the inherent technologically stagnant nature of the service sector implies that expansion of the service sector inevitably leads to stagnation in the economy is no longer apt considering the widespread application of modern ICTs in various services. Further, the negative relationship between growth and the expansion of service sector in Baumol (1967) is driven largely by the assumption of full employment of resources, as pointed out by Dutt (1992). Second, a *ceteris paribus* increase in labour productivity of industry increases both the relative size of the industrial sector and the growth rate of the economy in the demand-constrained two-sector model of Dutt (1992). On the other hand, a *ceteris paribus* increase in the labour productivity of the service sector increases the relative size of the service sector but decreases the growth rate of the economy. Nonetheless the implications of sector specific technology shocks in models such as that of Dutt (1992) are sensitive to specification of demand interlinkages between the two sectors and the form of the demand function for industrial products. In a model similar to the demand constrained model of Dutt (1992) it can be shown that if demand for services per unit of industrial output increases with a fall in the relative price of services, then a *ceteris paribus* increase in the labour productivity of the service sector can increase both the growth rate of the economy and the relative size of the service sector. It can be argued that in modern times not only have many services have been extremely receptive towards adoption of ICTs but more and more of such services are being jointly purchased along with various industrial goods. So, if demand for services is sufficiently elastic with respect to its relative price then improvements in labour productivity of the service sector can provide sufficient boost to demand for both sectors and increase the growth rate of economy. The growth rate of the economy can increase even if adoption of ICTs in the service sector not only increases the labour productivity of the sector but also its 'degree of monopoly'.

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Appendix A

Proof of Proposition 4. Suppose $\Omega_1 > 0$. Setting the right hand sides of the two equations in (13) equal to zero yields the following system of linear equations.

$$\begin{bmatrix} -(\frac{sz_i}{1+z_i} - \beta_i) & (\frac{sp}{1+z_s} + \beta_s) \\ \lambda_1 & -1 \end{bmatrix} \begin{bmatrix} X_i \\ X_s \end{bmatrix} = \begin{bmatrix} -(\alpha_i K_i + \alpha_s K_s) \\ 0 \end{bmatrix} \tag{44}$$

Now, $\Omega_1 = \frac{sz_i}{1+z_i} - \beta_i - \lambda_1(\frac{sp}{1+z_s} + \beta_s)$ is the determinant of 2×2 matrix in (44). Since $\Omega_1 \neq 0$, *Cramer's rule* yields the unique solution of (44) as $X_i = \frac{\alpha_i K_i + \alpha_s K_s}{\Omega_1} = X^*_{i1}$ and $X_s = \frac{\lambda_1(\alpha_i K_i + \alpha_s K_s)}{\Omega_1} = X^*_{s1}$. $\Omega_1 > 0$ implies $X^*_{i1} > 0$ and $X^*_{s1} > 0$ as α_i, α_s, K_i, K_s and λ_1 are all positive. For stability, notice that the Jacobian matrix for (13) is

$$\begin{bmatrix} -\psi_i(\frac{sz_i}{1+z_i} - \beta_i) & \psi_i(\frac{sp}{1+z_s} + \beta_s) \\ \lambda_1 & -\psi_s \end{bmatrix}$$

with determinant $\psi_i\psi_s\Omega_1 > 0$ and trace $-\psi_i\Omega_1 + \lambda_1(\frac{sp}{1+z_s} + \beta_s) - \psi_s < 0$ when $\Omega_1 > 0$, as ψ_i, ψ_s, λ_1, s, p, z_s and β_s are all positive. □

Proof of Proposition 5. We can re-arrange (16) as $\dot{k} = \dfrac{a_1 k^2 + b_1 k + c_1}{\Omega_1}$ where $a_1 = -\alpha_i\beta_s\lambda_1$, $b_1 = (\alpha_i - \alpha_s)\Omega_1 + \alpha_i\beta_i + \lambda_1\alpha_s\beta_s$ and $c_1 = \alpha_s\beta_i$. In the steady state $a_1k^2 + b_1k + c_1 = 0$ as $\Omega_1 > 0$. Now $a_1 < 0$ and $c_1 > 0$ as α_i, α_s, β_i, β_s and λ_1 are all positive. $a_1 < 0$ and $c_1 > 0$ imply $b_1^2 - 4a_1c_1 > 0$. Thus $a_1k^2 + b_1k + c_1 = 0$ has two distinct real roots, $\frac{-b_1 \pm \sqrt{b_1^2 - 4a_1c_1}}{2a_1}$. Since $a_1 < 0$, the steady state value of k is $k^*_1 = \frac{-b_1 - \sqrt{b_1^2 - 4a_1c_1}}{2a_1}$. For stability, define a function $V_1 : \mathbb{R}_{++} \mapsto \mathbb{R}$ such that $V_1(k) = (g_i - g_s)^2$, where g_i and g_s are given by (14) and (15) respectively. Note that, by definition, $V_1(k^*) = 0$ and $V_1(k \neq k^*) > 0$ for all $k \in \mathbb{R}_{++}$. Also, using (11), (14) and (15), $\dot{V}_1 = -2k(g_i - g_s)^2(\frac{\beta_1\alpha_s}{\Omega_1 k^2} + \frac{\beta_s\lambda_1\alpha_i}{\Omega_1}) < 0$ for all $k \in \mathbb{R}_{++}$ and $k \neq k^*_1$ as α_i, α_s, β_i, β_s, λ_1 and Ω_1 are all positive. Thus, V_1 is a *strict Liapunov function* for k^*_1. □

Proof of Proposition 7. Suppose $\Omega_2 > 0$. Setting the right hand sides of the two equations in (19) equal to zero yields the following system of linear equations.

$$\begin{bmatrix} -(1-c-\beta_i) & \beta_s \\ 0 & -1 \end{bmatrix} \begin{bmatrix} X_i \\ X_s \end{bmatrix} = \begin{bmatrix} -(\alpha_i K_i + \alpha_s K_s) \\ -\lambda K_i \end{bmatrix} \tag{45}$$

$\Omega_2 = 1 - c - \beta_i$ is the determinant of 2×2 matrix in (45). Since $\Omega_2 \neq 0$, *Cramer's rule* yields the unique solution of (45) as $X_i = \frac{\alpha_i K_i + \alpha_s K_s + \beta_s \lambda K_i}{\Omega_2} = X_{i2}^*$ and $X_s = \lambda K_i = X_{s2}^*$. $\Omega_2 > 0$ implies $X_{i2}^* > 0$ and $X_{s2}^* > 0$ as α_i, α_s, K_i, K_s and λ are all positive. For stability, notice that the Jacobian matrix for (19) is

$$\begin{bmatrix} -\psi_i(1-c-\beta_i) & \beta_s \\ 0 & -\psi_s \end{bmatrix}$$

with determinant $\psi_i \psi_s \Omega_2 > 0$ and trace $-\psi_i \Omega_2 - \psi_s < 0$ when $\Omega_2 > 0$, as ψ_i and ψ_s are both positive. □

Proof of Proposition 8. We can re-arrange (22) as $\dot{k} = \dfrac{a_2 k^2 + b_2 k + c_2}{\Omega_2}$ where $a_2 = -\beta_s \lambda$, $b_2 = \{(\alpha_i - \alpha_s)\Omega_2 + \alpha_i \beta_i + \beta_i \beta_s \lambda\}$ and $c_2 = \alpha_s \beta_i$. In the steady state $a_2 k^2 + b_2 k + c_2 = 0$ as $\Omega_2 > 0$. Now $a_2 < 0$ and $c_2 > 0$ as α_s, β_i, β_s an λ are all positive. $a_2 < 0$ and $c_2 > 0$ imply $b_2^2 - 4a_2 c_2 > 0$. This means that $a_2 k^2 + b_2 k + c_2 = 0$ has two distinct real roots, $\frac{-b_2 \pm \sqrt{b_2^2 - 4a_2 c_2}}{2a_2}$. Since $a_2 < 0$, the steady state value of k is $k_2^* = \frac{-b_2 - \sqrt{b_2^2 - 4a_2 c_2}}{2a_2}$. For stability, define a function $V_2 : \mathbb{R}_{++} \mapsto \mathbb{R}$ such that $V_2(k) = (g_i - g_s)^2$, where g_i and g_s are given by (20) and (21) respectively. Note that, by definition, $V_2(k_2^*) = 0$ and $V_2(k \neq k_2^*) > 0$ for all $k \in \mathbb{R}_{++}$. Also, using (11), (20) and (21), $\dot{V}_2 = -2k(g_i - g_s)^2(\frac{\beta_i \alpha_s}{\Omega_2 k^2} + \beta_s \lambda) < 0$ for all $k \in \mathbb{R}_{++}$ and $k \neq k_2^*$ as α_s, β_i, β_s, λ and Ω_2 are all positive. Thus, V_2 is a *strict Liapunov function* for k_2^*. □

Proof of Proposition 10. Suppose $\Omega_3 > 0$. Setting the right hand sides of the two equations in (25) equal to zero yields the following system of linear equations.

$$\begin{bmatrix} -(1-c-\beta_i) & (cp+\beta_s) \\ \frac{\theta}{p} & -1 \end{bmatrix} \begin{bmatrix} X_i \\ X_s \end{bmatrix} = \begin{bmatrix} -(\alpha_i K_i + \alpha_s K_s) \\ 0 \end{bmatrix} \tag{46}$$

$\Omega_3 = 1 - c(1+\theta) - \beta_i - \frac{\theta \beta_s}{p}$ is the determinant of 2×2 matrix in (46). Since $\Omega_3 \neq 0$, *Cramer's rule* yields the unique solution of (46) as $X_i = \frac{\alpha_i K_i + \alpha_s K_s}{\Omega_3} = X_{i3}^*$ and $X_s = \frac{\theta(\alpha_i K_i + \alpha_s K_s)}{p\Omega_3} = X_{s3}^*$. $\Omega_3 > 0$ implies $X_{i3}^* > 0$ and $X_{s3}^* > 0$ as α_i, α_s, K_i, K_s, p, and θ are all positive. For stability, notice that the Jacobian matrix for (25) is

$$\begin{bmatrix} -\psi_i(1-c-\beta_i) & (cp+\beta_s) \\ \frac{\theta}{p} & -\psi_s \end{bmatrix}$$

with determinant $\psi_i\psi_s\Omega_3 > 0$ and trace $-\psi_i(\Omega_3 + \theta c + \frac{\theta\beta_s}{p}) - \psi_s < 0$ when $\Omega_3 > 0$, as ψ_i, ψ_s, θ, c, p and β_s are all positive. □

Proof of Proposition 11. We can re-arrange (28) as $\dot{k} = \frac{a_3k^2+b_3k+c_3}{p\Omega_3}$ where $a_3 = -\alpha_i\beta_s\theta$, $b_3 = (\alpha_i - \alpha_s)p\Omega_3 + \alpha_i\beta_i p - \alpha_s\beta_s\theta$ and $c_3 = \alpha_s\beta_i p$. In the steady state $a_2k^2 + b_2k + c_2 = 0$ as $p > 0$ and $\Omega_3 > 0$. Now, $a_3 < 0$ and $c_3 > 0$ as α_i, α_s, β_i, β_s, θ and p are all positive. This means that $a_3k^2 + b_3k + c_3 = 0$ has two distinct real roots since $a_3 < 0$ and $c_3 > 0$ imply $b_3^2 - 4a_3c_3 > 0$. These are $\frac{-b_3\pm\sqrt{b_3^2-4a_3c_3}}{2a_3}$. Since $a_3 < 0$, the steady state value of k is $k_3^* = \frac{-b_3-\sqrt{b_3^2-4a_3c_3}}{2a_3}$. For stability, define a function $V_3 : \mathbb{R}_{++} \mapsto \mathbb{R}$ such that $V_3(k) = (g_i - g_s)^2$, where g_i and g_s are given by (26) and (27) respectively. Note that, by definition, $V_3(k_3^*) = 0$ and $V_3(k \neq k_3^*) > 0$ for all $k \in \mathbb{R}_{++}$. Also, from (11),(26) and (27), $\dot{V}_3 = -2k(g_i - g_s)^2(\frac{\beta_i\alpha_s}{\Omega_3k^2} + \frac{\beta_s\theta\alpha_i}{p\Omega_3}) < 0$ for all $k \in \mathbb{R}_{++}$ and $k \neq k_3^*$ as α_i, α_s, β_i, β_s, θ, p and Ω_3 are all positive. Thus, V_3 is a *strict Liapunov function* for k_3^*. □

Proof of Proposition 13. Suppose $\Omega_4 > 0$. Setting the right hand sides of the two equations in (31) equal to zero yields the following system of linear equations.

$$\begin{bmatrix} -(1-c-\beta_i) & (cp+\beta_s) \\ \theta(p) & -1 \end{bmatrix} \begin{bmatrix} X_i \\ X_s \end{bmatrix} = \begin{bmatrix} -(\alpha_iK_i + \alpha_sK_s) \\ 0 \end{bmatrix} \tag{47}$$

$\Omega_4 = 1 - c - \beta_i - \theta(p)(cp + \beta_s)$ is the determinant of 2×2 matrix in (47). Since $\Omega_4 \neq 0$, *Cramer's rule* yields the unique solution of (47) as $X_i = \frac{\alpha_iK_i+\alpha_sK_s}{\Omega_4} = X_{i4}^*$ and $X_s = \frac{\theta(p)(\alpha_iK_i+\alpha_sK_s)}{\Omega_4} = X_{s4}^*$. $\Omega_4 > 0$ implies $X_{i3}^* > 0$ and $X_{s3}^* > 0$ as α_i, α_s, K_i, K_s and $\theta(p)$ are all positive. For stability, notice that the Jacobian matrix for (31) is

$$\begin{bmatrix} -\psi_i(1-c-\beta_i) & (cp+\beta_s) \\ \theta(p) & -\psi_s \end{bmatrix}$$

with determinant $\psi_i\psi_s\Omega_4 > 0$ and trace $-\psi_i(\Omega_4+\theta(p)(cp+\beta_s)-\psi_s < 0$ when $\Omega_4 > 0$, as ψ_i, ψ_s, $\theta(p)$, c, p and β_s are all positive. □

Proof of Proposition 14. We can re-arrange (34) as $\dot{k} = \frac{a_4k^2+b_4k+c_4}{\Omega_4}$ where $a_4 = -\alpha_i\beta_s\theta(p)$, $b_4 = (\alpha_i - \alpha_s)\Omega_4 + \alpha_i\beta_i - \alpha_s\beta_s\theta(p)$ and $c_4 = \alpha_s\beta_i$. In the steady state $a_4k^2+b_4k+c_4 = 0$ as $\Omega_4 > 0$. Now, $a_4 < 0$ and $c_4 > 0$ as α_i, α_s, β_i, β_s and $\theta(p)$ are all positive. This means that $a_4k^2 + b_4k + c_4 = 0$ has two distinct real roots since $a_4 < 0$ and $c_4 > 0$ imply $b_4^2 - 4a_4c_4 > 0$. These are $\frac{-b_4 \pm \sqrt{b_4^2-4a_4c_4}}{2a_4}$. Since $a_4 < 0$, the steady state value of k is $k_4^* = \frac{-b_4 - \sqrt{b_4^2-4a_4c_4}}{2a_4}$. For stability, define a function $V_4 : \mathbb{R}_{++} \mapsto \mathbb{R}$ such that $V_4(k) = (g_i - g_s)^2$, where g_i and g_s are given by (32) and (33) respectively. Note that, by definition, $V_4(k_4^*) = 0$ and $V_4(k \neq k_4^*) > 0$ for all $k \in \mathbb{R}_{++}$. Also, from (11), (32) and (33), we have $\dot{V}_4 = -\,2k(g_i - g_s)^2(\frac{\beta_i\alpha_s}{\Omega_4k^2} + \frac{\beta_s\theta(p)\alpha_i}{\Omega_4}) < 0$ for all $k \in \mathbb{R}_{++}$ and $k \neq k_4^*$ as α_i, α_s, β_i, β_s and $\theta(p)$ are all positive. Thus, V_4 is a *strict Liapunov function* for k_4^*. $\square$

Proof of Proposition 16. Suppose $\Omega_5 > 0$. Setting the right hand sides of the two equations in (37) equal to zero yields the following system of linear equations.

$$\begin{bmatrix} -(1-c-\beta_i) & (cp+\beta_s+\gamma_s p h_s) \\ \frac{\theta}{p} & -1 \end{bmatrix} \begin{bmatrix} X_i \\ X_s \end{bmatrix} = \begin{bmatrix} -(\alpha_i K_i + \alpha_s K_s) \\ 0 \end{bmatrix} \tag{48}$$

$\Omega_5 = 1 - c - \beta_i - \theta c - \frac{\theta\beta_s}{p} - \gamma_s h_s$ is the determinant of 2×2 matrix in (48). Since $\Omega_5 \neq 0$, *Cramer's rule* yields the unique solution of (48) as $X_i = \frac{\alpha_i K_i + \alpha_s K_s}{\Omega_5} = X_{i5}^*$ and $X_s = \frac{\theta(\alpha_i K_i + \alpha_s K_s)}{p\Omega_5} = X_{s5}^*$. $\Omega_5 > 0$ implies $X_{i5}^* > 0$ and $X_{s5}^* > 0$ asα_i, α_s, K_i, K_s, p, and θ are all positive. For stability, notice that the Jacobian matrix for (37) is

$$\begin{bmatrix} -\psi_i(1-c-\beta_i) & (cp+\beta_s) \\ \theta(p) & -\psi_s \end{bmatrix}$$

with determinant $\psi_i\psi_s\Omega_5 > 0$ and trace $-\psi_i(\Omega_4 + \theta c + \frac{\theta\beta_s}{p} + \gamma_s h_s - \psi_s < 0$ when $\Omega_4 > 0$, as ψ_i, ψ_s, θ, c, p, β_s, h_s and γ_s are all positive. $\square$

Proof of Proposition 17. We can re-arrange (40) as $\dot{k} = \frac{a_5k^2+b_5k+c_5}{p\Omega_5}$ where $a_5 = -\alpha_i\theta(\beta_s + \gamma_s p h_s)$, $b_5 = (\alpha_i - \alpha_s)p\Omega_5 + \alpha_i\beta_i p - \alpha_s\theta(\beta_s + \gamma_s p h_s)$ and $c_5 = \alpha_s\beta_i p$. Since $p > 0$ and Ω_5, $a_5k^2 + b_5k + c_4 = 0$ in the steady state. Now, since α_i, α_s, β_i, β_s, θ, γ_s, p and h_s are all positive, $a_5 < 0$ and c_4, which imply $b_5^2 - 4a_5c_5 > 0$. Thus, $a_5k^2+b_5k+c_5 = 0$ has two distinct real roots, $\frac{-b_5 \pm \sqrt{b_5^2-4a_5c_5}}{2a_5}$. Since $a_5 < 0$, the steady state value of k is $k_5^* = \frac{-b_5 - \sqrt{b_5^2-4a_5c_5}}{2a_5}$. For stability, define a function $V_5 : \mathbb{R}_{++} \mapsto \mathbb{R}$ such that $V_5(k) = (g_i - g_s)^2$, where g_i and g_s are given by (38) and (39) respectively.

Note that, by definition, $V_5(k_5^*) = 0$ and $V_5(k \neq k_5^*) > 0$ for all $k \in \mathbb{R}_{++}$. Also, from (11), (38) and (39), we have $\dot{V}_5 = -2k(g_i - g_s)^2\{\frac{\beta_i \alpha_s}{\Omega_5 k^2} + \frac{(\beta_s + \gamma_s p h_s)\theta \alpha_i}{p \Omega_5}\} < 0$ for all $k \in \mathbb{R}_{++}$ and $k \neq K_5^*$ as α_i, α_s, β_i, β_s, γ_s, p, h_s, θ and Ω_5 are all positive. Thus, V_5 is a *strict Liapunov function* for k_5^*. $\square$