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ABSTRACT

This paper examines the effect of monetary policy on the liquidity premium, i.e., the market value of the liquidity services that financial assets provide. To guide the empirical analysis, I set up a monetary search model in which bonds provide liquidity services in addition to money. The theory predicts that money supply and the nominal interest rate are positively correlated with the liquidity premium, but the latter is negatively correlated with the bond supply. The empirical analysis over the period from 1946 and 2008 confirms the theoretical findings. This indicates that liquid bonds are substantive substitutes for money and the opportunity cost of holding money plays a key role in asset price determination. The model can rationalize the existence of negative nominal yields, when the nominal interest rate is low and liquid bond supply decreases.

JEL Classification: E31, E41, E50, E52, G12

Keywords: asset price, money search model, liquidity, liquidity premium, money supply

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1 Introduction

Investors value financial assets not only for their intrinsic value, i.e., their expected dividend or payment stream, but also for their liquidity: their ability to help agents facilitate transactions. For instance, U.S Treasuries are often used as collateral in a secured credit market through repurchase agreements, they are easily sold for cash in secondary asset markets, and, oftentimes, they are used directly as means of payment. Accordingly, many liquid financial assets are priced above their respective fundamental value, and their prices are higher than those of illiquid assets with comparable safety and maturity characteristics. Also, the liquidity premia account for a large part of the variation in the liquid asset prices observed in financial markets.

The objective of this paper is to examine the determinants of such liquidity premia. First, I set up a monetary search model and use it as a guide to my empirical exercise. My model builds upon Lagos and Wright (2005), but I extend the baseline framework in order to include a risk-free government bond in addition to fiat money. The government bond is liquid in the sense that it is useful in the exchange process, which is explicitly incorporated into the model. Due to its liquidity, its equilibrium price exceeds the fundamental value, and its supply affects the price (or the yield) through changing the liquidity premium. The ability of the bond to facilitate trade in a goods market characterized by frictions (such as anonymity and limited commitment) makes it a substitute for money to some degree, so that money supply, and not only bond supply, affects the liquidity premium and, consequently, the bond's price. The key mechanism that links money supply and the liquidity premium is the opportunity cost of holding money. An increase in money supply raises the inflation rate and, hence, the cost of holding money, implying a higher nominal interest rate through the Fisher effect. As a result, agents substitute the more costly fiat money with the liquid bond, which, in turn, leads to a higher liquidity premium and, ultimately, a higher bond price. In the extreme case in which the nominal interest rate (on an illiquid bond) is close to zero, and the supply of liquid assets is scarce, the model predicts the existence of a very high liquidity premium, and a potentially negative nominal yield on the liquid asset. Hence, my model can help us understand the emergence of negative yields could in several developed countries, such as Switzerland, the United States, and Germany, since 2008.

Next, I move on to the empirical exercise that tests the primary results of the theoretical model. In particular, I test whether money supply is positively correlated with the liquidity premium on liquid bonds, and whether the latter is negatively correlated with bond supply. In addition, I examine empirically whether the existence of liquidity premia can be an important factor in explaining the aforementioned negative yields on liquid bonds. In my empirical exercise, I use US Treasuries to capture the liquid bonds introduced in the model. To guarantee robustness of the empirical analysis, I employ various measures of the liquidity premia for these bonds: the spread of AAA-rated corporate bonds against the long term Treasury bonds, the TED spread, and the spreads of AA-rated Commercial Papers and Federal Deposit Insurance Corporation (FDIC) insured Certificates of deposits against Treasury bills.[1] The choice of these financial assets is justified by the fact that they are comparably safe but not as liquid as Treasury bonds of similar maturities; therefore one can reasonably argue that any spread between the yield on these assets and Treasury bonds (of similar maturities) reflects differences in liquidity premia. Next, I use a monetary aggregate, Narrow Money, as a proxy of money because it only includes components which can be used as a direct medium of exchange implied by the theory unlike other broader monetary aggregates such as M1 and M2. Furthermore, its demand is stable against its holding cost, or nominal interest rates in that it displays a downward sloping curve over the sample period as the theory presents later.

The theoretical and empirical analysis can explain the emergence of negative nominal yields. According to the theory, a reduction in bond supply can drive down its yield into the negative territory, in situations where the monetary authority is setting a low (slightly above or around zero) nominal interest rate, as has been the case recently in Switzerland and in the United States. The empirical exercise confirms that a reduction in the bond supply increases the liquidity premium, and decreases the yield. In fact, a big drop in the supply of liquid government bonds was markedly observed in Switzerland during the financial crisis, starting in the last quarter of 2008. It should be pointed out that the existence of negative nominal yields is often considered anomalous, because it is hard to reconcile through the lens of traditional monetary models. However, my model of asset liquidity can help rationalize this observation.[2]

From the theoretic point of view, a large money search literature presents that the liquidity premium is a primary factor of variation in the prices of liquid financial assets, and that its supply is negatively correlated with the liquidity premium, whereas money supply is positively. Similarly, the key mechanism in the literature is the opportunity cost of holding money. As a pioneer theoretical paper, Geromichalos, Licari, and Suarez-Lledo (2007) set up a Lagos-Wright type of money search framework with a real asset, and theoretically present that the money growth rate increases the liquidity premium in the economy where neither money nor assets are plentiful. They derive this result from the model where assets are a perfect substitute to money in transactions in a decentralized market, and money supply leads to an increase in the opportunity cost of holding money. Similarly, several papers with this substitution relationship between money and financial assets in the literature deliver the more or less similar results. Examples include Rocheteau and Wright (2005a), Lagos (2010b), Lester, Postlewaite, and Wright (2012), Jacquet and Tan (2012), Williamson (2012), Carapella and Williamson (2015), Geromicha-

[1] I choose the measures which Krishnamurthy and Vissing-Jorgensen (2012) use in their paper for comparison as well as an additional measure such as the TED spread. Also, the quarterly data are used here, unlike the yearly data are used in Krishnamurthy and Vissing-Jorgensen (2012) to increase the sample size of the measures.

[2] One of the important lessons we've learned from asset liquidity is that it can shed light on existing asset-related puzzles from a new perspective and provide a liquidity-based theory of asset pricing. Examples include Lagos (2010a), Geromichalos, Herrenbrueck, and Salyer (2013), Geromichalos and Simonovska (2011), and Jung and Lee (2015).

los and Herrenbrueck (2016), Rocheteau, Wright, and Xiao (2016), and Geromichalos, Lee, Lee, and Oikawa (2016). Also, Aruoba, Waller, and Wright (2011) calibrate money search models to examine how money supply affect capital formation. However, the literature has not tested empirically its aforementioned results, and not investigated much how liquidity premia cause the negative yields on liquid assets, either. Of course, it is worth noticing that this type of money search model is well fitted into the study mentioned above because it can delivers sharp predictions for the effects of money and bond supply on the liquidity premium. For example, one time injection of money does not affect the liquidity premium, but its growth does. Hence, money is neutral but not superneutral. Unlike money, one time injection of bonds has a substantial impact.[3] A model without money, or without explicit exchange processes would not deliver these results precisely.

To my best knowledge, while the results about the effects of money and bond supply on the liquidity premium has not been tested empirically yet in the money search literature, there are a few papers in the finance literature which study the supply effect on the liquidity premium so as to present that bond supply has a negative impacts on the liquidity premium, or the convenience yield.[4] For example, Krishnamurthy and Vissing-Jorgensen (2012) show a strong negative relationship between the U.S. Treasury supply and its convenience yield. Greenwood, Hanson, and Stein (2015) show that the T-bill supply has a negative impact on its liquid premium. Longstaff (2004), Vayanos (2004), Brunnermeier (2009) and Krishnamurthy (2010) investigate liquidity premia, but focus on the short time period such as financial crises. They all set up the models without money; therefore, money supply does not affect liquidity premia at all even if liquid bonds play a role as substitutes with money in reality, and the opportunity cost of holding money does not work to account for it, either.

Nagel (2014) shows how this substitution relationship between money and liquid bonds affect the liquidity premium through variation in the opportunity cost of holding money, which is represented by the federal funds rate. The paper shows that federal funds rate is positively correlated with the liquidity premium. Then, it concludes that bond supply does not have a 'persistent' effect on the liquidity premium unlike Krishnamurthy and Vissing-Jorgensen (2012) and Greenwood, Hanson, and Stein (2015). However, this result is derived from the fact that changes in the federal funds rate are involved with changes in supply in the Treasury supply, because the open market operation by the Federal Reserve is associated with the buying of selling of the Treasuries in the open market, even though it does not account for all the changes in the Treasury supply. Also, it does not allow to distinguish the effect of bond supply from that of money supply, to the effect of changes in money growth from changes in money level, or to study specific reasons why the negative yields have been observed in the situation where the

[3] Rocheteau, Wright, and Xiao (2016) argue, for the first time in the literature, that this is why the open market operations by the fed have effects on the interest rates in the market.

[4] The convenience yield counts for the premia from both safety and liquidity attributes of a financial assets such as the U.S. Treasuries.

nominal interest rate is hovering around zero. Importantly, the money search model I set up allows me to separate these effects theoretically, and to provide a guidance to analyze why the negative yields on liquid bonds can emerge in the aforementioned situation.

The rest of the paper is organized as follows. Section 2 lays out a theoretic model to be tested in Section 3. In Section 3, I provide a description of the data which are used in the empirical work and test the results from the theory. In Section 4, I discuss negative yields on liquid bonds with the theoretical and empirical results from the previous sections. Section 5 concludes.

2 Model

2.1 Physical Environment

Time is discrete and continues forever. There is a discount factor between periods $\beta \in (0, 1)$. Each period is divided into two sub-periods. A decentralized market (henceforce DM) with frictions opens in the second sub-period, and a perfectly competitive or centralized market (henceforth CM) follows. The frictions are characterized by anonymity among agents and bilateral bargaining trade. As a result, unsecured credit is not allowed in transactions, and exchange must be $quid\ pro\ quo$ or needs secured credit. There are two divisible and nonstorable consumption goods: goods produced and consumed in the CM (henceforth $CM\ goods$) and special goods in the DM (henceforth $DM\ goods$). There are two types of agents; buyers and sellers, whose measures are normalized to the unit, respectively. They live forever. Their identities are determined by the roles which they play in the DM and permanent. While sellers produce, sell and do not consume DM goods, buyers consume and do not produce. Their preferences in period t are given by

$$\begin{aligned} \text{Buyers}: \quad \mathcal{U}(x_t, h_t, q_t) =& u(q_t) + U(x_t) - h_t \\ \text{Sellers}: \quad \mathcal{V}(x_t, h_t, q_t) =& - c(q_t) + U(x_t) - h_t \end{aligned}$$

where x_t is consumption of CM goods, q_t consumption of DM goods, h_t hours worked to produce CM goods, and $c(q_t)$ a cost of production of q_t. As usual, U and u are twice continuously differentiable with with $U' > 0, u' > 0, U'' < 0, u'' < 0, u(0) = 0, u'(0) = \infty, u'(\infty) = 0$. Also, I assume that $c(q_t) = q_t$. Let $q^* \equiv \{q : u'(q) = 1\}$, i.e., it denotes the optimal consumption level in the DM. Also, assume that there exists $x^* \in (0, \infty)$ such that $U'(x^*) = 1$ and $U(x^*) > x^*$.

There are two types of assets; fiat money and a 1-period real government bond. They are perfectly divisible and storable. Agents can purchase any amount of money and government bonds at the ongoing price ϕ_t and ψ_t in the CM, respectively. Money grows at the rate of μ: $M_{t+1} = (1+\mu)M_t$. I assume that $\mu > \beta - 1$, but also consider the limit case where $\mu \to \beta - 1$, i.e., the case where the money growth rate approaches closely the Friedman rule. If μ is positive, it implies that new money is injected, but if μ negative, withdrawn through lump-sump transfers to buyers in the CM. A government bond issued in period t delivers one unit of CM good in period $t+1$, and its supply in period t is A_t. Since we focus on stationarity equilibria, A_t is fixed at A. The government (a consolidate authority) budget constraint is

$$G_t + T_t - \phi_t \mu M_t + A(1 - \psi_t) = 0,$$

where G_t is government expenditure, T_t is a lump-sum transfer or tax, $\phi_t \mu M_t$ is seigniorage of new money injection, and $A(1 - \psi_t)$ is government debt service.

Now, I describe more details about activities which occur in each sub-period. First, I start with the description of the second sub-period, where a CM opens. Both buyers and sellers consume and produce a CM good. They work or use their assets, money (m) and government bonds (a), which they are holding from the previous period in order to consume, so as to pay back the credit made in the previous period and to adjust their portfolios which they will bring to the next DM. They have access to technology that turns one unit of labor into one unit of general goods. Also, they trade money, and bonds among all agents to re-balance their portfolio they will bring to the next period.

Next, a DM opens in the first sub-period. All of the buyers are matched with a seller in a bilateral fashion and vice versa. Buyers make a take-it–or-leave-it (henceforth $TIOLI$) offer to a seller to determine the terms of trade.[5] Since buyers are anonymous and have limited commitment, a medium of exchange (henceforth MOE) is required in their transactions. Both money and government bonds can serve as media of exchange. Specifically, the DM is divided into two sub-markets, DM1 and DM2, depending on what type of medium of exchange can be used. In the DM1, sellers accept only a direct medium of exchange. Both assets are used as a direct medium of exchange, but, unlike money, only a fraction $g \in (0, 1)$ of bonds can serve as a direct medium of exchange. g is an illiquidity parameter of government bonds, and reflects the fact that the government bonds are not as liquid as money as a direct medium of exchange. Intuitively speaking, it can take time and cost in playing a role as money do in exchange in the DM1. On the other hand, in the DM2, sellers accpet only collateralized credit (or loans), i.e., secured credit as a MOE, and bonds are used as collateral for credit. The credit is repaid back in CM goods in the forthcoming CM. Also, a portion $h \in (0, 1)$ of bonds can only be used as collateral; therefore, buyers always have incentives to pay back their credits. This is so-called the Loan to Value (henceforth LTV) ratio, and also is related to the haircut since it is defined by 1 minus the LTV, following the standard approach in finance. The model will focus on cases of

[5] I could assume that they negotiate the terms of trade through a Kalai bargaining protocol, where the buyers' bargaining power is less than one. However, since the bargaining protocol is not critical to derive most of interesting results of the paper, I use the simplest setup here by assuming that buyers make a TIOLI offer to their trading partnert.

incentive compatible contracts. All buyers and sellers visit $DM1$ and $DM2$ with probabilities θ and $1-\theta$, respectively. Figure 1 summarizes the events within each period.

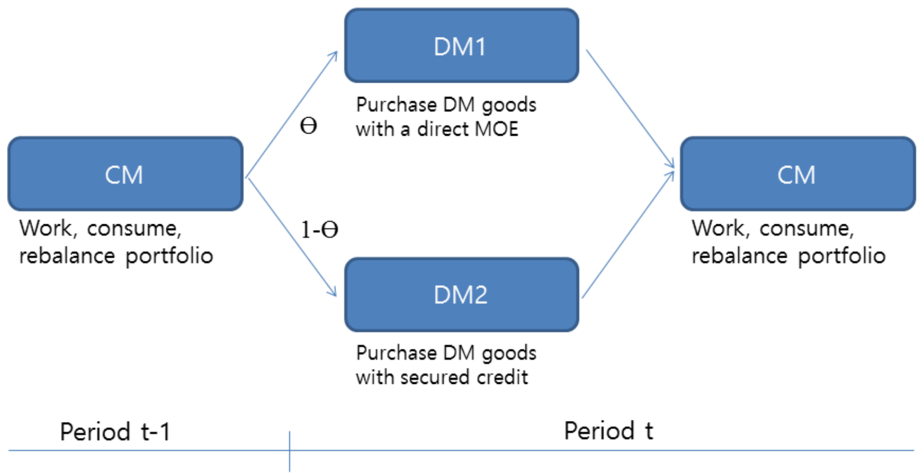

Figure 1: Market Timing

2.2 Value Functions

First, I describe the value function of a representative buyer who enters the CM with money (m), bonds (a) and the collateralized credit (ℓ) made last period, since it is the buyer that makes primary decisions for interesting results from the model. The value function of the buyer is

$$W^B(\mathbf{w}_t, \ell_t) = \max_{x_t, h_t, \mathbf{w}_{t+1}} \left\{ U(x_t) - h_t + \beta \mathbb{E}\left[V^B(\mathbf{w}_{t+1}) \right] \right\} \quad (1)$$
$$\text{s.t.} \;\; x_t + \boldsymbol{\phi}_t' \mathbf{w}_{t+1} = h_t + \boldsymbol{\phi}_t \mathbf{w}_t - \ell_t + T$$

where $\mathbf{w}_t = (m_t, a_t)$, $\boldsymbol{\phi}_t' = (\phi_t, \psi_t)$, and $\boldsymbol{\phi}_t = (\phi_t, 1)$. ℓ_t stands for the collateralized loan which is made last sub-period, and so must be paid back in the form of general goods. T_t is a lump-sum transfer to the buyer. V^B represents the buyer's value function in the next period DM. It can be easily verified that $x_t = x^*$ at the optimum. Substituting h_t in the budget constraint into the value function W^B yields

$$W^B(\mathbf{w}_t, \ell_t) = \boldsymbol{\phi}_t \mathbf{w}_t - \ell_t + \Lambda_t^B \quad (2)$$

where $\Lambda_t^B \equiv U(x^*) - x^* + T_t + \max_{x_t, \mathbf{w_{t+1}}} \{ -\boldsymbol{\phi}_t' \mathbf{w}_{t+1} + \beta \mathbb{E}\left[V^B(\mathbf{w}_{t+1}) \right] \}$. Notice that the value function in the CM is linear in the choice variables due to the quasi-linearity of $\mathcal{U}$, as in the standard models which are based on Lagos and Wright (2005). Consequently, the optimal choices of the buyer do not depend on the current state variables.

Next, consider a representative seller with money, bonds, and the collateralized loan who enters the CM. The loan is paid back by the counterpart buyer who she met in the previous DM.

$$W^S(\mathbf{w}_t, \ell_t) = \max_{x_t, h_t} \left\{ U(x_t) - h_t + \beta \mathbb{E}\left[V^S(\mathbf{0})\right] \right\}$$
$$\text{s.t. } x_t = h_t + \boldsymbol{\phi}_t \mathbf{w}_t + \ell_t$$

where V^S denotes the seller's value function in the DM. Notice that $\mathbf{w}_{t+1} = \mathbf{0}$ for the seller. Since the seller does not consume any good in the DM, there is no incentive to bring money and bonds to the next period DM, when the money holding cost is strictly positive due to $\mu_t > \beta - 1$.[6] It is also easily verified that $x_t = x^*$ at the optimum as in the case of the buyer. Replacing h_t into the value function yields

$$W^S(\mathbf{w}_t, \ell_t) = \boldsymbol{\phi}_t \mathbf{w}_t + \ell_t + \Lambda_t^S \tag{3}$$

where $\Lambda_t^S \equiv U(x^*) - x^* + \beta \mathbb{E}[V^S(0)]$.

Next, the DM opens. Buyers visit the DM1 with the probability θ and the DM2 with the probability $1 - \theta$. Also, all agents match in each DM. Hence, the expected value function of a buyer with portfolio $\mathbf{w}_t$ in the DM is given by

$$V^B(\mathbf{w}_t) = \theta \left[u(q_t^1) + W^B(\mathbf{w}_t - \boldsymbol{p}_t, 0)\right] + (1 - \theta) \left[u(q_t^2) + W^B(\mathbf{w}_t, \ell_t)\right] \tag{4}$$

where $\boldsymbol{p}_t = (p_t^m, p_t^a)$ is a portfolio exchanged for DM goods in a meeting with a seller in the DM1, and ℓ_t is the collateralized loan made in the DM2. q_t^1 (q_t^2) represents the quantity that are traded in the DM1 (DM2). The terms of trades in each market are determined by bargaining in pairwise meetings which Section 2.3 describes.

The value function of a seller is similar except for the fact that the seller does not bring any money and bonds to the DM for transactions.

$$V^S(\mathbf{0}) = \theta \left[-q_t^1 + W^S(\boldsymbol{p}_t, 0)\right] + (1 - \theta) \left[-q_t^2 + W^S(\mathbf{0}, \ell_t)\right]$$

2.3 Bargaining Problems in the DM

There are two sub-markets in the DM: DM1 and DM2, depending on what type of means of payment can be used in transactions. First, consider a meeting in the DM1 where a buyer with portfolio $\mathbf{w}_t$ meets with a seller. Sellers accept both money and bonds as a medium of exchange. However, a fraction g of bonds can only be accepted. The terms of trade is determined by the proportional bargaining over the quantity of DM goods, and a total payment of money

[6] See Rocheteau and Wright (2005b) for the precise and careful proof.

and bonds exchanged between them. A buyer makes a take-it-or-leave-it offer to a seller to maximize her surplus under the seller's participation constraint and her budget constraint. Then, the bargaining problem is expressed by

$$\max_{q_t^1, \boldsymbol{p}_t} \left\{ u(q_t^1) + W^B(\mathbf{w}_t - \boldsymbol{p}_t, 0) - W^B(\mathbf{w}_t, 0) \right\} \tag{5}$$
$$\text{s.t.} - q_t^1 + W^S(\boldsymbol{p}_t, 0) - W^S(\mathbf{0}, 0) = 0,$$

and the effective budget constraint $\boldsymbol{p}_t \leq \widetilde{\mathbf{w}}_t$, and $\widetilde{\mathbf{w}}_t = (m_t, g \cdot a_t)$. Notice that, since bonds are not as liquid as money in the DM1, the effective budget is less than the total budget. Only a fraction $g \in (0,1)$ of bonds can be used as a MOE here. Of course, I will consider an extreme case where $g \to 1$ later to discuss how negative interest yields emerge. Substituting (2) and (3) into (5) simplifies the above problem as follows.

$$\max_{q_t^1, \boldsymbol{p}_t} \left\{ u(q_t^1) - \boldsymbol{\phi}_t \boldsymbol{p}_t \right\} \tag{6}$$
$$\text{s.t.} - q_t^1 + \boldsymbol{\phi}_t \boldsymbol{p}_t = 0,$$

and $\boldsymbol{p}_t \leq \widetilde{\mathbf{w}}_t$, and $\widetilde{\mathbf{w}}_t = (m_t, g \cdot a_t)$. The following lemma summarizes the terms of trade which are determined by the solutions to bargaining problem.

Lemma 1. *The real balances of a representative buyer are denoted as $z(\mathbf{w}_t) \equiv \boldsymbol{\phi}_t \mathbf{w}_t$. Define $q^* = \{q : u'(q_t) = 1\}$, and z^* as the real balances of the portfolio (m_t, a_t) such that $\phi_t m_t + g a_t = q^*$. Also, $\boldsymbol{p}^*$ is the pairs of (m_t, a_t) in z^*. Then, the terms of trade are given by*

$$q_t^1(\mathbf{w}_t) = \begin{cases} q^*, & \text{if } z(\mathbf{w}) \geq z^*, \\ z(\widetilde{\mathbf{w}}_t), & \text{if } z(\mathbf{w}_t) < z^*. \end{cases} \quad \boldsymbol{p}_t(\mathbf{w}_t) = \begin{cases} \boldsymbol{p}^*, & \text{if } z(\mathbf{w}_t) \geq z^*, \\ \mathbf{w}_t, & \text{if } z(\mathbf{w}_t) < z^*. \end{cases} \tag{7}$$

Proof. See the appendix □

Similarly, in the DM2, a buyer makes a take-it-or-leave-it offer to a seller as in the DM1. However, she maximize her surplus subject to a different constraint, which is the credit limit constraint, unlike the effective budget constraint in DM1. Then, the bargaining problem is described as follows.

$$\max_{q_t^2, \ell_t} \left\{ u(q_t^2) + W^B(\mathbf{w}_t, \ell_t) - W^B(\mathbf{w}_t, 0) \right\} \tag{8}$$
$$\text{s.t.} - q_t^2 + W^S(\mathbf{0}, \ell_t) - W^S(\mathbf{0}, 0) = 0,$$

and the credit limit constraint $\ell_t \leq h a_t$. Substituting (2) and (3) into (8) yields the following expression.

$$\max_{q_t^2, \ell_t} \left\{ u(q_t^2) - \ell_t \right\} \tag{9}$$

$$-q_t^2 + \ell_t = 0, \tag{10}$$

and $\ell_t \leq ha_t$. The solution to the bargaining problem is described by the following lemma.

Lemma 2. *Define the total real value of a buyer's bond holdings as $z^a(\mathbf{w}_t) \equiv ha_t$. Also, define $z^{a*} \equiv q^*$. The terms of trade are given by*

$$q_t^2(\mathbf{w}_t) = \begin{cases} q^*, & \text{if } z^a(\mathbf{w}_t) \geq z^{a*}, \\ z^a(\mathbf{w}_t), & \text{if } z^a(\mathbf{w}_t) < z^{a*}, \end{cases} \quad \boldsymbol{\ell}(\mathbf{w}) = \begin{cases} z^{a*}, & \text{if } z^a(\mathbf{w}_t) \geq z^{a*}, \\ z^a(\mathbf{w}_t), & \text{if } z^a(\mathbf{w}_t) < z^{a*}. \end{cases} \tag{11}$$

Proof. See the appendix □

Since buyers make a TIOLI offer, i.e., they take all the bargaining power, the solution is straightforward. The main variables to determine the level of DM goods produced are the real balances, or the bond holdings of buyers in each transaction. For example, if the real balances are enough to get the optimal consumption level q^*, i.e., if $z(\mathbf{w}_t) \geq z^*$, then the optimal q^* level will be exchanged with the corresponding payment, z^*, which can be less than $z(\mathbf{w}_t)$. On the other hand, if the real balances are not enough in the same sense, then the buyers will hand over all of their real balances to sellers to purchase as many DM goods as possible. The sellers will produce the quantity that her participation constraint implies. The similar interpretation can be applied to the DM2.

2.4 Buyers' Optimal Choices

Now, I describe the objective function which a buyer maximizes by choosing money and bonds (m_{t+1}, a_{t+1}) in the DM. Substituting (4) into the inside of the maximization operator in (1) and using linearity of the value functions yield the following objective function J.

$$J = -\boldsymbol{\phi}_t'\mathbf{w}_{t+1} + \beta\Big\{\theta\big[u(q_{t+1}^1) + \boldsymbol{\phi}_{t+1}(\mathbf{w}_{t+1} - \boldsymbol{p}_{t+1})\big] + (1-\theta)\big[u(q_{t+1}^2) + \boldsymbol{\phi}_{t+1}\mathbf{w}_{t+1} - \ell_{t+1}\big]\Big\} \tag{12}$$

The first term stands for the cost of choosing money (m_{t+1}) and bonds (m_{t+1}) which buyers bring to the forthcoming DM. The terms in the curly bracket present the benefits they can obtain from transactions in the DM subject to their portfolios. Then, the Euler equations are given by

$$\phi_t = \beta \left[(1-\theta) + \theta u' \left(min\{ \boldsymbol{\phi}_{t+1} \widetilde{\mathbf{w}}_{t+1}, q^* \} \right) \right] \phi_{t+1}, \tag{13}$$

$$\psi_t = \beta \Big\{ \theta \big[(1-g) + g u' \left(min\{ \boldsymbol{\phi}_{t+1} \widetilde{\mathbf{w}}_{t+1}, q^* \} \right) \big] + (1-\theta) \big[(1-h) + h u' \left(min\{ h a_{t+1}, q^* \} \right) \big] \Big\}, \tag{14}$$

The left-hand side on each Euler equation presents the marginal cost of buying a unit of money or government bond. It is equal to its price to be paid when a buyer purchase it in the CM. On the other hand, the right-hand side is the marginal benefit from holding it in the DM. Buyers can use them as a medium of exchange to purchase DM goods produced by sellers. If they are used in the DM, i.e., if u' is zero on the right-hand side, its price should be equal to its fundamental value, $\beta\phi_{t+1}$ or β, respectively.

Figure 2 presents the continuous and decreasing money demand against the cost of holding money captured by $\phi_t/(\phi_{t+1}\beta)$, which comes from equation (13). Similarly, inserting equation (13) into (14) shows the inverse bond demand curve against its price. Their inverse relationship makes sense because the bond price implies the cost of holding the bonds, given the fixed dividend in the forthcoming CM. Also, the bond demand curve depends on the cost of holding money. It is easily found that the curve shifts out (in) as the money holding cost increases (decreases) as in Figure 3. This relationship is intuitively straightforward to understand. If the money holding cost increases, agents become less willing to hold money, i.e., the money demand will decrease. However, since the government bonds can also play a role in relaxing the liquidity constraint in the DM to some extent as money does, even if not perfectly, the demand on the government bonds will increase. Notice that both demand curves are flat in the regions where $m > m^*$ and $ha_t > q^*$, respectively. This is because one extra unit of money or government bond is not useful in the DM transactions any more. In these territories, buyers already hold money or bonds enough to purchase q^*.

2.5 Equilibrium and Characterization

I focus on stationarity equilibria, in which both real money and bond balances are constant over time. It implies that $\phi_t M_t = \phi_{t+1} M_{t+1}$ and $A_t = A$. Then, the money growth rate is equal to the inflation rate in the CM, i.e., $1 + \mu = \phi_t/\phi_{t+1} = 1 + \pi$.

Definition 1. A steady state equilibrium is a list of real balances of buyers, $\tilde{z}_t = \phi_t M_t + gA$, and bond holdings $\tilde{z}^a = hA$, money and bond prices $\boldsymbol{\phi}_t'$, bilateral terms of trade in $DM1$: $q(\mathbf{w}_t)$ and $\boldsymbol{p}(\mathbf{w}_t)$ which are given by Lemma 1, and bilateral terms of trade in $DM2$: $q(\mathbf{w}_t)$ and $\ell(\mathbf{w}_t)$ which are given by Lemma 2 such that:

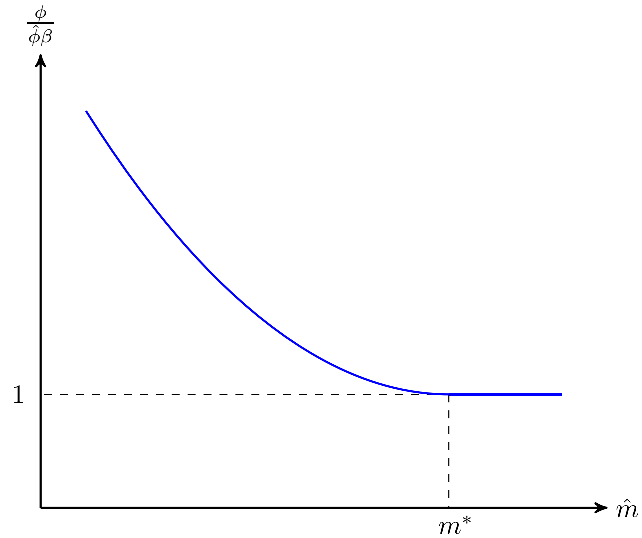

Figure 2: Money Demand

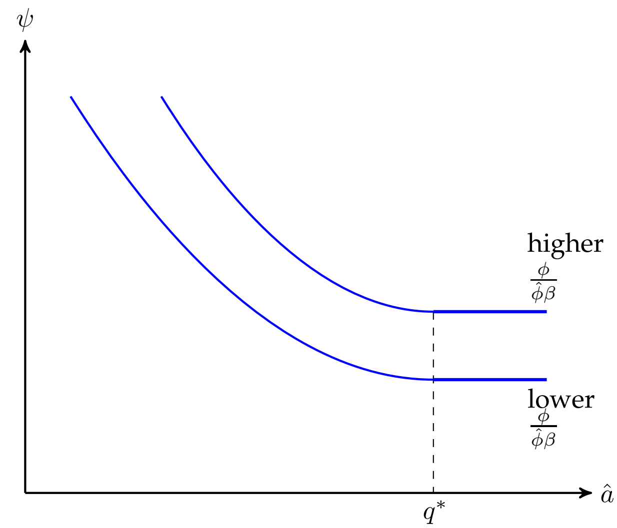

Figure 3: Asset Demand

(i) the decision rule of a representative buyer solves the individual optimization problem (1), taking prices $\boldsymbol{\phi}_t'$ and $\phi_t/\phi_{t+1} = 1+\mu$ as given;

(ii) the terms of trade in the DM satisfy (7) and (11);

(iii) prices are such that the CM clears, i.e., $\mathbf{w}_{t+1} = [\mu M_t, A]$ for buyers.

Then, the following lemma summarizes the equilibrium objects.

Lemma 3. *There exists a unique steady state equilibrium with four different cases.*

(i) If $\tilde{z}_t \geq z^*$ *and* $\tilde{z}^a \geq z^*$*, then,* $q_t^1 = q_t^2 = q^*$*,* $\phi_t = (z^* - gA)/M_t$*, and* $\psi_t = \beta$*;*

(ii) If $\tilde{z}_t \geq z^*$ *and* $\tilde{z}^a < z^*$*, then,* $q_t^1 = q^*$*,* $q_t^2 = \tilde{z}_t^a$*,* $\phi_t = (z^* - gA)/M_t$*, and* $\psi_t = \beta\{\theta + (1-\theta)\big[(1-h) + hu'(q_t^2)\big]\}$*;*

(iii) If $\tilde{z}_t < z^*$ *and* $\tilde{z}^a \geq z^*$*, then,* $q_t^1 = \tilde{z}_t$*,* $q_t^2 = q^*$*,* $\phi_t = (q_t^1 - gA)/M_t$*, and* $\psi_t = \beta\big\{\theta\big[(1-g) + gu'\left(q_t^1\right)\big] + (1-\theta)\big\}$*;*

(iv) If $\tilde{z}_t < z^*$ *and* $\tilde{z}^a < z^*$*, then,* $q_t^1 = \tilde{z}_t$*,* $q_t^2 = \tilde{z}_t^a$*,* $\phi_t = (q_t^1 - gA)/M_t$*, and* $\psi_t = \beta\big\{\theta\big[(1-g) + gu'\left(q_t^1\right)\big] + (1-\theta)\big[(1-h) + hu'\left(q_t^2\right)\big]\big\}$*.*

Proof. See the appendix. □

It is straightforward to understand the definition of equilibrium. The fact that the real money balances and the bond supply are constant over time in the steady state implies that both $\tilde{z}_t$ and $\tilde{z}^a$ are constant. Then, given the market clearing condition, $\tilde{z}_t$ and $\tilde{z}^a$ determine the quantities and real money and bond balances exchanged in the DM, following Lemma 1 and 2.

Now, the Euler equations, (13) and (14), for the optimal money and bond holdings with the above definition can be reexpressed as follows.

$$\phi_t = \beta\left\{1 + \theta\left[u'\left(min\{\tilde{z}_{t+1}, q^*\}\right) - 1\right]\right\}\phi_{t+1} \tag{15}$$

$$\psi_t = \beta\left\{1 + \theta \cdot g\left[u'\left(min\{\tilde{z}_{t+1}, q^*\}\right) - 1\right] + (1-\theta)h\left[u'\left(min\{\tilde{z}^a, q^*\}\right) - 1\right]\right\} \tag{16}$$

In order to examine how the equilibrium bond price responds to changes in money and bond supply, let's plug (15) into (16), then the price is as follows.

$$\psi = \beta\left\{1 + g\left(\frac{1+\mu}{\beta} - 1\right) + (1-\theta)h\left[u'\left(\tilde{z}^a\right) - 1\right]\right\} \tag{17}$$

$$= \beta\left\{1 + gi + (1-\theta)h\left[u'\left(\tilde{z}^a\right) - 1\right]\right\} \tag{18}$$

where $i \equiv (1+\mu)/\beta - 1$. There are several interesting points to notice here. First of all, the last equation is obtained by the Fisher equation, because $\mu = \pi$ in the stationary equilibrium and $1/\beta = 1+r$ when r stands for the yield on a real bond which is not useful in the DM exchange in the sense that it is not accepted by sellers. Hence, i represents a nominal interest rate of a totally illiquid real bond. Its real price (β), which is the inverse of the real interest rate $1/\beta$, is exactly equal to asset prices which are derived in the traditional asset pricing models where assets are only used as a store of value: the asset prices equal the present discount value of their future stream of consumption dividends. Second, the price of a liquid bond (ψ) is always higher than that of a illiquid bond (β) when the asset supply is not high enough in the sense that $hA < q^*$, i.e., $u'(\tilde{z}^a) > 1$. In other words, the rate of return on a liquid bond (ρ) is lower than that on an illiquid (i).[7] Hence, the difference between them can be used to measure the market price of the liquidity service that the liquid bond provides, i.e., the liquidity premium. Lastly, the zero net nominal interest rate ($i = 0$) implies that the money growth equals the Friedman rule, i.e., $\mu = \beta - 1$.

More importantly, the equations, (17) and (18), present that not only bond supply but also money supply determine the equilibrium bond price together with the bond demand when $0 < g < 1$, i.e., when they are substitutes to some extent in the sense that bonds help to relax the liquidity constraint in the DM1. This implies that the demands on money and bonds are interconnected because both of them are useful in exchange process, to a greater and lesser extent, as in the papers in the money search literature such as Geromichalos, Licari, and Suarez-Lledo (2007) in which money and real assets are perfect substitutes, and Lester, Postlewaite, and Wright (2012) in which the illiquid parameter g is endogenized. In this case, the price of the illiquid bond is fixed at β over time, whereas the price of the liquid bond is affected by both

[7] To distinguish nominal yields between an illiquid bond and an liquid bond, let ρ denote the latter.

money and bond supply. Hence, the price variation of the liquid bond is equal to the variation of the liquidity premium, and the two words can be used interchangeably here when the bond price exceed the fundamental value.

Next, consider that not all $\mu \in (\beta - 1, \infty)$ are consistent with a monetary equilibrium. In fact, a monetary equilibrium is supported for the range of $(\beta - 1, \bar{\mu})$, where $\bar{\mu} \equiv \{\mu : \mu = \beta [1 + \theta [u'(\tilde{z}^a) - 1]]\}$. If we allow for the case where $\mu = \beta - 1$, it implies $\tilde{z} \geq q^*$, and so will be the lower bound for a monetary equilibrium. In this case, the marginal change in money supply never affect the liquidity premium of a liquid bond. On the other hand, the upper bound $\bar{\mu}$ decreases in bond supply A. It implies that agents are less patient with high inflation, so that less willing to hold money given the supply of money, as the supply of bond increases.

The following proposition describes how the real equilibrium bond price, or the liquidity premium, is associated with money and bond supply. As mentioned in the introduction, we focus on the monetary equilibria, where $\mu \in (\beta - 1, \bar{\mu})$.

Proposition 1. *The real bond price exceeds the fundamental value, i.e, $\psi > \beta$, and is increasing in μ. Also,*

(i) if $\tilde{z}^a \geq q^*$, $\psi = \psi(\mu)$, i.e., the bond price is only affected by the money supply.

(ii) if $\tilde{z}^a < q^*$, $\psi = \psi(\mu, A)$, i.e., the bond price is affected by both money and bond supply. In addition, it is decreasing in A, i.e., $\psi'(A) < 0$.

The proof is straightforward. Notice in Proposition 1 that μ is also replaced with i because they are linear by the definition as in equation (18). The real bond price exceeds its fundamental value because a liquid bond plays a role in facilitating transactions in the DM; otherwise would not occur. Hence, the liquid bond bears a liquidity premium. If $\tilde{z}^a > q^*$, i.e., the bond supply is plentiful in that it allows agents to purchase the optimal quantity, q^*, in the DM2. The marginal increase in the bond supply does not allow buyers to purchase additional goods in the DM anymore. In other words, changing the bond supply does not affect transactions in the DM2; therefore it does not affect the liquidity premium. However, money supply changes the liquidity premium. For example, increasing μ raises up the opportunity cost of holding money, so that it increases the demand on bonds and lowers the rate of return on bonds. On the other hand, if $\tilde{z}^a < q*$, i.e., the bond supply is scarce, not only money supply but also bond supply affect the liquidity premium. This is because the marginal change in the bond supply has an impact on relaxing the liquidity constraint in the DM exchange.

Next, consider now some extreme cases where money and bonds are perfect substitutes or not substitutes at all so as to understand intuitively how the parameters, g and $1 - \theta$, can affect bond prices, or the liquidity premium. Moreover, I will empirically test which case is well supported by the U.S. data in Section 3. As mentioned in subsection 2.1, g is an illiquid parameter, implying how liquid bonds are, comparing with money in DM transactions. This parameter can be interpreted as a parameter of how well developed or how liquid a secondary asset market is, where bonds are exchanged for money. Less friction in the secondary market implies higher g because less friction means that bonds are more easily converted to money, vice versa. For example, if there are more investors or buyers for bonds due to the developed institution, including high-quality trading platform technology, in the secondary market, assets are more likely to be liquidated easily, and so to provide liquidity services easily. On the other hand, the parameter $1-\theta$ can be interpreted as how well the collateralized credit market functions when money is scarce. More collateralized transactions in the financial market implies higher $1-\theta$.

Now, consider the four cases as follows, depending on the different combinations of g and θ (or 1-θ). All the results are delivered by equations (17) and (18).

(*Case 1: Perfectly illiquid bonds*) Bonds are totally illiquid in the sense that they are useless in the DM exchange, i.e., $g \to 0$ and $\theta \to 1$. This is the case where the bonds only function as a store of value. Hence, the real bond price ψ is equal to the fundamental value, β, i.e., the present value of the dividend that the bonds deliver next period, and so they do not carry the liquidity premium at all. As a result, it is not affected by money and bond supply at all.

(*Case 2: Perfect substitutes to money*) Bonds are perfect substitutes to money, i.e., $g \to 1$ and $\theta \to 1$. The bond prices are equal to $1+\mu$. Then its nominal yield ρ is give by

$$1+\rho = (1+\pi)\frac{1}{\psi} = \phi_t/\phi_{t+1} \times \phi_{t+1}/\phi_t = 1. \tag{19}$$

The gross nominal interest rate $(1+\rho)$ equals 1 and the net nominal interest rate equals *zero* all the times. In this case, since the bonds are identical with money in terms of ability of facilitating transactions in the DM and also deliver dividends, their real prices are higher than the fundamental value β. Moreover, an increase (a decrease) in money supply raises (lowers) the real prices, or the liquidity premia.

(*Case 3: Liquid bonds but not substitutes to money*) Bonds are liquid in the DM, but not substitutes to money at all, i.e., $g \to 0$ and $0 < \theta < 1$. This is the case where the bonds are perfectly illiquid in the DM1, and so the two decentralized markets are totally separated. Hence, the supply of each does not affect each other, so that the liquidity premia which the bonds carry is only affected by their supply, not by money supply.

(*Case 4: Liquid bonds and perfect substitutes*) Bonds are liquid in the DM2 and also perfect substitutes to money in the DM1, i.e., $g \to 1$ and $0 < \theta < 1$. They hold extra values in exchange process. Then, equation (17) yield the net nominal interest rate as follows.

$$\rho = \frac{-\beta(1-\theta)h\left[u'(hA)) - 1)\right]}{(1+\mu) + \beta(1-\theta)h\left[u'(hA)) - 1)\right]} \tag{20}$$

$$= \frac{(1-\theta)h\left[u'(hA)) - 1)\right]}{i + (1-\theta)h\left[u'(hA)) - 1)\right]} < 0 \tag{21}$$

In this case, the numerator is always negative only if $u'(hA) > 1$, i.e., only if bond supply is scarce. Also, the liquidity premium is affected by both money and asset supply. The nominal rate of return on a liquid bond is negative, irrespective of money and bond supply, or the nominal rate of return on a illiquid bond i. It implies that lenders are willing to pay interests even though they lend money, because the bonds provide extra liquidity services in transactions. I will discuss more details about under what conditions negative interest rates emerge in a generic case, unlike this extreme case, in Section 4.

3 Data and Empirical Results

The theory in the previous section predicts that the real price of a liquid bond, or the liquidity premium is affected by both money and bond supply in a general case where $0 < g, \theta < 1$, whereas that of an illiquid bond is unaffected, as summarized in Proposition 1. In this section, I test the predictions of the theory here with the U.S. data: whether the liquidity premium is positively associated with the rate of money supply, but negatively with bond supply in reality. Basically, equations (22) and (23) are used as a guide for the empirical analysis.

3.1 Data

First of all, it is important to discuss how to measure all of the variables mentioned in the theory such as the liquidity premium, money supply (or, the nominal interest rate of an illiquid bond), and bond supply from the data before we move on to the empirical results. As well known, the real rates of return are not observed in reality. Also, there exist various types of monetary aggregates, such as Monetary Base, Narrow Money, M1, and M2, which we can use to measure money supply, and several types of government bonds are traded in the financial market.

First, I describe how to measure the liquidity premium. As the theory presents in the previous section, a change in the real yield on a liquid bond is equal to a change in the liquidity premium which the bond carries, only if the bond is default-free, except for some extreme cases. The problem is that neither the real yield nor the liquidity premium is observable as an index. Only the nominal yield is observable. For this reason, I use the yield spread between a liquid bond and an illiquid bond as a proxy of the liquidity premium as in the relevant literature. This measure is not only measurable, but also consistent with the theory. The theory presents the nominal yield of an illiquid real bond, a liquid real bond, and the spread between them as follows.

$$
\begin{aligned}
1+i &= (1+\pi)(1+r) = (1+\pi)\frac{1}{\beta} \\
1+\rho &= (1+\pi)\frac{1}{\psi}
\end{aligned}
$$

$$
s = i - \rho \approx \psi - \beta = \left[1 + g(\frac{1+\mu}{\beta} - 1)\right] + (1-\theta)h\left[u'\left(\tilde{z}^a\right) - 1\right] \tag{22}
$$

$$
= 1 + gi + (1-\theta)h\left[u'\left(\tilde{z}^a\right) - 1\right], \tag{23}
$$

where r stands for the real yield on an illiquid bond, and ρ on a liquid bond. As we used in deriving equation (17), the first two equations present the nominal yields of an illiquid bond and a liquid bond by the Fisher equation. Then, subtracting the latter from the former delivers the approximate yield spread between them, which is given by equations (22) and (23). This subtraction eliminates the effect of the inflation on the nominal yields of both bonds at the same time and, therefore, isolates the liquidity premium alone from other components such as the risk premium and the term premium in bond prices in the case where both bonds have the same maturity and default risk. As a result, this spread can be used as a measure of the liquidity premium meant in the theory. Also, it is important that it can be easily computable as the yield spread of two different bonds in terms of liquidity. Of course, it should be emphasized that they should have same or, at least, similar maturities and default risks.

Then, what should liquid and illiquid bonds be in reality? First, when it comes to a liquid bond, we define a liquid bond in the model as a bond which is useful in exchange process. In reality, it implies that the liquid bond should be easy to be sold for cash in the secondary market, to be accepted directly as a medium of exchange, or to be used for credit (or loans) in the credit market such as the Repurchase Agreement market (or REPO in short) and the federal funds market. Moreover, it should be safe in the sense that it is sure to deliver its dividend at maturity, i.e., there is no probability to default until maturity. For this reason, here, I use the yields of all types of Treasuries such as Treasury bonds, notes and bills, as the nominal yields of the liquid bond meant by the model.

On the other hand, an illiquid bond in the model implies a bond which can not be used in exchange, and its holder should keep it in his hand until maturity for cash unless he accepts a huge discount for the secondary trade. Hence, it is inferior to the liquid bond only in terms of being liquidating. Of course, they should be exactly or, at least, similarly as safe as the liquid bond to avoid the case where their yield difference includes the risk premium. In the case where there is only a small difference in terms of the default risk, it can be controlled in regressions by adding variables to explain it. Similarly, the maturities should be the same for both bonds.

[8] Taking all of these into account, I use the yields of Aaa-rated corporate bonds, 3-month Commercial Papers, Federal Deposit Insurance Corporation (FDIC) insured Certificates of deposits (henceforth FDIC CDs) as measures of the yields of illiquid bonds. All of these are available in Krishnamurthy and Vissing-Jorgensen (2012)[9]. I will match each of those yields with Treasuries with consideration for maturity in order to compute the liquidity premium on each maturity.

Lastly, I also use the TED spread as a measure of the liquidity premium on three-month Treasury bills, which is the difference between three-month LIBOR based on US dollars and three-month Treasury bill. It is true that 3-month LIBOR based on US dollars bears the risk premium, because the contracts between banks are not default free. However, it is also true that it is small, and so it can be controlled and absorbed by a variable to represent default risk in the regression.

Next, consider which of monetary aggregates should be used to measure money supply in the data. As mentioned above, there are several monetary aggregates which are complied by the Fed: Monetary Base, Narrow Money, M1, M2, and M3. There are two criteria to think about which one among them is appropriate. First, what the theory regards as money is perfectly liquid in exchange process, or it is a perfect medium of exchange, comparing to bonds. Hence, the monetary aggregate meant by the theory should not include any type of illiquid financial assets such as savings deposits including money market deposit accounts and small-denomination time deposits, which is precisely time deposits in amounts of less than $100,000.[10] Then, this criterion excludes M2, and also broader monetary aggregates such as M3. Secondly, its demand against the opportunity cost of holding it, represented by the nominal interest rate (or the inflation rate), should be stable. If it is not, the mechanism through which the theory works can not be applied to explain the relationship between money supply and the liquidity premium. Specifically, it should be true that, when the opportunity cost of holding money rises up, the demand declines. Otherwise, it will never lead to an increase in the demand on liquid bonds as a substitute to money, and neither does the liquidity premium the bonds bear in the end. However, the demand on M1 against the nominal interest rate has not been stable, so that M1 is excluded in the regressions shown later.[11]

Based on the aforementioned criteria, I use Narrow Money as a measure of money. Narrow Money is well suited to the theory in the sense that it is absolutely used as a medium of ex-

[8]In reality, since there exist secondary trade markets for almost all of bonds only if sellers of bonds accept more or less, or even considerable losses, there would not exist a perfectly illiquid bond. In other words, it is hard to find totally illiquid bonds, even if they are almost as safe as Treasuries and have the same or similar maturities. However, it can not be denied that the yield spread reflects the liquidity premium, even if we consider it.

[9]I updated the yields of Aaa-rated corporate bonds and 3-month Commercial Papers in their dataset by using the data FRED Economic Data (https://fred.stlouisfed.org/) provides because some values are revised.

[10]Source: http://www.federalreserve.gov/releases/h6/current/default.htm

[11]See Lucas Jr. and Nicolini (2015) for details about the stability of M1. The paper investigate why monetary aggregates become unstable over time.

change in transactions.[12] Also, it presents a stable demand curve over the sample period, i.e., a unambiguously negative relationship with the nominal interest rate over the period from 1946 to 2008. In addition, notice that I use the Federal Funds rate as a proxy of the nominal interest rate on illiquid financial bonds. I could use other interest rates such as 3 month commercial paper rate, but they deliver the same relationship because they show the strong co-movement historically. Figure 4 displays the ratio of Narrow Money to nominal GDP against its holding cost (i), which implies $L = M/PY$ in order to look at the real demand on money or real money balances proportional to Y implied by equation 15. In the case where bond supply is not plentiful, it can be re-expressed to present the money demand as follows.

$$\frac{\phi_t}{\phi_{t+1}\beta} = 1 + \theta\left[u'(\tilde{z}) - 1\right]$$
$$\Leftrightarrow 1 + i = 1 + \theta\left[u'(\tilde{z}) - 1\right],$$

where $\frac{\phi_t}{\phi_{t+1}\beta} = 1 + i$.[13] Obviously, the graph on the left panel presents that the real balances $\tilde{z}$, or money demand $\hat{m}$ is negatively associated with the nominal interest rate of an illiquid bond, or the opportunity cost of holding money.

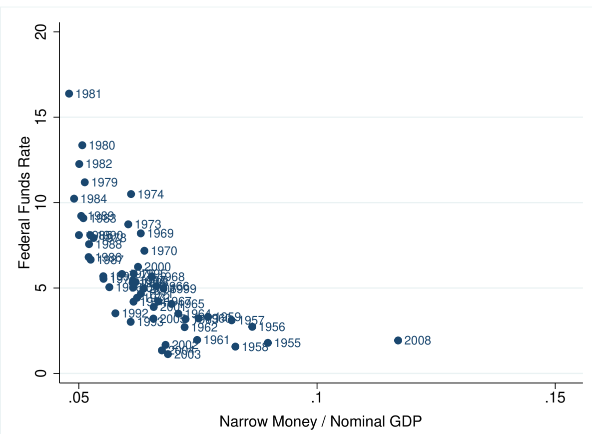

Figure 4: Money Demand

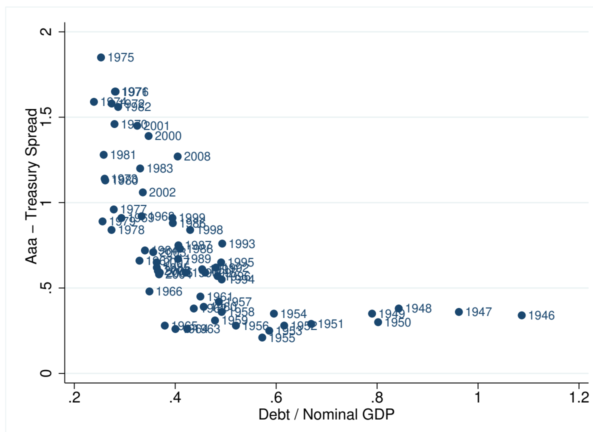

Figure 5: Bond Demand

Also, notice that money growth rate has a one to one relationship with the nominal yield of an illiquid bond in the stationary equilibrium, which is implied by the Fisher equation, i.e., $1+i = (1+\pi)(1+r) = (1+\mu)/\beta$, as seen in Equations (22) and (23). They theoretically stand for the same economic notion: the opportunity cost of holding money. The theory does not distinguish between them. The empirical result can show which variable is more suitable to explain changes in the liquidity premium. The nominal interest rate as an index of the opportunity cost

[12]Narrow Money includes nonbank public currency used as a medium of exchange, deposits held at Federal Reserves Banks, and reserve adjustment magnitude, which is"adjustments made to the monetary base due to changes in the statutory reserve requirements". See http://research.stlouisfed.org/publications/review/03/09/0309ra.xls for more details.

[13]Since $\tilde{z}_t = \tilde{z}_{t+1}$ in stationary equilibria, the time subscript is omitted.

of money holding has a positive impact on the liquidity premium through the same mechanism in which money growth works. In reality, there are a variety of interest rates in the financial market, and it is also difficult to find the yields of totally illiquid bonds. However, as well known, they have the strong co-movement relationship among them. Here, I use the Federal Funds rate as a proxy of the nominal interest rate which the model presents. The Federal Funds rate can reflect the money holding cost better than any other, because it is highly correlated with other short term interest rates which agents in an economy can consider as substitutes for cash, even if it is not perfectly substitutes. Moreover, since it is the policy interest rate which the Fed has been using, it is comparable to money supply as a policy variable.

When it comes to bond supply, I use the ratio of the outstanding stock of the public debt to the nominal GDP for the liquid bond supply meant by the theory, as in Krishnamurthy and Vissing-Jorgensen (2012). The ratio of debt to GDP is measured as the market value of the public debt at the end of a fiscal year divided by the GDP of the same year.[14] Using the same data as in Krishnamurthy and Vissing-Jorgensen (2012) and Nagel (2014) allows for comparison of my empirical results with the results in the literature.

Last but not least, Figure 5 looks at the bond demand against the yield spread between the Aaa-rated corporate bond and the long term Treasury bond. Notice that, as Krishnamurthy and Vissing-Jorgensen (2012) points out, it is the bond demand for not only liquidity but also safety, but it is mainly driven by the demand for the liquidity services Treasuries provide. It had been stable up until 2008 in the sense that it is an unambiguous downward sloping curve. However, after 2009, the demand seems to shift out after the recent financial crisis. This is similar to the money demand. For this reason, I only use the data over the period from 1945 up to 2008.

3.2 Empirical Results

Now, I describe the details about the empirical test for the theoretical predictions. As mentioned in the previous section, I first compute several different yield spreads between them and Treasuries, depending on maturities: the yield spreads between Aaa-rated corporate bonds and the long term Treasuries, between Aa-rated commercial papers and 3-month Treasury bills, and between 6-month FDIC insured Certificates of Deposit (henceforth CD) and 6-month Treasury bills, which are short-term bonds because their maturities are shorter than one year.[15] Each of the spreads stands for the liquidity premium which each of different maturities of Treasuries bears. In other words, all of those spreads reflect the market values for the liquidity services the Treasuries provide, but different in the sense that the former is the liquidity premium on

[14] See Henning Bohn's website for more details: http://econ.ucsb.edu/ bohn/data.html

[15] I use the spreads which are used in Krishnamurthy and Vissing-Jorgensen (2012) but slightly different because they are updated from the original source.

the long term Treasuries and the latter on the short term Treasuries.[16]

Next, it is important that I use the growth rate of Narrow Money for money supply in the regressions. As seen in Equation (15), what affects liquidity premia is the growth rate of money supply, not the absolute level of money supply. The theory predicts that one-time change in money supply does not affect the liquidity premia and other real variables such as real balances, quantities traded in the DM, but its growth rate does. The theory delivers that money supply is not super-neutral but neutral. A one-time injection or withdraw of money to an economy is ineffective because its relative price is adjusted to keep the real variables unchanged. On the other hand, I use the level of the debt to the nominal GDP ratio for bond supply in the regressions, because it is not neutral unlike money. Even one-time change in the case of bond supply affects the real variables.

Also, in order to control the risk premium, associated with default risks, which can be included in the yield spreads, a stock market volatility index is used as a default control variable which is used in Krishnamurthy and Vissing-Jorgensen (2012).[17] Including this measure in the regressions makes sure that changes in the yield spreads are driven mainly by changes in the liquidity premia, given the low default rate on Aaa-rated corporate bonds and Aa-rated Commercial Papers.[18] However, unlike these two securities, FDIC insured CDs are as safe as Treasuries, given FDIC insurance, so that its spread against the same maturities of Treasuries can be used as a good proxy of the liquidity premium.

Before the regressions, I graphically look at how money growth rate had evolved with the different measures of the liquidity premium over the sample period from Figures 6 and 7, and it is consistent with the predictions from the theory. The figures show that the movement of money growth is almost positively related to the liquidity premium, and their variations are also similar in terms of frequency and width.

Now, let's move onto the empirical results from the regressions. Table 1 presents the impacts of money growth, bond supply, and the Federal Funds rate on the liquidity premium, which is measured by the yield spread between Aaa-rated corporate bond and the long term Treasury bond. Specifically, Regressions (1) to (3) look at the impact of money growth with bond supply on the liquidity premium which the long-term Treasuries carry. In particular, Regression (2)

[16]Notice that there are two indexes which represents the opportunity cost of holding money meant by the theory: money growth rate and nominal interest rates. Even though they are theoretically equivalent by the Fisher equation and the definition of the stationary equilibrium, I will show that there can be cases where only one of them significantly affect long or short term liquidity premia in the data. In fact, Nagel (2014) only presents the cases where the Federal Funds rate has a positive effect on the liquidity premia, which are measured monthly only by some short term bonds. However, here I show the liquidity premia are also affected by money growth. and also how it is robust to the liquidity premium of the long term bond such as the long term Treasuries.

[17]See Krishnamurthy and Vissing-Jorgensen (2012) for the details about why this measure can be a proxy for default risk. In short, they argues that this measure have a high correlation with another default risk measure such as the median expected default frequency credit measure from Moody's Analytics.

[18]See Krishnamurthy and Vissing-Jorgensen (2012) for details. According to them, "there have never been a default on high-grade CP." Also, they use the spread of Aaa-rated bonds against Treasuries to estimate the market value of the liquidity convenience, assuming that the default risk of the Aaa-rated bonds is low.

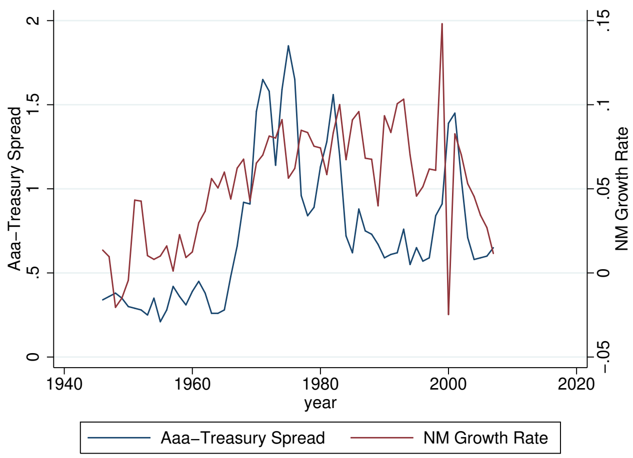

Figure 6: Aaa - Treasury Spread

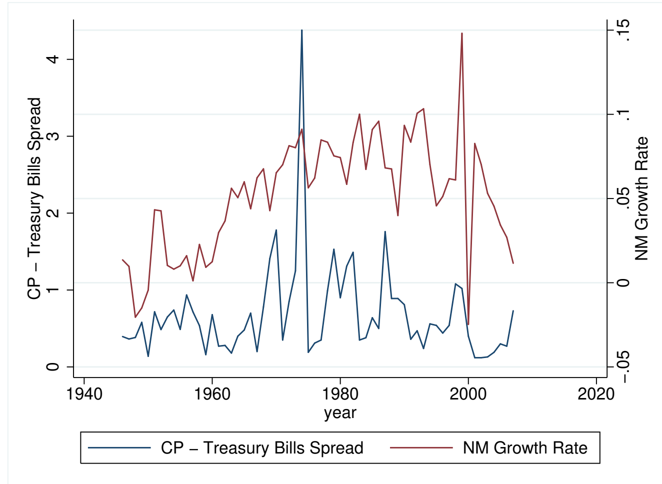

Figure 7: CP - T-Bill Spread

and (3) test whether Treasury bonds are substitutes with money or not. The theory predicts that if they are not substitutes, money growth does not affect the liquidity premium of Treasuries at all, because the money-traded market are totally separate from the bond-collaterlized market. However, the regression results deliver that money growth has a significant and positive impact on the liquidity premium. Also, bond supply is negatively correlated with the liquidity premium. An increase in bond supply reduces its market price for the liquidity services which the bonds provide. All of these results imply that Treasury bonds are substantive substitutes with money to some degree in that they provide liquidity services like money does. The negative effect of bond supply is consistent with Krishnamurthy and Vissing-Jorgensen (2012) even without the log specification[19], and also is robust to default risk. A control variable for default risk is included as an explanatory variable in Regression (3), and the coefficients of money growth and bond supply are still significant. This implies that changes in the spread, or the liquidity premium, are significantly affected by both of them even if money and bonds are not perfectly substitutes. The coefficient of the risk premium control variable is also significant. On the other hand, Regression (4) to (6) show the impact of the Federal Funds rate on the liquidity premium. Unlike money supply, its impact on the liquidity premium is not significant in the regressions with bond supply.

Next, Table 2 uses different measures of the liquidity premia: the yield spread of AA-rated Commercial Papers, and FDIC CDs against Treasury Bills, and the TED spread. All the regressions present that the impact of money growth is significantly positive and is robust with bond supply and default risk controls. Also, these results strongly support the theoretical prediction that Treasury Bills are substitutes with money like Treasury bonds, again.[20] Notice that

[19] They use the log specification in their regressions because it provides a good fit and there is only one parameter they are interested in.

[20] The regressions with the quarterly data over the same period present the similar result. See Appendix B for the regression results.

Table 1: Impact of money growth on Aaa Cor. - Treasury Bond Spread

	(1) 1946-2008	(2) 1946-2008	(3) 1946-2008	(4) 1955-2008	(5) 1955-2008	(6) 1955-2008
NM Growth	1.648**	0.917***	0.557***			
	(0.735)	(0.244)	(0.160)			
Debt to GDP		-1.496***	-1.326***		-3.795***	-3.188***
		(0.421)	(0.295)		(0.710)	(0.745)
Volatility			4.495***			3.484***
			(0.893)			(0.903)
Federal funds rate				0.0564***	0.000219	-0.00253
				(0.0147)	(0.0162)	(0.0152)
Constant	0.654***	1.348***	0.690***	0.509***	2.258***	1.567***
	(0.0833)	(0.205)	(0.211)	(0.120)	(0.366)	(0.434)
Observations	63	63	63	54	54	54
Adjusted R-squared	0.092	0.417	0.610	0.166	0.542	0.661

Notes: Coefficients are estimated by Newey-West estimators with lag(1) and its standard errors are presented in parenthesis. The dependent variables are the yield spreads between private and Treasury bonds, which are measured in a percentage unit. Explanatory variables are the growth rate of Narrow Money and the ratio of the market value of Treasury debt outstanding to nominal GDP. A control variable for the default risk on private assets is $Volatility$, which is measured by annualized standard deviation of weekly log stock returns on the $S\&P$ 500 index (Source: Krishnamurthy and Vissing-Jorgensen (2012)). *** p<0.01, ** p<0.05, * p<0.1.

Regressions (5) to (7) do not include a control variable for default risk because the FDIC CDs as an illiquid financial security are as safe as Treasuries, and so its spread with Treasuries only reflects the difference between liquidity services which they provides.

Regressions (4), (7), and (11) present whether Federal Funds rate affects the liquidity premium in the case where bonds are substitutes with money as in Regressions (3), (6) and (10). The Federal Funds rate as a proxy of the nominal interest rate is equivalent theoretically to the money growth, because both of them stand for the opportunity cost of holding money. In Regression (4) and (11), the Federal Funds rate has a significantly positive impact on the liquidity premia, which is measured by the yield spread between AA CPs and Treasury Bills, and the TED spread, whereas it does not in Regression (7). Also, bond supply still has a significant and strong negative effect on the liquidity premium.[21]

To summarize, money growth has a significant and positive impact on the liquidity premia, and also the nominal interest rate has a positive effect on it in some regressions. Even if money growth and the nominal interest rate are theoretically equivalent because both changes in both variables mean changes in the opportunity cost of holding money, they cannot be not in the

[21] In fact, this is different from the results which Nagel (2014) delivers. The paper argues that there is no impact of the bond supply on the liquidity premium. However, it does not look at the measures of the liquidity premium which are used here: the AAA cor. - Treasury bond spread, and the 6-month FDIC CDs - Treasury Bill spread. even though they also reflect the liquidity premia.

data. For example, this difference may come from the fact that short and long term bonds are traded in different institutions, by different agents, or different regulations, even though they are similar in terms of liquidity. However, notice that this does not change the economic mechanism of how money or nominal interest rates affect the liquidity premia which liquid bonds carry in the financial market. All the results provide strong support the theoretical predictions: bond prices bear the liquidity premium, bonds are substantive substitutes with money even if not perfect, and the money holding cost is a primary factor in the mechanism which deliver the aforementioned results.

Table 2: Impact of money growth on the spreads (Yearly)

Variables	AA CP - T-Bills				FDIC CDs - T-Bills			TED spread			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
	1946-2008	1946-2008	1946-2008	1955-2008	1984-2008	1984-2008	1984-2008	1986-2008	1986-2008	1986-2008	1986-2008
NM Growth	1.681***	1.256***	1.168***		2.378***	2.535***		1.341***	1.343***	1.209***	
	(0.543)	(0.247)	(0.298)		(0.249)	(0.231)		(0.194)	(0.187)	(0.224)	
Debt to GDP		-0.871*	-0.829*	-0.564		-5.471***	-5.074***		-0.0543	0.980	0.212
		(0.471)	(0.430)	(1.037)		(1.584)	(1.777)		(1.535)	(1.665)	(1.127)
Volatility			1.097	0.271						2.010	1.608
			(1.722)	(1.344)						(1.464)	(1.521)
Federal Funds Rate				0.0947***			0.0366				0.0923*
				(0.0323)			(0.0463)				(0.0468)
Constant	0.586***	0.990***	0.829***	0.362	-0.0569	2.151***	2.026**	0.565***	0.588	-0.120	-0.0824
	(0.0784)	(0.272)	(0.254)	(0.435)	(0.134)	(0.633)	(0.785)	(0.107)	(0.668)	(0.808)	(0.447)
Observations	63	63	63	54	25	25	25	23	23	23	23
Adjusted R-squared	0.037	0.076	0.066	0.210	0.272	0.551	0.206	0.180	0.139	0.147	0.233

Notes: Coefficients are estimated by Newey-West estimators with lag(1) and its standard errors are presented in parenthesis. The dependent variables are the yield spreads between private and Treasury bonds, which are measured in a percentage unit. Explanatory variables are the growth rate of Narrow Money and the ratio of the market value of Treasury debt outstanding to nominal GDP. A control variable for the default risk on private assets is $Volatility$, which is measured by annualized standard deviation of weekly log stock returns on the $S\&P$ 500 index (Source: Krishnamurthy and Vissing-Jorgensen (2012)). *** p<0.01, ** p<0.05, * p<0.1.

4 Discussion on Negative Interest Rates

Negative interest rates have been observed in some countries such as the United States, Switzerland, Japan and Germany, in particular, for the recent years after 2008. For example, in Switzerland, the yields of almost all the government bonds have been negative after 2008. In addition, the yield of 3 month Treasury Bills in the United States had been negative during several days in September, 2015, even if the federal fund rates were slightly positive.

The negative yields, or interest rates imply that lenders pay borrowers interests on their borrowings. It wouldn't make sense if interest rates were considered as the risk premium which is compensated for borrowers' default risk, because it is the lenders that take the default risk of the loans. Since cash is unambiguously as safe as any other government bonds, the lenders could hoard physical cash in their safes, whose interest rate is 0%, i.e., can never be negative. Then, why have we observed the negative interest rates in reality? Or, why do not investors in the financial market choose to hold cash, instead of the government bonds?

First, consider a financial market where investors are always willing to pay for the liquidity service the governments bonds provide. For example, in some financial markets such as Repurchase Agreement markets and the collateralized federal funds market, liquid bonds such as the government bonds are necessary as collateral in transactions. Theoretically, this can be regarded as the case where the value of $1-\theta$ in the model is not small. As shown before, the yield of a liquid bond from the model is given by

$$\rho = \frac{(1-g)\left[\frac{1+\mu}{\beta}-1\right]-(1-\theta)h\left[u'(z^a)-1\right]}{(1-g)+g\frac{1+\mu}{\beta}+(1-\theta)h\left[u'(z^a)-1\right]} \tag{24}$$

$$= \frac{(1-g)i-(1-\theta)h\left[u'(z^a)-1\right]}{1+gi+(1-\theta)h\left[u'(z^a)-1\right]}. \tag{25}$$

The theory predicts that there is a high chance that the negative yield would emerge, in particular, when bond supply decreases, and the nominal interest rate is low: money is not in short. This is because the liquidity services which liquid bonds provide are valued high relatively when the money holding cost is low. It implies that the bond buyers would be willing to accept the negative yields of the bonds to hold them in their portfolios for transactions. In fact, it is supported by the comments from the market participants during the periods of the low nominal interest rate. For example, according to *Bloomberg* (September 25, 2015), Kenneth Silliman, head of U.S. short-term rates trading in New York at TD Securities unit, one of 22 primary dealers that trade with the Fed said,

"Yields on U.S. Treasury bills fell below zero as an influx of cash and pent-up appetite for safe assets led investors to accept negative returns after the Federal Reserve decided not to raise its short-term interest rate. ...... Investors will have additional funds totaling about \$100 billion returned to them in

the next month as the government cuts bill supply heading into negotiations with Congress about the statutory debt limit", [22][23]

To summarize in brief, this article says that the main factors which drove down the negative interest rate below zero were 'an influx of cash and a cut in bill supply.' Also, remember that the policy rate of the Fed, has been hovering around zero for more than 7 years since 2008. In other words, when the negative interest rate occurred, the money holding cost was low, and also the bill supply was expected to decrease. However, there have existed strong demand on the government bonds. Both factors were at work together in the direction to raise up the liquidity premium of liquid bonds such as Treasury bills, so that the interest rate seemed to fall down to the negative territory.

Interestingly, this negative yield can also be explained by 'an influx of cash' in addition to 'a cut in bill supply'. As shown theoretically and empirically in the previous sections, money supply (or growth) increases the liquidity premium, whereas bond supply decreases it. Hence, with a slight abuse of the theory, this fall of the yield to the negative territory can be interpreted as an increase in the liquidity premium which the government bonds bear.

Moreover, we can find another example which the theory can be applied to in Switzerland. As shown in Figure 8, the negative yields on government bonds have been observed for a substantial period of time since 2008. Also, it looks at how money and the government bond supply have been evolving. The ratio of the government bond supply relative to GDP shrank from around 50% to around 30%, whereas the money supply, measured by M1[24], relatively increased more than twice during the same period. If we apply the empirical result to this example, it is highly likely that the relative scarcity of the liquid government bonds against money in the market have led to an increase in the high liquidity premia on the government bonds. Moreover, the interest target range of the Swiss National Bank was 0-1.00% at then end of 2008[25], and, after then, continued to decrease, and fell into the negative target. Accordingly, it can be inferred that the main factor for the negative interest rates is the high liquidity premia on the liquid government bonds due to their short supply.[26]

[22]Source: http://www.bloomberg.com/news/articles/2015-09-17/treasury-bill-yields-turn-negative-as-fed-leaves-rates-unchanged

[23]See the following comment in *The Wall Street Journal* (Sept 23, 2014) for another example: "Short-term debt trading at negative yields was essentially unheard of before the 2008 financial crisis. But since then, the condition has cropped up at times of market stress, reflecting extraordinarily expansive central-bank policy and anemic growth in much of the world. Yields on some U.S. bills traded below zero at the end of each of the past three years amid strong demand for liquid assets, according to analysts." Source: http://www.wsj.com/articles/treasury-bill-yield-tips-into-negative-territory-1411516748

[24]It includes currency in circulation, sight deposits and deposits in transaction accounts.

[25]It is fixed at 0 - 1.00% on 12/11/2008, 0 - 0.75% on 3/12/2009, 0 - 0.25% on 8/3/2011, -0.75 - 0.25% on 12/18/2014, and -1.25 - -0.25% on 1/15/2015.

[26]Also, according to Aleks Berentsen, Swiss government bonds can be used as collateral in some markets outside of Switzerland but where the Swiss franc cannot. It implies Swiss government bonds have higher $(1-\theta)$, so that there are higher possibility that their yields would be negative in the case where liquid bonds are scarce.

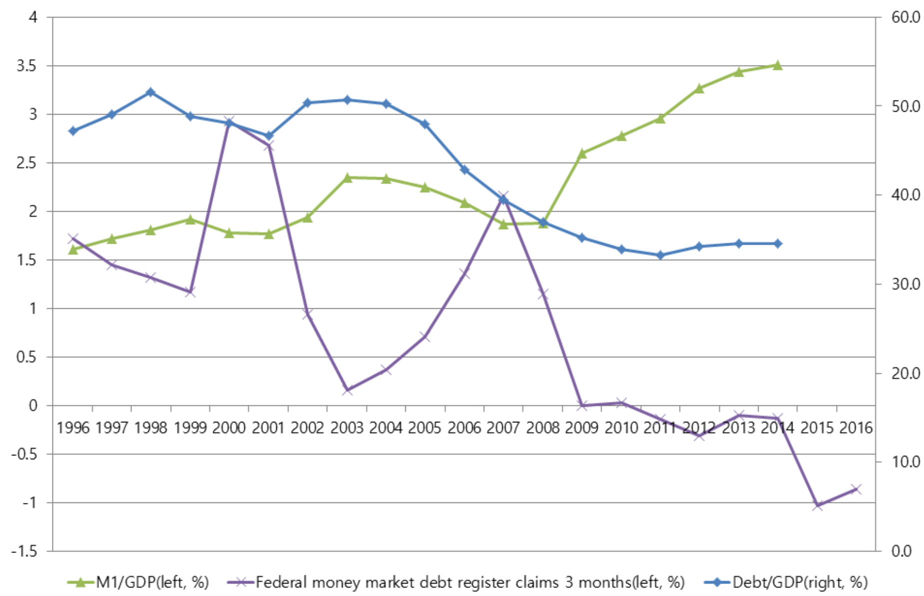

Figure 8: Interest Rates, Money and Gov't Bond Supply in Switzerland

5 Conclusion

This paper explores the effects of money and bond supply on the liquidity premia in the prices of liquid financial assets such as government bonds. The theory delivers elaborate predictions about under which conditions money supply can affect the liquidity premia. For example, it has a positive impact on them by changing the opportunity cost of holding money and so affecting the demand on liquid bonds only when liquid bonds are substitutes with money, even if partial. Moreover, if they are perfectly substitutes, the negative yields on liquid bonds can appear in the equilibrium. On the other hand, the bond supply also directly affects the liquidity premia by changing relative scarcity of the bonds. Lastly, the empirical analysis presents a strong support for the theoretical findings. The US data display that money supply as a proxy of the money holding cost has a positive impact on the liquidity premium, whereas bond supply has a negative impact. Also, it describes how the liquidity premia are associated with negative nominal yield on liquid bonds which were observed in the US and Switzerland.

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A Appendix

Proof. Proof of Lemmas 1 and 2.

First, consider Lemma 1. Substituting $\boldsymbol{\phi}_t \boldsymbol{p}_t$ into the objective function in Equation 6 re-express the bargaining problem as

$$\max_q \{u(q) - q\}$$

subject to $q = \boldsymbol{\phi}\boldsymbol{p}$, and $\boldsymbol{p} \leq \widetilde{\mathbf{w}}_t$. If $\boldsymbol{\phi}\widetilde{\mathbf{w}}_t \geq q^*$, the optimal choice of q will be the first best quantity q^*, i.e., $q = q^*$. Then, $\boldsymbol{p} = (p^m, p^a)$ such that $\phi p^m + g p^a = q^*$. However, if $\boldsymbol{\phi}\widetilde{\mathbf{w}}_t < q^*$, the effective budget constraint is binding. Accordingly, the buyer will give up all her real balances in order to purchase as many as possible. Then, the optimal choice of q will be the same as her real balances $\boldsymbol{\phi}\widetilde{\mathbf{w}}_t$. Also, $\boldsymbol{p} = (m, a)$. When it comes to Lemma 2, the same steps above can be taken for proof. Since it is straightforward, it is omitted. □

Proof. Proof of Lemma 3

First, consider whether the real balances are as a direct medium of exchange or as collateral to borrow credit enough to obtain the optimal quantity $q*$ in each of the two DM markets. If $\widetilde{z} \geq q^*$ or $\widetilde{z}^a \geq q^*$, $q^1 = q^*$ or $q^2 = q^*$; otherwise, $q^1 = \widetilde{z}$ or $q^2 = \widetilde{z}^a$ by lemmas 1 or 2. Then, plugging these result into the first order conditions (**??**) and (**??**) for the maximum of the objective function will yield the equilibrium prices ϕ and ψ. Also, the marginal utility function u' is monotonically decreasing in its argument, so that the equilibrium is uniquely determined. □

B Impact of money growth on the liquidity premium: Quarterly Data

Table 3: Impact of money growth on the liquidity premium (Quarterly, 1946Q1-2008Q4)

Variables	AAA Cor. - Treasury Bond			AA CP - T-Bills			TED Spread		
	(1)	(2)	(3)	(5)	(6)	(7)	(8)	(9)	(10)
VARIABLES	1946Q1-20008Q4				cpbill1	cpbill1	tedspread	tedspread	tedspread
NM Growth	3.060***	2.353***		1.144***	1.740***		3.116***	3.863***	
	(0.999)	(0.222)		(0.170)	(0.230)		(0.211)	(0.353)	
Dept to GDP		-1.708***	-1.969***		-1.742***	-0.0345		-2.940***	-0.0337
		(0.250)	(0.367)		(0.418)	(0.308)		(0.922)	(1.321)
Federal Funds Rate			-0.0235*			0.0926***			0.0914***
			(0.0136)			(0.0122)			(0.0193)
Constant	0.709***	1.716***	2.038***	0.587***	1.424***	0.0312	0.616***	2.314***	0.264
	(0.0406)	(0.135)	(0.254)	(0.0538)	(0.236)	(0.185)	(0.0565)	(0.539)	(0.731)
Observations	252	172	172	151	151	151	92	92	92
Adjusted R-squared	0.064	0.320	0.276	0.005	0.209	0.422	0.184	0.329	0.201

Notes: Coefficients are estimated by Newey-West estimators with lag(1) and its standard errors are presented in parenthesis. The dependent variables are the yield spreads between private financial assets and Treasuries. They are measured in a percentage unit. Explanatory variables are the growth rate of Narrow Money and the ratio of the market value of Treasury debt outstanding to nominal GDP. *** $p<0.01$, ** $p<0.05$, * $p<0.1$.