Task-Specific Abilities in Multi-Task Agency Relations

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Abstract

This paper analyzes a multi-task agency framework where the agent exhibits task-specific abilities. It illustrates how incentive contracts account for the agent's task-specific abilities if contractible performance measures do not reflect the agent's contribution to firm value. This paper further sheds light on potential ranking criteria for performance measures in multi-task agencies. It demonstrates that the value of performance measures in multi-task agencies cannot necessarily be compared by their respective signal/noise ratios as in single-task agency relations. It is rather pivotal to take the induced effort distortion and measure-cost efficiency into consideration – both determined by the agent's task specific abilities.

Keywords: Task-specific human capital, performance measurement, distortion, multi-task agencies, congruence, incentives.

JEL classification: D23, D82, J24

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1 Introduction

Empirical investigations have offered an abundance of evidence suggesting that individuals are highly responsive to monetary incentives (see e.g. Asch [1990], Paarsch and Shearer [1999] and Lazear [2000a]). Nevertheless, the specific effects of reward schemes are somewhat ambiguous when individuals are required to perform a collection of different tasks. In such situations, Kerr [1975] cautioned against the consequences of a reward system that inefficiently overemphasizes some tasks while underemphasizing others. An illustrative example cited by Kerr [1975] is the difficult trade-off between research and teaching responsibilities encountered by faculties at universities. Since teaching quality is harder to assess relative to research output, and prospective promotion decisions mainly hinge on research performance, it is a common phenomenon for faculty members to focus on research at the expense of teaching.[1] In general, inefficient effort allocations occur when available performance measures do not reflect employees' true contribution to firm value [Feltham and Xie, 1994]. In this case, employees focus on less or even non-valuable tasks, and disregarding more beneficial ones [Feltham and Xie, 1994].[2]

Previous multi-task agency literature such as Feltham and Xie [1994], Banker and Thevaranjan [2000], and Datar, Kulp, and Lambert [2001] focus on performance measure congruity and its effects on the efficiency of incentive contracts, but absent from these studies is the possibility that agents may perform some tasks more efficiently than others.[3] Recent literature however, emphasizes the role of acquiring human capital for specific tasks (see e.g. Lindbeck and Snower [2000], Gibbons and Waldman [2006] and Gibbons and Waldman [2004]).[4] Since individuals differ substantially in their learning aptitudes, which inevitably lead to discrepancies in skills and abilities [Gibbons and Waldman, 2006], it is reasonable to infer that different individuals might perform different tasks with varying degrees of ease.[5] For example, Sapienza and Gupta

[1]See Brickley and Zimmerman [2001] for an empirical study of this example.

[2]See as well the discussion in Gibbons [1998].

[3]Schnedler [2006] is an exception. However, his focus is different in the sense that he investigates the consequences of different marginal effort costs on the relative value of incongruent performance measures for the provision of incentives.

[4]For empirical evidence see Baker, Gibbs, and Holmström [1994].

[5]Maher, Ramanathan, and Peterson [1979] conceive the term 'congruence of perception with preferences' to indicate the phenomenon that even if an individual possesses the correct perception of different tasks, there might still be a preference on specific tasks.

[1994] indicate in their study of principal-agent relations within venture capital-backed firms that the frequency of venture capitalist (principal) - CEO (agent) interaction is partially dependent on the CEOs' venture experience. They provide evidence that CEOs with prior experiences (i.e. greater proficiency) in start-up ventures would have a lesser tendency of consulting with their venture capitalist.

In order to understand the nature of contracts in multi-task agency relations, it is essential to investigate whether and how task-specific abilities influence the agent's preferences for her effort allocation and the optimal provision of incentives in response to these abilities. This paper thus focuses on multi-task agencies in order to gain new insights into the provision of incentives if performance measures are incongruent with the principal's objective and the agent exhibits different abilities for performing relevant tasks.

This paper investigates how incentive contracts respond to individual task-specific abilities combined with incongruent performance measures. It further demonstrates how the value of performances measures can be compared in multi-task agencies. The analysis indicates that the signal/noise ratio—sufficient to rank performance measures in single-task agencies—can only be applied if all available measures provide the same information about the agent's relative effort allocation. The proposed ranking criteria is in general contingent on the agent's specific abilities such that different agents may imply various orderings of performance measures.

This paper proceeds as follows. In section 2, I give an overview of the model and derive the first-best contract in section 3. I provide in section 4 the second-best contract and focus on the relation between performance measure congruity and effort distortion in section 5. In section 6, I investigate how performance measures can be ranked in multi-task agencies, in particular when agents are characterized by task-specific abilities. Section 7 concludes.

2 The Model

Consider a single-period agency relationship between a risk-neutral principal and a risk-averse agent. The principal owns an asset and requires the agent's productive effort. Once employed, the agent is in charge of performing $n \geq 2$ tasks (multi-tasking). These tasks are tied together, i.e. the principal cannot split and allocate them to different agents.[6] The agent implements an effort vector $\boldsymbol{e} = (e_1, ..., e_n)^t$, $\boldsymbol{e} \in \mathbf{E} \subseteq \mathbb{R}^{n+}$, where e_i is the agent's effort allocated to task i.[7] Effort is non-verifiable and all activities $e_i \in \mathbf{E}$ are measured in the same unit.

To incorporate task-specific abilities for the agent, I adapt Lazear's [2000b] approach for a single-task agency model to this multi-task framework. In this sense, the abilities differ across tasks and determine the absolute and marginal effort costs borne by the agent. Let $\boldsymbol{\Psi}$ be an $n \times n$ matrix representing the agent's task-specific abilities. The agent's effort costs are contingent on $\boldsymbol{\Psi}$ and take the form $C(\boldsymbol{e}) = \boldsymbol{e}^t \boldsymbol{\Psi} \boldsymbol{e}/2$, where $\boldsymbol{\Psi}$ is a diagonal $n \times n$ matrix defined by $\boldsymbol{\Psi} = \operatorname{diag}(\psi_1, ..., \psi_n)$, $\psi_i > 0$, $i = 1, .., n$. A higher ability for performing task i is characterized by a lower ψ_i, $i = 1, ..., n$, and vice versa.[8]

The agent's preferences are represented by the negative exponential utility function

$$U(w, \boldsymbol{e}) = -\exp\left[-\rho\left(w - C(\boldsymbol{e})\right)\right], \tag{1}$$

where ρ denotes the Arrow-Pratt measure of absolute risk-aversion and w as the agent's wage. For parsimony, let $\bar{w} = 0$ be her reservation wage implying a reservation utility $\bar{U} = -1$.

By conducting effort $\boldsymbol{e}$, the agent contributes to the principal's non-verifiable gross payoff $V(\boldsymbol{e}) = \boldsymbol{\mu}^t \boldsymbol{e} + \varepsilon_V$, where ε_V is a normally distributed random component with zero mean and variance σ_V^2, representing firm-specific and economy wide risk. The n-dimensional vector $\boldsymbol{\mu} = (\mu_1, ..., \mu_n)^t$, $\mu_i \geq 0$, $i = 1, ..., n$, characterizes the marginal effect of $\boldsymbol{e}$ on gross payoff $V(\boldsymbol{e})$. Since $V(\boldsymbol{e})$ is non-verifiable, it cannot be part of an explicit single-period incentive contract. The only verifiable information about $\boldsymbol{e}$, however, is provided by the performance measure

$$P(\boldsymbol{e}) = \boldsymbol{\omega}^t \boldsymbol{e} + \varepsilon, \tag{2}$$

where $\boldsymbol{\omega} = (\omega_1, ..., \omega_n)^t \in \mathbb{R}^{n+}$ is the vector of performance measure sensitivities. The random component ε is normally distributed with zero mean and variance σ^2, and represents potential effects on the performance measure beyond the agent's control.

[6] For considerations on how multiple tasks are efficiently split among several agents, refer e.g. to Holmström and Milgrom [1991], Corts [2007], and Schöttner [2006].

[7] All used vectors are column vectors where 't' denotes the transpose.

[8] A similar approach is used by MacLeod [1996], where ψ_i, $i = 1, ..., n$, are random variables. However, his work is different in the sense that he focuses on the relationship between explicit and implicit incentive contracts rather than on the effort distortion induced by incongruent performance measurement.

As pointed out by Feltham and Xie [1994], the performance measure does not necessarily capture the agent's contribution to the gross payoff perfectly. Formally, if there exists a constant $\lambda \neq 0$ satisfying $\boldsymbol{\mu} = \lambda \boldsymbol{\omega}$, performance measure $P(\boldsymbol{e})$ is congruent with the gross payoff $V(\boldsymbol{e})$.[9] Otherwise, the performance measure is incongruent and its application in an incentive contract motivates the agent to implement an inefficient effort allocation across tasks [Feltham and Xie, 1994, Baker, 2002].

Baker [2002] provided a geometric measure for performance measure congruity. Since his result is fundamental to the subsequent analysis, it is summarized in the following definition.

Definition 1. *The congruence of performance measure $P(\boldsymbol{e})$ to gross payoff $V(\boldsymbol{e})$ with respect to the marginal effect of $\mathbf{e}$ is measured by $\Upsilon^C(\varphi) = \cos\varphi$, where φ is the angle between the vector of gross payoff sensitivities $\boldsymbol{\mu}$ and the vector of performance measure sensitivities $\boldsymbol{\omega}$.*

Accordingly, as long as vector $\boldsymbol{\mu}$ and vector $\boldsymbol{\omega}$ are linearly independent, the performance measure does not reflect the agent's contribution to gross payoff, and therefore, is incongruent. Formally, there exists no constant $\lambda \neq 0$ satisfying $\boldsymbol{\mu} = \lambda \boldsymbol{\omega}$, thereby implying $\varphi \neq 0$. A more congruent performance measure thereby implies a smaller angle φ and leads to a higher measure of congruity $\Upsilon^C(\varphi)$ due to the definition of the cosine. Finally note that $\varphi \in [0, \pi/2]$ since $\mu_i, \omega_i \geq 0$, $i = 1, ..., n$, where φ is represented in radian measure.

In line with previous multi-task literature, I restrict my analysis to a compensation scheme w which is linear in performance measure $P(\boldsymbol{e})$. The payment w takes therefore the form

$$w(\boldsymbol{e}) = \alpha + \beta P(\boldsymbol{e}), \tag{3}$$

where α denotes the fixed payment and β denotes the incentive parameter. The transfer α is utilized to split the surplus between the principal and the agent, whereas β is used to provide the agent with incentives for implementing effort.

Since the compensation scheme is linear, the agent's utility is exponential, and the error term is normally distributed, maximizing the agent's expected utility is analogous to maximizing her

[9] This phenomenon is described by several terms in the multi-task agency literature: *performance measure congruity* [Feltham and Xie, 1994, Bushman, Indjejikian, and Penno, 2000, Hughes, Zhang, and Xie, 2005], *non-distorted performance measure* [Baker, 2000, 2002], and *goal congruence* [Anthony and Govindarajan, 1995, Banker and Thevaranjan, 2000]. For the sake of consistence, I use the term *performance measure congruity* throughout this paper.

certainty equivalent

$$CE(\boldsymbol{e}) = \alpha + \beta\boldsymbol{\omega}^t\boldsymbol{e} - \frac{1}{2}\boldsymbol{e}^t\boldsymbol{\Psi}\boldsymbol{e} - \frac{\rho}{2}\beta^2\sigma^2, \tag{4}$$

where $\rho\beta^2\sigma^2/2$ is the required risk premium in order to compensate the agent for the uncertainty in her incentive payment $\beta P(\boldsymbol{e})$.

The timing of this problem is as follows. First, the principal offers the agent a contract (α^*, β^*). If this contract guarantees the agent at least the same expected utility as her best alternative, she accepts. After the agent implemented $\boldsymbol{e}$ and the random variables ε and ε_V are realized, the payments take place.

For clarification, I subsequently illustrate the distinction between effort intensity and effort allocation. Formally, let two arbitrary activities e_k and e_j vary to $\hat{e}_k$ and $\hat{e}_j$, respectively. If the ratio between both activities remains identical such that $e_k/e_j = \hat{e}_k/\hat{e}_j$, $k, j = 1, ..., n$, $k \neq j$, the relative effort allocation remains the same. In contrast, if $e_k/e_j \neq \hat{e}_k/\hat{e}_j$ for at least one pair $(k, j) \in \{1, ..., n\}$, $k \neq j$, the relative effort allocation varies. The overall effort intensity, however, changes without affecting the effort allocation, if there exists a constant $\lambda > 0$ satisfying $\boldsymbol{e} = \lambda\hat{\boldsymbol{e}}$, where $\hat{\boldsymbol{e}}$ is the modified effort vector.

For the ease of comparing different effort allocations, it is useful to commit to the subsequent definition throughout this paper.

Definition 2. *The agent implements a distorted effort allocation if there exists no constant $\lambda \neq 0$ satisfying $\boldsymbol{\mu} = \lambda\boldsymbol{e}$.*

The implemented effort allocation is referred to be distorted if it does not reflect the agent's marginal contribution to gross payoff $V(\boldsymbol{e})$. Note, however, that non-distortion is not necessarily optimal since this concept does not incorporate the corresponding costs for implementing an arbitrary effort vector.

3 The First-Best Contract

Before I move on to the second-best contract, it is useful to derive the first-best solution of this problem as a benchmark for the subsequent analyzes. Then, the first-best effort allocation and intensity can be compared to the second-best environment, where the agent's effort is non-contractible so that moral hazard occurs.

Suppose the principal can specify a desired effort intensity and allocation in an enforceable contract. In this case, she appoints the effort vector $\boldsymbol{e}$ which maximizes the difference between the expected gross payoff $V(\boldsymbol{e})$ and costs $w = C(\boldsymbol{e})$:

$$\max_{\boldsymbol{e}} \;\; \Pi(\boldsymbol{e}) = \boldsymbol{\mu}^t \boldsymbol{e} - \frac{1}{2}\boldsymbol{e}^t \boldsymbol{\Psi} \boldsymbol{e}. \tag{5}$$

Let $\boldsymbol{\phi} \equiv \boldsymbol{\Psi}^{-1}\boldsymbol{\mu} = (\mu_1/\psi_1, ..., \mu_n/\psi_n)^t$ be the vector of the payoff-cost sensitivity ratios. Then, the first-best effort vector is

$$\boldsymbol{e}^{fb} = \boldsymbol{\phi}. \tag{6}$$

The principal maximizes her expected profit by assigning each activity e_i in accordance to its payoff-cost sensitivity ratio μ_i/ψ_i, $i = 1, ..., n$. Activities with high ratios are consequently more intensively conducted relative to activities with low ratios.

Recall that $\boldsymbol{e}^{fb}$ is distorted if there exists no constant $\lambda \neq 0$ satisfying $\boldsymbol{\mu} = \lambda \boldsymbol{e}^{fb}$, see definition 2. In contrast, if the agent has different abilities across tasks, it is optimal to implement a distorted effort allocation in order to balance the benefits and costs of all relevant tasks.

By substituting $\boldsymbol{e}^{fb}$ in (5) and using the relation $\boldsymbol{\mu}^t\boldsymbol{\phi} = \|\boldsymbol{\mu}\|\|\boldsymbol{\phi}\| \cos\kappa$ for vector products, the expected first-best profit becomes

$$\Pi^{fb} = \frac{1}{2}\|\boldsymbol{\mu}\|\|\boldsymbol{\phi}\| \cos\kappa, \tag{7}$$

where κ is the angle between vector $\boldsymbol{\mu}$ and vector $\boldsymbol{\phi}$, and $\|\cdot\|$ denotes the length of the respective vector.

The agent's task-specific abilities affect the expected first-best profit in two ways. The first effect is a result of the overall cost intensity for implementing an arbitrary effort vector. To illustrate this effect, consider two agents A and B characterized by $\boldsymbol{\Psi}^A$ and $\boldsymbol{\Psi}^B$, respectively. If $\boldsymbol{\Psi}^A = \lambda\boldsymbol{\Psi}^B$, $\lambda > 1$, agent A exhibits a less overall cost intensity than agent B for the implementation of an arbitrary effort vector. Observe, however, that both agents share the same relative task-specific abilities across tasks. Therefore, $\lambda\|\boldsymbol{\phi}^A\| = \|\boldsymbol{\phi}^B\|$, whereas $\kappa^A = \kappa^B$. The second effect follows from the relation between the payoff sensitivities $\boldsymbol{\mu}$ and the agent's relative task-specific abilities $\boldsymbol{\Psi}$. Consider for instance the agent's ability ψ_i to perform task i. If this ability is increasing (i.e. ψ_i decreases) relative to the other abilities, the agent could implement the same effort vector, but suffers less disutility of effort for performing task i. In this case, $\|\boldsymbol{\phi}\|$ increases. However, the effect on κ is ambiguous. Particularly, decreasing ψ_i

leads to a higher angle κ if $\psi_i < 1$, and to a lower κ, otherwise. For the principal, however, it is optimal to enhance e_i^{fb} until the marginal benefit of task i is equal to its marginal costs, i.e. $\mu_i = \psi_i e_i$. Consequently, Π^{fb} increases. This eventually implies that a potential decline in $\cos\kappa$ is preponderated by an increase of $\|\boldsymbol{\phi}\|$.

4 The Second-Best Contract

If the principal cannot directly contract over $\boldsymbol{e}$, she faces an incentive problem for motivating the agent to implement appropriate effort. Since the gross payoff $V(\boldsymbol{e})$ is non-verifiable, the only contractible information is the performance measure $P(\boldsymbol{e})$. However, the application of $P(\boldsymbol{e})$ in an incentive contract may cause two inefficiencies. First, the performance measure—and therefore the agent's compensation—is uncertain such that the risk-averse agent requires a risk premium for accepting a contract dependent on $P(\boldsymbol{e})$. Second, the performance measure can be incongruent and, therefore, motivate the agent to inefficiently allocate her effort across tasks. The subsequent analysis focuses on the second inefficiency since the trade-off between incentive risk and the agent's desire for insurance is intensively analyzed by previous literature.[10]

In a second-best environment, the principal's problem is to design a contract (α^*, β^*) that maximizes her expected profit $\Pi = \mathrm{E}[V(\boldsymbol{e}) - w(\boldsymbol{e})]$ while ensuring the agent's participation. The optimal linear contract therefore solves

$$\max_{\alpha,\beta,\boldsymbol{e}} \ \Pi \equiv \boldsymbol{\mu}^t\boldsymbol{e} - \alpha - \beta\boldsymbol{\omega}^t\boldsymbol{e} \tag{8}$$

s.t.

$$\boldsymbol{e} = \arg\max_{\tilde{\boldsymbol{e}}} \alpha + \beta\boldsymbol{\omega}^t\tilde{\boldsymbol{e}} - \frac{1}{2}\tilde{\boldsymbol{e}}^t\boldsymbol{\Psi}\tilde{\boldsymbol{e}} - \frac{\rho}{2}\beta^2\sigma^2 \tag{9}$$

$$\alpha + \beta\boldsymbol{\omega}^t\boldsymbol{e} - \frac{1}{2}\boldsymbol{e}^t\boldsymbol{\Psi}\boldsymbol{e} - \frac{\rho}{2}\beta^2\sigma^2 \geq 0, \tag{10}$$

where (9) is the agent's incentive condition and (10) her participation constraint.

First, observe that (9) can be replaced by $\boldsymbol{e} = \boldsymbol{\Psi}^{-1}\boldsymbol{\omega}\beta$. For the subsequent analysis, let $\boldsymbol{\Gamma} \equiv \boldsymbol{\Psi}^{-1}\boldsymbol{\omega} = (\omega_1/\psi_1, ..., \omega_n/\psi_n)^t$ be the vector of measure-cost sensitivity ratios. Thus, the agent implements

$$\boldsymbol{e}^* = \boldsymbol{\Gamma}\beta. \tag{11}$$

[10] For a detailed analysis in a LEN-setting, see e.g. Spremann [1987], Baker [1992], and Prendergast [1999]; and for a general approach Shavell [1979], Holmström [1979], Grossman and Hart [1983], and Rees [1985].

In contrast to the first-best scenario, the agent's effort e_i for performing task i depends on the measure-cost sensitivity ratio ω_i/ψ_i and the incentive parameter β.

In order to maximize her expected profit, the principal sets α such that the agent's participation constraint is binding. By solving (10) for α and substituting the resulting expression together with $\boldsymbol{e}^*$ in the principal's objective function (8), the maximization problem simplifies to

$$\max_{\beta} \ \Pi \equiv \boldsymbol{\mu}^t \boldsymbol{\Gamma} \beta - \frac{\beta^2}{2} \left[\boldsymbol{\omega}^t \boldsymbol{\Gamma} + \rho \sigma^2 \right]. \tag{12}$$

The first-derivative of Π with respect to β gives the optimal incentive parameter

$$\beta^* = \frac{\boldsymbol{\mu}^t \boldsymbol{\Gamma}}{\boldsymbol{\omega}^t \boldsymbol{\Gamma} + \rho \sigma^2}. \tag{13}$$

Besides the precision of the performance measure, $1/\sigma^2$, with the agent's risk tolerance, $1/\rho$, the optimal incentive parameter is a function of the gross payoff sensitivities $\boldsymbol{\mu}$, the performance measure sensitivities $\boldsymbol{\omega}$, and the measure-cost sensitivity ratios $\boldsymbol{\Gamma}$. Recall that $\boldsymbol{\Gamma} = \boldsymbol{\Psi}^{-1}\boldsymbol{\omega}$, i.e. $\boldsymbol{\Gamma}$ comprises the agent's task-specific abilities $\boldsymbol{\Psi}$. Hence, β^* incorporates $\boldsymbol{\Psi}$ in two ways: (i) by its relation to the gross payoff sensitivities $\boldsymbol{\mu}$ in the numerator; and (ii), by its relation to the performance measure sensitivities $\boldsymbol{\omega}$ in the numerator and denominator. It can therefore be inferred that agents with different task-specific abilities may obtain diverse incentive contracts, even if they are in charge of performing an identical set of tasks and evaluated by the same information system.

Substituting β^* in (12) and using geometric representations give the principal's expected second-best profit

$$\Pi^* = \frac{\|\boldsymbol{\mu}\|^2 \|\boldsymbol{\Gamma}\|^2 \cos^2 \theta}{2(\|\boldsymbol{\omega}\| \|\boldsymbol{\Gamma}\| \cos \xi + \rho \sigma^2)}, \tag{14}$$

where θ denotes the angle between the vector of payoff sensitivities $\boldsymbol{\mu}$ and the vector of measure-cost sensitivity ratios $\boldsymbol{\Gamma}$. The angle between the vector of performance measure sensitivities $\boldsymbol{\omega}$ and vector $\boldsymbol{\Gamma}$ is denoted by ξ.

5 Performance Measure Congruity and Effort Distortion

In this section, I focus more intensively on performance measure congruity and its effect on effort distortion if the agent performs different tasks with varying degrees of ease.

Performance measure congruity refers to the degree of alignment between the agent's marginal effect on her performance measure and on the expected gross payoff [Feltham and Xie, 1994]. Performance measure congruity can thus be characterized by the angle φ between the vector of payoff sensitivities $\boldsymbol{\mu}$ and the vector of performance measure sensitivities $\boldsymbol{\omega}$, as emphasized by Baker [2002]. In contrast, effort distortion refers to the relation between an implemented effort vector $\boldsymbol{e}$ and the vector of the payoff sensitivities $\boldsymbol{\mu}$. If the agent's effort allocation reflects its relative contribution to $V(\boldsymbol{e})$, her effort is non-distorted, see definition 2. However, as shown in section 3, effort distortion is not necessarily inefficient. Even the first-best effort is distorted if the agent has comparative advantages in performing some tasks relative to others. Nevertheless, a distorted effort allocation is inefficient if it deviates from the one implemented under first-best. The agent implements an efficient (first-best) effort allocation if there exists a constant $\lambda > 0$ satisfying $\boldsymbol{e}^{fb} = \lambda \boldsymbol{e}^{*}$. Recall that $\boldsymbol{e}^{fb} = \boldsymbol{\Psi}^{-1}\boldsymbol{\mu}$ and $\boldsymbol{e}^{*} = \beta\boldsymbol{\Psi}^{-1}\boldsymbol{\omega}$. This leads to the first observation.

Corollary 1. *Only a congruent performance measure with $\boldsymbol{\mu} = \lambda\boldsymbol{\omega}$, $\lambda \in \mathbb{R}^{*}$, leads to a first-best effort allocation. If in addition $\psi_i = \hat{\psi} > 0$, $i = 1, ..., n$, the second-best effort vector $\mathbf{e}^{*}$ is non-distorted.*

Observe that the first part of this corollary is independent of the agent's task-specific abilities. Consequently, I achieve the same observation as Feltham and Xie [1994] even for a more general setting with task-specific abilities. If the applied performance measure is incongruent, we can infer that the agent is motivated to implement an inefficient effort allocation, regardless of her characteristics. However, the extent of this inefficiency is determined by $\boldsymbol{\Psi}$. Finally, identical task-specific abilities additionally lead to non-distorted effort if the applied performance measure is congruent. The rationale for this observation is that identical abilities for performing all relevant tasks imply that the agent's preference for her effort allocation is only determined by the relative contribution of her tasks to the performance measure. If this measure reflects the agent's relative contribution to firm value, i.e. it is congruent, she is motivated to implement non-distorted effort.

As we know from previous literature, the principal can motivate the agent to implement any desired effort intensity by providing an appropriate incentive parameter β. In contrast, the effort allocation cannot be controlled by the principal, as long as the underlying information system generates only one performance measure. It can be deduced from previous observations that $\mathbf{\Gamma}$ plays an important role for the induced effort allocation.

Proposition 1. *If $\psi_k \neq \psi_j$ for at least one pair $(k, j) \in \{1, ..., n\}$, $k \neq j$, then $\Upsilon^D(\theta) = \cos\theta$ measures effort distortion under second-best.*

Proof See appendix.

Note that the measure $\Upsilon^D(\theta)$ is negatively related to effort distortion. The less distorted the agent's effort allocation with respect to $\boldsymbol{\mu}$, the smaller is θ, and consequently, the higher is $\Upsilon^D(\theta)$. If $\theta = 0$, the application of performance measure $P(\boldsymbol{e})$ motivates non-distorted effort. Observe, however, that an incongruent performance measure induces non-distorted effort if $\boldsymbol{\mu} = \lambda\beta\mathbf{\Gamma}$, $\lambda \in \mathbb{R}^*$, or equivalently,

$$\boldsymbol{\omega} = \mathbf{\Psi}\boldsymbol{\mu}\,(\lambda\beta)^{-1}. \tag{15}$$

In this case, the performance measure sensitivities $\boldsymbol{\omega}$ are a transformation of the agent's marginal contribution to gross payoff $\boldsymbol{\mu}$ and her task-specific abilities $\mathbf{\Psi}$. However, as pointed out by corollary 1, a non-distorted effort allocation can only be optimal if $P(\boldsymbol{e})$ is perfectly congruent and the agent experiences identical abilities for performing all relevant tasks.

Suppose the available performance measure $P(\boldsymbol{e})$ changes such that the agent is motivated to implement a less distorted effort allocation. Formally, θ decreases. This implies, *ceteris paribus*, a higher expected profit Π^*. Note, however, that there is a second effect on Π^* captured by ξ as the angle between $\boldsymbol{\omega}$ and $\mathbf{\Gamma}$. To illustrate this effect, we can re-formulate the agent's effort costs by substituting $\boldsymbol{e}^*$:

$$C(\cdot) = \frac{1}{2}\beta^2\|\boldsymbol{\omega}\|\|\mathbf{\Gamma}\|\cos\xi. \tag{16}$$

The properties of the agent's task-specific abilities affect her effort costs in two ways. The first effect is a result of the effort cost intensity over all tasks. For illustrative purposes, assume that the effort costs take the form $C(\boldsymbol{e}) = \boldsymbol{e}^t\lambda\mathbf{\Psi}\boldsymbol{e}/2$ with $\lambda > 0$. Increasing λ implies that all tasks become more costly to perform, thereby leading to a higher $\|\mathbf{\Gamma}\|$ without affecting $\cos\xi$. The second effect is caused by the relation between the performance measure sensitivities $\boldsymbol{\omega}$ and the agent's task-specific abilities $\mathbf{\Psi}$. The relative abilities across tasks thereby affect $\|\mathbf{\Gamma}\|$ and $\cos\xi$. Recall that $\|\mathbf{\Gamma}\|$ determines the effort intensity without affecting the allocation. In contrast, $\cos\xi$

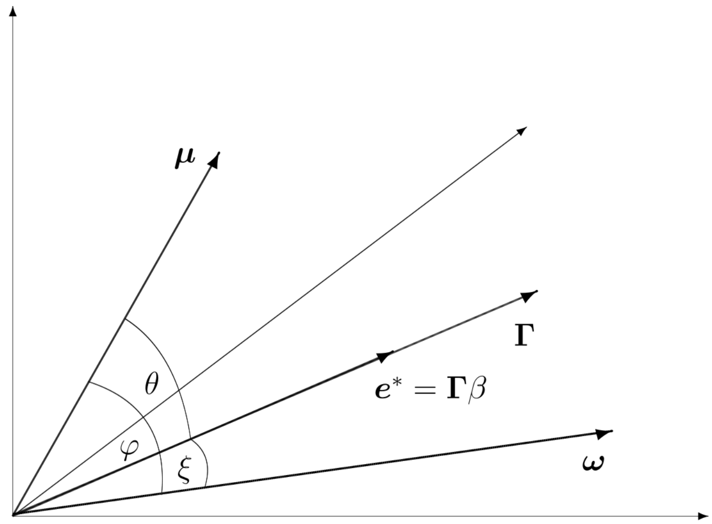

Figure 1: Performance Measure Congruity and Effort Distortion for $n = 3$

measures the agent's effort costs (in utility terms) for a particular effort allocation motivated by $P(\boldsymbol{e})$.

Corollary 2. *If $\psi_k \neq \psi_j$ for at least one pair $(k, j) \in \{1, ..., n\}$, $k \neq j$, then $\Upsilon^{M/C}(\xi) = \cos \xi$ characterizes the measure-cost efficiency.*

The previous results are illustrated in figure 1 for the three-dimensional case ($n = 3$). Besides the second-best effort vector $\boldsymbol{e}^*$, it depicts the vectors of the gross payoff sensitivities $\boldsymbol{\mu}$, performance measure sensitivities $\boldsymbol{\omega}$, and measure-cost sensitivity ratios $\boldsymbol{\Gamma}$. The effort vector $\boldsymbol{e}^*$ has the same direction as $\boldsymbol{\Gamma}$, only their lengths differ, depending on β. Observe that $\boldsymbol{e}^*$ is not necessarily on the plane spanned by $\boldsymbol{\mu}$ and $\boldsymbol{\omega}$. The location of $\boldsymbol{e}^*$ relative to $\boldsymbol{\mu}$ characterizes the induced effort distortion (angle θ), whereas the relation between $\boldsymbol{\mu}$ and $\boldsymbol{\omega}$ measures the congruity of performance measure $P(\boldsymbol{e})$ (angle φ). Finally, the measure-cost efficiency is characterized by the relation of $\boldsymbol{\Gamma}$ to $\boldsymbol{\omega}$ (angle ξ).

If vector $\boldsymbol{\mu}$ and vector $\boldsymbol{\omega}$ point in the same direction, then $\boldsymbol{e}^{fb} = \lambda \boldsymbol{e}^*$, $\lambda > 0$, i.e. the incentive contract motivates the agent to implement the first-best effort allocation, see corollary 1. Nevertheless, inducing a first-best effort intensity by adjusting β can only be optimal if the agent is either risk-neutral or the performance measure is perfectly precise. Otherwise, the principal imposes to much incentive risk on the agent which requires the payment of a higher risk premium to ensure her participation.

Now consider the case where the agent has identical abilities for all tasks, i.e. $\psi_i = \hat{\psi} > 0$, $i = 1, ..., n$. As a consequence, $\boldsymbol{\Gamma} = \boldsymbol{\omega}/\hat{\psi}$ so that vector $\boldsymbol{\Gamma}$ and vector $\boldsymbol{\omega}$ point in the same direction. This additionally implies that $\boldsymbol{e}^* = \boldsymbol{\omega}\beta/\hat{\psi}$ and $\xi = 0$. Thus, $\boldsymbol{e}^*$ and $\boldsymbol{\omega}$ are identical with respect to their direction, only their lengths differ, depending on β and $\hat{\psi}$. Accordingly, the measure of congruity is now identical to the measure of distortion. This observation is summarized and proofed by the subsequent proposition.

Proposition 2. *If* $\psi_i = \hat{\psi} > 0$, $i = 1, ..., n$, *then* $\Upsilon^D(\varphi) = \Upsilon^C(\varphi) = \cos\varphi$.

Proof See appendix.

If agents do not exhibit different task-specific abilities, performance measure congruity and effort distortion are captured by the same measure. However, if we allow the agent to possess different abilities across tasks, it becomes pivotal to distinguish between both concepts. The application of incongruent performance measures in incentive contracts leads to inefficient effort allocations, but the extent of these inefficiencies are further determined by the agent's relative abilities for performing the relevant tasks.

Consider again the expected second-best profit Π^* from section 4. According to the previous observations, it depends on three elements: (i) the measure of distortion $\Upsilon^D(\theta)$ in the numerator; (ii) the measure-cost efficiency $\Upsilon^{M/C}(\xi)$ in the denominator; and (iii), the agent's risk aversion ρ in conjunction with the variance σ^2 of the applied performance measure in the denominator. It is common knowledge that the trade-off between incentive risk and the agent's desire for insurance affects optimal incentive contracts. Moreover, as demonstrated by Feltham and Xie [1994] and Baker [2002], incentive contracts in multi-task agency relations are adjusted to the congruity of applied performance measures. However, the previous analysis indicates that the measure-costs efficiency is a third crucial factor whenever the agent performs some tasks more efficiently than others due to task-specific abilities.

6 Ranking Performance Measures

As Feltham and Xie [1994] emphasized, performance measures may differ with respect to their congruity and precision. The previous analysis additionally indicates that task-specific abilities play a crucial role for the contract efficiency. This section therefore focuses on how the attributes of performance measures and agents eventually determine the relative value of measures in multi-task agencies.

Consider a set $\mathbf{P}$ of $m \geq 2$ performance measures $P_i(\boldsymbol{e}) = \boldsymbol{\omega}_i^t \boldsymbol{e} + \varepsilon_i$, with $P_i(\boldsymbol{e}) \in \mathbf{P} \subseteq \mathbb{R}^m$ and $\varepsilon_i \sim N(0, \sigma_i^2)$.[11] To illustrate the relative value of individual performance measures, we can compare the expected profits each of them would induce if applied in the agent's incentive contract. Then, performance measure $P_k(\boldsymbol{e})$ is referred to be strictly superior, if it provides the principal a strictly higher expected profit than all other available measures $P_i(\boldsymbol{e}) \in \mathbf{P}$, $i \neq k$.

For single-task agency relations, Kim and Suh [1991] have shown that the value of performance measures can be compared by their respective signal/noise ratio. Schnedler [2006] generalized their signal/noise ratio to a setting, where the agent is in charge of conducting multiple tasks. By applying the formulation proposed by Schnedler [2006] (see Definition 2), the signal/noise ratio of performance measures $P_i(\boldsymbol{e})$ is

$$\Lambda_i = \frac{\left(\nabla P_i(\boldsymbol{e}^*)\right)^t \left(\nabla P_i(\boldsymbol{e}^*)\right)}{\sigma_i^2}, \tag{17}$$

where $\nabla P_i(\boldsymbol{e}^*)$ is the gradient of performance measure $P_i(\boldsymbol{e})$ with respect to $\boldsymbol{e}$. In single-task agencies, performance measures with higher signal/noise ratios provide more precise information about the implemented effort and are therefore strictly preferred to measures with lower ratios. In this multi-task setting, the signal/noise ratio of performance measures $P_i(\boldsymbol{e})$ is

$$\Lambda_i = \frac{\|\boldsymbol{\omega}_i\|^2}{\sigma_i^2}. \tag{18}$$

One can infer from the previous analysis that signal/noise ratios are not necessarily sufficient to rank performance measures in multi-task agencies, especially, when agents differ in their task-specific abilities.[12] This deduction is supported by the next proposition.

Proposition 3. *Performance measure $P_k(\boldsymbol{e})$ is strictly superior to any other performance measure $P_j(\boldsymbol{e}) \in \mathbf{P}$, $j \neq k$, if and only if,*

$$\frac{\|\boldsymbol{\omega}_k\|}{\|\boldsymbol{\Gamma}_k\|} \frac{\Upsilon^{M/C}(\xi_k)}{(\Upsilon^D(\theta_k))^2} + \frac{\rho\sigma_k^2}{\|\boldsymbol{\Gamma}_k\|^2(\Upsilon^D(\theta_k))^2} < \frac{\|\boldsymbol{\omega}_j\|}{\|\boldsymbol{\Gamma}_j\|} \frac{\Upsilon^{M/C}(\xi_j)}{(\Upsilon^D(\theta_j))^2} + \frac{\rho\sigma_j^2}{\|\boldsymbol{\Gamma}_j\|^2(\Upsilon^D(\theta_j))^2}, \tag{19}$$

where $\Upsilon^D(\theta_i)$ is the measure of distortion induced by $P_i(\boldsymbol{e})$, and $\Upsilon^{M/C}(\xi_i)$ is the related quantification for the measure-cost efficiency, $i = \{j, k\}$.

[11] Subscript i refers henceforth to performance measure $P_i(\boldsymbol{e}) \in \mathbf{P}$.

[12] Schnedler [2006] comes to the same conclusion by analyzing a multi-task agency framework, where the agent enjoys cost synergies across tasks.

Proof Follows directly by rearranging $\Pi^*(P_k(\boldsymbol{e})) > \Pi^*(P_j(\boldsymbol{e}))$ and substituting $\Upsilon^{M/C}(\xi_i) = \cos\xi_i$ and $\Upsilon^D(\theta_i) = \cos\theta_i$, $i = k, j$.

The value of a performance measure in comparison to any other measure is contingent on two ratios: (i) the normalized ratio between the measure-cost efficiency $\Upsilon^{M/C}(\cdot)$ and the induced effort distortion $\Upsilon^D(\cdot)$; and, (ii) the normalized inverse of the distortion measure $\Upsilon^D(\cdot)$ with the precision $1/\sigma_k^2$ of the performance measure and the agent's risk tolerance $1/\rho$. Observe finally that performance measure congruity does not directly enter into this ranking criteria. It, however, affects indirectly the measure of effort distortion $\Upsilon^D(\theta_i)$ and the measure-cost efficiency characterized by $\Upsilon^{M/C}(\xi_i)$.

In fact, the value of performance measures in multi-task agencies cannot necessarily be compared by their respective signal/noise ratios. It is rather pivotal to take the induced effort distortion and measure-cost efficiency into consideration—both determined by the performance measure sensitivities $\boldsymbol{\omega}_i$ relative to the agent's task specific abilities $\boldsymbol{\Psi}$. Therefore, comparing the value of performance measures requires specific knowledge about the agent's characteristics, which is not necessary for ranking performance measures in single-task agencies. In multi-task agencies, however, the agent's characteristics eventually determine the principal's preference for a specific information system.

Corollary 3. *Suppose $\psi_i = \hat{\psi} > 0$, $i = 1, ..., n$. Then, performance measure $P_k(\boldsymbol{e})$ is strictly superior to any other performance measure $P_j(\boldsymbol{e}) \in \mathbf{P}$, $j \neq k$, if and only if,*

$$\frac{1}{\Upsilon^C(\varphi_k)}\left[1 + \hat{\psi}\rho\Lambda_k^{-1}\right]^{\frac{1}{2}} < \frac{1}{\Upsilon^C(\varphi_j)}\left[1 + \hat{\psi}\rho\Lambda_j^{-1}\right]^{\frac{1}{2}}, \tag{20}$$

where Λ_i, $i = \{j, k\}$, is the signal/noise ratio of performance measure $P_i(\boldsymbol{e})$, and $\Upsilon^C(\varphi_i)$ its congruity measure.

Proof See appendix.

If the agent's preference for an effort allocation depends only on the characteristics of her performance evaluation since her abilities are identical for all tasks, we can use adjusted signal/noise ratios to rank performance measures in multi-task agencies. Nevertheless, it is still required to know $\hat{\psi}$ and ρ in order to assess the relative value of performance measures.

The subsequent proposition offers a sufficient condition ensuring that performance measures can be ranked exclusively by their respective signal/noise ratios, and therefore, independent of the agent's characteristics.

Proposition 4. *Suppose there exist constants $\lambda_j \neq 0$ satisfying $\boldsymbol{\omega}_i = \lambda_j \boldsymbol{\omega}_j$ for all $i, j = 1, ..., m$, $i \neq j$. Then, performance measure $P_k(\boldsymbol{e})$ is strictly superior to any other performance measure $P_j(\boldsymbol{e}) \in \mathbf{P}$, $j \neq k$, if and only if, $\Lambda_k > \Lambda_j$.*

Proof See appendix.

Accordingly, the signal/noise ratio is sufficient to rank performance measures in multi-task agencies, if all measures provide the same information about the agent's relative effort allocation. In this case, observe that $\Upsilon^C(\varphi_i) = \Upsilon^C(\varphi_j)$, $i, j = 1, ..., m$, i.e. all performance measures share the same measure of congruity.[13] As a consequence, every available performance measure—if applied in the agent's incentive contract—would imply the same effort distortion and measure-cost efficiency. Then, their relative value is defined by their precision and scale, which in turn is represented by their respective signal/noise ratio.

To investigate the effects of task-specific abilities on the ordering of performance measures, it is insightful to eliminate effects related to their precision. By setting $\rho = 0$, condition (19) simplifies to

$$\nu \, \frac{\cos^2 \theta_k}{\cos^2 \theta_j} > \frac{\cos \xi_k}{\cos \xi_j}, \qquad \nu = \frac{\|\boldsymbol{\omega}_j\|}{\|\boldsymbol{\omega}_k\|} \frac{\|\boldsymbol{\Gamma}_k\|}{\|\boldsymbol{\Gamma}_j\|}. \tag{21}$$

The value of performance measure $P_k(\boldsymbol{e})$ relative to $P_j(\boldsymbol{e})$ depends—besides on their precision and scaling as previously emphasized—on their relative effort distortion ($\cos \theta_i$) and relative measure-cost efficiency ($\cos \xi_i$) weighted by the multiplier ν, $i = k, j$. In order to make both measures comparable, it is essential to normalize their scale $\|\boldsymbol{\omega}_i\|$, and exclude their effect on $\|\boldsymbol{\Gamma}_i\|$, $i = k, j$. Accordingly, if either the agent is risk-neutral or the realization of performance measures is not influenced by random effects, the relative value of performance measures depends on two factors: (i) the motivated effort allocation and its contribution to gross payoff $V(\boldsymbol{e})$; and, (ii) the imposed costs to motivate this effort allocation.

7 Conclusion

Applying incongruent performance measures in incentive contracts motivates agents to implement an inefficient effort allocation across relevant tasks. This paper incorporates task-specific

[13] Note that the reversed inference cannot be made, i.e. if $\Upsilon^C(\varphi_i) = \Upsilon^C(\varphi_j)$, it is not necessarily true that $\boldsymbol{\omega}_i = \lambda_j \boldsymbol{\omega}_j$, $\lambda_j \neq 0$, $i, j = 1, ..., m$, $i \neq j$. In this case, the signal/noise ratio is not sufficient to rank performance measures in multi-task agencies.

abilities in a multi-task agency framework and investigates their effects on the provision of incentives. As demonstrated, incentive contracts are tailored to the agent's task-specific abilities and, particularly, depend on three factors: (i) the inefficiency of effort distortion as a result of applying incongruent performance measures in incentive contracts, relative to the agent's task-specific abilities (*distortion effect*), (ii) the agent's effort costs associated with the motivated effort allocation (*measure-cost efficiency*); and (iii), the precision of the information system with the agent's risk-aversion (*risk effect*).

This paper further proposes a ranking criteria for performance measures in multi-task agencies. One important observation is that the signal/noise ratio, commonly used to assess performance measures in single-task agencies, is not a sufficient ranking criteria in multi-task agencies. The relative value of performance measures depends—besides on their precision—on their congruity relative to the agent's task-specific abilities, thereby implying that their ranking is tied to the agent's characteristics. Hence, we can infer that the selection of 'suitable' agents for a given information system provides the principal some latitude to improve the contract efficiency.

This paper is part of a larger research agenda. Previous multi-task literature focused primarily on performance measure congruity and its effect on incentive contracts. As this paper illustrates, we can shed more light on the nature of incentive contracts in multi-task agency relations, when we keep in mind that agents may differ in their skills and abilities to perform particular tasks.

8 Appendix

Proof of Proposition 1.

Effort distortion refers to the relation of $\boldsymbol{e}^*$ to $\boldsymbol{\mu}$ and can be therefore measured by the vector product $\boldsymbol{\mu}^t\boldsymbol{e}^*$. Since $\boldsymbol{e}^* = \boldsymbol{\Gamma}\beta$,

$$\boldsymbol{\mu}^t\boldsymbol{e} = \beta\sum_{i=1}^{n}\mu_i\Gamma_i = \beta\|\boldsymbol{\mu}\|\|\boldsymbol{\Gamma}\|\cos\theta. \tag{22}$$

First note that $\|\boldsymbol{\mu}\|$ does not affect the relative importance of tasks for $V(\boldsymbol{e})$. Furthermore, $\beta\|\boldsymbol{\Gamma}\|$ determines the lengths of vector $\boldsymbol{e}^*$, but not its direction in the n-dimensional space. The length is arbitrary in the sense that it can be adjusted by β. Consequently, $\Upsilon^D(\theta) = \cos\theta \in [0,1]$ measures the induced effort distortion under second-best.

Q.E.D.

Proof of Proposition 2.

To measure effort distortion, we can use the vector product $\boldsymbol{\mu}^t\boldsymbol{e}^*$. If $\psi_i = \hat{\psi} > 0$, $i = 1, ..., n$, then $\boldsymbol{e}^* = \beta\boldsymbol{\omega}/\hat{\psi}$. This leads to

$$\boldsymbol{\mu}^t\boldsymbol{e} = \frac{\beta}{\hat{\psi}}\sum_{i=1}^{n}\mu_i\omega_i = \frac{\beta}{\hat{\psi}}\|\boldsymbol{\mu}\|\|\boldsymbol{\omega}\|\cos\varphi. \tag{23}$$

Again, $\|\boldsymbol{\mu}\|$ does not affect the relative importance of tasks for $V(\boldsymbol{e})$, and $\beta\|\boldsymbol{\omega}\|$ determines the lengths of vector $\boldsymbol{e}^*$ but not its direction in the n-dimensional space. Thus, $\bar{\Upsilon}^D(\varphi) = \cos\varphi \in [0,1]$ measures distortion under second-best if $\psi_i = \hat{\psi} > 0$, $i = 1, ..., n$. Consequently, $\bar{\Upsilon}^D(\varphi) = \Upsilon^C(\varphi)$.

Q.E.D.

Proof of Corollary 3.

If $\psi_i = \hat{\psi} > 0$, $i = 1, ..., n$, then $\boldsymbol{\Gamma}_i = \boldsymbol{\omega}_i/\hat{\psi}$ and $\|\boldsymbol{\Gamma}_i\| = \|\boldsymbol{\omega}_i\|/\hat{\psi}$, $i = \{j,k\}$. Consequently, $\Upsilon^{M/C}(\xi = 0) = 1$ and $\bar{\Upsilon}^D(\varphi_i) = \Upsilon^C(\varphi_i)$, see proposition 2. By substituting $\Lambda_i = \|\boldsymbol{\omega}_i\|^2/\sigma_i^2$, $i = \{j,k\}$, the ranking criteria of proposition 3 can be reformulated to the one stated in the corollary.

Q.E.D.

Proof of Proposition 4.

Observe first that the expected profit on the basis of $P_i(\boldsymbol{e})$ can be written as

$$\Pi^* = \frac{(\boldsymbol{\mu}^t \boldsymbol{\Gamma}_i)^2}{2(\boldsymbol{\omega}_i^t \boldsymbol{\Gamma}_i + \rho\sigma_i^2)}. \tag{24}$$

Recall that $\boldsymbol{\Gamma}_i = \boldsymbol{\Psi}^{-1}\boldsymbol{\omega}_i$. Consequently, performance measure $P_k(\boldsymbol{e})$ is strictly superior to any other performance measure $P_j(\boldsymbol{e}) \in \mathbf{P}$, $\forall j \neq k$, if and only if,

$$\frac{\left(\boldsymbol{\mu}^t \boldsymbol{\Psi}^{-1}\boldsymbol{\omega}_k\right)^2}{2(\boldsymbol{\omega}_k^t \boldsymbol{\Psi}^{-1}\boldsymbol{\omega}_k + \rho\sigma_k^2)} > \frac{\left(\boldsymbol{\mu}^t \boldsymbol{\Psi}^{-1}\boldsymbol{\omega}_j\right)^2}{2(\boldsymbol{\omega}_j^t \boldsymbol{\Psi}^{-1}\boldsymbol{\omega}_j + \rho\sigma_j^2)}. \tag{25}$$

If $\boldsymbol{\omega}_k = \lambda\boldsymbol{\omega}_j$, we can re-scale $P_j(\boldsymbol{e})$ such that it is characterized by the same sensitivity in $\boldsymbol{e}$ as $P_k(\boldsymbol{e})$. Accordingly,

$$\bar{P}_j(\boldsymbol{e}) = \boldsymbol{\omega}_j^t \boldsymbol{e} + \frac{\varepsilon_j}{\lambda}, \tag{26}$$

where $\mathrm{Var}\left[\bar{P}_j(\boldsymbol{e})\right] = \sigma_j^2\lambda^{-2}$. Let $\boldsymbol{\omega} \equiv \boldsymbol{\omega}_i$, $i = j, k$. This leads to

$$\frac{\left(\boldsymbol{\mu}^t \boldsymbol{\Psi}^{-1}\boldsymbol{\omega}\right)^2}{2(\boldsymbol{\omega}^t \boldsymbol{\Psi}^{-1}\boldsymbol{\omega} + \rho\sigma_k^2)} > \frac{\left(\boldsymbol{\mu}^t \boldsymbol{\Psi}^{-1}\boldsymbol{\omega}\right)^2}{2(\boldsymbol{\omega}^t \boldsymbol{\Psi}^{-1}\boldsymbol{\omega} + \rho\sigma_j^2\lambda^{-2})}, \tag{27}$$

which can be re-arranged to

$$\frac{1}{\sigma_k^2} > \frac{\lambda^2}{\sigma_j^2}. \tag{28}$$

Recall that after re-scaling, $\boldsymbol{\omega}_k = \boldsymbol{\omega}_j$. Thus, (28) can be written as

$$\frac{\|\boldsymbol{\omega}_k\|^2}{\sigma_k^2} > \frac{\lambda^2\|\boldsymbol{\omega}_j\|^2}{\sigma_j^2}, \tag{29}$$

which is identical to $\Lambda_k > \Lambda_j$.

Q.E.D.

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