Wavelet Based Analysis Of Major Real Estate Markets

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Abstract

Wavelet coherence of time series provide valuable information about dynamic correlation and its impact on time scales. Here, we analyze the wavelet coherence of major real estate markets data. Our paper is the first to link co-movement in terms of wavelet coherence. Here we consider USA, Canada, Japan, China and Developed Europe real estate market prices as time series.Wavelet coherence results reveal interconnected relationships between these markets and how these relationships vary in the time-frequency space. These relationships allow us to build VARMA models of real estate data which yield forecast results with small errors.

Keywords: Real Estate Markets, REIT, Co-movement, Wavelet Coherence, VARMA

JEL Classification: C60, F21, G11, G15.

1. Introduction

Features of inter-relationships between financial assets as a consequence of their nature and mutual parameters affecting them together provide valuable information for the forecast of time series. The degree of these relationships vary over time according to ever changing market conditions and dissimilar intensity of parameters' effects.

Wavelet analysis enables one to detect dynamic correlations between time series.Although there are many methods, for this purpose, the most notably Fourier analysis, however, wavelet analysis distinguishes by extending on time domain as well as frequency domain. Subsequently it allows one to examine co-movements in both domains in detail. As a result of such properties, Wavelet analysis has found wide application areas ranging from geophysics to biomedical.

Quantitative efficiency in applications of wavelet analysis, especially, was accelerated by the works of Lau and Weg and Torrence and Compo. While Lau and Weg (1995). New statistical significance tests were developed by Torrence and Compo (1998) by setting up significance levels and confidence intervals as well as describing cross-wavelet spectra and coherence on their study based on atmospheric time series.

Grinsted et al. (2004) expanded the applications on wavelet coherence and cross wavelet transform as focusing on wavelet analysis for two time series. They also have demonstrated the use of phase angle statistics in order to test confidence in casual relationships by employing extended wavelet software packages for geophysical time series.

Since then many researches from variety of fields have implemented wavelet analysis to identify dynamic correlations in time series. Almasri, A., Shukur, G. (2003) exercises wavelet analysis on the causality relationship between government spending and revenue, Aguiar-Conraria and Soares (2008) employs wavelet coherency on monetary policy, Rua, A. and L. C. Nunes (2009) on co-movement of international stock market returns and most recently, Tiwari (2012) analyses the relationship between oil prices, industrial production and inflation in German economy and interest rates and share prices in Indian economy through the cross wavelet analysis. Barunik et al. (2012, 2013) examines co-movement of energy commodities, traded assets like gold, oil and stocks and European stock markets studying its dynamics in the time-frequency domain and compares its results with standard econometric tools.

As a forecasting tool, Vector Autoregressive Moving Average (VARMA) model is employed for this study considering its powerful qualities in a multivariate setting. Basically, the vector ARMA is a multivariate generalization of the univariate ARMA model. Because it relies on both information in the past values and the information on other related variables. VARMA has been quite successful on forecasting economic data in multivariate case with a remarkable performance over similar models (Lutkepohl and Poskitt (1996b)). Athanasopoulos and Vahid (2008) suggested that VARMA model forecast performances are better than VAR for macroeconomic forecasting. Gustavo and Kapetanios (2010) also conclude that VARMA models perform better than VAR and AR models, especially, on forecasting medium and large datasets.

In our study, we examine the relationship between Real Estate Markets by wavelet coherence. We use some major REIT daily prices which are; Creit Canadian REIT (REF-UN.TO) – Canada , LINK REIT – Hong Kong and China, iShares US Real Estate ETF(IYR) – USA, iShares International Developed Real Estate ETF(IFGL) - Japan, USA, developed Europe, Canada , iShares Europe Developed Real Estate ETF(IFEU) – Developed Europe, to obtain better conclusions about this study.

A real estate investment trust (REIT) is a trust that owns, and in most cases operates, income-producing real estate. REITs own many types of commercial real estate. Because of higher liquidity, the lower transaction costs, lower capital requirements, investors usually prefer to invest indirect ways like real estate investment trusts (REITs), real estate exchange traded funds (ETFs) etc. In order to be able to observe daily co-movements between real estate market prices, we used REIT stock prices which closely follow monthly real estate benchmark indices.

In general, a rise in one market correlates with a rise in the other one. We want to show that the relationship between these markets by the mathematical methods. Our aim is to contribute literature by providing an effective way of applying wavelet coherency analysis and VARMA forecasting method jointly to obtain significant results for practical usage.

We use wavelet coherence to detect dynamic correlations in REIT time series. Phase angle statistics provide additional information in causal relationships. These observed dynamic dependencies and interconnections identified by wavelet coherence analysis set a basis for the application of VARMA forecast. VARMA models produce linear forecasts and takes advantage of the information of the past values of two time series productively (Lutkepohl, H., 2004). Mainly, we compare forecast results of the intervals according to their degree of dependencies. Taking into account the most coherent area, the application of VARMA forecast results are observed to be remarkably successful and comparison with the least coherent area’s weak forecast results also support the effectiveness of applied method.

After having introduced the main theoretical concepts, the paper continues with Part 2 which we cover the methodology and brief introduction to wavelet analysis and the cross wavelet transform. Subsequent Part 3 presents empirical results of wavelet coherency and then forecast results. Part 4 carries on with forecast methodology and forecast outcomes and the last part concludes with the remarks of the paper's findings.

2. Methodology

2.1. Wavelet Analysis

Wavelet analysis have advantages over Fourier analysis. The wavelets have zero mean and are localized in frequency and time. So, the wavelet analysis can be used to analyze time series that contain non-stationary power at many different frequencies. Heisenberg uncertainty principle suggests that there is always a tradeoff between localization in time and frequency. Uncertainty can be minimized by wavelet transform methods.

In our study, we use wavelet package written by Grinsted et al. (2004). With this package, we use Morlet Wavelet since it provides a good balance between time and frequency. In time frequency space, we examine phase relationships of two time series which are not normally distributed. CWT (Continuous Wavelet Transform) time series can be perfectly decomposed and then reconstructed. CWT is particularly useful for feature extraction purposes. Then, the Cross Wavelet Transform (XWT) is constructed from two CWTs. XWT exposes these two time series' common power and relative phase in time-frequency space. Phase relationships between the series illuminate the causality links.

CWT acts as a band pass filter to the time series $x(t)$ and is defined by the convolution;

$$W_x(\tau, s) = \frac{1}{\sqrt{s}} \sum_{t=1}^{N} x(t) \varphi^* \left(\frac{t-\tau}{s}\right),$$

where complex conjugate is represented by *.

We also use the Morlet Wavelets which is introduced in Goupillaud, Grossman and Morlet (1984) and defined as;

$\varphi(\eta) = \pi^{-1/4} e^{i\omega_0 \eta} e^{-1/2\eta^2}$,

where η represents time and ω_0 represents frequency which is chosen 6 since it provides a good balance between time and frequency localization (Grinsted et al. (2004)). Morlet wavelets are especially favored for several reasons like numerical advantages, scale to frequency conversion facility, low Heisenberg box area and exceptional compromise between time and frequency concentration.

2.2.The Cross Wavelet Transform

The Cross Wavelet Transform of two time series X and Y, given wavelet transforms $W_n^X(s)$ and W_n^Y(s), then;

Cross-wavelet spectrum is defined as; $W_n^X(s)$=$W_n^X(s)$. W_n^{Y*}(s). Here, W_n^{Y*}(s) is complex conjugate of W_n^Y(s), and cross wavelet power is $|W_n^{XY}(s)|$. So, Cross Wavelet Distribution is given as;

$$\frac{|W_n^X(s).W_n^{Y*}(s)|}{\sigma_X \sigma_Y},$$

Moreover, given the background wavelet spectra, P_k^X and P_k^Y, the confidence level $Z_v(p)$ is;

$$\frac{|W_n^X(s).W_n^{Y*}(s)|}{\sigma_X \sigma_Y} \Rightarrow \frac{Z_v(p)}{v} \sqrt{P_k^X P_k^Y},$$

(Torrence and Compo, 1998).

Phase

Definition of wavelet coherence phase difference is;

$$\varphi_{xy}(\mathrm{u, s}) = tan^{-1}\left(\frac{I\{S((s^{-1}W_{xy}(u,s)))\}}{R\{S((s^{-1}W_{xy}(u,s)))\}}\right), \varphi_{xy} \in [-\pi,\ \pi],$$

I and R represent imaginary and real parts of the power spectrum. Phase differences between two time series indicate how they co-move at different scales. If two time series are in phase and phase difference is between 0 and $\pi/2$ it means that second series lead first one. Similarly, if the phase difference is between 0 and $-\pi/2$, then the first time series lead.

The Cone of Influence

At the edges, discontinuity causes the wavelet power to fall e^{-2} of the value. Data numbers of time series are brought to power of two and padded with zeros. Especially, the larger the scale, the more zeroes are necessary. As a result, wavelet power is decreased by a factor e^{-2} and area of edges are named as the cone of influence (Torrence and Compo, 1998).

3. Empirical Results

3.1.Data sources

Creit Canadian REIT (REF-UN.TO), LINK REIT (0823.HK), iShares US Real Estate ETF (IYR), iShares International Developed Real Estate ETF (IFGL), iShares Europe Developed Real Estate ETF (IFEU) daily price datum were drawn from Yahoo Finance website. The time series are examined for almost 6.5 year time period between 20/11/2009 and 28/6/2016 for wavelet coherency analysis.

3.2.Wavelet Coherence Results

Our intention with wavelet coherence to observe how correlations vary over certain frequency and time interval. In wavelet coherence plot, significant dependence level is shown by colors from red to blue which are ranged from the most dependent relationship to the least dependent. Black lines on each sides represent the area affected by edge effects where the number of data are not adequate. Phases are indicated by arrows. If arrows point right, two series are positively correlated. If they point left, series are not in phase. Besides, if arrows point down, it means that the first time series leads the second. Here, horizontal axis and vertical axis represent frequency and time respectively.

3.2.1. IYR-REF-UN.TO

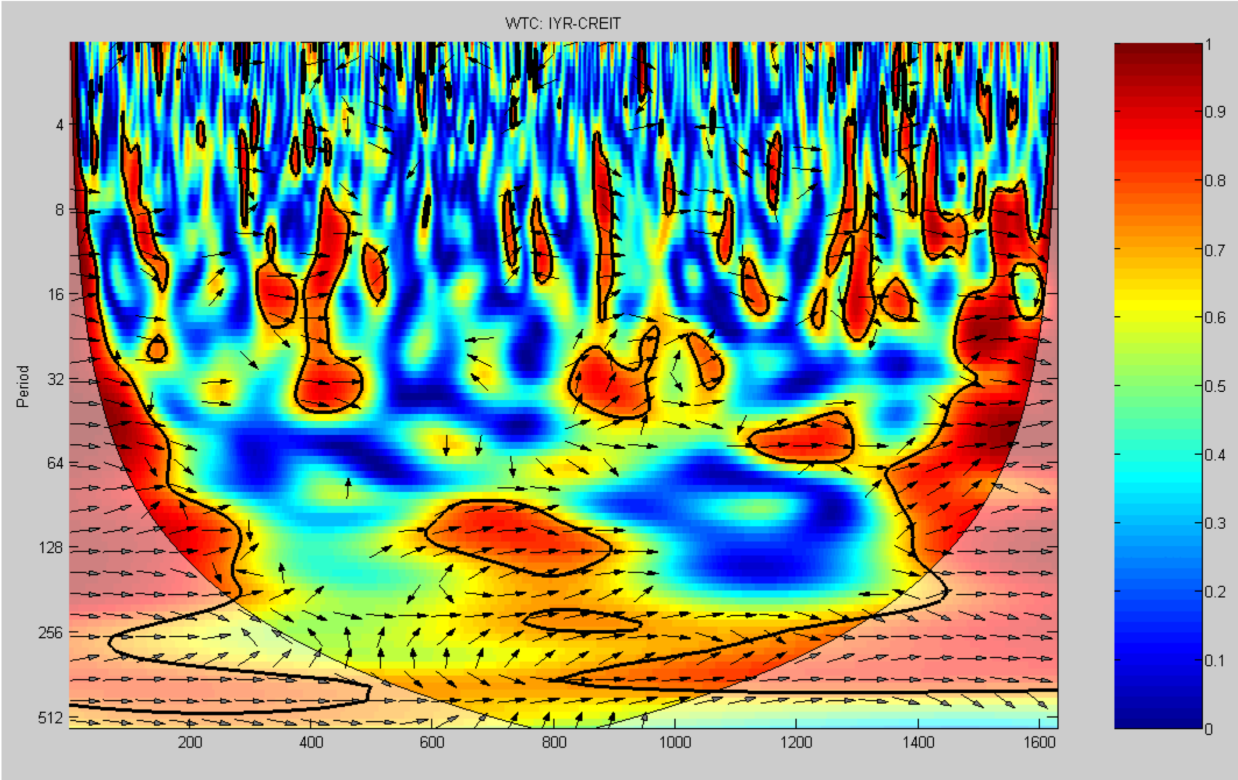

Figure 1: IYR-REF UN.TO Wavelet coherence

IYR-REF UN.TO wavelet coherence plot shows that throughout the practiced time interval, significant areas exist in various frequencies. The most coherent area is especially consistent at 8-32 days frequencies and pairs move together in small time periods stretched examined time period in which arrows' direction to the right indicates that two time series are positively correlated.

3.2.2. IYR-IFEU

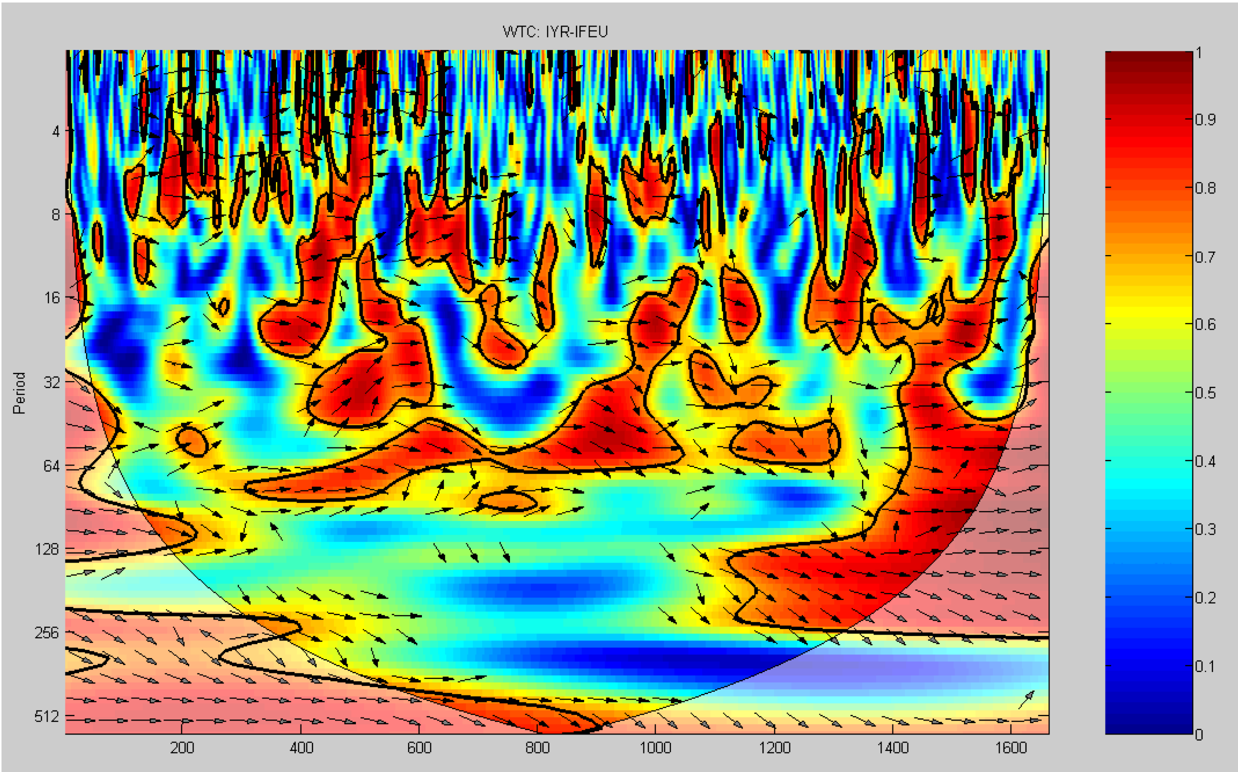

Figure 2: IYR-IFEU Wavelet coherence

Wavelet coherence analysis results also show large significantly coherent areas for IYR and IFEU which are indicated inside the black lines plotted in red colors. Especially, frequencies between 4 and 64, co-movements are appear to in large areas. While the most coherent areas lie between 6/9/2011 and 11/11/2013 at different, the least correlated areas are spotted between 6/28/2010 and 1/31/2011. Considering phase relationships, wavelet coherence indicates that almost all arrows point to the right and so, it is concluded that two time series are in phase.

3.2.3. IYR-IFGL

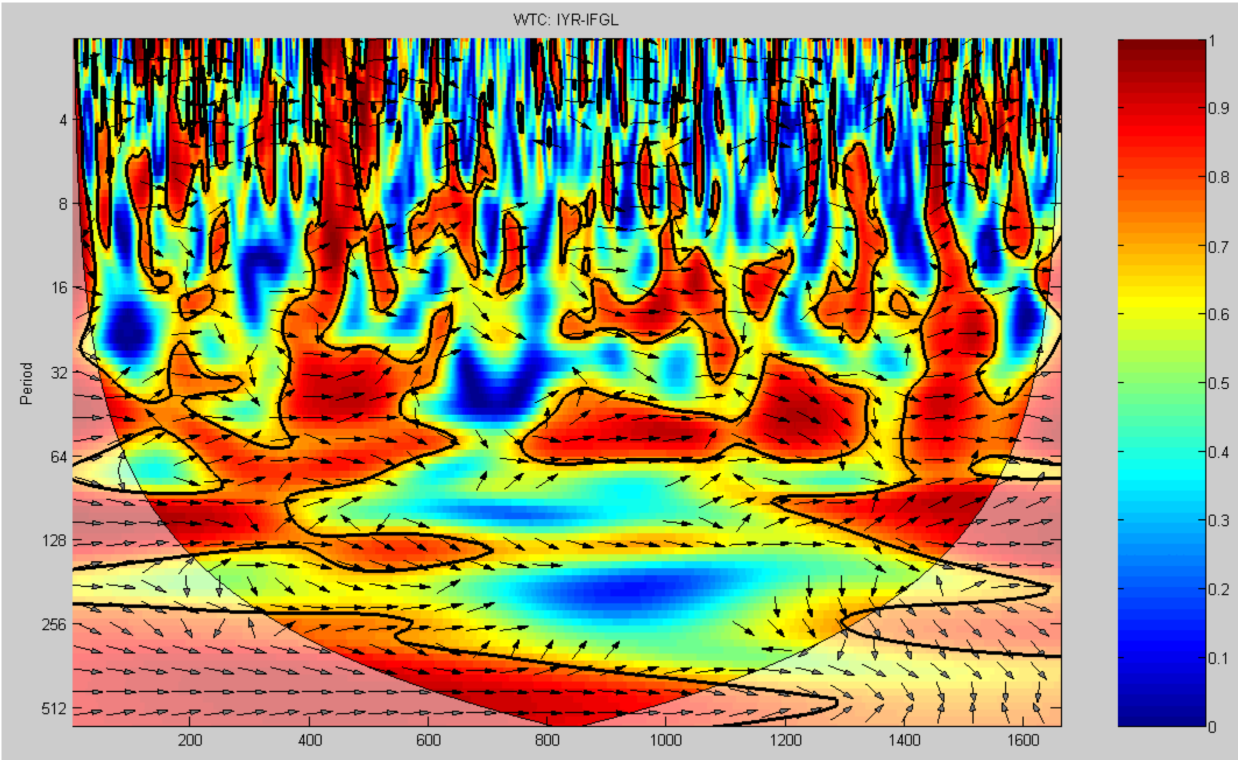

Figure 3: IYR-IFGL Wavelet coherence

IYR-IFGL plot shows that two time series are correlated in almost all time intervals and in frequencies, while at most between 9/8/2010 and 4/10/2012. Although, it is very small time interval the least coherent area lies between 4/11/2012 and 1/28/2013. In almost all significant areas, right pointed arrows suggest positive correlation.

3.2.4. IYR-LINK

Seeing that previous two pairs acts intensely interconnected, the wavelet coherence graph indicate that correlations are less powerful between IYR and LINK by suggesting that paired time series don't relate each other significantly.

Overall, analysis of wavelet coherence allow us observe many exciting interrelationships in a much detailed way than other methods. Relationships among pairs are all positively correlated. Remarkably, co-movements are the strongest between IYR-IFGL and IYR-IFEU.

Multivariate ARMA Process

For our study, Multivariate ARMA Process is applied as the forecasting instrument, aiming to reflect and use the uncovered information about interconnected relationships between pairs to obtain high precision performance over traditional methods.

It is expected that Varma Process improve the forecast accuracy by including more interconnected variables (Granger 1969). In forecasting of selected paired time-series variables, Multivariate ARMA Model processes the information in the past values of interconnected variables, simultaneously, by constructing a vector, consequentially, enabling more precise forecast performance.

Historically, it is turned out to be harder to apply Varma. On the other hand, for two and three variable cases, Varma demonstrated to produce clearer results (Lutkepohl H., 2004).

Multivariate Arma process is represented as;

$A_0 y_t$=$A_1 y_{t-1}$+$A_2 y_{t-2}$+…+$A_p y_{t-p}$+$M_0 \varepsilon_t + M_1 \varepsilon_{t-1}$+$M_2 \varepsilon_{t-2} + \cdots + M_q \varepsilon_{t-q}$

As a vector of y_t of m dimensional time series, the equation satisfies multivariate ARMA process of orders p and q, in which $A_0, A_1, A_2, \dots, A_p$ and $M_0, M_1, M_2, \dots, M_q$ matrices of order m*m and ε_t is an disturbance vector.

If it is written in summary notation by using lag operators,

A(L)y_t=M(L) ε_t, wherein,

A(z)= A_0-$A_1 z$-…-$A_p z^p$ and M(z)= M_0+$M_1 z$+…+$M_q z^q$ are matrix valued polynomials.

The zero order matrices are often be identical and they will be equal to the identity matrix,

A_0=$M_0 = I_m$ (Lutkepohl H., 2004).

4.1. Forecast Results

4.1.1. IYR-IFEU

As wavelet coherence results suggest, the most coherent area was 6/9/2011 and 11/11/2013. Forecasting for the relevant time interval has been made for following 30 days between 12/11/2013 and 24/12/2013 and illustrated below.

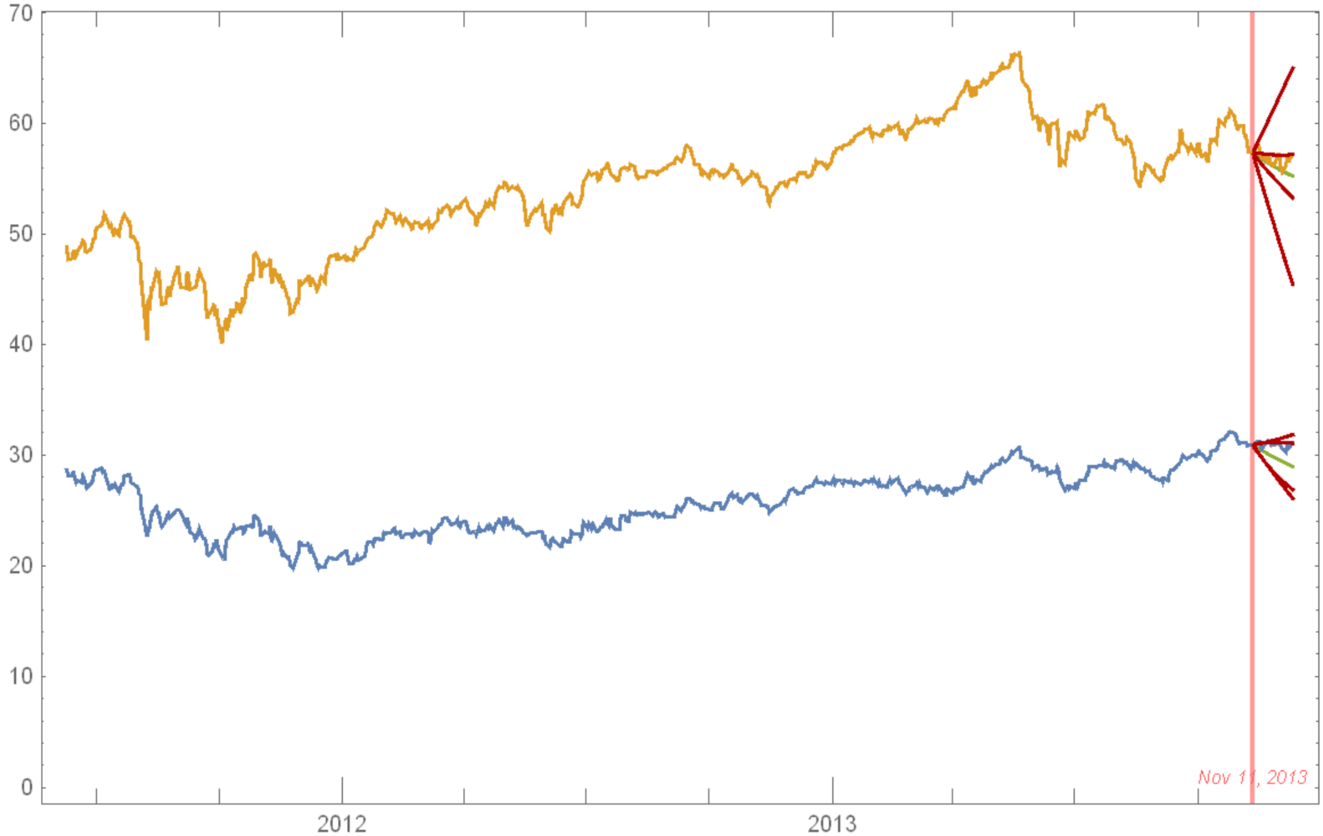

Figure 4: IYR-IFEU 30 Days with error bands (the most coherent)

In figure, IFEU is represented in orange color, IYR in blue and forecast line in green. As it is seen, forecast results are produced very close to actual outcome. Especially for IFEU, in forecast interval in which Varma method combined with Wavelet Coherence demonstrates such remarkable forecast outcomes in very small error bands which are represented with red color.

Having examined the most coherent area, the least coherence is observed to be formed in the area between dates of 6/28/2010 and 1/31/2011. Forecast for this area has been made for following 30 days between 2/1/2011 and 3/15/2011.

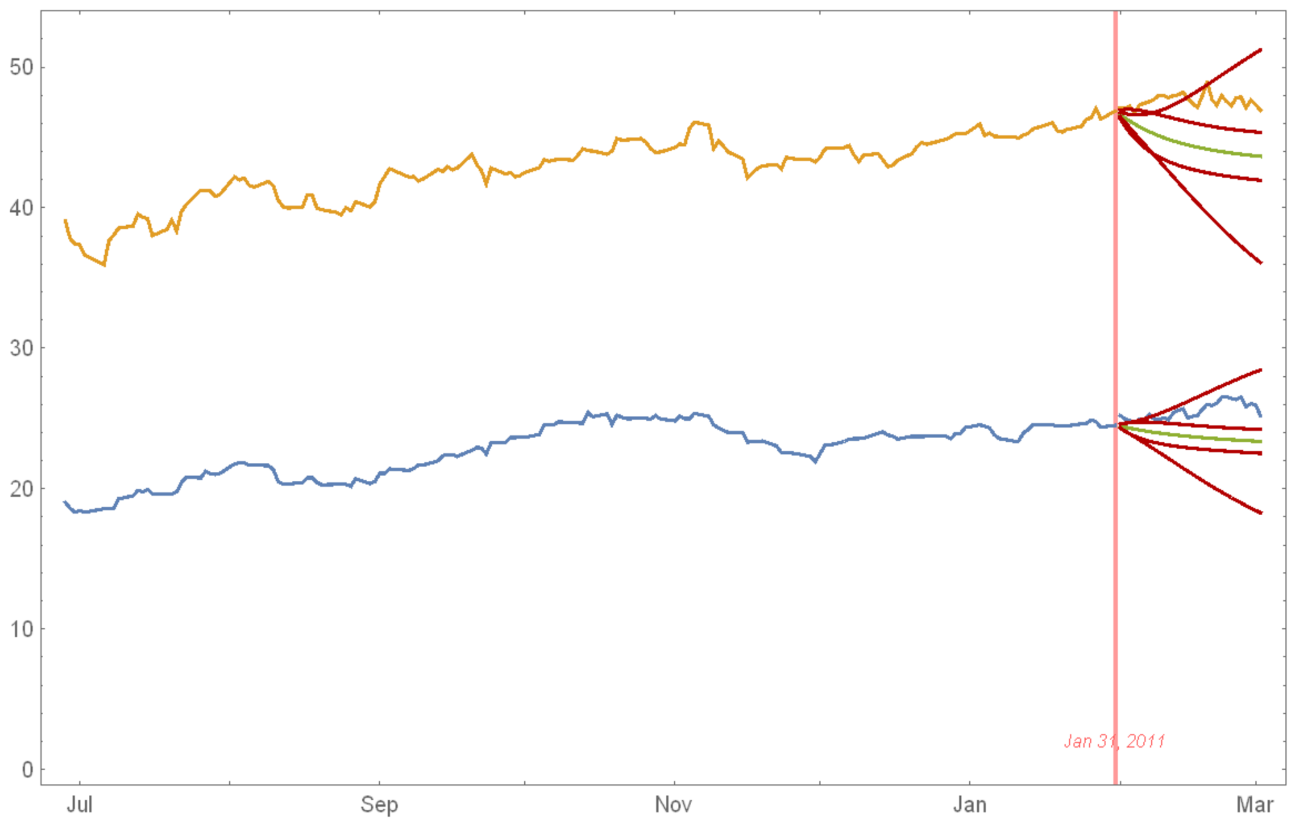

Figure 5: IYR-IFEU 30 Days with error bands (the least coherent)

As a result of less correlation between IYR and IFEU pair, it is expected from the Varma method forecast, the least coherent area results indicate significant divergence from the actual values, for both time series. These two cases for IYR and IFEU strongly suggest that intensity of wavelet coherence directly affects accuracy of forecast results.

4.1.2. IYR-IFGL

The same forecasting practice is applied for IYR and IFGL data. Previously, wavelet coherence results revealed that the most coherent area appeared to be between 9/8/2010 and 4/10/2012.

Following 30 days between 4/11/2012 and 5/22/2012 time interval was the forecast time period.

Figure 6: IYR-IFGL 30 Days with error bands (the most coherent)

In figure above, IYR is represented in blue color, IFGL in orange and forecast line in green. In this case, for IYR forecast line for the area in which the most coherent relationships occur, follow the real rate line successfully.

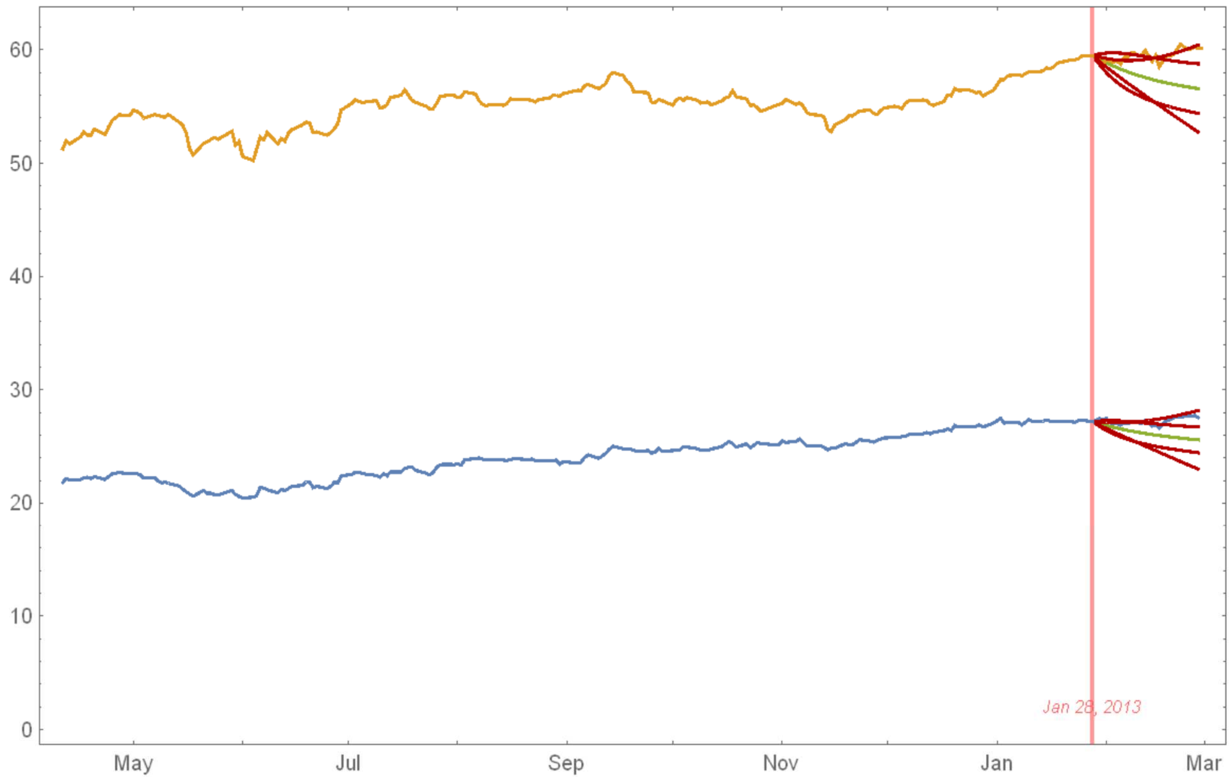

Figure 7: IYR-IFGL 30 Days with error bands (the least coherent)

When it comes to forecast result of the least coherent area which was identified by wavelet coherency analysis as between 4/11/2012 and 1/28/2013, it is clearly noticeable that forecast line and actual both price value lines for 30 days period between 1/29/2013 and 3/13/2013 differs markedly, considering the scale.

5. Conclusion

In this paper we have obtained wavelet coherence of REIT time series. We have determined the time intervals and time scales in which dynamic correlation are high. Furthermore we have developed Vector Autoregressive Moving Average (VARMA) models for these time intervals.

Forecasting based on VARMA models exhibit less errors than the forecasting obtained from individually modelled time series. This result not only testify the validity of the coherence results but also the correct modeling of time series for the carefully determined time intervals.

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