Increasing Returns in a Model With Creative and Physical Capital: Does a Balanced Growth Path Exist?

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by

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Abstract

In this note we study aspects of economic growth in a region that produces a final consumption good with creative and physical capital. This consumption good is manufactured with a production function that exhibits increasing returns to scale. Our analysis leads to three results. First, we compute the growth rate of creative capital in our regional economy. Second, we show that despite the presence of increasing returns, the regional economy under study converges to a balanced growth path (BGP). Finally, we compute the growth rates of physical capital and output on the BGP.

Keywords: Balanced Growth Path, Creative Capital, Creative Region, Economic Growth, Increasing Returns

JEL Codes: R11, D20

1. Introduction

Economists and regional scientists are now very familiar with the twin concepts of the *creative class* and *creative capital*. According to Richard Florida (2002, p. 68), the creative class "consists of people who add economic value through their creativity." This class consists of professionals such as doctors, lawyers, scientists, engineers, university professors, and, notably, bohemians such as artists, musicians, and sculptors. The distinguishing feature of these people is that they possess creative capital which is defined to be the "intrinsically human ability to create new ideas, new technologies, new business models, new cultural forms, and whole new industries that really [matter]" (Florida, 2005, p. 32).

Florida maintains that the creative class is salient because this group of people gives rise to ideas, information, and technology, outputs that are very important for the growth of cities and regions. Hence, cities and regions that want to succeed in the global arena must attempt to attract and retain members of this creative class who are, we are told, the fundamental drivers of economic growth.

Recently, several studies have analyzed the nature of *production* in regions that use creative capital—possessed by members of the creative class—to produce one or more final consumption goods. For instance, one of the two sectors in the model of a trading regional economy in Batabyal and Nijkamp (2010) uses creative and physical capital to produce a final good. However, these authors do not specify the scale properties of the underlying production functions they work with. Batabyal and Nijkamp (2013) study unbalanced growth in an urban economy with a creative capital possessing creative class. Even so, these authors work with general production functions and they too omit any discussion of the scale properties of these functions.

Usman and Batabyal (2014) analyze goods production, learning by doing, and economic growth in a region possessing both creative and physical capital. The analysis here involves production functions that display decreasing returns to scale. Donovan and Batabyal (2015) study investment in a creative region and the impact that investment income taxation has on the well-being of a creative region. Nevertheless, the production function for consumption goods in their paper also displays decreasing returns to scale. Capello *et al.* (2009) have pointed to the important role played by *increasing* returns to knowledge in shaping economic growth in a region but these authors do not formally study the connection between increasing returns and economic growth.

To close this discussion, to the best of our knowledge, the existing literature has paid *no* attention to theoretically analyzing regional economic growth when the underlying creative capital using production function displays *increasing* returns to scale. Given this lacuna in the literature, in this note we study aspects of economic growth in a region that produces a final consumption good with creative and physical capital. This consumption good is manufactured with a production function that exhibits increasing returns to scale. Our analysis leads to three results delineated in sections 3 through 5 below. Section 2 describes the theoretical framework which is adapted from Lucas (1988). Section 3 computes the growth rate of creative capital in our regional economy. Section 4 shows that despite the presence of increasing returns, our regional economy converges to a balanced growth path (BGP). Section 5 computes the growth rates of physical capital and output on the BGP. Section 6 concludes and then suggests two ways in which the research described in this note might be extended.

2. The Theoretical Framework

Consider a regional economy that is creative in the sense of Richard Florida. This creative region is populated by infinitely lived members of the creative class and these members collectively own the region's initial (time $t=0$) stock of physical capital. Output of the final consumption good at any time t or $Q(t)$ is produced in accordance with the production function

$$Q(t)=K(t)^{\gamma}\{(1-a_R)R(t)\}^{\delta}, \tag{1}$$

where $K(t)$ is the physical capital input, $R(t)$ is the creative capital input, and $(1-a_R)$ is the fraction of the creative capital stock that is used to produce the final consumption good $Q(t)$. The exponents $\gamma\in(0,1)$ and $\delta\in(0,1)$. The production function in equation (1) displays increasing returns to scale and hence we have $\gamma+\delta>1$.

The evolution of the stock of creative capital in our region is given by the differential equation

$$\frac{dR(t)}{dt}=\dot{R}(t)=Ba_R R(t), \tag{2}$$

where $B>0$ is a shift parameter and a_R is the fraction of the creative capital stock that is used to propagate this stock. The dynamics of the stock of physical capital is described by the differential equation

$$\frac{dK(t)}{dt}=\dot{K}(t)=sQ(t), \tag{3}$$

where s>0 is the savings rate in our creative region. This completes the description of our theoretical framework. Our next task is to derive an analytic expression for the growth rate of creative capital in the regional economy under study.

3. Growth Rate of Creative Capital

Let us denote the growth rate of creative capital $R(t)$ by g_R. Then, to obtain g_R, we divide the left-hand side (LHS) and the right-hand side (RHS) of equation (2) by $R(t)$. This gives us

$$g_R = \frac{\dot{R}(t)}{R(t)} = Ba_R > 0. \tag{4}$$

Inspecting equation (4), we see that because the growth rate of creative capital g_R is the product of two positive constants, it itself is also a positive *constant*. We now use this information to show that in spite of the presence of increasing returns, our regional economy converges to a balanced growth path (BGP).

4. Existence of a BGP

We begin our demonstration by substituting the production function in equation (1) into equation (3) which describes the dynamics of the physical capital stock. This yields

$$\dot{K}(t) = sK(t)^{\gamma}\{(1-a_R)R(t)\}^{\delta}. \tag{5}$$

Following the procedure adopted in section 3, to obtain the growth rate of the physical capital stock g_K, we divide the LHS and the RHS of equation (5) by $K(t)$. This gives us

$$g_K = \frac{\dot{K}(t)}{K(t)} = sK(t)^{\gamma-1}\{(1-a_R)R(t)\}^{\delta}. \tag{6}$$

The next step is to analyze the intertemporal behavior of the growth rate of physical capital g_K. To do so, let us first take the logarithm of equation (6) and then differentiate the resulting expression with respect to time. Doing this, we get

$$\frac{\dot{g}_K}{g_K} = (\gamma-1)\frac{\dot{K}(t)}{K(t)} + \delta\frac{\dot{R}(t)}{R(t)} = (\gamma-1)g_K + \delta g_R. \tag{7}$$

Multiplying the LHS and the RHS of equation (7) by $g_K(t)$ gives us

$$\dot{g}_K(t) = (\gamma-1)g_K(t)^2 + \delta g_R g_K(t). \tag{8}$$

Note that because $\gamma \in (0,1)$, equation (8) tells us that the production of the final consumption good in our creative region exhibits *decreasing* returns to physical capital alone.

Now, using equation (8), we can draw a phase diagram in which the change in the growth

Figure 1 about here

rate of physical capital $\dot{g}_K(t)$ is a function of the growth rate of physical capital $g_K(t)$. Figure 1 shows that the growth rate of physical capital $g_K(t)$ is constant when $\dot{g}_K(t)=0$ or when

$(\gamma-1)g_K(t)+\delta g_R=0$. Solving this last expression for $g_K(t)$ gives us a closed-form expression for the optimal growth rate of physical capital or g_K^*. We get

$$g_K^*=\{\frac{\delta}{1-\gamma}\}g_R=\frac{\delta Ba_R}{1-\gamma}. \tag{9}$$

Observe that $g_K^*>g_R$ because $\gamma+\delta>1$ and hence the numerator δ is greater than the denominator $1-\gamma$ in equation (9).

Inspecting equation (9), it is straightforward to confirm that g_K^* is constant. This can also be confirmed by referring to the phase diagram in figure 1. In figure 1, we see that to the left of g_K^*, we have $\dot{g}_K(t)>0$ and hence $g_K(t)$ rises toward g_K^*. In contrast, to the right of g_K^*, we have $\dot{g}_K(t)<0$ and therefore $g_K(t)$ falls toward g_K^*. Putting these last two sentences together, we see that the growth rate of physical capital converges to a *constant* value of g_K^* and hence a BGP *exists* in our creative region. Our final task in this note is to compute the growth rates of physical capital and output on the BGP.

5. Growth Rates of Physical Capital and Output

We begin with the output $Q(t)$ of the final consumption good. As in section 4, we differentiate the logarithm of equation (1) with respect to time. This gives us an expression for the growth rate of output. That expression is

$$\frac{\dot{Q}(t)}{Q(t)} = \gamma\frac{\dot{K}(t)}{K(t)} + \delta\frac{\dot{R}(t)}{R(t)} = \gamma g_K(t) + \delta g_R. \tag{10}$$

We know that on the BGP, $g_K(t)$ is constant and is given by equation (9). Therefore, we can rewrite equation (10) as

$$\frac{\dot{Q}(t)}{Q(t)} = \{\frac{\gamma\delta}{1-\gamma}\}g_R + \delta g_R = \{\frac{\gamma\delta + \delta - \gamma\delta}{1-\gamma}\}g_R = \{\frac{\delta}{1-\gamma}\}g_R = g_K^*. \tag{11}$$

From equation (11) it is clear that the output of the final consumption good grows at the *same* constant rate as the stock of physical capital and this rate is *higher* than the constant rate at which the stock of creative capital in our region grows. This completes our analysis of increasing returns to scale in output production in a region that utilizes creative and physical capital to produce this output.

6. Conclusions

In this note we analyzed aspects of economic growth in a region that produced a final consumption good with creative and physical capital. This consumption good was manufactured with a production function that displayed increasing returns to scale. Our analysis led to three results. First, we calculated the growth rate of creative capital in our regional economy. Second, we showed that despite the presence of increasing returns, the regional economy being studied converged to a BGP. Finally, we computed the growth rates of physical capital and output on the BGP.

The analysis in this note can be extended in a number of different directions. In what follows, we suggest two possible extensions. First, it would be useful to extend the analysis conducted here by letting the fraction a_R and the savings rate s be endogenous and potentially time varying. Second, it would also be instructive to study the effects that increasing returns to scale have on an aggregate economy of two creative regions where the two regions under consideration trade with each other in either the final consumption good or in inputs. Studies that analyze these aspects of the underlying problem will provide additional insights into the nexuses between increasing returns to scale in production and the economic growth of creative regions.

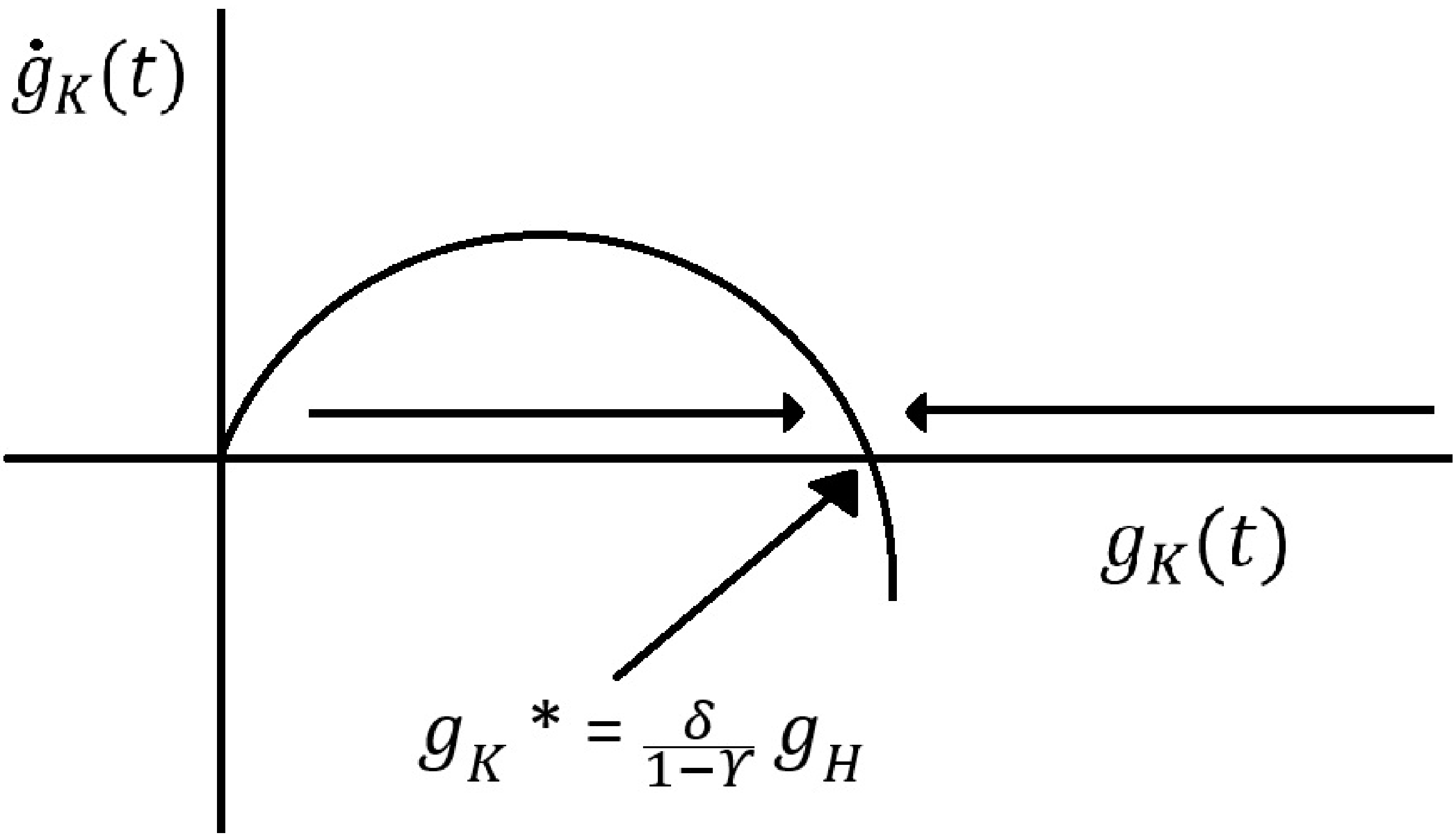

Figure 1: Phase diagram implied by equation (8)

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