Residual income and value creation: An investigation into the lost-capital paradigm

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13. November 2007

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November 13, 2007

Abstract

This paper presents a new way of measuring residual income, originally introduced by Magni (2000a, 2000b, 2000c, 2001a, 2001b, 2003). Contrary to the standard residual income, the capital charge is equal to the *capital lost* by investors. The lost capital may be viewed as (a) the foregone capital, (b) the capital implicitly infused into the business, (c) the outstanding capital of a *shadow project*, (d) the claimholders' credit. Relations of the lost capital with book values and market values are studied, as well as relations of the lost-capital residual income with the classical standard paradigm; many appealing properties are derived, among which a property of earnings aggregation. Different concepts and results, provided by different authors in such different fields as economic theory, management accounting and corporate finance, are considered: O'Hanlon and Peasnell's (2002) unrecovered capital and Excess Value Created; Ohlson's (2005) Abnormal Earnings Growth; O'Byrne's (1997) EVA improvement; Miller and Modigliani's (1961) investment opportunities approach to valuation; Keynes's (1936) user cost; Drukarczyk and Schueler's (2000) Net Economic Income, Fernández's (2002) Created Shareholder Value, Anthony's (1975) profit. They are all conveniently reinterpreted within the theoretical domain of the lost-capital paradigm and conjoined in a unified view. The results found make this new theoretical approach a good candidate for firm valuation, incentive compensation, capital budgeting decision-making.

Keywords. Management accounting, corporate finance, residual income, abnormal earnings, paradigm, value creation, incentive compensation, outstanding capital, lost capital, net present value, book value, market value.

Acknowledgements. The author wishes to thank Stephen O'Byrne and Richard Brief, whose invaluable remarks have stimulated the addition of section 5.

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Introduction

Residual income is income in excess of an income that could be obtained if investor invested their funds at the opportunity cost of capital. Introduced in the first half of the past century (e.g. Preinreich, 1936, 1938) on the ground of some intuition by early microeconomists (e.g. Marshall, 1947), the term "residual income" has been first used by Solomons (1965). The terms "abnormal earnings" and "excess profit" are also used in management accounting and business economics, respectively, to mean earnings (profit) in excess of normal earnings (profit). While it was a minor area of research in the '50s and '60s, the massive literature developed on projet and firm valuation in the last forty years have induced a renewed interest on residual income, both as a valuation tool and as a basis for incentive compensation. In particular, important works such as Peasnell's (1981, 1982) and Ohlson's (1989, 1995) in management accounting, and the proposal of Economic Value Added in applied corporate finance (Stewart, 1991) have triggered a considerable amount of contributions striving to obtain a link between performance measurement and value creation as well as to construct appropriate compensation plans for managers.

A major element in residual income is played by the opportunity cost (capital charge), which represents the income that could be obtained by investing funds at the cost of capital. While the counterfactual feature of the opportunity cost as a foregone income is well known (Coase, 1968; Buchanan, 1969), no debate has ever taken place in the literature about possible alternative ways of computing such a counterfactual income. The traditional accepted formalization of opportunity cost rests on the assumption of investment of the *actual* capital at the cost of capital.

In recent years, a new definition of residual income, called *Systemic Value Added*, has been proposed in Magni (2000a, 2000b, 2000c, 2001a, 2001b, 2003), derived from the comparison between two alternative dynamic systems: The first one describes the net worth's evolution in case of project acceptance, the second one refers to project rejection. Rather than a particular metric, the Systemic Value Added is a paradigm, on the basis of which one can construct infinite possible metrics. The paradigm has been thoroughly studied by the author from several points of view: Conceptual, formal, theoretical, cognitive, empirical (see Magni, 2004, 2005, 2006, forthcoming; Ghiselli Ricci and Magni, 2006).

This paper revisits the Systemic-Value-Added paradigm, which is here renamed *lost-capital* paradigm. The purpose is to show that this new paradigm may be useful for both valuation and management compensation, and that it is capable of encompassing seemingly disparate perspectives and conjoining them in one single theoretical domain. To this end, the lost-capital paradigm is thoroughly investigated in two senses: (i) Formal results are provided aimed at clarifying both the link between performance and value creation and the link between residual income and compensation plan; in addition, the formal and conceptual relations that the two paradigms bear one another are studied; (ii) several notions, models and results in the literature are considered, spanning from the '30s up to most recent years, ranging from microeconomics to management accounting and corporate finance. Different as they are in aims and scope, they are here unified in the comprehensive theoretical domain of the lost-capital paradigm.

In particular, after a brief introduction of the standard paradigm (section 1)

- the lost-capital paradigm is presented and the role of the capital foregone by investors is highlighted (section 2)
- the two paradigms are connected via a cumulation procedure according to which the lost-capital residual income equals the interest earned on cumulated conventional residual incomes (section 3)
- it is shown that, if the lost-capital paradigm is followed, (abnormal) earnings aggregation applies (section 4)
- it is underlined that Ohlson's (2005) Abnormal Earnings Growth is the equity counterpart of O'Byrne's (1997) Economic-Value-Added improvement, and that the fundamental EVA equation is equivalent to Miller and Modigliani's (1961) equity valuation inclusive of the future goodwill; it is also shown that the cumulated future value of the Abnormal Earnings Growth is equal to the lost-capital residual income (section 5)
- the Net Present Value of a project is shown to equal the market value minus the lost capital; this implies that the aggregated earnings measure the difference between book value and the capital infused into the business, as well as the differences between Net Present Value and Market Value Added (section 6)
- the lost capital is shown to coincide with the notion of *unrecovered capital* introduced by O'Hanlon and Peasnell (2002); the link they provide between residual income and value creation is mirrored by analogous results in the lost-capital paradigm, where the Net Present Value (=Excess Value Created) is split in past and prospective lost-capital residual incomes, and the property of earnings aggregation is preserved (section 7)
- a subclass of lost-capital residual incomes, called *Net Value Added*, is presented which is aligned in sign with the Net Present Value at each time t (section 8)
- Keynes's (1967) notion of user cost and Coase's (1968) notion of depreciation through use are retrieved and explained as basic constituents of the Net Value Added (section 9)
- Drukarczyk and Schueler's (2000) *Net Economic Income* and Fernández's (2002) *Created Shareholder Value* are shown to be related: The former and the lost-capital companion of the latter are the Net Value Added computed from an entity approach and a proprietary approach, respectively (section 10)
- a final unification of the two paradigms is shown to be implied by Anthony's (1975) notion of profit: The use of his argument leads to a subclass of residual income models that belong to both paradigms. This subclass represents the only intersection of the two paradigms. Anthony's argument, consistent with the realization concept and therefore with accounting, is opposite to Fernández's perspective, which considers value creation as a 'windfall gain', consistently with a theory-of-finance perspective (section 11)
- the main results of the paper are briefly summarized (section 12)

Some caveats are worth underlining:

- the analysis is meant to be valid for projects, firms, divisions, businesses. We will interchangeably use the terms 'project', 'firm', 'business', 'enterprise'.
- the entity approach and the equity approach are merged. According to the perspective adopted, capital will refer to either equity capital or total liabilities, and claimholders will be equityholders only or equityholders *and* debtholders
- the terms 'outstanding balance' and 'outstanding capital' are used as synonyms and refer to the *actual* capital employed (which will be distinguished from the capital *infused* into the business)
- the cost of debt (required return to debt) equals the nominal rate on debt. This (usual) assumption is made to avoid discrepancy between the equity Net Present Value (discounted equity cash flows) and the entity Net Present Value (discounted free cash flows[1]), and to guarantee that discounted residual incomes lead to the market value irrespective of the outlook adopted (equity vs. entity)
- we will be concerned with project (firm) $\vec{d}$, described by the cash-flow stream $(-d_0, d_1, \dots d_n)$. With no loss of generality, we will assume that the final cash flow d_n is inclusive of the project's terminal value (a finite-time horizon is assumed)
- for the sake of notational convenience, cost of capital is constant. Generalizing to variable costs of capital is just a matter of symbology
- time subscripts are sometimes omitted, if no ambiguity arises
- prospective values should be intended as expected values (from the point of view of the valuation date)
- cash flow available for distribution is assumed to be entirely distributed, unless otherwise indicated
- all notational conventions are collected in Table 0 at the end of the paper.

1 The standard paradigm

Consider a project (firm) $\vec{d}$=$(-d_0, d_1, d_2, \dots, d_n)$ and a sequence of n uniperiodic subprojects $\vec{d_t}$ such that

$$\vec{d_t} = (0, \dots, 0, -y_{t-1}, y_t + d_t, 0, 0, \dots, 0) \in \mathbb{R}^{n+1}, \qquad t = 1, \dots, n;$$

the vector $\vec{y}$=$(y_0, y_1, y_2, \dots, y_n)$ is such that $y_0 := d_0$, and y_t, $= 1, 2, \dots n-1$, is arbitrary in $\mathbb{R}$. We may interpret y_t as the capital employed in $\vec{d}_{t+1}$. Let r_t be a rate of return such that

$$y_{t-1} = \frac{y_t + d_t}{1 + r_t} \tag{1}$$

for $t \geq 1$. From eq. (1), one finds the recurrence equation linking successive capitals:

$$y_t(r) = y_{t-1}(r)(1 + r_t) - d_t \tag{2}$$

(Soper, 1959; Teichroew et al. 1965a, 1965b. See also Peasnell, 1982, p. 366), where the functional dependence of the capital on the return rates is highlighted. While $y_t(r)$ may be any number, from a financial point of view it is possible to interpret it as the actual capital employed in $\vec{d}$ at the beginning of the $(t+1)$-th

[1] By free cash flow we mean the equity cash flow that stockholders would receive if the firm were unlevered (see Fernández, 2002).

period and redefine "income" as a general term representing the product of capital invested $y_{t-1}(r)$ and rate of return r_t.[2] The final $y_n(r)$ is determined by picking $t = n$ in eq. (2). If $y_n(r)$=0, then $\vec{d} = \sum_{t=1}^{n} \vec{d_t}$. The initial condition $y_0(r) := d_0$ says that the initial outstanding capital employed to undertake the project coincides with the capital infused by the investors (it is a negative dividend). The net present value (NPV) of subproject $\vec{d_t}$ is $-\frac{y_{t-1}(r)}{(1+i)^{t-1}} + \frac{y_t(r)+d_t}{(1+i)^t}$, which becomes, owing to eq. (2),

$$\frac{y_{t-1}(r)(r_t - i)}{(1+i)^t} \qquad t = 1, \dots, n. \tag{3}$$

It is widely known that the sum of these uniperiodic NPVs is just the project's NPV:

$$\text{NPV} = \sum_{t=1}^{n} \frac{y_{t-1}(r)(r_t - i)}{(1+i)^t}$$

(see Peasnell, 1981, 1982; Peccati, 1992; Martin and Petty, 2000; Lundholm and O'Keefe, 2001; Martin et al. 2003; Vélez-Pareja and Tham, 2003; Ohlson, 2005).

Remark 1. It is worth noting that:

- Solving eq. (2) one finds

  $$d_0(1+r)^{0,n} - \sum_{t=1}^{n} d_t(1+r)^{t,n} = y_n(r) \tag{4}$$

  where $(1+r)^{\tau,h}$:=$(1+r_{\tau+1})\dots(1+r_h)$ is the capitalization factor from τ to h. The vector $\vec{r} = (r_1, r_2, \dots r_n)$ is a discount function that generalizes the notion of internal rate of return. It is therefore an *internal discount function* (IDF) (see also Peasnell, 1981, p. 367). We stress that eq. (4) holds for any choice of $\vec{r}$ satisfying eq. (1). Eq. (4) holds irrespective of the choice of the outstanding balances $y_t(r)$ as well (IDF and outstanding capitals are in a biunivocal correspondence: Once selected either of the two, the other one is univocally determined). This means that any such discount function $\vec{r}$ is an IDF for the cash-flow stream $(-d_0, d_1, \dots, d_n + y_n(r))$ (see also Ohlson, 2005).

  Let $\vec{r} = (r_1^*, r_2^*, \dots, r_n^*)$ be such that $y_n(r^*) = 0$. Then, $\vec{r}^*$ is an IDF for the cash-flow stream $(-d_0, d_1, \dots, d_n)$.

- if $y_t(r)$ is the equity book value B_t^e, then r_t is the Return On Equity (ROE), which determines an IDF for firm $\vec{d}$. Therefore, the ROE is an index with a genuine economic meaning (for relation between ROEs and internal rate of return, see also Peasnell, 1982; Brief and Lawson, 1990). The amount $r_t B_{t-1}^e$ is obviously the shareholders' net profit

- recurrence equation (2) is a familiar relation in finance, used in the construction of amortization plans, and is consistent with the clean surplus relation often advocated in management accounting (Peasnell, 1982; Ohlson, 1989, 1995):

  $$\text{cash flows} = \text{income} + \text{capital's depreciation}.$$

  This sets a link between accounting and loan theory: The time-t outstanding balance is, in an amortization plan, the residual principal debt at time t; the IDF represents the contractual rate(s), the variation of the outstanding balance is the principal repayment, the cash flows are the instalments, and the product $ry_{t-1}(r)$ is the interest charge (see also Kellison, 1991; Promislow, 2006). The idea

[2] This is because any project may be conveniently assimilated to (interpreted as) a financial loan. See Remark 1.

of income as interest is unambiguous and already recognized in the relevant literature (see Forker and Powell, 2000, p. 237). This analogy is perfectly fulfilled in Anthony's perspective, where equity is seen as a shareholders' credit (section 11). (See also Table 1).

Let $\vec{r} = (r_1^*, r_2^*, \ldots, r_n^*)$ be an IDF for project $\vec{d}$, so that $y_n(r^*) = 0$. We give the following definition:

Definition 1. The classical paradigm of residual income is formally represented by the set $\{x_t^a\}$ of those models such that

$$x_t^a = y_{t-1}(r^*)(r_t^* - i). \tag{5}$$

In the standard definition of residual income a capital charge $iy_{t-1}(r^*)$, representing counterfactual income, is deducted from the actual income $r_t^* y_{t-1}(r^*)$. The set $\{x_t^a\}$ of the standard paradigm consists of many infinite residual income (RI) models, depending on the choice of $\vec{r}^*$ and the choice of the cost of capital i. The former automatically determines the choice of $\vec{y}(r^*)$, the latter depends on the perspective taken: Cost of equity if equity cash flows are considered, weighted average cost of capital if free cash flows are used, pre-tax weighted average cost of capital if capital cash flows are employed (see Ruback, 2002, and Fernández, 2002, for the notion of capital cash flow).

To name a few metrics, the following ones belong to the set of the standard RI models:

Entity approach

- *Economic Value Added* (EVA) (Stewart, 1991). It is found by selecting i=$wacc$, r^*=ROA, and d_t=free cash flow (consequently, $y(r^*)$ is the book value of total liabilities).
- *Cash Flow Return On Investment* (CFROI) (Madden, 1999). Strictly speaking, the CFROI is not a RI measure, but an internal rate of return. However, given an IRR, it is rather natural to construct the relative RI metric. The CFROI is the (inflation-adjusted) internal rate of return of the business, obtained by equating to zero the sum of the discounted free cash flows. Therefore, d_t=free cash flows and r_t^*=IRR (the outstanding balance $y(r^*)$ is automatically determined by eq. (2)). Choosing i=$wacc$ one constructs the relative RI measure.

Equity approach

- *Edwards-Bell-Ohlson* (EBO) (Edwards and Bell, 1961; Ohlson, 1989, 1995). It is obtained by choosing i=k_e, d_t=equity cash flow, and r^*=ROE (therefore $y(r^*)$ is the book value of equity. See also Arnold, 2005).
- *Created Shareholder Value* (CSV) (Fernández, 2002). It is found by picking $y_t(r^*)$=V_t^e ($\vec{r}^*$ is automatically determined by eq. (2)) and i=k_e.
- *Cash Flow Return On Equity* (CFROE). It is the internal rate of return obtained by equating to zero the sum of the discounted equity cash flows, i.e. d_t=equity cash flow ($y(r^*)$ is automatically determined by the usual recursive equation). The resulting RI model is found by selecting i=k_e.

2 The lost-capital paradigm

In this section we revisit the *Systemic Value Added* model, relabelling it the "lost-capital paradigm".

In Magni (2000a, 2000b, 2001a, 2001b, 2003) attention is drawn on shareholders' wealth. It is assumed that, in case of acceptance of the project, shareholders reinvest the equity cash flows at the cost of capital

i (this is the standard assumption of the Net Present Value rule). Therefore, in each period shareholders' wealth is a portfolio of the project and the proceeds of the reinvestments. The all-comprehensive profit (inclusive of income from the project and earned interest from the reinvestments) is given by

$$r^* y_{t-1}(r^*) + iC_{t-1},$$

where C_{t-1} is the value, at time $t-1$, of the reinvestment proceeds, which evolves according to the dynamic system $C_t = C_{t-1}(1+i) + d_t$. Suppose, instead, that the project is not undertaken and the amount d_0 is invested at the cost of capital: Letting C^t be its compounded value at time t (C^0=C_0+d_0 is the initial investor's wealth), wealth is such that $C^t = C^{t-1}(1+i)$, so that the periodic income is

$$iC^{t-1}.$$

The residual income is given by the difference of the two alternative incomes, and is called *Systemic Value Added* because it is deduced from the two dynamic systems:

$$\text{Systemic Value Added} = \big(r^* y_{t-1}(r^*) + iC_{t-1}\big) - iC^{t-1}. \tag{6}$$

This residual income consists of three parts: $r^* y_{t-1}(r^*)$ represents income from investment in the business, iC_{t-1} represents earned interest from reinvestment proceeds, iC^{t-1} is the income that shareholder forgo if project is undertaken. Note that, in Magni's model, C_{t-1} is part of the investor's actual wealth, whereas C^{t-1} is a foregone capital.

We here revisit this paradigm by adopting an arbitrage-type perspective, which enables us to dispense with the reinvestment assumption of interim cash flows. To this end, one can construct a twin asset that replicates the project's payoff. This is accomplished by assuming that d_0 is invested at the cost of capital and that, at the end of each period, cash flow d_t is withdrawn from the asset's balance. So doing, the cash-flow stream of the project is replicated and, at the end of the n-th period, the residual capital $y_n(i)$ is obtained as an arbitrage gain (or loss). In other terms, the two alternatives are represented by a double application of eq. (2) with two different IDFs: The first one is an arbitrary vector $\vec{r} = (r_1^*, r_2^*, \ldots, r_n^*)$ such that $y_n(r^*) = 0$, the second one is the vector of the costs of capital (which, we remind, are here assumed to be constant: $\vec{i} = (i, i, \ldots, i)$):

$$y_t(r^*) = y_{t-1}(r^*)(1 + r_t^*) - d_t \tag{7}$$

$$y_t(i) = y_{t-1}(i)(1 + i) - d_t; \tag{8}$$

the first dynamic system represents the evolution of the actual outstanding balance, the second one represents the path the balance would follow if investors invested their funds at the cost of capital while withdrawing, at each period, the cash flow d_t from the balance. Under this interpretation, $y_t(r^*)$ is the actual capital employed by investors, whereas $y_t(i)$ is the capital that would be (or have been) employed if, at time 0, investors decided (or had decided) to invest funds at the cost of capital. The amount $y_t(i)$ is therefore the capital sacrificed by investors: The lost capital. Thus, $r^* y_{t-1}(r^*)$ represents the actual income in the t-period, whereas $iy_{t-1}(i)$ represents the lost income.

The difference between actual income and lost income gives the lost-capital (LC) residual income.

Definition 2. The lost-capital paradigm is formally represented by the set $\{\xi_t^a\}$ of those models such that

$$\xi_t^a = r_t^* \, y_{t-1}(r^*) - i \, y_{t-1}(i) \tag{9}$$

Remark 2. Eq. (9) is just eq. (6) disguised in a different shape, given that $C^t - C_t = y_t(i)$ for every t (see Magni, 2000a, 2003, 2005): The lost capital may be decomposed into an actual capital C_t and a foregone capital C^t. In his papers Magni shows that the lost capital is just the outstanding capital of a *shadow* project whose classical residual income coincides with the lost-capital residual income of project $\vec{d}$. This interpretation makes the lost capital take on a non-counterfactual interpretation.[3]

Remark 3. The difference between the two alternative paradigms of RI lies in the counterfactual feature of the opportunity cost (capital charge). In the former, the counterfactual state (the alternative course of action) is obtained with a *ceteris paribus* argument: The rate of return changes, while other things (in particular, the capital invested) are held fixed. In the latter the counterfactual state is obtained by adopting a *mutatis mutandis* formalization: Both rate and capital change, because investing d_0 at the cost of capital entails a change in subsequent incomes and outstanding balances.

Remark 4. As noted, we dispense with the reinvestment assumptions of interim cash flows. The same cash-flow stream $\vec{d}$ may be obtained by investing d_0 either at the rate r^* or at the rate i. The difference between the terminal outstanding balances $y_n(r^*) - y_n(i)$ indicates possible existence of arbitrage (remember that we assume that the terminal value is included in the final cash flow, so that $y_n(r^*){=}0$).

Remark 5. Eq. (9) may be conveniently derived from an accounting perspective. Consider two mutually exclusive courses of action: Investing funds at the corporate rate of return, as opposed to investing funds at the corporate cost of capital. The two alternative courses of action give rise to two alternative clean-surplus type relations:

$$\begin{aligned} d_t &= r_t^*\, y_{t-1}(r^*) - \Delta\, y_t(r^*) \\ d_t &= i\, y_{t-1}(i) - \Delta\, y_t(i). \end{aligned}$$

Subtracting the latter from the former, we have

$$\Delta\, y_t(r^*) - \Delta\, y_t(i) = \xi_t^a.$$

Given that depreciation is capital's variation changed in sign, the latter equality informs that periodic performance is positive if and only if the depreciation of the firm's capital is higher upon investing funds at the cost of capital rather than at the corporate actual rate of return.

Remark 6. The LC residual income is linked to depreciation in two different senses:

- *depreciation through time*: eq. (2) and eq. (9) imply

$$\xi_t^a = \underbrace{[y_{t-1}(i) - y_t(i)]}_{\text{rejection}} - \underbrace{[y_{t-1}(r^*) - y_t(r^*)]}_{\text{acceptance}} \tag{10}$$

  where each depreciation charge refers to time, in the two alternative cases of project rejection and acceptance, respectively

- *depreciation through use*: eq. (10) may be rewritten as

$$\xi_t^a = \underbrace{[y_{t-1}(i) - y_{t-1}(r^*)]}_{\text{time } t-1} - \underbrace{[y_t(i) - y_t(r^*)]}_{\text{time } t} \tag{11}$$

  where each depreciation charge refers to different uses of the funds, at time $t{-}1$ and time t respectively.

[3] We do not deal with this result for reasons of space. The interested reader may turn to Magni (2000a, 2004, 2005, 2006).

Therefore, the LC paradigm encompasses both depreciation through time and depreciation through use (see section 9).

Whenever a metric in the classical paradigm is constructed, a corresponding metric in the LC paradigm is univocally determined. We will therefore use the following definition:

Definition 3. Let L the mathematical operator that transforms standard metrics in LC metrics:

$$L : x_t^a \to \xi_t^a.$$

If ξ_t^a is the image of x_t^a via L, i.e. $\xi_t^a = L(x_t^a)$, then ξ_t^a is said to be the LC-companion of x_t^a.

For example, the LC companions of EVA, EBO, and CSV are, respectively,

$$L(\mathrm{EVA}_t) = \mathrm{ROA} \cdot B_t^l - wacc \cdot y_t(wacc)$$

$$L(\mathrm{EBO}_t) = \mathrm{ROE} \cdot B_t^e - k_e \cdot y_t(k_e)$$

$$L(\mathrm{CSV}_t) = \begin{cases} r_t^* d_0 - k_e\, d_0 & t = 1 \\ k_e\, V_t^e - k_e\, y_t(k_e) & t > 1 \end{cases}$$

where $r_1^* = (V_1^e + d_1 - d_0)/d_0$ (see also Table 2).

3 Cumulations of residual incomes

It is rather common in the value-based management literature to find recommendations regarding multi-period cumulation of residual incomes (e.g. Stewart, 1991; Ehrbar, 1998; Young and O'Byrne, 2001), given that residual income as classically formalized does not measure value creation (see Martin et al. 2003): Performance bonuses should be banked and paid out over time. A compensation plan should therefore somehow consider capitalization of previous residual incomes. While section 8 shows that there exists a subclass of LC residual incomes that do measure value creation, this section shows that a cumulation of past residual incomes is intrinsically incorporated in the definition of LC residual income.

Proposition 1. *The lost-capital RI is equal to the sum of the standard RI plus accumulated interest on past standard RIs.*

$$\begin{aligned} \xi_1^a &= x_1^a \\ \xi_t^a &= x_t^a + i \sum_{k=1}^{t-1} x_k^a u^{t-1-k} \qquad \textit{for } t > 1 \end{aligned} \tag{12}$$

where $u := 1 + i$.

Proof. The first equation is obvious, given that $y_0(r^*) = y_0(i)$. Using the usual recursive (clean surplus) relation $d_k = y_{k-1}(r^*)(1 + r_k^*) - y_k(r^*)$ one finds

$$\begin{aligned} y_{t-1}(i) &= y_0(i) u^{t-1} - \sum_{k=1}^{t-1} d_k u^{t-1-k} \\ &= y_0(i) u^{t-1} - \sum_{k=1}^{t-1} \big(y_{k-1}(r^*)(1 + r_k^*) - y_k(r^*)\big) u^{t-1-k}. \end{aligned}$$

Upon algebraic manipulations,

$$y_{t-1}(i) = y_{t-1}(r^*) - x_1^a u^{t-2} - x_2^a u^{t-3} - \ldots - x_{t-1}^a.$$

Therefore,

$$\begin{aligned}\xi_t^a &= r_t^* y_{t-1}(r^*) - i y_{t-1}(i) \\ &= r_t^* y_{t-1}(r^*) - i(y_{t-1}(r^*) - x_1^a u^{t-2} - x_2^a u^{t-3} + \ldots - x_{t-1}^a)\end{aligned}$$

which is eq. (12). □

Remark 7. Eq. (12) shows that the notion of LC residual income may be reconstructed from the classical paradigm by compounding the past classical residual incomes and calculating interest on them at the cost of capital.

Remark 8. Using induction on eq. (12) it is easily proved that

$$\sum_{k=1}^{t} \xi_k^a = \sum_{k=1}^{t} x_t^a u^{t-k} \qquad \text{for every } t \geq 1 \tag{13}$$

(see Magni, 2005, Lemma 2.4 and Theorem 2.2, for a generalization of eqs. (12) and (13)). Applying both eqs. (12) and (13) we find

$$\xi_t^a = x_t^a + i \sum_{k=1}^{t-1} \xi_k^a \qquad \text{for every } t > 1 \tag{14}$$

which expresses the LC residual income in terms of cumulations of past LC residual incomes.

Eqs. (12) and (13) enables one to show that (i) the LC paradigm is consistent with market values and Net Present Values, (ii) a significant property of (residual) income aggregation holds: Next section is just devoted to these issues.

4 Net present value, book values, market values

Eq. (13) implies that projects and firms can be appraised through the LC paradigm by reversing the role of summing and discounting: The standard-type residual income model is tied to the net present value via a *discount-and-sum* procedure, whereas the LC paradigm employs a *sum-and-discount* procedure. Letting $v{:=}u^{-1}{=}(1+i)^{-1}$ and reminding that $\sum_{k=1}^{n} x_k^a v^k{=}\text{NPV}$, if one picks $t{=}n$ in eq. (13) one obtains

$$\begin{aligned} v^n \sum_{k=1}^{n} \xi_k^a &= v^n \sum_{k=1}^{n} x_k^a u^{n-k} \\ &= \sum_{k=1}^{n} v^k x_k^a \\ &= \text{NPV}. \end{aligned} \tag{15}$$

Residual incomes are first summed, and then discounted: The reverse of the classical procedure. In terms of Net Terminal Value one gets, at time n,

$$N_n = \text{NPV}(1+i)^n = \sum_{k=1}^{n} \xi_k^a. \tag{16}$$

The Net Terminal Value is given by the *uncompounded* sum of all residual incomes ξ_t^a. This means that the LC residual income is additively coherent.[4] Note also that, replacing r with i in eq. (4), the terminal lost capital is just the project's Net Terminal Value (changed in sign): $y_n(i) = -N_n$. Thus, the terminal lost capital may be found by summing the past residual incomes: $y_n(i) = -\sum_{t=1}^{n} \xi_t^a$.

The additive coherence, far from being a mere elegant formal property, unfolds the powerful property of income aggregation, as opposed to discounting. That is, equations (15) and (16) show that capital budgeting problems may be solved by dispensing with forecasting each and every cash flow and, in addition, by dispensing with forecasting each and every residual incomes. If the lost-capital paradigm is used, only the total residual incomes that a firm (project) releases within the fixed horizon is relevant. One does not have to worry about timing. This additive coherence reflects the aggregation property of accounting. Given that NPV=V_0−d_0= with d_0=$y_0(r^*)$, one can express the firm's market value as a function of the outstanding capital and the total residual incomes:

$$V_0 = y_0(r^*) + v^n \sum_{k=1}^{n} \xi_k^a. \tag{17}$$

The above equation is particularly useful if book values are used for the outstanding capital: Picking $y_k(r^*)$=B_k^e and i=k_e one may write

$$V_0^e = B_0^e + \frac{1}{(1+k_e)^n} \sum_{k=1}^{n} \text{abnormal earnings} \tag{18}$$

$$= B_0^e + \frac{1}{(1+k_e)^n} \sum_{k=1}^{n} \big(\text{earnings} - \text{normal earnings}\big). \tag{19}$$

Lost-capital abnormal earnings aggregate in a value sense and avoid prediction in each of the following years. Value is derived from knowledge about total abnormal earnings in a span of n years, no matter how they distribute across periods. The notion of *indicated average future earning power* stated by Graham, Dodd, and Cottle (see Penman, 1992, p. 471) may be now referred to *abnormal* earnings: One may estimate an average abnormal earning for a future span of years and multiply by the number of years to obtain the Net Terminal Value. By discounting back and adding the equity book value one gets the equity market value. Section 7 provides a generalization of eqs. (18) and (19) when the analysis starts at time t>0.

Remark 9. The Net Terminal Value N_n may be reexpressed in a further fashion, where no capitalization process is involved for the standard RIs, while the lost-capital RIs are only linearly compounded. Expanding eq. (14),

$$\begin{aligned}
\xi_1^a &= x_1^a \\
\xi_2^a &= x_2^a + i\xi_1^a \\
\xi_3^a &= x_3^a + i(\xi_1^a + \xi_2^a) \\
\ldots &= \ldots \\
\xi_n^a &= x_n^a + i(\xi_1^a + \xi_2^a + \ldots \xi_{n-1}^a)
\end{aligned}$$

and, summing by column,

$$\sum_{t=1}^{n} \xi_t^a = \sum_{t=1}^{n} x_t^a + \sum_{t=1}^{n} i(n-t)\xi_t^a. \tag{20}$$

[4]See Magni (2003b) for the property of antisymmetry of the LC residual income and its implications.

Because $\sum_{t=1}^{n}\xi_t^a{=}N_n$, we have

$$N_n = \sum_{t=1}^{n} x_t^a + \sum_{t=1}^{n} i(n-t)\xi_t^a.$$

The project's Net Terminal Value may therefore be viewed as a double sum of residual incomes: A sum of uncompounded conventional RIs plus a sum of linearly compounded LC residual incomes.

5 Ohlson's Abnormal Earnings Growth, O'Byrne's EVA improvement, and LC residual income

The notion of Abnormal Earnings Growth (AEG), recently proposed by Ohlson (2005) as a method of firm valuation, is arousing interest among management accounting scholars (see Ohlson and Juettner-Nauroth, 2005; Penman, 2005; Brief, 2007). AEG is the difference between two (standard) consecutive residual earnings (equity perspective): Denoting AEG with z_t, we define

$$z_t = \text{Residual earnings}_{t+1} - \text{Residual Earnings}_t \qquad t = 1, 2, \ldots, n.$$

This very concept has been previously used and studied for value-based management purposes by O'Byrne (1996, 1997) and Young and O'Byrne (2001). The so-called EVA improvement is just the AEG in an entity perspective. In their 2001 book, Young and O'Byrne illustrate a numerical example (p. 29) where the future value of EVA improvement is calculated period by period.[5] They explain the way they compute the future value of EVA improvement as follows: "We do this by multiplying the prior-year future value by 1.10 (1+the WACC of 10 percent) and then adding current-year excess EVA improvement" (p. 40). Formalizing their algorithm and denoting with F_t the future value of EVA improvement,

$$F_t = F_{t-1}(1+i) + \text{EVA}_t - \text{EVA}_{t-1}. \tag{21}$$

Let us generalize the above equation by replacing EVA with the generic residual income x_t^a and redefine AEG to include both equity and entity perspective:

$$z_{t-1} = x_t^a - x_{t-1}^a \qquad t = 1, 2, \ldots, n \tag{22}$$

with $z_0{=}x_1^a$. The future value of cumulated AEGs may be formalized as

$$F_t = F_{t-1}(1+i) + z_{t-1}. \tag{23}$$

We may interpret the above equation as representing the growth in the "AEG account". As the account starts from zero (at the beginning of the project, no residual income has been generated), it is natural to take the boundary condition $F_0{:=}0$. The account grows by a normal return iF_{t-1} plus an abnormal return

[5] Rigorously speaking, the authors compute the future value of the *Excess* EVA improvement but, given their assumptions of no excess future growth value, excess EVA improvement equals EVA improvement (see O'Byrne, 1997, for relations among excess EVA improvement, future growth value, and excess return).

z_{t-1}.[6]. Using eq. (23), one finds

$$\begin{aligned}
F_1 &= 0(1+i) + z_0 \\
F_2 &= z_0(1+i) + z_1 \\
F_3 &= z_0(1+i)^2 + z_1(1+i) + z_2 \\
\vdots &= \quad \vdots \\
F_t &= z_0(1+i)^{t-1} + z_1(1+i)^{t-2} + z_2(1+i)^{t-3} + \ldots + z_{t-1}
\end{aligned} \tag{24}$$

We may then prove the following

Proposition 2. *The future value of cumulated AEGs is equal to the lost-capital residual income*

$$\xi_t^a = F_t = \sum_{k=1}^{t} z_{k-1} u^{t-k} \tag{25}$$

Proof. Reminding that $z_0 = x_1^a$ and using eqs. (22) and (24), simple manipulations lead to

$$\begin{aligned}
F_t &= x_1^a u^{t-1} + (x_2^a - x_1^a) u^{t-2} + \ldots + (x_t^a - x_{t-1}^a) \\
F_t &= i x_1^a u^{t-2} + i x_2^a u^{t-3} + \ldots + i x_{t-1}^a + x_t^a \\
F_t &= x_t^a + i \sum_{k=1}^{t-1} x_k^a u^{t-1-k}
\end{aligned}$$

From eq. (12), $x_t^a + i \sum_{k=1}^{t-1} x_k^a u^{t-1-k} = \xi_t^a$, so that $F_t = \xi_t^a$. □

Remark 10. Young and O'Byrne (2001, p. 42) illustrate a numerical example where the notions of Adjusted Invested Capital and Adjusted EVA are introduced. In the example, they assume earnings=dividends. It is easy to show that the two notions correspond to the notions of lost capital and LC residual income. The recurrence equations for the two notions, inferred from the authors' explanations at p. 42 and the numbers in the Table, are as follows:

$$\begin{aligned}
\mathrm{AIC}_t &= \mathrm{AIC}_{t-1} - \mathrm{AE}_t \\
\mathrm{AE}_t &= \mathrm{Earnings}_t - wacc * \mathrm{AIC}_{t-1}.
\end{aligned} \tag{26}$$

where *wacc* coincides with the cost of equity, given their assumption of zero debt. The two equations yield

$$\begin{aligned}
\mathrm{AIC}_t &= \mathrm{AIC}_{t-1} - \mathrm{Earnings}_t + wacc * \mathrm{AIC}_{t-1} \\
&= \mathrm{AIC}_{t-1} * (1 + wacc) - \mathrm{Earnings}_t
\end{aligned} \tag{27}$$

If one assumes $\mathrm{Earnings}_t$=dividends, eq. (27) corresponds to the recurrence equation for $y_t(wacc)$ (see eq.(8)), so that $\mathrm{AIC}_t = y_t(wacc)$. As a result: (i) AE_t in eq. (26) is equal to the lost-capital EVA as well as to the future value of cumulated AEGs: $\mathrm{AE}_t = L(\mathrm{EVA}_t) = F_t$.

[6] The notion of future value of cumulated AEGs is quite natural, given that AEG measures the growth of abnormal earnings (for this reason GAE might be a better acronym. See Brief, 2007, p. 433)

Remark 11. Reminding that $y_0(r^*){=}d_0{=}B_0$, eqs. (17) and (25) imply

$$\begin{aligned} V_0 = B_0 + \text{NPV} &= B_0 + v^n \sum_{t=1}^{n} \xi_t^a = B_0 + v^n \sum_{t=1}^{n} F_t \\ &= B_0 + v^n \sum_{t=1}^{n} \sum_{k=1}^{t} z_{k-1} u^{t-k} = B_0 + \sum_{t=1}^{n} \sum_{k=1}^{t} z_{k-1} v^{n-t+k} \end{aligned} \tag{28}$$

Disentangling the double sum in eq. (28), one finds

$$\begin{array}{lllllll} \sum_{t=1}^{n} \sum_{k=1}^{t} v^{n-t+k} z_{k-1} & = z_0 v^n & & & & & \\ & + z_0 v^{n-1} & + z_1 v^n & & & & \\ & + z_0 v^{n-2} & + z_1 v^{n-1} & + z_2 v^n & & & \\ & \vdots & \vdots & \vdots & & & \\ & + z_0 v & + z_1 v^2 & + z_2 v^3 & + z_3 v^4 \quad \dots & + z_{n-2} v^{n-1} & + z_{n-1} v^n \end{array}$$

The t-th column of the above sum may be written as $\sum_{k=t}^{n} z_{t-1} v^k$. Summing the n columns,

$$\sum_{t=1}^{n} \sum_{k=t}^{n} z_{t-1} v^k = \sum_{t=1}^{n} \sum_{k=1}^{t} z_{k-1} v^{n-t+k}.$$

Hence,

$$V_0 = B_0 + \sum_{t=1}^{n} \sum_{k=t}^{n} z_{t-1} v^k. \tag{29}$$

Therefore, the lost-capital paradigm gives us the opportunity of viewing AEG with the book value as the anchoring value.[7] The generalization for infinite-lived firms is straightforward:

$$
\begin{aligned}
V_0 &= B_0 + \lim_{n\to\infty} \sum_{t=1}^{n}\sum_{k=t}^{n} z_{t-1}v^k \\
&= B_0 + \sum_{t=1}^{\infty}\sum_{k=t}^{\infty} z_{t-1}v^k \\
&= B_0 + \sum_{t=1}^{\infty} z_{t-1}\frac{v^t}{1-v} \\
&= B_0 + \frac{1}{i}\sum_{t=1}^{\infty} z_{t-1}v^{t-1} = B_0 + \frac{z_0}{i} + \frac{1}{i}\sum_{t=1}^{\infty} z_t v^t. \qquad (31)
\end{aligned}
$$

The latter is just the fundamental EVA equation. O'Byrne (1996, p. 117) introduces this equation by making use of Miller and Modigliani's (1961) investment opportunities approach to valuation; Miller and Modigliani's approach is substantiated in their equation (12), where they include the excess profit generated by the increase in physical assets. Such an excess profit, in the language of EVA, is just the EVA improvement. As a result, our eq. (29) is the lost-capital companion (in a finite-time setting) of Miller and Modigliani's valuation formula (12) based on earnings plus the value of the future opportunities.[8]

6 Tying lost capital to value creation

This section studies some relations among the notions of firm value, net present value, market value added, and the link with the notion of capital.

The net present value of an asset is commonly defined as the difference between the market value of the asset and the capital infused into it at a certain time. This implies that the capital infused may defined as follows:

Definition 4. *At each time t, the capital infused by an investor into an asset is given by the difference between the market value of the asset and its Net Present Value.*

Armed with the above definition, we show the following

[7] If one is willing to highlight the first-period earnings as anchoring value (as is done in Ohlson, 2005), one finds

$$\sum_{t=1}^{n} v^t z_{t-1} = \sum_{t=1}^{n} v^t x_t^a - v(\sum_{t=1}^{n} v^t x_t^a) + v^{n+1}x_n^a = N_0 - vN_0 + v^{n+1}x_n^a = ivN_0 + v^{n+1}x_n^a$$

where N_0:=NPV. Reminding that x_{n+1}^a=0 (the project ends at time n), so that z_n=$-x_n^a$, one finds

$$N_0 = \frac{(1+i)}{i}\left(\sum_{t=1}^{n} v^t z_{t-1} + v^{n+1}z_n\right) = \frac{1}{i}\left(\sum_{t=1}^{n} v^{t-1} z_{t-1} + v^n z_n\right) = \frac{1}{i}\left(\sum_{t=0}^{n} v^t z_t\right).$$

Using the fact that z_0=x_1^a=$(r^*-i)y_0(r^*)$ with $r^* y_0(r^*)$ being the first-period income, one gets

$$V_0 = N_0 + y_0(r^*) = \frac{r^*}{i}y_0(r^*) + \left(\sum_{t=1}^{n} v^t z_t\right) = \frac{\text{Income}_1}{i} + \frac{1}{i}\left(\sum_{t=1}^{n} v^t z_t\right). \qquad (30)$$

Obviously, eq. (30) is equivalent to eq. (29).

[8] An equivalent formulation of Miller and Modigliani's equation (12) is anticipated in Bodenhorn (1959) and in Walter (1956).

Proposition 3. *For every t, the lost capital $y_t(i)$ is the capital infused at time t into the project:*

$$y_t(i) = V_t - N_t. \tag{32}$$

Proof. Reminding that $y_0(r){:=}d_0$ for any return rate r_t, using eq. (8) one finds

$$y_t(i) = d_0 u^t - \sum_{k=1}^{t} d_k u^{t-k}; \tag{33}$$

however, $V_t = \sum_{k=t+1}^{n} d_k u^{t-k}$ and $N_t{=}\text{NPV}u^t{=}\sum_{k=1}^{n} d_k u^{t-k} - d_0 u^t$, whence

$$V_t - N_t = d_0 u^t - \sum_{k=1}^{t} d_k u^{t-k}. \tag{34}$$

Eqs. (33) and (34) coincide. □

While the notion of lost capital has been previously introduced as a foregone capital, Proposition 3 allows us to reinterpret it as the capital infused by investors into the firm at the beginning of each period: The net present value N_t just measures by how much the (market) value of the firm exceeds (if positive) the capital infused into the enterprise. Such a capital is not $y_t(r^*)$, as could erroneously be expected: It is just the lost capital. If one deducts $y_t(r^*)$ from V_t, one obtains what may be called the *generalized* Market Value Added (gMVA). If book values are selected for $\vec{y}$, the gMVA boils down to the well-known Market Value Added (MVA).

$$N_t = V_t - y_t(i) \tag{35}$$

$$\text{gMVA}_t = V_t - y_t(r^*) \tag{36}$$

Proposition 4. *For every t≥1, the difference between the net present value and the market value added is given by the (uncompounded) past lost-capital residual incomes:*

$$N_t - \text{MVA}_t = \sum_{k=1}^{t} \xi_k^a \tag{37}$$

Proof. From eq. (10) we have

$$\begin{aligned}\sum_{k=1}^{t} \xi_k^a &= \sum_{k=1}^{t} [y_{k-1}(i) - y_k(i)] - [y_{k-1}(r^*) - y_k(r^*)] \\ &= y_t(r^*) - y_t(i).\end{aligned} \tag{38}$$

Picking $y_t(r^*) = B_t^e$, eq. (36) becomes

$$\text{MVA}_t = V_t - B_t^e. \tag{39}$$

Deducting the latter from eq. (35) and using eq. (38) one gets eq. (37). □

Proposition 4 says that if one uses the Market Value Added to measure value creation, one forgets the past residual incomes. In other words, value creation is obtained by adding to the firm's Market Value Added the LC residual incomes generated in the past. This very Proposition highlights the major role played by the LC residual income as a measure of excess variation of net present value upon Market Value Added.

Corollary 1. *The LC residual income is the difference between NPV's variation and MVA's variation:*

$$\xi_t^a = \Delta N_t - \Delta MVA_t. \tag{40}$$

Proof. From eq. (37) we have $N_{t-1}{-}\text{MVA}_{t-1}{=}\sum_{k=1}^{t-1}\xi_t^a$. Subtracting the latter from eq. (37) one gets eq. (40). □

Proposition 5. *The firm's outstanding balance is given by the sum of the capital infused and the (uncompounded) past lost-capital residual incomes:*

$$y_t(r^*) = y_t(i) + \sum_{k=1}^{t} \xi_k^a. \tag{41}$$

Proof. Straightforward from eq. (38) □

The above Proposition provides the relation among the outstanding balance, the lost capital and past residual incomes. The relation holds for any $y_t(r^*)$, in particular for $y_t(r^*) = B_t$, so one is given the link connecting book value, lost capital and past residual incomes.

Propositions 3-5 show that the investors' commitment to the business is the lost capital, not the actual outstanding capital, and, in particular, not the book value. The relation between $y_t(r^*)$ and $y_t(i)$ unveils the relation between the MVA and the NPV. At each date, the net present value N_t is an overall measure taking account of the entire life of the project. Therefore, it comprises both a forward-looking and a backward-looking perspective. In contrast, the Market Value Added erases the past and limits its perspective to prospective cash flows: In its view the firm incorporates (the project begins) at time t.

Net Present Value and Market Value Added may be seen as different ways of splitting the market value of equity: From eqs. (35) and (36),

$$V_t = N_t + y_t(i) \tag{42}$$
$$V_t = g\text{MVA}_t + y_t(r^*). \tag{43}$$

Eq. (42) determines an unambiguous partition of V_t, given a cash-flow $\vec{d}$ and a cost of capital i. Eq. (43) originates a set of infinite partitions, one for any choice of $\vec{r}^*$.[9]

7 O'Hanlon and Peasnell's approach and the lost capital

This section shows that the approach of O'Hanlon and Peasnell (2002) perspective is consistent with the LC paradigm. In their paper, O'Hanlon and Peasnell (OP) introduce the notion of *Excess Value Created* (EVC), which is based on the notion of "unrecovered capital". They define EVC as the difference

$$EVC_t = V_t^e - U_t^0 \tag{44}$$

where U_t^0 is the unrecovered capital:

$$U_t^0 = d_0(1+k_e)^t - \sum_{k=1}^{t} d_k(1+k_e)^{t-k}.$$

[9]To be rigorous, one should write $g\text{MVA}_t(r^*)$ rather than $g\text{MVA}_t$, because the generalized MVA changes as $\vec{r}^*$ changes.

Owing to eq. (33), the unrecovered capital is just the capital lost by shareholders: $U_t^0 = y_t(k_e)$. The EVC, which OP acknowledge as analogous to Young and O'Byrne's (2001) excess return, actually coincides with the Net Present Value N_t, and eq. (44) is the equity version of our eq. (35):

$$U_t^0 = y_t(k_e)$$
$$N_t = EVC_t.$$

In their Proposition 1 (p. 233), OP show that the book value of equity may be written as the sum of the unrecovered capital and the compounded past residual incomes, and in their Proposition 2 (pp. 233-234) they show that the EVC equals the sum of compounded residual incomes and the Market Value Added. Using our symbols, OP show that

$$B_t^e = y_t(k_e) + \sum_{k=1}^{t} x_t^a (1+k_e)^{t-k} \tag{45}$$

$$N_t = \sum_{k=1}^{t} x_k^a (1+k_e)^{t-k} + \sum_{k=t+1}^{n} x_k^a (1+k_e)^{t-k} \tag{46}$$

It is worth noting that our Propositions 5 and 4 are, respectively, the LC-companions of OP (2002)'s Propositions 1 and 2. In particular, to pass from eq. (41) to eq. (45) and from eq. (37) to eq. (46) one just has to use eq. (13) with $i = k_e$ and $r^* = \text{ROE}$.

However, the following Propositions directly tie the LC paradigm to value creation, dispensing with the notion of market value added (and, therefore, dispensing with the standard RI models).

Proposition 6. *For every $t \geq 1$, the time-t Net Present Value is given by the sum of all LC residual incomes, discounted at time t:*

$$N_t = v^{n-t} \sum_{k=1}^{n} \xi_k^a$$

Proof. We have $N_t = v^{n-t} \sum_{k=1}^{n} x_k^a u^{n-k}$. Using eq. (13) with $t=n$ the thesis follows. □

Consider now the project generated by the truncation of $\vec{d}$ from time 0 to time $t-1$, or, which is the same, generated by the sum of subprojects $\vec{d}_{t+1}, \vec{d}_{t+2}, \ldots \vec{d}_n$. Denote this project by $\vec{d}_{t,n}$. Then,

$$\vec{d}_{t,n} = \sum_{k=t+1}^{n} \vec{d}_k = (0, 0, \ldots, 0, -y_t(r^*), d_{t+1}, \ldots, d_n) \in \mathbb{R}^{n+1}.$$

In other words, $\vec{d}_{t,n}$ is the future part of project $\vec{d}$. Letting

$$\vec{d}_{0,t} = \sum_{k=1}^{t} \vec{d}_k = (-d_0, d_1, d_2, \ldots, d_t + y_t(r^*), 0, 0, \ldots, 0) \in \mathbb{R}^{n+1}$$

be the first part of project $\vec{d}$, then project $\vec{d}$ is the sum of the two parts: $\vec{d} = \vec{d}_{0,t} + \vec{d}_{t,n}$.

The following Proposition holds.

Proposition 7. *The Net Present Value of project $\vec{d}$ is decomposed into two shares: (i) the sum of the LC residual incomes of project $\vec{d}$'s first part, and (ii) the discounted sum of the LC residual incomes of project $\vec{d}$'s future part:*

$$N_t = \sum_{k=1}^{t} \xi_k^a + v^{n-t} \sum_{k=t+1}^{n} \xi_{k,(\vec{d}_{t,n})}^a \tag{47}$$

where $\xi^a_{k,(\vec{d}_{t,n})}$ is the LC residual income from $\vec{d}_{t,n}$.

Proof. Project $\vec{d}_{t,n}$ begins at time t with initial outstanding capital equal to $y_t(r^*)$. The initial boundary condition is $y_t(r^*)=y^\circ_t(i)$, where $y^\circ_t(i)$ denotes the initial lost capital of project $\vec{d}_{t,n}$; its evolution is given by $y^\circ_k(i) = y^\circ_{k-1}(i)(1+i) - d_k$ for $k > t$. Therefore, any result previously found for project $\vec{d}$ holds for project $\vec{d}_{t,n}$ as well. In particular, eq. (13) applied to project $\vec{d}_{t,n}$ becomes

$$\sum_{k=t+1}^{\tau} \xi^a_{k,(\vec{d}_{t,n})} = \sum_{k=t+1}^{\tau} x^a_{k,(\vec{d}_{t,n})} u^{\tau-k} \qquad \text{for every } \tau > t$$

where $x^a_{k,(\vec{d}_{t,n})}$ is the standard RI for project $\vec{d}_{t,n}$. However, the right-hand side holds for both $\vec{d}$ and $\vec{d}_{t,n}$, because cash flows, outstanding capitals, rates of return of the two projects coincide ($\vec{d}_{t,n}$ is the future part of $\vec{d}$). Therefore, $x^a_{k,(\vec{d}_{t,n})} = x^a_k$. This implies

$$\sum_{k=t+1}^{\tau} \xi^a_{k,(\vec{d}_{t,n})} = \sum_{k=t+1}^{\tau} x^a_k u^{\tau-k} \qquad \text{for every } \tau > t.$$

Picking τ=n, and using the fact that $v^{n-t}\sum_{k=t+1}^{n} x^a_k(1+i)^{n-k}$=$gMVA_t$, one gets

$$v^{n-t} \sum_{k=t+1}^{n} \xi^a_{k,(\vec{d}_{t,n})} = g\text{MVA}_t.$$

Eq. (47) is finally derived by using eq. (37) with gMVA$_t$ replacing MVA$_t$.[10] □

Remark 12. In the proof above we make use of the initial boundary condition according to which the initial outstanding capital equals the initial lost capital. If the analysis is made at time t, time t is the 'new' time 0, and the capital initially infused into the project at the new time 0 coincides, by assumption, with both the outstanding capital and the lost capital of project $\vec{d}_{t,n}$. This implies that, using book values for outstanding capitals, book value represent the *initial* lost capital of project $\vec{d}$'s future part.

Proposition 7 says that the Net Present Value (the Excess Value Created, in OP's words) is reached by summing the lost-capital RIs of the first part of $\vec{d}$ and by discounting the aggregated lost-capital RIs of the future part of $\vec{d}$. Picking i=k_e and r^*=ROE in eq. (47) one finds the equivalent of OP's eq. (46) expressed in genuine LC terms.

The same Proposition induces a generalization of eq. (17). Using the equality gMVA$_t$=$V_t - y_t(r^*)$ and the fact that gMVA$_t$=$v^{n-t}\sum_{k=t+1}^{n}\xi^a_{k,(\vec{d}_{t,n})}$ (see proof of Proposition 7), one finds

$$V_t = y_t(r^*) + v^{n-t} \sum_{k=t+1}^{n} \xi^a_{k,(\vec{d}_{t,n})}. \tag{48}$$

Choosing the equity perspective and selecting book values as outstanding capitals, the above equality becomes

$$V^e_t = B^e_t + \frac{1}{(1+k_e)^{n-t}} \sum_{k=t+1}^{n} \xi^a_{k,(\vec{d}_{t,n})} \qquad \text{for every } t. \tag{49}$$

Setting t=0 one finds back eq. (17), given that $\vec{d}_{0,n} = \vec{d}$, which implies $\xi^a_{k,(\vec{d}_{0,n})}$=$\xi^a_k$ for all k. Eq. (49) says that to get the equity market value one does not need to forecast dividends nor residual incomes: Only the

[10] Obviously, eq. (37) does hold if MVA$_t$ is replaced by gMVA$_t$.

total amount of prospective residual incomes is relevant. The result here shown is related to Ohlson's (1989, 1995) famous result: Ohlson deals with an infinite horizon and shows that, if abnormal earnings follow an autoregressive process,

$$V_t^e \approx \frac{1}{\big((1+k_e)^T-1\big)}\Big[\sum_{k=t+1}^{T}\text{earnings}+\sum_{k=t+1}^{T}\big((1+k_e)^{T-k}-1\big)d_k\Big] \tag{50}$$

if T is sufficiently large; the difference between the left-hand side and the right-hand side is independent of dividends d_k.

We have just found, assuming a finite horizon,

$$V_t^e = B_t^e + \frac{1}{(1+k_e)^{n-t}}\Big(\sum_{k=t+1}^{n}\text{earnings}-\sum_{k=t+1}^{n}\text{normal earnings}\Big) \tag{51}$$

It is noteworthy that the difference between market value and book value (the MVA) is exactly equal to total abnormal earnings aggregated, multiplied by the proper capitalization factor. Further, both eqs. (50) and (51) include aggregated earnings, but the latter includes two types of earnings, the firm's expected earnings and the expected earnings of a normal firm. And, recalling that $V_t^e = \sum_{k=t+1}^{n} d_k(1+k_e)^{t-k}$, it is also worth noting that dividends appear in the left-hand side of eq. (51) but not in the right-hand side, where only earnings appear.

From a practical point of view, forecasts of cash flows may be replaced by forecasts of earnings. Earning aggregations make forecast of each and every residual income irrelevant: Only the total residual incomes is of concern. In particular, the concept of normal earning may turn to be useful. In real-life applications (and in theory as well), normal earnings are often referred to as the average earnings of a class of firms; typically, accounting offers a huge amount of information on both the firm's past earnings and the average earnings of the sector where the firm operates. If these data are reliable indicators, then they may represent the basis for the determination of two average earning powers: One refers to the firm, the other one concerns a normal (average) firm operating in the same sector.[11] Multiplying both earning powers by the relevant horizon and deducting the latter from the former, the project's Net Terminal Value is found. Discounting back, the Net Present Value is reached.[12] Formally, considering $t = 0$ as the valuation date and assuming ξ is the expected

[11] Normal earnings must obviously refer to an average firm whose equity book value is, at time, t, the same as the equity book value of the firm under consideration: This is just the meaning of the initial boundary condition $y_t^\circ(i) = y_t(r^*)$.

[12] 'Dividends irrelevance' is often misinterpreted. Eq. (51) obviously holds regardless of the dividend policy, given that it is an identity, but both sides of the equation are constant with respect to changes in dividend policies *only if* extra distribution of dividends or retained equity cash flow are used for zero-NPV activities. Suppose dividends differ from the prospective equity cash flows d_t by an amount h_t, $t = 1, 2, \ldots, n$, $h_t \neq 0$; one gets

$$V_t^e = \sum_{t=1}^{n}\frac{d_t}{(1+k_e)^t} = \sum_{t=1}^{n}\frac{d_t+h_t}{(1+k_e)^t}$$

for any vector $(h_1, h_2, \ldots, h_{n-1}) \in \mathbb{R}^{n-1}$ if and only if $\sum_{t=1}^{n}\frac{h_t}{(1+k_e)^t} = 0$. This means that V_t^e is a constant function with respect to $h_1, h_2, \ldots, h_{n-1}$. if the difference between the cash flow available for distribution and the dividend is always reinvested (if positive) or financed (if negative) at the equity cost of capital. In this case, the right-hand side of eq. (51) is constant as well. See DeAngelo and DeAngelo (2006) and Magni (2007) on the relevance or irrelevance of dividends in Miller and Modigliani's (1961) theorem.

average (abnormal) earnings of the firm, eq. (51) becomes

$$V_0^e = B_0^e + \frac{1}{(1+k_e)^n}\sum_{t=1}^{n}\xi_n = B_0^e + \frac{n\xi}{(1+k_e)^n}.$$

8 Aligning RI and NPV: The Net Value Added

As Young and O'Byrne (2001) remind, while excess return is the ultimate goal of the firm, "we need flow measures, not stock measures" (p. 34). And, in addition, we need a metric aligned with excess return, or, which is the same, with the net present (or terminal) value. A possible route to this end is "to develop a modified measure of residual income for which each period's RI_t has the same sign as the present value of all future RI_t or net present value" (Martin et al., 2003, p. 14).[13] Grinyer (1985, 1987, 1995) selects a metric named *Earned Economic Income* (EEI) which does comply with the sign of the Net Present Value (N_0). His index is defined as

$$\mathrm{EEI}_t = d_t - Dep_t$$

where $Dep_t{=}d_t\frac{d_0}{V_0^e}$. Substituting, and reminding that $N_0{=}V_0^e - d_0$,

$$\mathrm{EEI}_t = \frac{d_t}{V_0^e}N_0.$$

Therefore, $\mathrm{EEI}_t{>}0$ if and only if $N_0{>}0$. Yet, the economic meaning of the EEI as a genuine residual income is not clear: "the relationship between EEI and RI appears not to be well understood" (Peasnell, 1995, p. 235). Formally, this ambiguity is confirmed by the fact that there is no easy way to rewrite EEI in the form described by eq. (5).[14] Furthermore, alignment with the NPV holds only under particular assumptions, viz. if and only if the project's cash flows have the same sign. However, "this latter constraint is very unrealistic as many positive NPV projects have a mixture of positive and negative cash flows throughout the project's life." (Martin et al., 2003, pp. 20–21).

Focussing on the set of all possible metrics $\{\xi_t^a\}$ within the LC paradigm, it is possible to single out, quite naturally, a subclass of RI models that manifests a perfect alignment with the Net Present Value, irrespective of the sign of the project's cash flows. We label this subclass *Net Value Added*:

Definition 5. The Net Value Added (NVA) is the subclass of lost-capital RIs generated by the choice of market values as outstanding capitals: That is, $y_t(r^*){=}V_t$, for $t = 1, \ldots, n-1$.

Proposition 8. *The NVA has the same sign as the net value N_t.*

Proof. Proposition 3 implies

$$\xi_t^a = r_t^*\, y_{t-1}(r^*) - i\,(V_{t-1} - N_{t-1}). \tag{52}$$

[13] As previously seen, the present value of all future RI_t is gMVA_t, which differs from the net present value N_t, as long as $t > 0$. Thus, to choose a metric aligned either to MVA or to NPV means to adopt different perspectives.

[14] Certainly, EEI does not belong to the class of lost-capital RI models, given that it is not additively coherent with respect to the Net Terminal Value: $\sum_{t=1}^{n}\mathrm{EEI}_t \neq N_n$. And if EEI belonged to the class of standard RI models, the vector of outstanding balances would be such that $y_{t-1}(r^*) = d_tN_0/[V_0^e(r_t^* - i)]$ and $y_{t-1} = (y_t(r^*) + d_t)/(1 + r_t^*)$, which is not true in general.

Definition 5 implies $\vec{r}=(\frac{V_1+d_1-d_0}{d_0},i,i,\dots i)$, so that eq. (52) becomes

$$\mathrm{NVA}_t=\begin{cases} i\big(V_{t-1}-d_0\big)+\big(V_{t-1}-d_0\big) & \text{if } t=1\\ i\big(V_{t-1}-y_{t-1}(i)\big) & \text{if } t>1\end{cases} \tag{53}$$

where we have used the equalities $V_0{-}N_0{=}d_0$ and $V_1+d_1{=}V_0(1+i)$.

Therefore, given that $N_t{=}V_t{-}y_t(i)$ for all t, we have, for $t>1$, $\mathrm{NVA}_t{>}0$ if and only if $N_{t-1}{>}0$; as for $t{=}1$, we have $\mathrm{NVA}_1{>}0$ if and only if $N_1{>}0$, given that $N_1{=}N_0(1+i){=}(V_0-d_0)(1+i)$.[15] □

As no assumption has been made on the signs of the cash flows, the above Proposition compellingly proposes a subclass of RI models that always signal a positive residual income if and only if Net Present Value is positive, i.e if and only value exceeds capital infused into the business. The uncompounded sum of all the NVAs is equal to the project's Net Terminal Value N_n.[16]

Remark 13. The significance of the LC paradigm is also appreciated in terms of evolutions of NPV and gMVA. From eq. (32) and eq. (53) we find

$$N_t=\begin{cases} \mathrm{NVA}_t & \text{if } t=1\\ N_{t-1}+\mathrm{NVA}_t & \text{if } t>1.\end{cases} \tag{54}$$

The NVA is the periodic addition to the net present value or, equivalently, the NVA is just the RI model generated by the NPV.
Using induction upon eq. (54),

$$N_t=\sum_{k=1}^{t}\mathrm{NVA}_k. \tag{55}$$

The above equation and eq. (37) imply $\mathrm{MVA}_t{=}0$. This is obvious, given that in the NVA model the outstanding balance $y_t(r^*)$ equals the market value for all $t{\geq}1$. Eq. (55) gives some insights on the Net Present Value. Having previously found that the NPV may be written as sum of *future* LC residual incomes and *past* LC residual incomes (eq. (47)), we have now rewritten the NPV by using only *past* LC residual incomes. This result shows that NPV and LC paradigm are strictly connected, and that the use of the NVA-class enables one us to dismiss the future LC residual incomes.[17] Also, we are left with an equality where no explicit capitalization process is given.

9 User cost, lost capital, and Net Value Added

In 1936, Keynes introduced the notion of *user cost* in *The General Theory of Employment, Interest and Money*. Referring to an entrepreneur, user cost is defined as the difference between "the value of his capital equipment at the end of the period ... and ... the value it might have had at the end of the period if he had refrained from using it" (Keynes, 1967, p. 66). Some years after, the same concept is investigated by Coase (1968), which relabels it "depreciation through use". User cost is equal to

$$G'-G \tag{56}$$

[15] It is worth noting that all net values N_t have the same sign.

[16] While this result just derives from eq. (16), we give a direct proof in the Appendix (see also eq. (55)).

[17] This does not imply that future data are not relevant: Every NVA_k, $k\leq t$, depends on N_t which, in turn, depends on future cash flows as well as past ones.

where "G' is the value of the entrepreneurial stock and equipment had they not been used and G is their value after use" (Scott, 1953, p. 370). Equation (56) compares two different choices: "The choice between ... using a machine for a purpose and using it for another" (Coase, 1968, p. 123) and the result represents a depreciation in the value of the asset. Such a depreciation represents the "opportunity cost of putting goods and resources to a certain use" (Scott, 1953, p. 369), and is therefore an economic measure of "the opportunity *lost* when another decision is carried through" (Scott, 1953, p. 375, italics added).

In this section we apply this concept to the situation where the entrepreneur may either put his resources in asset $\vec{d}$ or invest them in an asset yielding return at the market rate i. To compute G and G', one must calculate "the present value of the net receipts ... by discounting them at a rate of interest" (Coase, 1968, p. 123). This "rate of discount coincides with that in the market" (Scott, 1953, p. 378).

Using the arbitrage-type description given in section 2, if project is undertaken the cash-flow stream is $(-d_0, d_1, d_2, \dots, d_n)$; if the entrepreneur abstains from investing in the projet, his cash-flow stream is $(-d_0, d_1, d_2, \dots, d_n + y_n(i))$. In the former case, the value of the entrepreneurial stock at time t is $G = \sum_{k=t+1}^{n} d_k v^{k-t}$. In the latter case, it is $G' = \sum_{k=t+1}^{n} d_k v^{k-t} + y_n(i)v^{n-t}$. User cost is therefore

$$\begin{aligned} G' - G &= \sum_{k=t+1}^{n} d_k v^{k-t} + y_n(i)v^{n-t} - \sum_{k=t+1}^{n} d_k v^{k-t} \\ &= y_n(i)v^{n-t} \end{aligned} \tag{57}$$

which, as Keynes acknowledges, represents "the discounted value of the additional prospective yield which would be obtained at some later date" (Keynes, 1967, p. 70). In other terms, reminding that the Net Terminal Value is the final lost capital changed in sign ($N_n = -y_n(i)$), user cost is the time-t NPV (changed in sign): $G' - G = -N_t$. It is worth noting that G=V_t by definition of market value. Also, G'=$y_t(i)$. To prove the latter, just note that, using eq. (4) with r=i,

$$d_0 u^n - \sum_{k=1}^{t} d_k u^{n-k} = \sum_{k=t+1}^{n} d_k u^{n-k} + y_n(i).$$

Dividing by $(1+i)^{n-t}$,

$$d_0 u^t - \sum_{k=1}^{t} d_k u^{t-k} = \sum_{k=t+1}^{n} d_k v^{k-t} + y_n(i)v^{n-t}. \tag{58}$$

The left-hand side is $y_t(i)$, the right-hand side is G'. Therefore, the lost capital is a fundamental ingredient of Keynes's user cost, which confirms the importance of such a notion. Furthermore, we have the following

Proposition 9. *The Net Value Added is equal to the periodic change in user cost*

Proof. From eq. (11)

$$\text{NVA}_t = [y_{t-1}(i) - V_{t-1}] - [y_t(i) - V_t].$$

But, as just seen, $G'_t = y_t(i)$ and $G_t = V_t$, where subscripts are added for calling attention to time. Thus,

$$\text{NVA}_t = (G'_{t-1} - G_{t-1}) - (G'_t - G_t).$$

□

User cost is the change in the value of the asset due to a different use of it, and, in turn, the NVA is the change in value of user cost due to time. Not only we do have a link between an important keynesian concept and the LC paradigm, but the notion of user cost enables us to present residual income in terms of periodic variation of user cost.

It is worth noting that one may write

$$\text{NVA}_t = [y_{t-1}(i) - y_t(i)] - [V_{t-1} - V_t]$$

or, equivalently,

$$\text{NVA}_t = [y_{t-1}(i) - V_{t-1}] - [y_t(i) - V_t];$$

equations (10) and (11) may be then interpreted as representing lost-capital residual income in terms of a *generalized* user cost. Scott (1953) observes that "economists cannot afford to lump together, as "depreciation", changes in present value caused by the passage of time, and by use" (p. 371). In fact, the above equalities just show that LC paradigm does enable one to lump together *depreciation through time* and *depreciation through use*.

10 Net Value Added, Created Shareholder Value, and Net Economic Income

The LC perspective gives us the opportunity of conjoining two seemingly disparate metrics in a unified view, introduced in a value-based management book and in a corporate finance book, respectively. The former is the *Net Economic Income* (NEI) and its use is suggested by Drukarczyk and Schueler (2000) for managerial purposes. The latter is the *Created Shareholder Value* (CSV) and is fostered by Fernández's (2002) for measuring value creation.

It is easy to see that NEI and the LC-companion of CSV belong to the class of NVA metrics. As for NEI, the authors define current invested capital IC_t as

$$IC_t = IC_\tau(1 + wacc)^{t-\tau} - \sum_{k=\tau+1}^{t} NCF_k(1 + wacc)^{t-\tau},$$

where $\tau < t$ is the time of the initial investment and NCF_k are the free cash flows. Evidently, setting τ=0, IC_t is just $y_t(i)$, and i=$wacc$ (which also means that their notion of invested capital coincides with the entity version of O'Hanlon and Peasnell's unrecovered capital).[18] Net Economic Income is defined as

$$\text{NEI}_t = NCF_t + (MV_t - MV_{t-1}) - wacc \cdot IC_{t-1} \tag{59}$$

with MV_t being the market value of the firm. It is evident that this perspective is consistent with the LC paradigm and that NEI is just an instantiation of the NVA measures in an entity approach:

Proposition 10. *Net Economic Income is an entity-approach version of NVA.*[19]

Proof. Pick i=$wacc$ and V=V^l in eq. (53), so that

$$\text{NVA}_t = \begin{cases} \left(V_{t-1}^l - d_0\right) + wacc \cdot \left(V_{t-1}^l - d_0\right) & \text{if } t = 1 \\ wacc \cdot \left(V_{t-1}^l - y_{t-1}(wacc)\right) & \text{if } t > 1. \end{cases}$$

[18]Note that, with $\tau > 0$, IC_τ is the initial lost-capital of the future part of project $\vec{d}$ starting at $t = t^*$, i.e. the initial lost capital of $\vec{d}_{\tau,n}$: IC_τ=$y_\tau^\circ(i)$.

[19]Another entity-approach version of NVA is found by considering capital cash flows instead of free cash flows, and using pre-tax *wacc* as the cost of capital.

Therefore eqs. (59) and (53) coincide, given that $wacc \cdot V^l_{t-1}{=}NCF_{t-1}{+}(MV_t - MV_{t-1})$. □

Net Economic Income is therefore the NVA from the point of view of all capital providers.[20]

As for Fernández's metric, it lies within the boundaries of the conventional paradigm, as seen in section 1. The author suggests the choice $y_t(r^*){=}V^e_t$, $t = 1, 2, \ldots, n-1$, so that

$$\mathrm{CSV}_t = \begin{cases} d_0(r^*_t - k_e) & \text{if} \quad t = 1 \\ V^e_{t-1}(r^*_t - k_e) & \text{if} \quad t > 1. \end{cases} \tag{60}$$

The author's choice of equity market values as outstanding capitals implies that, in the first period, the internal rate of return is $r^*_1{=}(V^e_1 + d_1)/d_0{-}1$ (see Fernández, 2002, p. 281) and $r^*_t{=}k_e$ otherwise. This in turn implies that the CSV model imputes value creation to the first period only (assuming expectations are met): $\mathrm{CSV}_1{=}d_0(\frac{V^e_1+d_1-d_0}{d_0} - k_e)$ and $\mathrm{CSV}_t{=}0$ for $t > 1$.[21] This metric is not aligned to N_t, because (if expectations are met) residual incomes after $t > 1$ are all zero, regardless of the sign of the Net Present Value. However, the LC-companion of CSV is aligned with N_t, because it is just the NVA in an equity approach.

Proposition 11. *The LC-companion of Fernández's CSV is the equity-approach version of the NVA.*

Proof. For t =1,

$$L(\mathrm{CSV}_1) = \mathrm{CSV}_1 = d_0(\frac{V^e_1 + d_1 - d_0}{d_0} - k_e) = (V^e_0 - d_0) + k_e(V^e_0 - d_0) = \mathrm{NVA}_1.$$

As for $t > 1$, to pass from CSV_t to $L(\mathrm{CSV}_t)$ we replace $k_e\, V^e_{t-1}$ with $k_e\, y_{t-1}(k_e)$ in eq. (60). One finds

$$\begin{aligned} L(\mathrm{CSV})_t &= r^*_t V^e_{t-1} - k_e\, y_{t-1}(k_e) = k_e\,(V^e_{t-1} - y_{t-1}(k_e)) \\ &= k_e\, N_{t-1} = N_t - N_{t-1} = \mathrm{NVA}_t \end{aligned}$$

□

Propositions 10 and 11 show that seemingly dissimilar metrics (CSV and NEI) share common conceptual and formal analogies if they are connected via a LC perspective: Both preserve the NPV sign (the NEI directly, the CSV after transforming it into its LC-companion).

11 Anthony's argument and the unification of the two paradigms

In his *Accounting for the Cost of Interest*, Anthony (1975) advocates the use of a charge on equity capital in accounting statements: The interest on the use of equity capital should be accounted for as an item of cost. Evidently, to record equity interest as a cost boils down to redefine the notion of profit: In this view, profit is earnings in excess of the equity interest. Anthony's profit is therefore just what management accountants call *residual* income, as he himself recognizes (Anthony, 1975, p. 3).

The idea of recording equity interest as a cost for accounting purposes implies that, for certain assets, the amounts recorded is higher, and shareholders' equity is correspondingly higher. Anthony describes an enlightening example that is worth quoting extensively:

[20]Rigorously speaking, coincidence holds for $t{>}1$. For $t{=}1$ the NEI is, so to say, ill-defined (because it compares return from d_0 with return from V^e_0 (i.e. the initial condition $d_0 = y_0(r^*) = y_0(i)$ is not fulfilled).

[21]We also have $\mathrm{CSV}_1{=}N_1{=}\mathrm{NPV}(1 + k_e)$.

> Consider, for example, a corporation that is formed to invest in land. It buys a parcel for \$1,000,000, holds it for five years, sells the land for \$2,000,000 at the end of the fifth year, and liquidates.
>
> ...
>
> In the proposed system, interest cost would be added to the cost of the land each year, and there would be a corresponding credit to shareholder's equity. At the end of the fifth year, there would be an additional entry to shareholders' equity, representing the net income realized from the sale; that is, the difference between the sales revenue and the accumulated cost of the land. Thus, the statements would show an increase in shareholders' equity in each of the five years. During the first four years, the company would report neither income nor loss; instead, the costs incurred in holding the land, here assumed to be only equity interest, would be added to the original cost of the land. In the fifth year, when the sale took place, net income would be reported as the difference between the selling price and the costs accumulated in inventory up to that time. (Anthony, 1975, p. 30)

As shown below, paraphrasing in a formal way Anthony's suggestion, an interesting residual income model is generated. Under Anthony's proposal, the book value of the land increases periodically by the cost of equity capital. This means that the depreciation charge for the land is negative (i.e. it is an increase in shareholders' equity) and is equal to the interest on equity. In other terms, the periodic rate of return in the first four years is set equal to the ROE, and the ROE is set equal to the cost of equity: Formally, the project is $\vec{d}$=$(-1, 0, 0, 0, 2)$ (in millions), and Anthony is choosing r^*_t=ROE=k_e and therefore $y_t(r^*) = B^e_t = y_t(k_e)$ for $t = 1, 2, 3, 4$. The lost capital coincides with the equity book value and the latter evolves according to

$$y_t(k_e) = y_{t-1}(k_e)(1 + k_e) \qquad \text{for all } 1 < t < 5, \tag{61}$$

which is just eq. (8) with i=k_e and d_t=0 for $t < 5$. Thus, the equity book values are

$$y_0(r^*) = 1, \quad y_1(r^*) = (1 + k_e), \quad y_2(r^*) = (1 + k_e)^2, \quad y_3(r^*) = (1 + k_e)^3, \quad y_4(r^*) = (1 + k_e)^4.$$

During the first four years, residual income (Anthony's profit) is neither positive nor negative, because net income is equal to the increase in shareholders' equity,[22] which is just equal to the capital charge $iy_{t-1}(i)$=$k_e y_{t-1}(k_e)$:

$$\begin{aligned}\text{Residual Income} &= \text{Net Income} - \text{equity capital charge}\\ &= k_e\, y_{t-1}(k_e) - k_e\, y_{t-1}(k_e) = 0.\\ &= k_e\,(1 + k_e)^{t-1} - k_e\,(1 + k_e)^{t-1} = 0.\end{aligned}$$

At time $t = 5$, the accumulated cost is $y_4(k_e)(1 + k_e)$=$(1 + k_e)^5$ and the net income is given by the sum of the negative depreciation (=appreciation) charge $k_e y_4(k_e)$ and the surplus generated by the sale of the land: $2 - y_4(k_e)(1 + k_e)$. Therefore,

$$\begin{aligned}\text{Residual Income} &= \text{Net Income} - \text{equity capital charge}\\ &= \big[k_e y_4(k_e) + 2 - y_4(k_e)(1 + k_e)\big] - k_e y_4(k_e)\\ &= 2 - y_4(k_e)(1 + k_e)\\ &= 2 - (1 + k_e)^5\end{aligned}$$

[22] This is because revenues are zero and the depreciation charge is negative (equity appreciates).

As Anthony acknowledges, last year's (residual) profit is just the difference between the selling price and the costs accumulated up to that time. This residual income may be written as

$$\text{Residual Income} = y_4(k_e)(r_5^* - k_e)$$

with $r_5^*{=}\big(k_e y_4(k_e)+2-y_4(k_e)(1+k_e)\big)/y_4(k_e){=}\big(k_e(1+k_e)^4+2-(1+k_e)^5\big)/(1+k_e)^4$. It is worth noting that this model provides zero residual incomes for all years except the last one, when residual income is equal to the project's Net Terminal Value:

$$\text{Residual Income} = 2 - (1+k_e)^5 = N_5$$

or, in terms of NPV,

$$\text{Residual Income} = \Big(-1 + \frac{2}{(1+k_e)^5}\Big)(1+k_e)^5 = \text{NPV}(1+k_e)^5.$$

Applying Anthony's argument to a generic project, the project's outstanding capital is set equal to the lost capital: $y_t(r^*){=}y_t(i)$ so that eqs. (7) and (8) coincide. Also, taking an equity approach, r^* is set equal to ROE and i is set equal to k_e for $t = 1, 2, \ldots, n-1$; a new RI model is thus generated, here named Anthony's Residual Income (ARI):

$$\text{ARI}_t = r_t^* y_{t-1}(k_e) - k_e y_{t-1}(k_e) \tag{62}$$

with $r_t^*{=}k_e$ if $t < n$, and $r_n^*{=}\frac{k_e y_{n-1}(k_e)+d_n-y_{n-1}(k_e)(1+k_e)}{y_{n-1(k_e)}}$.

By suggesting that the lost capital be directly recorded in accounting statements, because it represents a real cost, Anthony implicitly maintains that the appropriate book value of assets should be given by the value assets would have had if the initial sum d_0 had been invested at the cost of equity. This is a conceptual shift: In his view the book value equals the lost capital, i.e. the capital shareholders renounce to when investing in the project (firm). However, this lost-opportunity interpretation is not given by Anthony, who, instead, considers the lost capital not lost at all: It is just the shareholders' credit. Therefore, he uses a metaphor from loan theory (see Table 1), and to him the clean surplus relation is derived by interpreting equity as a shareholders' credit.

This conceptual shift brings about some interesting consequences:

(a) The lost capital may be interpreted as the capital which is "borrowed" from claimholders

(b) Anthony's residual income is a mirror-image of Fernández's CSV: According to the latter value is created in the first period, according to the former value is created in the last period. Therefore, the latter is, so to say, finance-derived, whereas the former is accounting-derived

(c) Anthony's RI model realizes a unification of the two paradigms. His argument is the only one that is consistent with both paradigms

As for claim (a), it gives us a fourth interpretation of the lost capital, besides the three previously found: The lost capital is the capital which is lost by investors (section 2), the outstanding capital of a shadow project whose standard RI coincides with the lost-capital RI (Magni, 2000a, 2005, 2006), the capital infused into the business (section 6), and the capital "borrowed" from shareholders, whose interest rate is the equity cost of capital. These four interpretations, while seemingly discordant, are coherently harmonized under the formal lens of the LC paradigm.[23]

[23] O'Hanlon and Peasnell's (2002) view of the lost capital as an unrecovered capital coincides with the first interpretation: Capital cannot be recovered, and thus it is definitively foregone.

Claim (b) is evident from Table 3, which uses the definition of CSV and ARI given in the previous and current section respectively: In Anthony's view, value is recorded only in the last period, whereas the previous RIs are zero. This is consistent with accounting principles: "In accordance with the realization concept, income would be reported only in the fifth year, when the land was sold" (Anthony, 1975, p. 31). In Fernández's view, value is created in the first period, when the project is undertaken, whereas the subsequent RIs are zero. This is consistent with a financial perspective, according to which market immediately recognizes value creation (see also Robichek and Myers, 1965, pp. 11-12). Referring to dates instead of periods: Fernández recognizes value creation at time 0 as a windfall gain (value creation=Net Present Value), Anthony recognizes value at time n (value creation=Net Terminal Value).
As for claim (c), looking at eqs. (5) and (9), the two sets of model intersect if and only if

$$y_{t-1}(r_t^* - i) = r_t^* y_{t-1} - i y_{t-1}(i) \qquad \text{for every} \quad t = 1, 2, \ldots, n.$$

The above equality implies $y_{t-1}(r^*) = y_{t-1}(i)$ for every $t = 1, 2, \ldots, n$, which is just Anthony's suggestion.

Thus, Anthony's argument gives rise to a theoretically significant subclass of RI models: They are the only models that belong to both paradigms. Putting it in equivalent terms, the notion of residual income is univocal if Anthony's argument is used, because the project's outstanding capital is made to coincide with the lost capital.[24,25]

12 Concluding remarks

This paper presents an investigation into an alternative non-standard notion of residual income (RI), originally introduced in Magni (2000a, 2000b, 2000c, 2001a, 2001b) with the name *Systemic Value Added*. The paradigm is here renamed "lost-capital" (LC) paradigm, owing to the central role played by the capital that investors *lose* by undertaking the project. The LC paradigm is a theoretical domain which enables one to embrace varied notions, results, and models which have been developed in different fields with disparate scopes and aims.

The main results of the paper are the following ones:

- the LC paradigm may be drawn from an arbitrage-type line of reasoning
- the LC paradigm may be derived from an accounting argument where a pair of clean surplus relations are involved
- the sum of LC residual incomes equal the capitalized sum of conventional residual incomes
- a property of (abnormal) earning aggregation holds: The Net Terminal Value is given by the (uncompounded) sum of LC residual incomes. Therefore, to get the value of the business time is not important: Only total aggregated earnings are relevant
- the capital (implicitly) infused into the business by the investors at the beginning of each period is not the outstanding capital of the business (in particular, it is not the book value), but the lost capital itself

[24] Strictly speaking, Anthony selects r^*=ROE=i=k_e, but obviously his argument also implies the possible choice of r^*=ROA=i=$wacc$, which means that an entity perspective is adopted.

[25] Anthony's example may be interpreted as a particular case of either EBO or L(EBO), where ROE is set equal to k_e in all periods but the last one.

- the value of the firm is equal to equity book value plus total abnormal earnings aggregated, multiplied by the appropriate capitalization factor.
- the total RIs aggregated measure the difference between time-t Net Present Value and time-t Market Value Added
- the lost-capital residual income measures the difference between variation in Net Present Value and variation in Market Value Added
- the project's (firm's) outstanding capital is equal to the lost capital plus the total aggregated residual incomes
- the LC paradigm is consistent with O'Byrne's (1997) EVA improvement, Ohlson's (2005) Abnormal Earnings Growth, and with Miller and Modigliani's investment opportunities approach to valuation. The future value of cumulated AEGs is equal to the LC residual income
- the LC paradigm is consistent with O'Hanlon and Peasnell's (2002) approach: Their unrecovered capital coincides with the lost capital and their Excess Value Created is just the time-t Net Present Value. Their results, expressed in a standard-residual-income perspective, may be rewritten in LC terms: In particular, the Excess Value Created is split into a backward-looking component (past LC residual incomes) and a forward-looking one (prospective LC residual incomes). Both components enjoy the property of earnings aggregation
- a subclass of LC residual income models (Net Value Added) guarantees that residual incomes have the same sign as the Net Present Value. This makes this class particularly interesting for incentive compensation purposes
- the economic notion of user cost, introduced by Keynes in 1936, is consistent with the LC residual income. In particular, the periodic variation in user cost is equal to the Net Value Added. The notion of user cost enables one to lump together two types of depreciation implicit in the LC paradigm: Depreciation through time and depreciation through use
- if an equity approach is adopted, the Net Value Added gives rise to the lost-capital companion of Fernández's (2002) Created Shareholder Value; if an entity approach is followed, the Net Value Added gives rise to Drukarczyk and Schueler's (2000) Net Economic Income
- Anthony's (1975) proposal of accounting for the cost of equity interest is such that capital charge is included in the very notion of profit. Anthony's argument refers to items such as inventory and plants: Generalizing and formalizing his line of reasoning, one obtains a model symmetric to Fernández's above mentioned model: To the latter, value is created in the first period, to the former value is generated in the last period; residual incomes are zero for both models in any other period
- the subclass of models derived by Anthony's argument is the only subclass belonging to either the set of standard RI models and the set of LC models. In particular, if an equity approach is adopted Anthony's model may be interpreted as derived from either the EBO model or its LC companion L(EBO); if an entity approach is taken the model may be interpreted as derived from either the EVA model or its LC companion L(EVA)
- the lost capital may equivalently be interpreted as (i) the capital foregone by investors, (ii) the outstanding capital of a shadow project whose standard residual income coincides with project $\vec{d}$'s LC residual income, (iii) the capital infused into the business, (iv) the claimholders' credit.

Future researches may be devoted to deepen the theoretical network originated by the LC paradigm, which seems to be susceptible of embracing several different notions and models and providing a fruitful integration among concepts in various fields such as economics, accounting, finance. An enrichment of this conceptual environment will possibly address in a more thorough way the issue of practical usefulness of this paradigm for value creation, incentive compensation, and capital budgeting decisions. From these points of view, and from the more general view of a fruitful integration of accounting and finance, the results found seem to be auspicious.

Appendix

We show that the sum of NVAs is equal to the Net Terminal Value N_n.

$$\begin{aligned}
\sum_{t=1}^{n} \mathrm{NVA}_t &= N_1 + iN_1 + \ldots + iN_{n-1} \\
&= N_0(1+i) + iN_1 + \ldots + iN_{n-1} \\
&= N_0 \left[1 + i\big(1 + u + u^2 + \ldots u^{n-1}\big) \right] \\
&= N_0 \left[1 + i\frac{u^n - 1}{u - 1} \right] \\
&= N_0 u^n \\
&= N_n
\end{aligned}$$

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Table 0. Notational Conventions

Symbol	means	is equal to
$\vec{d}$	project, firm, business	
d_t	cash flow from $\vec{d}$ available for distribution	equity/free/capital cash flow
$\vec{d}_t$	uniperiodic project	
y_{t-1}	capital invested	
r_t	uniperiodic rate of return	
$y_{t-1}(r)$	capital growing at rate r	
i	(opportunity) cost of capital	
$\vec{r}^*$	internal discount function for project $\vec{d}$	
r_t^*	periodic (internal) rate of return	
B_t^e	book value of equity	$\sum_{k=t+1}^n \frac{d_k}{\prod_{k=t+1}^n (1+\mathrm{ROE}_k)}$
ROE	Return On Equity	
x_t^a	standard residual income	
$y_{t-1}(r^*)$	outstanding capital of project $\vec{d}$	
RI	residual income	
EVA	Economic Value Added	
$wacc$	weighted average cost of capital	
ROA	Return On Assets	
CFROI	Cash Flow Return On Investment	
EBO	Edwards-Bell-Ohlson	
k_e	cost of equity (required return on equity)	
CSV	Created Shareholder Value	
V_t^e	market value of equity	$\sum_{k=t+1}^n \frac{d_k}{(1+k_e)^{k-t}}$
CFROE	Cash Flow Return On Equity	
C_t	value of the reinvestment proceeds at time $t-1$	
C^t	amount of wealth at time t if project is not undertaken	
LC	lost capital	
ξ_t^a	lost-capital residual income	
$\Delta y_t(\cdot)$	capital's variation	$y_t(\cdot) - y_{t-1}(\cdot)$
$L(x_t^a)$	LC-companion of x_t^a	
$L(\mathrm{EVA})$	LC companion of EVA	
B_t^l	book value of total liabilities	
$L(\mathrm{EBO})$	LC companion of EBO	
$L(\mathrm{CSV})$	LC companion of CSV	
u	compounding factor	$1+i$
v	discount factor	$(1+i)^{-1}$

(The Table is continued on the next page)

continued - Table 0. Notational Conventions

Symbol	means	is equal to
NPV	Net Present Value	$\sum_{k=1}^{n} \frac{a_k}{(1+i)^k} - a_0$
N_n	Net Terminal Value	NPV$(1+i)^n$
AEG	Abnormal Earnings Growth	
z_t	Abnormal Earnings Growth	Residual earnings$_{t+1}$ $-$ Residual Earnings$_t$
F_t	future value of cumulated AEGs	
AIC	Adjusted Invested Capital	
AE	Adjusted EVA	
V_t	market value	$\sum_{k=t+1}^{n} \frac{d_k}{(1+i)^{k-t}}$
N_t	time-t Net Present Value	NPV$(1+i)^t$
gMVA	generalized Market Value Added	$V_t - A_t(x)$
MVA	Market Value Added	$V_t^e - B_t^e$
ΔN_t	Net Present Value's variation	$N_t - N_{t-1}$
ΔMVA_t	Market Value Added's variation	MVA$_t-$MVA$_{t-1}$
B_t	book value	
EVC_t	Excess Value Created	N_t
U_t^0	unrecovered capital	$y_t(i)$
$\vec{d}_{t,n}$	second part of project $\vec{d}$	
$\vec{d}_{0,t}$	first part of project $\vec{d}$	
$\xi^a_{k,(\vec{d}_{t,n})}$	lost-capital RI of project $\vec{d}$'s second part	
$x^a_{k,(\vec{d}_{t,n})}$	standard RI of project $\vec{d}$'s second part	
$y_t^{\circ}(i)$	initial lost capital of project $\vec{d}_{t,n}$	$y_t(r^*)$
EEI$_t$	Earned Economic Income	$d_t N_0/V_0^e$
NVA$_t$	Net Value Added	
G	value of assets if they are used	V_t
G'	value of assets if they had not been used	$y_t(i)$
$G'-G$	user cost	N_t
IC_t	current invested capital	$y_t(i)$
NCF_k	Net Cash Flow	free cash flows
NEI$_t$	Net Economic Income	
MV_t	market value	V_t
V_t^l	market value of total liabilities	$\sum_{k=t+1}^{n} \frac{d_k}{\prod_{k=t+1}^{n}(1+wacc_k)}$
ARI$_t$	Anthony's Residual Income	

Table 1. The firm and the loan

FIRM		LOAN	
cash flow	$\longrightarrow$	instalment	d_t
capital employed	$\longrightarrow$	residual debt (outstanding balance)	y_{t-1}
capital's depreciaton	$\longrightarrow$	principal repayment	$y_{t-1} - y_t$
periodic rate of return	$\longrightarrow$	contractual rate	r_t
income	$\longrightarrow$	interest	$r_t A_{t-1}$

Table 2. Constructing residual incomes in the two paradigms

	IDF	$y_t(r^*)$	i	$y_t(i)$	d_t
Entity approach					
EVA	ROA	B_t^l	$wacc$	$y_t(wacc)$	FCF
RI^{CFROI}	CFROI	$y_t(\text{CFROI})$	$wacc$	$y_t(wacc)$	FCF
NEI	$\begin{cases} r_1^* = \frac{V_1^l + d_1 - d_0}{d_0} \\ r_t^* = wacc \end{cases}$	V_t^l	$wacc$	$y_t(wacc)$	FCF
Equity approach					
EBO	ROE	B_t^e	k_e	$y_t(k_e)$	ECF
RI^{CFROE}	CFROE	$y_t(\text{CFROE})$	k_e	$y_t(k_e)$	ECF
CSV	$\begin{cases} r_1^* = \frac{V_1^e + d_1 - d_0}{d_0} \\ r_t^* = k_e \end{cases}$	V_t^e	k_e	$y_t(k_e)$	ECF

Table 3. Anthony's and Fernández's symmetric residual incomes

	period					
	1	2	3	...	$n-1$	n
Anthony's Residual Income	0	0	0	...	0	$\text{NPV}(1+k_e)^n$
Created Shareholder Value	$\text{NPV}(1+k_e)$	0	0	...	0	0