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Abstract

We study a credence goods market in which an expert holds private information about his treatment cost besides his superior knowledge about the nature of the consumer's problem. Under the assumption of *liability*, cheating may occur through overcharging—a price for major treatment is charged while a minor treatment is provided, while under *liability* and *verifiability*, cheating can only occur through costly overtreatment of minor problems. Neither liability nor liability and verifiability achieves socially efficient outcome. Adding verifiability improves social welfare because it increases the probability that a major problem is repaired and the associated overtreatment cost is dominated by the gain from more problems being repaired.

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1 Introduction

In a credence goods market, the expert knows more about the nature of the consumer's problem, that is, whether it is a major problem that demands a major treatment or a minor one that demands a minor treatment. Even after consumption, the consumer may still have no idea whether the goods or services she has received was appropriate (Darby and Karni (1973)). For example, the water pump in a consumer's car is leaking, and the mechanic suggests replacing the pump (a major treatment). After the repair, the pump functions well but the consumer never knows whether replacing a valve (a minor treatment) would have been sufficient to solve the problem or a new pump was indeed necessary.

The information advantage of the expert—the expert knows more about the nature of the consumer's problem than the consumer herself—leads to cheating incentive for the expert: he may provide the consumer a major treatment for a larger profit even if a minor treatment is sufficient to resolve the consumer's problem, or provide the consumer a minor treatment which does not solve the consumer's problem, or charge the consumer a price for major treatment even though a minor treatment has been provided.

The expert may also have private information about his treatment cost, besides his superior knowledge about the nature of the consumer's problem. Private information about production cost on the side of goods/service supplier is an important issue in industrial organization theory and has been well studied in the analysis of firm behavior in regular goods market. Firms choose their outputs and prices differently when their production costs are private information rather than common knowledge. A monopoly with private information on production cost may use "limit pricing" to threaten potential market entrants. Uncertainty about a firm's production cost is a primary concern for a government in the regulation of natural monopoly. Such private information naturally exists in the provision of credence goods as well but has so far been largely ignored in the literature.[1]

A direct implication of the additional private information on the expert's treatment cost is that "equal-margin principle" fails to implement the efficient outcome. In the literature (see, for example, Dulleck and Kerschbamer (2006)), the equal-margin principle has been an important rule in solving the expert's cheating incentive: if the expert receives the same profit margin from providing different types of treatment, he has no incentive to provide the wrong treatment or charge the wrong price. The rule no longer achieves efficiency

[1]For example, a mechanic has private information on how long it takes him to fix a consumer's engine problem. A surgeon knows privately how skilled he is in an operation. See Hilger (2016) for more examples.

when the expert holds private information about his treatment cost—if a pricing scheme generates equal profit margin for one type of expert in the provision of different types of treatment, it must generate unequal profit margin for some other type of the expert, thus distorting his incentive in the provision of services.

We introduce the expert's private information about his treatment cost into an otherwise standard credence goods model. The consumer's problem is either minor or major. An expert can always diagnose and repair a minor problem at zero cost. However, to repair a major problem, a low-cost expert incurs a lower treatment cost than a high-cost expert. The expert knows whether he is a high-cost type or low-cost type, but the consumer only knows the prior distribution of types. In our expert-consumer game the expert is a price setter, and the prices he posts may convey information about his types. We are interested in how the private information of the expert on his treatment cost affects the equilibrium outcome and how different market institutions perform given this new dimension of private information.

We focus on two types of market institutions/assumptions: i) liability (L), ii) liability and verifiability (LV). The expert is liable for the outcome of the treatment under the assumption of liability. The type of treatment is costlessly verifiable by the consumer under the assumption of verifiability. These two institutions are important tools that are very often explored in the literature for the regulation of the experts' behavior. See, for example, Dulleck and Kerschbamer (2006), and Fong, Liu, and Wright (2014). Both institutions can be implemented or enforced through legal systems. Liability can be enforced through harsh punishment of the expert if he does not resolve the consumer's problem. Verifiability can be implemented through creating hard evidence of the treatment process. For example, some service providers videotype or record the treatment process for file. Some mechanics return the consumers' replaced parts as a way to verify the type of treatment provided.

Under liability, an expert can not provide a minor treatment when a major one is necessary because a minor one does not repair the consumer's problem. Nevertheless, the expert can overcharge —charging the consumer a price for major treatment while a minor treatment is provided. The expert also has the option to overtreat the consumer—providing major treatment when only minor treatment is necessary. But overtreating is dominated for the expert because overcharging brings the same price of major treatment without actually incurring the cost of major treatment. Thus under L, cheating may occur in the form of overcharging. If verifiability is in place, the expert can not charge the consumer a price for major treatment while providing a minor one because the consumer can costlessly verify the type of treatment she receives, but the expert can provide a major treatment when a minor treatment is needed. Therefore, under LV cheating may occur in the form of overtreatment.

When there are multiple equilibria outcomes, we focus on the one in which both types of the experts' payoffs are maximized, that is, the Pareto dominating equilibrium for the experts.[2] Under liability, in the experts' Pareto dominating equilibrium, both types of experts behave honestly but a major problem remains unrepaired with positive probability because the consumer disciplines the expert by rejecting with positive probability a recommendation to repair her problem at the price of major treatment.

Under liability and verifiability, different types of equilibrium outcome prevail depending on how likely the expert is a low-cost type. If the expert is very likely to be a low-cost type, the prevailing equilibrium is similar in nature to the one under liability: the experts behave honestly but the consumer disciplines the expert by rejecting a recommendation of major treatment with positive probability. If the expert is more likely to be a high-cost type, the prevailing equilibrium is a cheating one—a low-cost expert always overtreats in case of a minor problem, a high-cost type may or may not overtreats depending on the parameters. In these cheating equilibria outcomes, the prices for major treatments are relatively low and the consumer always accepts a recommendation of major treatment.

One may conjecture that imposing verifiability on top of liability decreases social welfare because overcharging under L is purely a monetary transfer between the consumer and the expert and does not hurt social welfare, while when verifiability is added overtreatment incurs wasteful treatment cost. We find that, in contrast, adding verifiability always improves social welfare. When the expert is more likely to be low-cost type, the experts do not cheat in equilibrium under both L and LV, but verifiability increases the consumer's probability of accepting a recommendation of major treatment. As a result, a major problem is repaired at a higher probability under LV and social welfare is improved. When the expert is more likely to be a high-cost type, the experts overtreat in equilibrium under LV but the price for major treatment is relatively low and the consumer always accepts a recommendation of major treatment. As a result, a major problem is always repaired. The gain from repaired major problem dominates the social loss from wasteful overtreatment cost, and thus imposing verifiability improves social welfare.

There is a large literature that analyses how different market structure and institutions

[2]If such an equilibrium exists, it is also the experts' optimal equilibrium. Pareto domination is used in the selection of equilibria in other papers as well, for example, in Anton and Yao (1989).

affect the behavior of experts in a credence goods market. See, for example, Pitchik and Schotter (1987), Wolinsky (1995), Emons (1997), Emons (2001), Fong (2005), Alger and Salanié (2006), Dulleck and Kerschbamer (2009). Dulleck and Kerschbamer (2006) offer a nice survey of the early contributions in a united framework. More recent works include Liu (2011) that analyses the credence goods market with the coexistence of conscientious and selfish experts, Fong, Liu, and Wright (2014) that compare the performance of verifiability vs. liability, Bester and Dahm (2014) that consider the impact of subjective evaluation, Frankel and Schwarz (2014) that analyse a repeated interaction, Dulleck and Wigger (2015) that model the services of politicians as credence goods, and Dulleck, Gong, and Li (2015) that compare the performance of auctions and sequential search in a procurement environment. Our paper is different from most of these contributions by taking into account a second dimension of the experts' private information and analysing how this additional private information affects the experts' behavior under different institutions.

There are a few papers that model additional private information besides the expert's information advantage on the nature of the consumer's problem. Fong (2005) analyses a model in which consumers differ in their losses from the same problem under the assumption of liability and asks when the experts cheat, and whom do they target. Liu (2011) analyses a credence goods market with the coexistence of conscientious and selfish experts under the assumption of liability. In her model, a conscientious expert benefits from repairing a consumer's problem—as a result, there exists a nonuniform price equilibrium in which different types of experts post different price lists. In our setting, the private information is on treatment cost, and there exists no separating equilibrium in which different types of experts reveal their types through different prices. Fong, Liu, and Wright (2014) discuss the experts' behavior if honest experts who never cheat coexist with selfish expert under the assumption of verifiability. Different from these papers, we analyse both the case of liability and the case of liability and verifiability, and this exercise allows us to assess the incremental benefit of imposing the assumption of verifiability in the credence goods market. Hilger (2016) accounts for the factor that consumers may not be able to observe expert cost functions and shows that the assumption of verifiability leads to mistreatment on the side of the expert. We analyse a more dynamic model by allowing the consumers to walk away after receiving a recommendation from the expert. We also show that mistreatment occurs in equilibrium if verifiability is in place but our focus is on a complementary question—since verifiability leads to mistreatment, is it socially beneficial to impose verifiability if it is a choice of institutions?

The remainder of the paper is organized as follows. In Section 2, we present the credence goods model with the coexistence of high-cost and low-cost experts. Section 3 analyses the equilibria under liability. Section 4 analyses the equilibria when both liability and verifiability hold. In Section 5 we discuss the social benefit of adding verifiability. In Section 6 we analyse the model under alternative parameter specifications. Section 7 concludes. All the proofs are in the appendix.

2 The Model

There is one monopoly expert and one consumer. The consumer has a problem k, either a minor one (m) or a major one (M). The consumer's utility is $-\ell_k$ with $\ell_M > \ell_m > 0$ if her problem remains unresolved and $-P$ if her problem is repaired at price P.

The expert provides two procedures of treatment $T \in \{m, M\}$. Treatment M repairs both types of problems but treatment m only repairs a minor problem. The expert can be of two cost types: a low-cost type (L) or a high-cost type (H). Experts of both types perform treatment m at zero cost. A low-cost expert performs treatment M at cost c while a high-cost expert performs treatment M at cost θc, with $\theta > 1$. Parameter θ measures the cost disparity between the two types of experts.

The consumer does not know the nature of her problem, thus the treatment she needs, nor does she observe the type $(t \in \{L, H\})$ of the expert. But she knows the population distribution

$$\Pr(k = M) = 1 - \Pr(k = m) = \beta; \qquad \Pr(t = L) = 1 - \Pr(t = H) = \gamma \tag{1}$$

in which $\beta \in [0, 1]$ and $\gamma \in [0, 1]$. It is the expert's private information whether his type is L or H. The expert privately learns the nature of consumer's problem after costless diagnosis.

We make the following assumptions regarding the parameters

$$\ell_m < \bar{\ell} < c < \theta c < \ell_M \qquad \text{in which} \qquad \bar{\ell} = \beta \ell_M + (1 - \beta)\ell_m, \tag{2}$$

$$\ell_M - \theta c > \ell_m. \tag{3}$$

Assumption (2) ensures that i) it is efficient for both types of experts to repair a major problem, ii) the cost of repairing a major problem is larger than the *ex ante* expected loss to the consumer. In section 6, we analyze the alternative case with parameter specifications $\ell_m < c < \bar{\ell} < \theta c < \ell_M$.[3] Assumption (3) ensures that repairing a major problem is more socially valuable than repairing a minor problem for both types of experts.

Following the literature (for example Dulleck and Kerschbamer (2006)), we define the market institutions of liability and verifiability as follows.

DEFINITION 1. **Liability**(L): the consumer can verify *ex post* whether his problem is resolved or not at zero cost and the expert is liable for the outcome of the treatment. **Verifiability**(V): the type of treatment the consumer receives is costlessly verifiable.

If the expert repairs the problem $k \in \{m, M\}$, under the institution of liability, the consumer knows that her problem is solved but does not know which treatment has been provided; under the institution of verifiability, the consumer learns the type of treatment she has received. Thus, under liability, the expert can not perform a treatment m if $k = M$ because a minor treatment can not repair a major problem—insufficient treatment is precluded by liability. Under the institution of verifiability, the expert can not charge the price of a minor (major) treatment when a major (minor) treatment is performed—mischarging is precluded.

In the following analysis, we consider a game in which the expert posts prices for different types of treatments after privately observing his cost type. The prices an expert posts reveal (partly) the experts' private information on his cost types. As in most signaling games, there exists a plethora of perfect Bayesian Nash equilibria. Whenever possible, we will focus on the pareto dominating equilibrium outcome for the experts—the equilibrium outcome that generates the (weakly) highest expected payoffs for both types of experts among all possible equilibrium outcomes.

3 Equilibrium under Liability

In this section, we analyse the expert-consumer game under the institution of liability. The game proceeds as follows:

1. Nature draws the expert's type t and reveals the information privately to the expert.
2. Knowing his type, the expert posts a price list (P_M^t, P_m^t), with $P_m^t \leq P_M^t$, $P_M^t \leq \ell_M$ and $P_m^t \leq \ell_m$.

[3]Note that the case $\ell_m < c < \theta c < \bar{\ell} < \ell_M$ is trivial because there always exists an efficient equilibrium in which the experts repair the consumers' problems at price $\bar{\ell}$.

3. Nature draws the consumer's type k and the consumer visits the expert. The expert learns the nature of the consumer's problem. The expert either declines to repair the consumer's problem or offers to repair the problem at price P_k^t, with $k \in \{m, M\}$. If the expert declines to treat the consumer, the game ends. Otherwise, the game proceeds to the next stage.

4. If the consumer accepts the expert's offer, the expert must repair the problem at the quoted price. If the consumer declines the offer, the consumer's problem remains unrepaired and the expert receives no payment.

The expert's strategy consists of a pair of prices (P_M^t, P_m^t) at stage 2 and a recommendation policy at stage 3. The recommendation policy specifies the probabilities that the expert offers to repair the consumer's problem at P_M^t, P_m^t, and rejects to treat the consumer, conditional on the nature of the problem $k \in \{M, m\}$. The subgame following each given price list (P_M^t, P_m^t) is referred to as a "recommendation subgame".

Note that due to the restriction $P_m^t \leq P_M^t$, an expert never offers to repair a major problem at P_m^t—he either offers to repair the problem at P_M^t or rejects to treat the consumer. Thus, we use ρ^t to denote the probability that a type t expert offers to repair a minor problem at price P_M^t. Further note that for any $P_m^t \geq 0$, an expert never rejects to repair a minor problem because the cost of repairing a minor problem is zero. We use ϕ^t as the probability that a type t expert rejects to repair a major problem.

The consumer's strategy is her probability of accepting an offer at P_M or P_m. Given the assumption of liability, on receiving an offer P_m, the consumer can infer that her problem is minor and thus always accepts the offer if $P_m \leq \ell_m$. For the expert, it is never optimal to post a price $P_m > \ell_m$ and being rejected when recommending a treatment at P_m. Thus, in the following analysis, we use λ for the consumer's probability of accepting an offer at price P_M.

A perfect Bayesian Nash equilibrium must satisfy the following three conditions:

1. Each type of the expert's strategy maximizes his expected payoff given the consumer's strategy.

2. The consumer's strategy maximizes her expected payoff given her beliefs about the nature of her problem and the expert's cost type.

3. The consumer's beliefs are correct on the equilibrium path.

The game is solved through backward induction. Cheating in the form of overcharging occurs if $\rho^t > 0$ for any t in equilibrium, that is, at least one type of the expert charges P_M^t with positive probability when $k = m$. The next proposition states the main result of this section.

Proposition 1 (No-cheating outcome). *In the experts' pareto-dominating equilibrium, both types of experts post the same price list* $(P_M, P_m) = (\ell_M, \ell_m)$*. Both types of experts truthfully reveal the nature of the consumer's problem* ($\rho^H = \rho^L = 0$) *and never reject to provide a treatment* ($\phi^H = \phi^L = 0$)*. An offer at* P_m *is always accepted and an offer at* P_M *is accepted by the consumer with probability* $\lambda = \frac{\ell_m}{\ell_M}$*. The experts' expected payoffs from this equilibrium outcome are respectively* $\Pi_L = \beta(\ell_M - c)\frac{\ell_m}{\ell_M} + (1-\beta)\ell_m$ *and* $\Pi_H = \beta(\ell_M - \theta c)\frac{\ell_m}{\ell_M} + (1-\beta)\ell_m$*.*

In the proof of Proposition 1, we show that in a recommendation subgame following a pooling price list $(P_M, P_m) \in [\theta c, \ell_M] \times [0, \ell_m]$, there is an equilibrium in which both types of experts play the same mixed strategies. The equilibrium strategy profile in each subgame is characterized by

$$\phi^H = \phi^L = 0, \quad \rho^H = \rho^L = \frac{\beta(\ell_M - P_M)}{(1-\beta)(P_M - \ell_m)}, \quad \lambda = \frac{P_m}{P_M}. \tag{4}$$

See Lemma 3 in the appendix for the proof. For any $P_M < \ell_M$, $\rho^t \in (0, 1)$, cheating (overcharging) does occur in these subgames. In the whole game, these cheating outcomes can also be sustained as an equilibrium outcome of the whole game through the construction of proper consumer belief systems. Nevertheless, the experts' expected payoffs from these equilibria are dominated by the payoffs of the experts if they set $(P_M, P_m) = (\ell_M, \ell_m)$. There are also other equilibria outcomes with both types of experts overcharging, for example, if $(P_M, P_m) \in [c, \theta c] \times [0, \ell_m]$ (see Lemma 2 in the appendix). But the experts' payoffs from these equilibria outcomes are dominated as well.

Furthermore, as we show in the proof of Lemma 6, there exists no separating equilibrium in which the experts post different price lists that reveal perfectly their cost types, nor does there exist a uniform price equilibrium in which the experts post a single price for both types of treatments.

The existence of no-cheating equilibrium does not mean that the outcome is efficient. In the equilibrium outcome in Proposition 1, although the experts do not cheat, the consumer declines an offer at price P_M with positive probability to discipline the expert's overcharging incentive. Rejecting an offer at P_M with positive probability leaves a major problem unresolved and causes social inefficiency. As a result, a major problem remains unresolved with probability $1 - \frac{\ell_m}{\ell_M}$ and in such case the consumer suffers a loss $-\ell_M$. When a major problem is resolved, it is resolved at an expected cost of $\gamma c + (1-\gamma)\theta c$. A minor problem is always repaired at zero cost. We summarize the expected social welfare level as follows.

Corollary 1. *The social welfare level under the institution of liability is given by*

$$\begin{aligned} W_L &= -\beta(1 - \frac{\ell_m}{\ell_M})\ell_M - \beta\frac{\ell_m}{\ell_M}(\gamma c + (1-\gamma)\theta c) \\ &= -\beta(\ell_M - \ell_m) - \beta\frac{\ell_m}{\ell_M}(\gamma c + (1-\gamma)\theta c) \end{aligned} \tag{5}$$

4 Equilibria under Liability and Verifiability

In this section, we assume that both the institutions of liability and verifiability are in place. As discussed before, under the assumption of verifiability, the type of treatment provided can be costlessly verified by the consumer. As a result, different from the case in Section 3, an expert can not charge the consumer a price $P_k \neq P_{k'}$ if treatment k' is performed, with $k \neq k' \in \{M, m\}$.

The game is slightly different from the one in Section 3. Given that the prices are announced at stage 2, the expert's action is to recommend a treatment to the consumer at stage 3 (instead of making an offer to repair the consumer's problem at one of the posted prices in Section 3). If the recommended treatment is accepted, the expert will perform the treatment and get paid the posted price for that treatment.

3. Nature draws the consumer's type. The consumer visits the expert. Through diagnosis, the expert learns the nature of the consumer's problem. The expert either rejects to treat the consumer or recommends a treatment T. If the expert rejects to treat the consumer, the game ends. Otherwise, the game proceeds to the next stage.

4. If the consumer accepts the recommendation, the recommended treatment $T \in \{m, M\}$ is performed in exchange for a price P^t_T. If the consumer declines the recommendation, the consumer's problem remains unrepaired and the expert receives no payment.

The expert's strategy consists of a list of prices $\{P^t_M, P^t_m\}$ at stage 2 and a recommendation policy at stage 3 in which the expert either rejects to treat the consumer or recommends a major treatment or a minor treatment. Note that if $k = M$, the expert can either choose to recommend a major treatment or rejects to treat the consumer, while providing a minor treatment does not resolve the consumer's problem and is not feasible due to the assumption of liability. Let ρ^t be the shorthand for the probability with which the expert of type t recommends major treatment when the problem is $k = m$. Again we use ϕ^t for the probability that a type t expert rejects to treat a consumer if $k = M$.

The consumer's strategy specifies the probability of accepting or rejecting the expert's recommendation. Due to the restriction $P_m \leq \ell_m$ and the assumption of liability, whenever the consumer is recommended a minor treatment, the consumer can infer that her problem must be minor and thus never declines a recommendation $T = m$. We denote λ as the probability that the consumer accepts a recommendation of major treatment.

Overtreating occurs if in equilibrium an expert prescribes major treatment with positive probability when $k = m$, that is, $\rho^t > 0$ for any $t \in \{L, H\}$. The magnitude of γ, the probability that an expert is of a low-cost type, affects the existence of different types of equilibria in the recommendation subgame following a given price list, thus, the equilibrium outcome of the whole game depends upon γ as well. We summarize the main results of this section in Propositions 2 to 4.

Proposition 2 (No-cheating outcome). *Suppose* $\gamma > \frac{\beta(\ell_M - c\theta)}{(1-\beta)(c\theta - \ell_m)}$. *In the experts' Pareto dominating equilibrium outcome, both types of experts post the pooling price list* $(P_M, P_m) = (\ell_M, \ell_m)$. *Given this price list, both types of experts recommend a treatment honestly (that is,* $\phi^t = 0$, $\rho^t = 0$*). A recommendation of major treatment is rejected by a consumer with positive probability* ($\lambda = \frac{\ell_m}{\ell_M - c}$) *and a recommendation of minor treatment is always accepted. The experts' expected payoffs from this equilibrium outcome are respectively* $\Pi^L = \ell_m$ *and* $\Pi^H = \ell_m(1 - \frac{\beta(\theta-1)c}{\ell_M - c})$.

In the proof of Proposition 2, we show that in the recommendation subgame following a pooling price list (P_M, P_m) such that $(P_M, P_m) \in [\max\{\theta c, P_m + c, \frac{\beta\ell_M + \gamma(1-\beta)\ell_m}{\gamma(1-\beta)+\beta}\}, \ell_M] \times [0, \ell_m]$, there is an equilibrium involving the following strategy profiles:

$$\phi^H = \phi^L = 0, \quad \rho^H = 0, \quad \rho^L = \frac{\beta(\ell_M - P_M)}{\gamma(1-\beta)(P_M - \ell_m)}, \quad \lambda = \frac{P_m}{P_M - c}. \tag{6}$$

See Lemma 7 in the appendix for the proof. In this equilibrium of the recommendation subgame, a high-cost expert treats the consumer honestly and a low-cost type expert partially overtreats (that is, $\rho^L \in (0,1]$). The experts' expected payoffs are respectively

$$\Pi^H = \beta(P_M - \theta c)\frac{P_m}{P_M - c} + (1-\beta)P_m, \qquad \Pi^L = P_m. \tag{7}$$

Because the experts' expected payoffs increase in P_M and P_m, any perfect Bayesian Nash equilibrium of the whole game with $P_M < \ell_M$ and $P_m < \ell_m$ is Pareto dominated by the equilibrium with $(P_M, P_m) = (\ell_M, \ell_m)$ for the experts. There are also other equilibria that involve price lists $(P_M, P_m) \notin [\max\{\theta c, P_m + c, \frac{\beta\ell_M + \gamma(1-\beta)\ell_m}{\gamma(1-\beta)+\beta}\}, \ell_M] \times [0, \ell_m]$. For example, suppose $(P_M, P_m) \in [\max\{P_m + c, \frac{\beta\ell_M+\gamma(1-\beta)\ell_m}{\gamma(1-\beta)+\beta}\}, \theta c) \times [0, \ell_m]$, if $\gamma > \frac{\beta(\ell_M - \theta c)}{(\theta c - \ell_m)(1-\beta)}$, the following strategies form an equilibrium of the recommendation subgame:

$$\phi^H = 1, \quad \phi^L = 0, \quad \rho^H = 0, \quad \rho^L = \frac{\beta(\ell_M - P_M)}{\gamma(1-\beta)(P_M - \ell_m)}, \quad \lambda = \frac{P_m}{P_M - c}. \tag{8}$$

See Lemma 9 in the appendix for the proof. These strategies can be supported as an equilibrium of the whole game by appropriate consumer beliefs. But the experts' payoffs from these equilibria are again dominated by those in the equilibrium outcome in Proposition 2.

Similar to Proposition 1, no cheating on the side of the experts does not mean that the market outcome is efficient. In the equilibrium outcome characterized in Proposition 2, both types of experts make honest recommendations in both states. Nevertheless, The consumer disciplines the expert by declining a recommendation of major treatment with strictly positive probability. This leads to market inefficiency. This stands in contrast to the result that verifiability and liability resolve the market inefficiency problem if the experts' costs are public information(see, for example, Dulleck and Kerschbamer (2006)). In our setting, private information about treatment costs makes it impossible for full efficiency to be restored through liability and verifiability.

Note that in the equilibrium outcome in Proposition 2, a minor problem is always repaired. If $k = M$, the problem is either repaired at an expected cost of $\gamma c + (1-\gamma)\theta c$ which occurs with probability $\lambda = \frac{\ell_m}{\ell_M - c}$, or remains unrepaired which occurs with probability $1 - \lambda$, and in the latter case no treatment cost occurs but the consumer suffers a loss ℓ_M. We summarize the social welfare level achieved in Proposition 2 in the next corollary.

Corollary 2. *If $\gamma > \frac{\beta(\ell_M - c\theta)}{(1-\beta)(c\theta - \ell_m)}$, the social welfare level under the institution of liability* *and verifiability is given by*

$$W_{LV} = -\beta \frac{\ell_m}{\ell_M - c} (\gamma c + (1-\gamma)\theta c) - \beta \frac{\ell_M - c - \ell_m}{\ell_M - c} \ell_M := \Delta_1. \tag{9}$$

We now turn to the case that the expert has a low probability to be of the cost efficient type. The next proposition summarizes the equilibrium outcome.

PROPOSITION 3 (Overtreating outcome). *Suppose* $\gamma \leq \frac{\beta(\ell_M - \ell_m - \theta c)}{(1-\beta)\theta c}$. *In the experts' Pareto dominating equilibrium outcome, both types of experts post the same price list* $(P_M, P_m) = (\ell_m + \theta c, \ell_m)$. *Given this price list, a low-cost type expert always overtreats* ($\rho^L = 1$). *A high-cost type expert partially overtreats (* $\rho^H = \frac{\beta(\ell_M - \ell_m - \theta c)}{(1-\gamma)(1-\beta)\theta c} - \frac{\gamma}{1-\gamma} \in [0,1)$). *The consumer accepts a recommendation of major treatment with probability one* ($\lambda = 1$). *The experts' expected payoffs are* $\Pi_L = \ell_m + (\theta - 1)c$ *and* $\Pi_H = \ell_m$.

The proof of Proposition 3 shows that in the recommendation subgame following a pooling price list (P_M, P_m) with $(P_M, P_m) \in [P_m + \theta c, \frac{\beta \ell_M + \gamma(1-\beta)\ell_m}{\beta + \gamma(1-\beta)}] \times [0, \min\{\ell_m, \frac{\beta \ell_M + \gamma(1-\beta)\ell_m}{\beta + \gamma(1-\beta)} - \theta c\}]$, the following strategies form an equilibrium of the subgame

$$\phi^H = \phi^L = 0, \quad \rho^L = 1, \quad \rho^H = \frac{\beta(\ell_M - P_M)}{(1-\gamma)(1-\beta)(P_M - \ell_m)} - \frac{\gamma}{1-\gamma}, \quad \lambda = \frac{P_m}{P_M - \theta c}. \tag{10}$$

See Lemma 10 in the appendix for the proof. The expected payoffs of the experts are $\Pi^L = P_m(1 + \frac{(\theta-1)c}{P_M - \theta c})$ and $\Pi^H = P_m$. In these subgames a low-cost expert always overtreats and a high-cost expert partially overtreats. Because both Π^L and Π^H increases in P_m and Π^L decreases in P_M, setting $(P_M, P_m) = (\ell_m + \theta c, \ell_m)$ is the Pareto dominating outcome for the two types of experts in the given price range. Again, there are possibly other perfect Bayesian Nash equilibria with $(P_M, P_m) \notin [P_m + \theta c, \frac{\beta \ell_M + \gamma(1-\beta)\ell_m}{\beta + \gamma(1-\beta)}] \times [0, \min\{\ell_m, \frac{\beta \ell_M + \gamma(1-\beta)\ell_m}{\beta + \gamma(1-\beta)} - \theta c\}]$. The experts' payoffs from those equilibria are also Pareto dominated by the one in Proposition 3 for the given set of γ.

In the equilibrium outcome in Proposition 3, both types of experts cheat in the form of overtreating but the price for major treatment is low relative to those in Proposition 2, the consumer always accepts a recommendation of major treatment. The inefficiency now comes from the unnecessary treatment cost in repairing a minor problem, not from unrepaired major problems as in Proposition 2.

In the equilibrium outcome in Proposition 3, a major problem is always repaired at an expected cost of $\gamma c + (1-\gamma)\theta c$, while a minor problem is always repaired at an expected cost of $\gamma c + (1-\gamma)\rho^H \theta c$, with $\rho^H = \frac{\beta(\ell_M - \ell_m - \theta c)}{(1-\gamma)(1-\beta)\theta c} - \frac{\gamma}{1-\gamma}$. The social welfare level achieved in Proposition 3 is summarized in the following corollary.

Corollary 3. *If $\gamma \le \frac{\beta(\ell_M - \ell_m - \theta c)}{(1-\beta)\theta c}$, the social welfare level achieved under the institution of liability and verifiability is given by*

$$\begin{aligned} W_{LV} &= -\beta\left(\gamma c + (1-\gamma)\theta c\right) - (1-\beta)\left(\gamma c + (1-\gamma)\rho^H \theta c\right) \\ &= -\beta(\ell_M - \ell_m) + \gamma(\theta - 1)c := \Delta_2. \end{aligned} \tag{11}$$

Finally, we consider the case with intermediate γ in the next proposition. For a subset of the parameter range, different types of experts prefer different equilibrium outcome.

PROPOSITION 4 (Multiple equilibrium outcomes). *Suppose $\frac{\beta(\ell_M - \ell_m - \theta c)}{(1-\beta)\theta c} < \gamma \le \frac{\beta(\ell_M - \theta c)}{(1-\beta)(\theta c - \ell_m)}$.*

If $\ell_m > (\theta - 1)c$, there exists unique $\hat{\gamma}_1 < \hat{\gamma}_2$ with $\hat{\gamma}_1, \hat{\gamma}_2 \in (\frac{\beta(\ell_M - \ell_m - \theta c)}{(1-\beta)\theta c}, \frac{\beta(\ell_M - \theta c)}{(1-\beta)(\theta c - \ell_m)}]$ such that

1. *if $\gamma \le \hat{\gamma}_1$, in the experts' Pareto dominating equilibrium outcome, both types of experts post the same prices $(P_M, P_m) = (\delta, \delta - \theta c)$ in which $\delta := \frac{\beta \ell_M + \gamma(1-\beta)\ell_m}{\beta + \gamma(1-\beta)}$. A low-cost type expert always overtreats ($\rho^L = 1$). A high-cost type expert behaves honestly ($\rho^H = 0$). The consumer accepts a recommendation of major treatment with probability one ($\lambda = 1$). The experts' expected payoffs are $\Pi^L = \delta - c$ and $\Pi^H = \delta - \theta c$. (Low-cost type overtreating outcome.)*

2. *if $\gamma \ge \hat{\gamma}_2$, in the experts' Pareto dominating equilibrium outcome, both types of experts post the price list $(P_M, P_m) = (\ell_M, \ell_m)$. Given this price list, both types of experts behave honestly (that is, $\phi^t = 0$, $\rho^t = 0$). A recommendation of major treatment is rejected by a consumer with positive probability ($\lambda = \frac{\ell_m}{\ell_M - c}$). The experts' expected payoffs are $\Pi^L = \ell_m$ and $\Pi^H = \ell_m(1 - \frac{\beta(\theta-1)c}{\ell_M - c})$. (No-cheating outcome.)*

3. *if $\hat{\gamma}_1 < \gamma < \hat{\gamma}_2$, a low-cost expert prefers the low-cost expert overtreating outcome in [1] with price list $(P_M, P_m) = (\delta, \delta - \theta c)$ while a high-cost expert prefers the no-cheating outcome in [2] with price list $(P_M, P_m) = (\ell_M, \ell_m)$. Either the outcome in [1] or the outcome in [2] may emerge as an equilibrium outcome of the whole game.*

If $\ell_m \le (\theta - 1)c$, there exists a unique $\hat{\gamma}_1 \in (\frac{\beta(\ell_M - \ell_m - \theta c)}{(1-\beta)\theta c}, \frac{\beta(\ell_M - \theta c)}{(1-\beta)(\theta c - \ell_m)}]$ such that if $\gamma \le \hat{\gamma}_1$, the Pareto-dominating outcome is the low-cost type overtreating outcome; if $\gamma > \hat{\gamma}_1$, *either the low-cost type overtreating outcome or the no-cheating outcome may emerge as an equilibrium in the whole game.*

Note that in the low-cost type overtreating outcome in Proposition 4, a minor problem is repaired at an expected cost of γc, and a major problem is repaired at an expected cost of $\gamma c + (1-\gamma)\theta c$. The social welfare level achieved this equilibrium outcome is $-(1-\beta)\gamma c - \beta(\gamma c + (1-\gamma)\theta c) = -\gamma c - (1-\gamma)\beta\theta c$. The no-cheating outcome of Proposition 4 is the same as that in Proposition 2, thus the social welfare achieved is the same as that in Corollary 2.

Corollary 4. *Suppose* $\frac{\beta(\ell_M - \ell_m - \theta c)}{(1-\beta)\theta c} < \gamma \leq \frac{\beta(\ell_M - \theta c)}{(1-\beta)(\theta c - \ell_m)}$. *The social welfare level achieved under the institution of liability and verifiability is given by*

$$W_{LV} = \begin{cases} -\gamma c - (1-\gamma)\beta\theta c := \Delta_3 \quad \text{if} \quad \frac{\beta(\ell_M - \ell_m - \theta c)}{(1-\beta)\theta c} < \gamma \leq \hat{\gamma}_1, \\ \Delta_1 \quad \text{if} \quad \hat{\gamma}_2 \leq \gamma \leq \frac{\beta(\ell_M - \theta c)}{(1-\beta)(\theta c - \ell_m)}, \\ \Delta_1 \text{ or } \Delta_3 \quad \text{if} \quad \hat{\gamma}_1 < \gamma < \hat{\gamma}_2 \end{cases} \tag{12}$$

for $\ell_m > (\theta - 1)c$. *The social welfare is given by*

$$W_{LV} = \begin{cases} \Delta_3 \quad \text{if} \quad \frac{\beta(\ell_M - \ell_m - \theta c)}{(1-\beta)\theta c} < \gamma \leq \hat{\gamma}_1, \\ \Delta_1 \text{ or } \Delta_3 \quad \text{if} \quad \hat{\gamma}_1 < \gamma \leq \frac{\beta(\ell_M - \theta c)}{(1-\beta)(\theta c - \ell_m)} \end{cases} \tag{13}$$

for $\ell_m \leq (\theta - 1)c$.

Summarizing the results in Proposition 2 to 4, we see that there are two critical values of parameter γ, the prior probability of an expert being a low-cost type, that are important for the equilibrium outcomes. One critical value, denoted as $\hat{\gamma}_3$, is such that $\hat{\gamma}_3 = \hat{\gamma}_2$ if $\ell_m > (\theta - 1)c$ and $\hat{\gamma}_3 = \frac{\beta(\ell_M - \theta c)}{(1-\beta)(\theta c - \ell_m)}$ if $\ell_m \leq (\theta - 1)c$. When γ is relatively large, that is, $\gamma \geq \hat{\gamma}_3$, the experts' Pareto dominating equilibrium is the no-cheating outcome, and there the social inefficiency comes from the consumer's rejection of a recommendation of major treatment. The other critical value is $\hat{\gamma}_1$ as defined in Proposition 4. When γ is relatively small, that is $\gamma \leq \hat{\gamma}_1$, the Pareto dominating equilibrium is a cheating one: a low-cost type always cheats but a high-cost type either cheats partially or does not cheat. Cheating occurs through overtreatment, therefore, the cost of social inefficiency is the treatment cost from repairing a minor problem through major treatment.

For a given price list, a high-cost expert benefits less from overtreatment than a low-cost expert because the incurred treatment cost is higher. As a result, in the cheating equilibria in Proposition 3 and 4, a high-cost type cheats with a lower probability than a low-cost expert.

Because a low-cost expert has a larger incentive to overtreat, when an expert is more likely to be a low-cost type (large γ), the consumer's best reaction is to discipline the experts by rejecting a major treatment with a positive probability. If an expert is more likely to be the high-cost type (a small γ), the incentive for overtreatment is relatively small, this paired with a relatively low price for major treatment leads to the outcome that a consumer always accepts a major treatment because the loss from being overtreated is relatively low in comparison to the benefit from getting her problem repaired.

5 The Role of Verifiability

It is not obvious whether imposing the assumption of verifiability on top of liability improves social welfare. Under the assumption L, the expert has no incentive to overtreat the consumer because overtreating is always dominated by overcharging for the expert—overtreating incurs the treatment cost while overcharging brings the same price without incurring the treatment cost. Cheating in the form of mischarging does not harm social welfare directly because it is a monetary transfer between the expert and the consumer. Nevertheless, it affects social welfare indirectly through the strategic behavior of the consumer—the consumer disciplines the experts by declining an offer at high price with positive probability. Thus, under L, social inefficiency comes from unrepaired consumer problem.

If verifiability is added on top of liability, mischarging is ruled out, and the expert can only cheat by overtreatment. The consumer may either discipline the experts by declining a major treatment, which leaves a problem unresolved with positive probability, or the experts may overtreat in case of a minor problem. Thus, under LV, social inefficiency comes either from unrepaired consumer problem, or from wasteful overtreatment cost.

Comparing the social welfare achieved under the two institutions we arrive at the following results.

PROPOSITION 5. *Imposing verifiability on top of liability improves social welfare. The benefit of verifiability weakly increases with γ if the equilibrium outcome under* LV *is the no-cheating outcome in Proposition 2 and 4 or the overtreating outcome in proposition*

3. How the benefit of verifiability changes with γ is indeterminate when the equilibrium outcome is the low-cost type overtreating outcome in Proposition 4.

When $\gamma \geq \hat{\gamma}_3$, the Pareto dominating equilibrium under LV is the no-cheating outcome. There the experts do not cheat but the consumer disciplines the expert by rejecting a recommendation of major treatment with positive probability. Under L, a major problem is repaired with probability $\frac{\ell_m}{\ell_M}$. Under LV, a major problem is repaired with probability $\frac{\ell_m}{\ell_M - c}$ which is larger than $\frac{\ell_m}{\ell_M}$. The main social benefit from adding verifiability comes from the increase in the probability that the consumer's major problem is repaired.

When $\gamma \leq \frac{\beta(\ell_M - \ell_m - \theta c)}{(1-\beta)\theta c}$, the Pareto dominating equilibrium under LV is the cheating outcome in Proposition 3. There, a recommendation is always accepted, and the consumer's problem is always repaired. Since a low-cost expert always overtreats and a high-cost expert partially overtreats, the main social inefficiency comes from the unnecessary treatment cost to repair a minor problem. Our comparison shows that this social loss is smaller than the social loss from unrepaired consumer problem under L.

When $\frac{\beta(\ell_M - \ell_m - \theta c)}{(1-\beta)\theta c} < \gamma < \hat{\gamma}_3$, the low-cost type overtreating outcome in Proposition 4 is either the Pareto dominating equilibrium for both types of experts or is preferred by one type of experts over other equilibrium outcomes. There, similar to the case in $\gamma \leq \frac{\beta(\ell_M - \ell_m - \theta c)}{(1-\beta)\theta c}$, a recommendation is always accepted, thus the consumer's problem is always repaired. In this equilibrium, only a low-cost expert overtreats, therefore the social welfare must exceed that in the case with $\gamma \leq \frac{\beta(\ell_M - \ell_m - \theta c)}{(1-\beta)\theta c}$. Hence, the social welfare level must also be higher than that under the institution of liability.

How the benefits of verifiability vary with parameter γ depends on the type of equilibrium outcome that prevails. Under institution L, increasing γ does not change the probability that a consumer's major problem is repaired ($\frac{\ell_m}{\ell_M}$), but it increases social welfare W_L through reducing the expected cost of repairing a major problem. Under LV, in the no-cheating outcome, the benefit from reduced expected cost of repairing a major problem is amplified because a recommendation of major treatment is accepted with a higher probability, thus increasing γ increases the benefits of verifiability.

In the overtreating outcome in Proposition 3, a larger γ reduces the expected cost of repairing a major problem, and the overtreatment cost when $k = m$. These two effects together makes a larger γ more socially appealing under LV.

In the low-cost type overtreating outcome, only a low-cost expert overtreats. When γ increases, the effect on the reduction of overtreatment cost is weaker than in the case with

$\gamma \leq \frac{\beta(\ell_M-\ell_m-\theta c)}{(1-\beta)\theta c}$. As a result, how the incremental benefit of adding verifiability changes with γ becomes ambiguous.

6 Discussions

In this section we discuss the effect of relaxing assumption (2). Suppose $c \leq \bar{\ell} < \theta c < \ell_M$ instead of assumption (2). Given this parameter set, the equilibrium we identified in Proposition 1 and Propositions 2 to 4 in which the two types of experts post the same price list are still equilibria under L and LV respectively.

Furthermore, if γ is neither too large nor too small, there exists a uniform price equilibrium in which the two types of experts post the same single price $\bar{p}$ under the two institutions. When a single price is posted for both types of treatment, the expert's strategy is either to recommend repairing the consumer's problem at price $\bar{p}$, or declining to treat the consumer. Therefore there is no difference whether verifiability is in place or not. Again we focus on the equilibrium outcome that is Pareto dominating for the experts within the class of uniform price equilibrium.

Proposition 6 (Uniform-price Equilibrium). *Suppose $c \leq \bar{\ell} < \theta c < \ell_M$. Under both institutions,* L *and* LV*, if $\gamma \in (\frac{(1-\beta)(c-\ell_m)}{\beta(\ell_M-c)}, \frac{(1-\beta)(\theta c-\ell_m)}{\beta(\ell_M-\theta c)}]$, there exists a uniform-price equilibrium in which both types of experts post a single price $\bar{p} = \frac{\gamma\beta\ell_M+(1-\beta)\ell_m}{\gamma\beta+1-\beta}$, a low-cost expert offers to repair the consumer's problem in both states at price $\bar{p}$, a high-cost expert offers to repair the consumer's problem at price $\bar{p}$ if $k = m$ and declines to treat a consumer if $k = M$. The consumer accepts the expert's recommendation with probability one.*

In the equilibrium in Proposition 6, the existence of the low-cost expert creates an incentive for the high-cost expert to cream skim the consumer with a minor problem and dump the consumer with a major problem. Under the uniform price equilibrium, the equilibrium outcome is the same under L and LV, therefore it does not change our comparison of social welfare in Section 5.

7 Conclusion

We study a credence goods market in which the expert holds private information about his treatment cost besides knowing more about the nature of the consumer's problem than the consumer. We analyze the market equilibria under the assumption of i) liability; ii) liability and verifiability. Under liability, cheating may occur in the form of overcharging, a pure monetary transfer between the consumer and the expert; under liability and verifiability, cheating may occur in the form of overtreatment, which involves wasteful treatment cost in repairing a minor problem. We show that neither institutions implement socially efficiently outcome. Adding verifiability improves social welfare because it increases the probability that a major problem is repaired and the associated overtreatment cost is dominated by the gain from more problems being repaired.

Appendix

Proof of Proposition 1

The proof proceeds as follows: 1) In Lemma 1 to 3, we characterize the equilibria of a recommendation subgame following an exogenous pooling price (P_M, P_m) in which the two types of experts have posted the same price list. 2) In Lemma 4, we show the experts' payoffs in Lemma 3 Pareto dominate the others. 3) In Lemma 5, we show that pooling price $(P_M, P_m) = (\ell_M, \ell_m)$ is supported as an equilibrium of the whole game. 4) In Lemma 6, we show that there exists no other equilibrium with separating prices or uniform single price.

LEMMA 1. *Suppose both types of experts have posted the price list* (P_M, P_m) *with* $(P_M, P_m) \in [0, \ell_m] \times [0, \ell_m]$. *The recommendation subgame has an equilibrium in which both types of experts adopt the following strategies*

$$\phi^H = \phi^L = 1, \quad \rho^H = \rho^L = 1, \quad \lambda = 1. \tag{14}$$

In this equilibrium, the expert's payoff is

$$\Pi^L = \Pi^H = (1 - \beta) P_M. \tag{15}$$

Proof. Suppose the prices indeed satisfy $(P_M, P_m) \in [0, \ell_m] \times [0, \ell_m]$. Given the strategies of the expert, if the expert offers to repair her problem at P_M, the consumer believes that she has a minor problem. Since $P_M \leq \ell_m$, the consumer accepts the offer. Since $P_M \leq \ell_m < c < \theta c$, both types of experts rejects to treat the consumer if $k = M$. The profits of the expert follow directly from the given strategies of the players. □

LEMMA 2. *Suppose both types of experts have posted a price list* (P_M, P_m) *with* $(P_M, P_m) \in [c, \theta c) \times [0, \ell_m]$. *The recommendation subgame has an equilibrium in which*

$$\phi^L = 0, \quad \phi^H = 1, \quad \rho^L = \rho^H = \frac{\gamma\beta(\ell_M - P_M)}{(1-\beta)(P_M - \ell_m)}, \quad \lambda = \frac{P_m}{P_M}. \tag{16}$$

The expected payoffs of the experts are respectively

$$\Pi^H = (1-\beta)P_m, \qquad \Pi^L = \beta(P_M - c)\frac{P_m}{P_M} + (1-\beta)P_m. \tag{17}$$

Proof. Suppose the posted prices indeed satisfy $(P_M, P_m) \in [c, \theta c) \times [0, \ell_m]$. If $k = M$, a low-cost expert is willing to repair the problem at P_M while a high-cost expert rejects to treat the consumer. If $k = m$, given the strategy of the consumer, offering P_M brings a payoff $\lambda P_M = P_m$ to both types of experts, thus any $\rho^t \in [0, 1]$ is optimal for either type of the expert. Given the expert's strategy, on receiving an offer at P_M, the consumer believes her problem is major with probability $\frac{\beta\gamma}{\beta\gamma+(1-\beta)(\gamma\rho^H+(1-\gamma)\rho^L)}$. Thus, rejecting such an offer brings her a payoff

$$\begin{aligned} u^R =& -\left(\frac{(1-\beta)(\gamma\rho^H + (1-\gamma)\rho^L)}{\beta\gamma + (1-\beta)(\gamma\rho^H + (1-\gamma)\rho^L)}\ell_m + \frac{\beta\gamma}{\beta\gamma + (1-\beta)(\gamma\rho^H + (1-\gamma)\rho^L)}\ell_M\right) \\ =& -P_M \end{aligned}$$

which is the same as her payoff from accepting an offer at P_M and having her problem repaired. Thus, any $\lambda \in [0, 1]$ is optimal for the consumer. The expected payoffs of the experts in the equilibrium follow from the given strategies. □

LEMMA 3. *Suppose both types have posted a price list* $(P_M, P_m) \in [\theta c, \ell_M] \times [0, \ell_m]$. *The recommendation subgame has an equilibrium in which both types of experts use the following strategies*

$$\phi^H = \phi^L = 0, \quad \rho^H = \rho^L = \frac{\beta(\ell_M - P_M)}{(1-\beta)(P_M - \ell_m)}, \quad \lambda = \frac{P_m}{P_M}. \tag{18}$$

The experts' expected profits are respectively

$$\Pi^H = \beta(P_M - \theta c)\frac{P_m}{P_M} + (1-\beta)P_m, \quad \Pi^L = \beta(P_M - c)\frac{P_m}{P_M} + (1-\beta)P_m. \tag{19}$$

Proof. Suppose the experts have indeed posted the same price list $(P_M, P_m) \in [\theta c, \ell_M] \times [0, \ell_m]$. Since $P_M \geq \theta c$, both types of experts offer to repair the problem at P_M if $k = M$. If $k = m$, given the consumer's strategy, proposing P_M brings a payoff of $\lambda P_M = \frac{P_m}{P_M} P_m = P_m$

for both types of experts. Thus, any $\rho^t \in [0,1]$ is optimal for the expert.

Given the expert's strategy, on receiving an offer P_M, the consumer believes that her problem is major with probability $\frac{\beta}{\beta+(1-\beta)(\gamma\rho^L+(1-\gamma)\rho^H)}$, thus rejecting the offer brings her an expected payoff

$$\begin{aligned} u^R &= -\left(\frac{(1-\beta)(\gamma\rho^L+(1-\gamma)\rho^H)}{\beta+(1-\beta)(\gamma\rho^L+(1-\gamma)\rho^H)}\ell_m + \frac{\beta}{\beta+(1-\beta)(\gamma\rho^L+(1-\gamma)\rho^H)}\ell_M\right) \\ &= -P_M \end{aligned} \tag{20}$$

which is the same as her payoff from accepting the offer at price P_M and getting her problem resolved. Thus, any $\lambda \in [0,1]$ is optimal for the consumer. Therefore, the proposed strategies are indeed mutually best responses. The expected payoffs of the experts follow from the strategies. □

LEMMA 4. *The experts' payoffs are weakly higher if the pooling price list satisfies* $(P_M, P_m) \in [\theta c, \ell_M] \times [0, \ell_m]$ *than if it satisfies* $(P_M, P_m) \in [0, \ell_m] \times [0, \ell_m]$ *or* $(P_M, P_m) \in [c, \theta c) \times [0, \ell_m]$.

Proof. With the subgame equilibrium outcome in Lemma 1, the experts' payoffs are maximized at $(1-\beta)\ell_m$ by setting $P_M = \ell_m$.

With the subgame equilibrium outcome in Lemma 2, the expected profits of both types of experts increase in P_m, and the low-cost type expert's payoff increases in P_M as well. Therefore, the maximal profits are

$$\Pi^H = (1-\beta)\ell_m, \quad \Pi^L < \ell_m(1-\frac{\beta}{\theta}), \tag{21}$$

achieved by setting $P_m = \ell_m$ and P_M infinitely close to θc.

With the subgame equilibrium outcome in Lemma 3, the maximal profits are

$$\Pi^H = \ell_m - \frac{\beta\theta c \ell_m}{\ell_M}, \quad \Pi^L = \ell_m - \frac{\beta c \ell_m}{\ell_M}, \tag{22}$$

achieved by setting $(P_M, P_m) = (\ell_M, \ell_m)$. Comparing the maximal payoffs immediately leads to our claim. □

LEMMA 5. *Both types of experts posting a price list* $(P_M, P_m) = (\ell_M, \ell_m)$ *indeed forms a perfect Bayesian Nash equilibrium of the whole game.*

Proof. We need to show that the experts have no incentive to unilaterally deviate from the price list $(P_M, P_m) = (\ell_M, \ell_m)$ at the price setting stage. There are two ways of possible unilateral deviations: i) a single price P'; ii) a price list $(P'_M, P'_m) \neq (\ell_M, \ell_m)$. Consider a deviation to a single price P'. We construct the off-equilibrium path belief as follows: the consumer believes the deviating expert is a high-cost type, if $P' \leq \theta c$ her problem is minor and if $P' > \theta c$ her problem is major with probability β. Given her beliefs, the consumer's optimal strategy following a price P' is to accept an offer if $P' \leq \ell_m$ and decline if $P' > \ell_m$ because $\bar{\ell} < c < \theta c$ and no expert offers to repair a major problem at price $\bar{\ell}$. Given the consumer's strategy, the expert's deviation profit is bounded from above by $(1-\beta)\ell_m$, which is lower than the expert's expected profit from proposing $(P_M, P_m) = (\ell_M, \ell_m)$. A similar argument shows that unilateral deviation to (P'_M, P'_m) is not profitable either. □

Lemma 6. *There exists no uniform price equilibrium in which the two types of experts post the same single price or separating equilibrium in which the two types post different prices.*

Proof. Suppose there is indeed a uniform price equilibrium with a single price P, due to the assumption $\bar{\ell} < c$, there is no equilibrium in which the experts are willing to repair both types of problems at a single price. In a uniform price equilibrium, if both types rejects to treat the consumer if $k = M$, the maximal profits for both types of experts are $(1-\beta)\ell_m$, which is obviously dominated, as shown in Lemma 4. Thus, the equilibrium strategy profile in a uniform price equilibrium must involve

$$\phi^H = 1, \quad \phi^L = 0, \quad \rho^L = \rho^H = 1, \quad \lambda = 1. \tag{23}$$

That is, a high-cost expert rejects to repair a major problem. A low-cost expert always offers to repair the consumer's problem at price P and a high-cost expert offers to repair the consumer's problem at price P if $k = m$. Given these strategies, if a consumer rejects an offer at price P, her expected payoff is

$$u^R = -\frac{\beta\gamma}{\beta\gamma + (1-\beta)}\ell_M - \frac{1-\beta}{\beta\gamma + (1-\beta)}\ell_m. \tag{24}$$

The consumer only accepts the offer with positive probability if

$$P \leq \frac{\beta\gamma}{\beta\gamma + (1-\beta)}\ell_M + \frac{1-\beta}{\beta\gamma + (1-\beta)}\ell_m.$$

Note that

$$\frac{\beta\gamma}{\beta\gamma + (1-\beta)}\ell_M + \frac{1-\beta}{\beta\gamma + (1-\beta)}\ell_m \leq \bar{\ell} < c, \tag{25}$$

a low-cost expert is not willing to repair a major problem at P. Thus there exists no feasible price such that the strategies in (23) form an equilibrium.

Two types of separating equilibria candidates are to be considered. i) The two types of experts post different price list (P_M^H, P_m^H) and (P_M^L, P_m^L) which perfectly reveals the type of the expert. The consumers accepts an offer at P_M^t with probability $\lambda^t = \frac{P_m^t}{P_M^t}$. Note that the payoffs of the experts are $\Pi^t = P_m^t$. To prevent imitation, it is necessary that $P_m^H = P_m^L$. Therefore, $P_M^H \neq P_M^L$ in a separating equilibrium. Suppose $P_M^H > P_M^L$. Then, for a low-cost expert, posting (P_M^L, P_m^L) brings a payoff equal to P_m^L. Posting (P_M^H, P_m^H) instead brings a payoff

$$\beta(P_M^H - c)\frac{P_m^H}{P_M^H} + (1-\beta)P_m^H > P_m^L \tag{26}$$

using $P_m^H = P_m^L$. Thus, there exists no separating equilibrium with $P_M^H > P_M^L$. The same is true if $P_M^H < P_M^L$.

ii) One type posts a single price and the other type posts a price list. Suppose a low-cost expert posts a single price $\bar{P}$ and a high-cost expert posts a price list (P_M^H, P_m^H). For the consumer to accept $\bar{P}$, $\bar{P} \leq \bar{\ell}$ has to hold. By our assumption, $\bar{\ell} < c$, thus a low-cost expert is not willing to repair the consumer's problem at a price $\bar{P}$ that is always acceptable to the consumer. The same logic applies if a high-cost expert posts a single price. □

Proof of Propositions 2 to 4

To prepare for the proofs of Propositions 2 to 4, we characterize the equilibria of the subgame following an exogenous pooling price list (P_M, P_m) in Lemma 7 to 10.

Lemma 7 (Low-cost expert partially overtreats, high-cost expert behaves honestly). *Suppose the two types of experts have posted the same price list* (P_M, P_m) *such that* $(P_M, P_m) \in [\max\{\theta c, P_m + c, \frac{\beta\ell_M + \gamma(1-\beta)\ell_m}{\gamma(1-\beta)+\beta}\}, \ell_M] \times [0, \ell_m]$. *The following strategies form an equilibrium of the recommendation subgame:*

$$\phi^H = \phi^L = 0, \quad \rho^H = 0, \quad \rho^L = \frac{\beta(\ell_M - P_M)}{\gamma(1-\beta)(P_M - \ell_m)}, \quad \lambda = \frac{P_m}{P_M - c}, \tag{27}$$

in which a high-cost expert treats the consumer honestly and a low-cost type expert partially overtreats. The experts' expected payoffs are respectively

$$\Pi^H = \beta(P_M - \theta c)\frac{P_m}{P_M - c} + (1-\beta)P_m, \qquad \Pi^L = P_m. \tag{28}$$

Proof. Suppose the experts have indeed posted the same price list that satisfies $(P_M, P_m) \in$

$[\max\{\theta c, P_m + c, \frac{\beta\ell_M+\gamma(1-\beta)\ell_m}{\gamma(1-\beta)+\beta}\}, \ell_M] \times [0, \ell_m]$. Since $P_M \geq \theta c$, both types of experts are willing to repair a major problem. $P_M \geq P_m + c$ ensures $\lambda \leq 1$. $P_M \geq \frac{\beta\ell_M+\gamma(1-\beta)\ell_m}{\gamma(1-\beta)+\beta}$ ensures $\rho^L \leq 1$.

Given $k = m$, a high-cost expert receives P_m by recommending $T = m$ and receives $\lambda(P_M - \theta c)$ by recommending $T = M$ and being rejected with probability $1 - \lambda$. Given the consumer's strategy, $P_m > \frac{P_m}{P_M - c}(P_M - \theta c)$ because $\theta > 1$, and thus $\rho^H = 0$ is the high-cost expert's optimal choice.

Given $k = m$, a low-cost expert receives P_m by recommending $T = m$ and receives $\lambda(P_M - c)$ by recommending $T = M$ and being declined with probability $1 - \lambda$. Given the consumer's strategy, the two payoffs are the same and thus any $\rho^L \in [0, 1]$ is optimal for the low-cost type expert.

Given the strategies of the expert, a consumer accepting $T = M$ treatment receives utility $u^A = -P_M$ and by declining the major treatment she expects a loss

$$
\begin{aligned}
u^R &= -\left(\frac{(1-\beta)\gamma\rho^L}{\beta + (1-\beta)\gamma\rho^L}\ell_m + \frac{\beta}{\beta + (1-\beta)\gamma\rho^L}\ell_M\right) \\
&= -P_M = u^A. \qquad (29)
\end{aligned}
$$

Therefore, any $\lambda \in [0, 1]$ is an optimal strategy for the consumer. Thus, given a price list satisfying $P_M \in [\max\{\theta c, P_m + c, \frac{\beta\ell_M+\gamma(1-\beta)\ell_m}{\gamma(1-\beta)+\beta}\}, \ell_M]$ and $P_m \leq \ell_m$, the strategies of the expert and the consumer in (27) are mutually best responses. The experts' expected payoffs follow from the given strategies. □

LEMMA 8 (Efficient Outcome). *Suppose $\ell_m \geq (\theta - 1)c$. Suppose both types of experts have posted the same price list with (P_M, P_m) with $(P_M, P_m) \in [\theta c, P_m + c] \times [(\theta - 1)c, \ell_m]$. The following strategies form an equilibrium of the recommendation subgame:*

$$\phi^H = \phi^L = 0, \quad \rho^L = \rho^H = 0, \quad \lambda = 1. \qquad (30)$$

Both types of experts treat the consumer honestly. The ex ante expected payoffs of the experts are respectively

$$\Pi^H = \beta(P_M - \theta c) + (1 - \beta)P_m, \qquad \Pi^L = \beta(P_M - c) + (1 - \beta)P_m \qquad (31)$$

Proof. Since the prices satisfy $\theta c \leq P_M \leq P_m + c$ and $P_m \geq (\theta - 1)c$, both types of experts are willing to treat a major problem.

Given the strategy of the consumer, if $k = m$, for a high-cost expert, recommending a major treatment brings a payoff $P_M - \theta c$ and recommending a minor treatment brings a payoff P_m. Note that $P_M - \theta c < P_M - c$. Since $P_M \leq P_m + c$, $P_M - \theta c < P_m$, and a high-cost expert will recommend $T = m$ truthfully, that is, $\rho^H = 0$.

For a low-cost expert, if $k = m$, recommending a major treatment brings a payoff $P_M - c$ while recommending a minor treatment brings a payoff P_m. Because $P_M - c \leq P_m$, a low-cost expert has no incentive to overtreat the consumer, that is, $\rho^L = 0$.

Given the strategies of the experts, if the consumer accepts a recommendation of major treatment, her payoff is $-P_M$, and rejecting the recommendation brings a payoff $-\ell_M$. Because $P_M \leq P_m + c \leq \ell_m + c \leq \ell_M$, accepting the recommendation is an optimal choice for the consumer. The expected payoffs of the experts follow from the strategies. □

Lemma 9 (High-cost expert dumps and low-cost type overtreats). *Suppose both types of experts have posted the same price list* (P_M, P_m) *such that* $(P_M, P_m) \in [\max\{P_m + c, \frac{\beta\ell_M + \gamma(1-\beta)\ell_m}{\gamma(1-\beta)+\beta}\}, \theta c) \times [0, \ell_m]$. *If* $\gamma > \frac{\beta(\ell_M - \theta c)}{(\theta c - \ell_m)(1-\beta)}$, *the following strategies form an equilibrium of the recommendation subgame:*

$$\phi^H = 1, \quad \phi^L = 0, \quad \rho^H = 0, \quad \rho^L = \frac{\beta(\ell_M - P_M)}{\gamma(1-\beta)(P_M - \ell_m)}, \quad \lambda = \frac{P_m}{P_M - c}. \tag{32}$$

A high-cost expert rejects to treat a major problem and a low-cost expert partially overtreats. The expected payoffs of the experts are

$$\Pi^H = (1-\beta)P_m, \qquad \Pi^L = P_m. \tag{33}$$

Proof. Given $\max\{P_m + c, \frac{\beta\ell_M + \gamma(1-\beta)\ell_m}{\gamma(1-\beta)+\beta}\} \leq P_M < \theta c$, a high-cost expert rejects to repair a major problem because $P_M - \theta c < 0$ and a low-cost expert is willing to to repair a major problem because $P_M \geq P_m + c > c$. $P_M \geq \frac{\beta\ell_M + \gamma(1-\beta)\ell_m}{\gamma(1-\beta)+\beta}$ ensures $\rho^L \leq 1$. The condition $\gamma > \frac{\beta(\ell_M - \theta c)}{(\theta c - \ell_m)(1-\beta)}$ ensures that $\frac{\beta\ell_M + \gamma(1-\beta)\ell_m}{\gamma(1-\beta)+\beta} < \theta c$ so that a feasible P_M indeed exists.

If $k = m$, a high-cost expert obviously has no incentive to overtreat because $P_M < \theta c$. For a low-cost expert, overtreating brings a payoff $\lambda(P_M - c) = P_m$, which is the same as his payoff from honest recommendation. Therefore, any $\rho^L \in [0, 1]$ is a best response for a low-cost expert.

For the consumer, accepting a recommendation of major treatment brings a payoff of

$-P_M$, and rejecting such a recommendation brings a payoff

$$\begin{aligned} u^R &= -\left(\frac{(1-\beta)\gamma\rho^L}{\beta+(1-\beta)\gamma\rho^L}\ell_m + \frac{\beta}{\beta+(1-\beta)\gamma\rho^L}\ell_M\right) \\ &= -P_M. \end{aligned} \tag{34}$$

Since the payoffs from rejecting and accepting a major treatment is the same, any $\lambda \in [0,1]$ is a best response for the consumer. The payoffs of the experts follow from the strategies. □

Lemma 10 (Low-cost type fully overtreats and high-cost type partially overtreats). *Suppose both types of experts have posted the same price with* $(P_M, P_m) \in [P_m + \theta c, \frac{\beta\ell_M+\gamma(1-\beta)\ell_m}{\beta+\gamma(1-\beta)}] \times [0, \min\{\ell_m, \frac{\beta\ell_M+\gamma(1-\beta)\ell_m}{\beta+\gamma(1-\beta)} - \theta c\}]$. *If* $\gamma \leq \frac{\beta(\ell_M - c\theta)}{(1-\beta)(c\theta-\ell_m)} \in [0,1]$, *the following strategies form an equilibrium of the recommendation subgame:*

$$\phi^H = \phi^L = 0, \quad \rho^L = 1, \quad \rho^H = \frac{\beta(\ell_M - P_M)}{(1-\gamma)(1-\beta)(P_M-\ell_m)} - \frac{\gamma}{1-\gamma}, \quad \lambda = \frac{P_m}{P_M - \theta c}. \tag{35}$$

The expected payoffs of the experts are $\Pi^L = P_m(1 + \frac{(\theta-1)c}{P_M-\theta c})$ *and* $\Pi^H = P_m$.

Proof. Suppose the prices indeed satisfy $P_m \leq \min\{\ell_m, \frac{\beta\ell_M+\gamma(1-\beta)\ell_m}{\beta+\gamma(1-\beta)} - \theta c\}$, and $P_m + \theta c \leq P_M \leq \frac{\beta\ell_M+\gamma(1-\beta)\ell_m}{\beta+\gamma(1-\beta)}$. Note that $\gamma \leq \frac{\beta(\ell_M-c\theta)}{(1-\beta)(c\theta-\ell_m)} \in [0,1]$ ensures that $\frac{\beta\ell_M+\gamma(1-\beta)\ell_m}{\beta+\gamma(1-\beta)} - \theta c \geq 0$ so that a feasible P_m indeed exists.

Note that both types of experts are willing to treat a major problem because $P_M \geq P_m + \theta c \geq \theta c$. Given the consumer's strategy, for a low-cost expert, if $k = m$, recommending $T = m$ brings him a payoff P_m, while recommending a major treatment brings a payoff $\lambda(P_M - c) = \frac{P_m}{P_M-\theta c}(P_M - c) > P_m$. Thus $\rho^L = 1$ is optimal for a low-cost expert.

For a high-cost expert, for $k = m$, recommending a major treatment brings a payoff $\lambda(P_M - \theta c) = P_m$, the same as his payoff from recommending a minor treatment. Thus, any $\rho^H \in [0,1]$ is optimal.

When recommended a major treatment, the consumer updates her belief that she is visiting either a high-cost expert who recommends M with probability $\frac{\beta(\ell_M-P_M)}{(1-\gamma)(1-\beta)(P_M-\ell_m)} - \frac{\gamma}{1-\gamma}$ when $k = m$ or a low-cost expert who always recommends M for her. By accepting a major treatment, her problem is resolved and her payoff is $-P^M$. By rejecting a major treatment, the consumer's expected payoff is

$$u^R = -\frac{\beta}{\beta+(1-\beta)(\gamma+(1-\gamma)\rho^H)}\ell_M - \frac{(1-\beta)(\gamma+(1-\gamma)\rho^H)}{\beta+(1-\beta)(\gamma+(1-\gamma)\rho^H)}\ell_m = -P_M$$

Thus any $\lambda \in [0,1]$ is optimal for the consumer. The expected payoffs of the experts follow from the given strategies. □

Proof of Proposition 2. We first show that the price list $(P_M, P_m) = (\ell_M, \ell_m)$ is optimal for both types of experts. We then show that $(P_M, P_m) = (\ell_M, \ell_m)$ is sustained as a perfect Bayesian Nash equilibrium of the whole game.

Note that if $\gamma > \frac{\beta(\ell_M - c\theta)}{(1-\beta)(c\theta - \ell_m)}$ holds, the subgame equilibrium in Lemma 10 does not exist. Thus it suffices to compare the subgame equilibria in Lemma 7 to Lemma 9 to characterize the experts' optimal choice of prices and the equilibrium outcome of the entire game.

In Lemma 7, Π^H and Π^L reach their maximal values

$$\Pi^H = \beta(\ell_M - \theta c)\frac{\ell_m}{\ell_M - c} + (1-\beta)\ell_m = \ell_m\left(1 - \frac{\beta(\theta-1)c}{\ell_M - c}\right), \qquad \Pi^L = \ell_m \tag{36}$$

with price list $(P_M, P_m) = (\ell_M, \ell_m)$.

In Lemma 8, the maximal values of the experts' payoffs are

$$\Pi^H = \beta(P_M - \theta c) + (1-\beta)P_m = \beta(\ell_m - (\theta-1)c) + (1-\beta)\ell_m,$$
$$\Pi^L = \beta(P_M - c) + (1-\beta)P_m = \ell_m,$$

achieved with price list $(P_M, P_m) = (\ell_m + c, \ell_m)$.

In Lemma 9, the maximal values of the experts' payoffs are

$$\Pi^H = (1-\beta)\ell_m, \qquad \Pi^L = \ell_m. \tag{37}$$

achieved with $P_m = \ell_m$.

Direct comparison shows that the expected payoffs of both types of experts are the highest in the equilibrium outcome in Lemma 7 with price list $(P_M, P_m) = (\ell_M, \ell_m)$.

Furthermore, as under the institution of liability, there exists no uniform price equilibrium in which the two types of experts post the same single price or separating equilibrium in which the two types post different prices. By constructing the consumer's beliefs in the same way as in Proposition 1, we can show that the experts have no incentive to unilaterally deviate from price list $(P_M, P_m) = (\ell_M, \ell_m)$ at the price setting stage.

□

Proof of Proposition 3. We first show that $(P_M, P_m) = (\ell_m + \theta c, \ell_m)$ is optimal choice of prices for the experts. We then confirm this price list can be sustained as an equilibrium of the whole game.

If $\gamma \leq \frac{\beta(\ell_M - c\theta)}{(1-\beta)(c\theta - \ell_m)} \in [0,1]$, the subgame equilibrium in Lemma 9 does not exist while the subgame equilibrium in Lemma 10 holds. Note that comparison of the experts' payoffs in Lemma 7 and 8 is the same as in Proposition 2. Thus, we need to compare the outcome in Lemma 10 with the outcome in Proposition 2 for the optimal price choice of the experts and the equilibria of the whole game.

Let's first consider the outcome in Lemma 10. Since $\gamma \leq \frac{\beta(\ell_M - \ell_m - \theta c)}{(1-\beta)\theta c}$, $P_m \leq \ell_m$ because $\ell_m \leq \frac{\beta\ell_M + \gamma(1-\beta)\ell_m}{\beta + \gamma(1-\beta)} - \theta c$. Further note that both $\Pi^L = P_m(1 + \frac{(\theta-1)c}{P_M - \theta c})$ and $\Pi^H = P_m$ increase in P_m, and Π^L decreases in P_M. Both types of the experts' payoffs are maximized by setting $P_m = \ell_m$ and $P_M = \ell_m + \theta c$. The experts' maximal payoffs are given by

$$\Pi^H = \ell_m, \qquad \Pi^L = \ell_m + (\theta - 1)c, \tag{38}$$

both of which are higher than those from Proposition 2.

Again, as under the institution of liability, there exists no uniform price equilibrium in which the two types of experts post the same single price or separating equilibrium in which the two types post different prices. Finally, since the profits of both types of experts in Proposition 3 are larger than those in Proposition 2, we can construct the consumer's off-equilibrium path beliefs in the same way to guarantee that the experts have no incentive to unilaterally deviate from the price list $(P_M, P_m) = (\ell_m + \theta c, \ell_m)$ at the price setting stage. □

Proof of Proposition 4. If $\frac{\beta(\ell_M - \ell_m - \theta c)}{(1-\beta)\theta c} < \gamma \leq \frac{\beta(\ell_M - c\theta)}{(1-\beta)(c\theta - \ell_m)}$, the recommendation subgame equilibrium in Lemma 10 exists. For the optimal prices of the experts, we need to compare the experts' payoffs in recommendation subgames in Lemma 7, 8 and 10. The optimal prices for the experts in Lemma 10 are $P_m = \delta - \theta c$ and $P_M = \delta$ in which $\delta := \frac{\beta\ell_M + \gamma(1-\beta)\ell_m}{\beta + \gamma(1-\beta)}$. The maximal payoffs of the experts are

$$\tilde{\Pi}^H = \delta - \theta c, \qquad \tilde{\Pi}^L = \delta - c. \tag{39}$$

The experts' payoffs in the subgame equilibrium in Lemma 8 are always weakly dominated by those in Lemma 7 due to our assumption $\ell_M > \ell_m + \theta c$. The experts' expected payoffs in Lemma 7 are

$$\Pi^H = \ell_m(1 - \frac{\beta(\theta - 1)c}{\ell_M - c}), \quad \Pi^L = \ell_m. \tag{40}$$

Therefore, which equilibrium is Pareto dominating for the experts depend on the comparison of $\tilde{\Pi}^t$ and Π^t.

Define $\hat{\gamma}_1$ as the solution to $\frac{\beta\ell_M+\hat{\gamma}_1(1-\beta)\ell_m}{\beta+\hat{\gamma}_1(1-\beta)} - \theta c = \ell_m(1-\frac{\beta(\theta-1)c}{\ell_M-c})$ and $\hat{\gamma}_2$ as the solution to $\frac{\beta\ell_M+\hat{\gamma}_2(1-\beta)\ell_m}{\beta+\hat{\gamma}_2(1-\beta)} - c = \ell_m$. Notice that $\frac{d\delta}{d\gamma} < 0$, so $\frac{d\tilde{\Pi}^H}{d\gamma} < 0$ and $\frac{d\tilde{\Pi}^L}{d\gamma} < 0$. Furthermore, $\theta c + \ell_m(1-\frac{\beta(\theta-1)c}{\ell_M-c}) > c + \ell_m$, then $\hat{\gamma}_2 > \hat{\gamma}_1$. Note that $\hat{\gamma}_1$ always falls in the range of $(\frac{\beta(\ell_M-\ell_m-\theta c)}{(1-\beta)\theta c}, \frac{\beta(\ell_M-c\theta)}{(1-\beta)(c\theta-\ell_m)}]$, while $\hat{\gamma}_2$ falls in the range of $(\frac{\beta(\ell_M-\ell_m-\theta c)}{(1-\beta)\theta c}, \frac{\beta(\ell_M-c\theta)}{(1-\beta)(c\theta-\ell_m)})$ if $(\theta-1)c < \ell_m$, and $\hat{\gamma}_2 \geq \frac{\beta(\ell_M-c\theta)}{(1-\beta)(c\theta-\ell_m)}$ if $\ell_m \leq (\theta-1)c$.

If $\ell_m > (\theta-1)c$, there exists unique $\hat{\gamma}_1$ and $\hat{\gamma}_2$ such that $\frac{\beta(\ell_M-\ell_m-\theta c)}{(1-\beta)\theta c} < \hat{\gamma}_1 < \hat{\gamma}_2 < \frac{\beta(\ell_M-c\theta)}{(1-\beta)(c\theta-\ell_m)}$. Then we have

1. If $\gamma \leq \hat{\gamma}_1$, then $\tilde{\Pi}^H = \delta - \theta c \geq \Pi^H = \ell_m(1-\frac{\beta(\theta-1)c}{\ell_M-c})$ and $\tilde{\Pi}^L = \delta - c > \Pi^L = \ell_m$. Both types of experts are better off in the most profitable equilibrium outcome in Lemma 10 with price list $(P_M, P_m) = (\delta, \delta-\theta c)$ than in the equilibrium outcome in Lemma 7.

   Plugging $P_M = \delta$ and $P_m = \delta - \theta c$ into (35) gives $\rho^H = 0$. Thus, in this Pareto dominating equilibrium, a low-cost expert cheats but a high-cost expert behaves truthfully. We refer to this case as the "low-cost type overtreating outcome".

2. If $\gamma \geq \hat{\gamma}_2$, then $\tilde{\Pi}^H = \delta - \theta c < \Pi^H = \ell_m(1-\frac{\beta(\theta-1)c}{\ell_M-c})$ and $\tilde{\Pi}^L = \delta - c \leq \Pi^L = \ell_m$. Both types of experts prefer the most profitable equilibrium outcome in Lemma 7 with price list $(P_M, P_m) = (\ell_M, \ell_m)$ over the equilibrium outcome in Lemma 10. This is the same "no-cheating outcome" as in Proposition 2.

3. If $\hat{\gamma}_1 < \gamma < \hat{\gamma}_2$, then $\tilde{\Pi}^H = \delta - \theta c < \tilde{\Pi}^H = \ell_m(1-\frac{\beta(\theta-1)c}{\ell_M-c})$ and $\tilde{\Pi}^L = \delta - c > \Pi^L = \ell_m$. No Pareto-dominating equilibrium exists. Either the most profitable outcome in Lemma 7 or 10 may emerge as an equilibrium in the whole game.

If $\ell_m \geq (\theta-1)c$, there exists a unique $\hat{\gamma}_1$ such that $\frac{\beta(\ell_M-\ell_m-\theta c)}{(1-\beta)\theta c} < \hat{\gamma}_1 < \frac{\beta(\ell_M-c\theta)}{(1-\beta)(c\theta-\ell_m)}$, but $\hat{\gamma}_2 \geq \frac{\beta(\ell_M-c\theta)}{(1-\beta)(c\theta-\ell_m)}$. As a result, if $\gamma \leq \hat{\gamma}_1$, the Pareto dominating equilibrium is the low-cost type overtreating outcome. If $\gamma > \hat{\gamma}_1$, the two types of experts prefer different equilibrium outcome. Either the low-cost type overtreating outcome or the no-cheating outcome may emerge in the whole game.

To make the proof complete, we can construct the off-equilibrium path belief of the consumers in the same way as that in Proposition 2 to guarantee that the experts have no incentive to unilaterally deviate from the optimal price list stated in the proposition at the price setting stage. □

Proof of Proposition 5. If $\gamma > \frac{\beta(\ell_M - c\theta)}{(1-\beta)(c\theta - \ell_m)}$, comparing the social welfare level in Corollary 2 and in Corollary 1 gives

$$\begin{aligned}
\Delta W &= \Delta_1 - W_L \\
&= -\beta \frac{\ell_m}{\ell_M - c} (\gamma c + (1-\gamma)\theta c) - \beta \frac{\ell_M - c - \ell_m}{\ell_M - c} \ell_M - \left(-\beta(\ell_M - \ell_m) - \beta \frac{\ell_m}{\ell_M} (\gamma c + (1-\gamma)\theta c) \right) \\
&= \frac{\beta \ell_m c}{(\ell_M - c)\ell_M} (\ell_M - (\gamma c + (1-\gamma)\theta c)) > 0 \qquad (41)
\end{aligned}$$

in which the inequality obtains because $\ell_M > \gamma c + (1-\gamma)\theta c$.

If $\gamma \leq \frac{\beta(\ell_M - \ell_m - \theta c)}{(1-\beta)\theta c}$, comparing the social welfare level in Corollary 3 and 1 gives

$$\begin{aligned}
\Delta W &= \Delta_2 - W_L \\
&= -\beta(\ell_M - \ell_m) + \gamma(\theta - 1)c - \left(-\beta(\ell_M - \ell_m) - \beta \frac{\ell_m}{\ell_M} (\gamma c + (1-\gamma)\theta c) \right) \qquad (42) \\
&= \gamma(\theta - 1)c + \beta \frac{\ell_m}{\ell_M} (\gamma c + (1-\gamma)\theta c) > 0 \qquad (43)
\end{aligned}$$

If $\frac{\beta(\ell_M - \ell_m - \theta c)}{(1-\beta)\theta c} < \gamma \leq \frac{\beta(\ell_M - \theta c)}{(1-\beta)(\theta c - \ell_m)}$, the social welfare level is either Δ_1 or Δ_3 (see Corollary 4). We have already shown that Δ_1 is larger than W_L. Further note that the equilibrium outcome that delivers social welfare level Δ_3 is such that only a low-cost expert overtreats, thus $\Delta_3 > \Delta_2$ must hold. Therefore, $\Delta_3 > W_L$.

In summary, the social welfare achieved in the equilibrium outcomes under LV is always higher than that under only liability. Therefore, adding verifiability improves social welfare.

Next we show how ΔW varies with γ. Notice that the social welfare level under LV is either Δ_1, Δ_2 or Δ_3. If $\Delta W = \Delta_1 - W_L$, we have

$$\frac{\partial \Delta W}{\partial \gamma} = \frac{\beta \ell_m c}{(\ell_M - c)\ell_M} (\theta - 1)c > 0. \qquad (44)$$

If $\Delta W = \Delta_2 - W_L$, we have

$$\frac{\partial \Delta W}{\partial \gamma} = (1 - \beta \frac{\ell_m}{\ell_M})(\theta - 1)c > 0. \qquad (45)$$

If $\Delta W = \Delta_3 - W_L$, we have

$$\frac{\partial \Delta W}{\partial \gamma} = \beta(1 - \frac{\ell_m}{\ell_M})\theta c - (1 - \beta \frac{\ell_m}{\ell_M})c, \qquad (46)$$

the sign of which is indeterminate. This is the case in the low-cost type overtreating outcome in Proposition 4. □

Proof of Proposition 6. Suppose $\gamma \leq (\frac{(1-\beta)(c-\ell_m)}{\beta(\ell_M-c)}, \frac{(1-\beta)(\theta c-\ell_m)}{\beta(\ell_M-\theta c)}]$ hold.

1. Given the expert's strategies, the consumer's strategy is optimal. If the expert offers to repair the problem at price $\bar{p}$, the consumer believes that her problem is major with probability $\frac{\gamma\beta}{\gamma\beta+1-\beta}$ and minor with probability $\frac{1-\beta}{\gamma\beta+1-\beta}$. Thus, the consumer's expected utility from rejecting the offer is

$$u^R = \frac{\gamma\beta}{\gamma\beta+1-\beta}\ell_M + \frac{1-\beta}{\gamma\beta+1-\beta}\ell_m \tag{47}$$

   which equals $\bar{p}$, the price she pays for accepting the offer and getting her problem resolved. Thus, the consumer accepts a repair offer at price $\bar{p}$ with probability 1.

2. We characterize the consumer's equilibrium strategy in the recommendation subgame following a single price $p' \neq \bar{p}$. The off-equilibrium path belief is such that a price deviation at the first stage is made by a high-cost type, and if $p' < \theta c$, her problem is minor and if $p' \geq \theta c$ her problem is major with probability β. Given her belief, the consumer only accepts a repair offer at price $p' \leq \ell_m$. Accepting a repair offer at price $p' \in (\ell_m, \theta c)$ will result in a loss $\ell_m - p'$. Accepting a repair offer at price $p' \in [\theta c, \ell_M]$ results in a loss $p' - \bar{\ell}$ because $\bar{\ell} < \theta c$ by assumption.

   We now characterize the consumer's equilibrium strategy in the recommendation subgame following a price list (P_m, P_M), with $P_m < P_M$. The consumer's off-equilibrium path belief is constructed similarly as before. She believes that the price deviation is made by a high-cost expert. At the recommendation stage, the expert either recommends to repair the consumer's problem at price $\tilde{p} \in \{P_m, P_M\}$ or declines to treat the consumer. Upon being recommended $\tilde{p}$, the consumer believes her problem is minor for $\tilde{p} < \theta c$ and major with probability β for $\tilde{p} \geq \theta c$. For the same argument in the last paragraph, the consumer only accepts an offer of repairing the problem at $\tilde{p} \leq \ell_m$.

3. A high-cost expert. In the recommendation subgame following $\bar{p}$, a high-cost expert will offer to repair the consumer's problem only in state $k = m$ because $\bar{p} < \theta c$. If he deviates to a different price p', since the consumer only accepts an offer with $p' \leq \ell_m$, the maximal profit the high-cost expert can achieve is $(1-\beta)\ell_m$ which is lower than

his profit from offering $\bar{p}$ and getting $(1-\beta)\bar{p}$. The same happens if the expert offers price list (P_m, P_M) with $P_M > P_m$.

4. A low-cost expert. Because $\bar{p} > c$, a low-cost expert receives a positive profit in both states by offering to repair the consumer's problem at price $\bar{p}$. Following the same argument in [3], a low-cost expert will offer the price $\bar{p}$ instead of other prices.

□

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