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Abstract: - This article is a first attempt to measure technical efficiency of the Greek Public Power Corporation, technical efficiency examined within the framework of stochastic production frontiers. The period 1970-'97 is examined and a Cobb-Douglas production function is used. The results demonstrate that the firm’s technical efficiency ranged between 82.5% and 100% and achieved its maximum performance in 1974 and 1992. These estimates are, in general terms, consistent with the findings of other researchers.

Keywords: Energy Sector, Electricity Generation, Technical Efficiency, Stochastic Frontier, D.E.A.

1. Introduction

When firms are not efficient and their inefficiency persists over time it is not easy for them to survive in competitive markets because it implies irrational allocation of resources. This is the reason why measuring technical efficiency is a very important task, especially for firms that already operate or are obliged to operate under such conditions (e.g. E.U. framework).

Farrell [1] was the first to provide us with the definition of technical (or productive) efficiency and until the late 1970s its empirical application was very limited. However, Aigner et al. [2] introduced the stochastic frontier production function, and Meeusen and van den Broeck [3] considered the Cobb-Douglas production function with a composed multiplicative disturbance term. Since then, Farrell’s idea became an important tool for estimating technical (in)efficiency of various sectors and industries.

There are two approaches to the construction of frontier production functions: On the one hand, there is the deterministic approach, which is alternatively called “Data Envelopment Analysis” (D.E.A.), and uses mathematical programming techniques. For a review see [4]. However, D.E.A. cannot discriminate between inefficiency and noise, and tends to produce overestimated (in)efficiency measures, while stochastic frontier models are based on the idea that the data are contaminated with noise. Consequently, on the other hand, there is the stochastic approach, which uses econometric techniques. For a survey of the literature see [5].

The present paper measures the extent of technical efficiency in the Greek Public Power Corporation (G.P.P.C.) in the 1970-1997 time span. The stochastic framework is used and the analysis is based on the assumption that the error term, in a statistically fitted production function, consists of two components: the conventional normal
distribution of random elements, and a one-sided distribution of non-random elements representing inefficiency. The results of this paper are compared with those obtained by Roboli and Tsolas [6] using the deterministic approach.

The paper is organized as follows: Section 2 gives a brief overview of the theoretical framework; Section 3 presents the methodological framework used. Section 4 presents the data used and the empirical results, comments on the findings and compares them with the results of the deterministic approach. Finally, section 5 concludes the paper.

2. Theory
As well known, the isoquant (II) of an economic activity X is the locus of all minimum combinations of capital (K/Y) and labor (L/Y) per unit of output required to produce one unit of X’s output, Y. Thus, the isoquant describes completely the technology of X. The relative prices of K and L are given by the line BC. As known, there exists one point A which expresses the least costly combination of inputs for producing the given quantity of output. The deviation of observed input-per-unit-of-output ratios from the isoquant, is considered to be associated with technical inefficiency of the firm involved.

For instance, if the input combination, in Figure 1 below, was D instead of A, then DG/OG would be a measure of technical inefficiency, defined as the proportional excess cost of inputs used over the feasible minimum cost G, using the input proportions indicated by OG. Note that G is technically efficient, because it lies on the isoquant II, but does not lie on the line BC meaning that it is not the least cost combination if factor prices are BC, i.e. it is price inefficient.

The ratio GF/OF measures price inefficiency (or allocative inefficiency) while the ratio OF/OD is the overall or economic efficiency of firm D and is the product of technical and price efficiency.

Thus, the presence of technical inefficiency implies that a greater amount of inputs is used for the production of a certain amount of output, than the amount of inputs that would have been required if the unit was technically efficient, under the assumption that production technology remains unchanged.

In other words, a greater than required amount of pollutants (embodied in the inputs) is used for the production of the same amount of output.
3. Methodology

Excellent reviews of the stochastic frontier model have been provided by various researchers, see [7]. The stochastic production frontier (SPF) is given by the following equation:

\[ y = f(x) \exp(v), \quad e = (v-u), \quad u \geq 0 \]

where: \( y \) is output, \( f(x) \) is the deterministic part of the frontier production function (FPF), \( v \) is a symmetrical random error and \( u \) is a one-sided positive error term representing technical efficiency. The elements of \( v \) represent the conventional normal distribution of random elements including measurement errors, minor omitted variables, and other exogenous factors beyond the firm’s control. The elements of \( u \) indicate shortfalls of the firm’s production units from the efficient frontier.

The economic logic of the so-called “composed error” specification is that production is subject to two random disturbances of different origin. The positive disturbance \( u \) expresses the fact that each firm’s output lies on or below its frontier. Any deviation is the result of factors controllable by the firm, such as technical efficiency, the capability of the producer and his employees, the defective and damaged products, etc. However, the frontier itself may vary randomly over time for the same firm and consequently the frontier is stochastic, with random disturbance \( v \), which expresses external to the firm events, such as luck, climate, as well as errors of observation and measurement of \( y \). Thus, productive efficiency may be measured by the following ratio:

\[ y / f(x) \exp(v) = \exp(-u). \]

Given a parametric functional form for \( f(x) \) and distributional assumptions about \( y \) and \( u \), equation (1) can be estimated by the Ordinary Least Squares (O.L.S.) method.

Equation (1) can be estimated using the Maximum Likelihood (ML) method [2]. However, the O.L.S. estimators have statistical properties at least as desirable as those of the ML estimators [8], are easier to obtain and tend to provide encouraging results [7].

More specifically, equation (1) is written as:

\[ \ln(y) = \ln[f(x)] + \nu - u \]
\[ \ln(y) = -\mu + \ln[f(x)] + (\nu-u+\mu) \]

where: \( \mu = E(u)>0 \).

Inserting \( \mu \), it is assumed that \( u \) and \( \nu \) are independent and identically distributed and are both independent of \( x \), so equation (2) satisfies the assumptions for the traditional O.L.S. except for the normality assumption of \( \nu-u+\mu \). Also, it is assumed that \( \ln[f(x)] \) is linear in the parameters, so that the O.L.S. would yield the Best Linear Unbiased Estimators (B.L.U.E.) of the parameters, except for the constant term, \( a_0 \), for which the bias will be \( -\mu \). Thus, the O.L.S. will give an unbiased estimator of \( a_0-\mu \). The estimation of the SPF by the O.L.S. leads to consistent estimators for all the parameters, \( \mu \) included, under the assumption that \( \nu \) is normally and \( u \) is half-normally distributed.

Half-normal and exponential distributions are (usually) employed for \( u \), however, these two assumptions lead to very similar estimates [9].

Estimation of equation (2) by O.L.S. gives the residuals \( e_i, \ i = 1, 2, \ldots, N \). The second and third central moments of the residuals, \( m_2(e) \) and \( m_3(e) \) respectively, are calculated, as known, as follows:

\[ m_2(e) = \frac{1}{N-k} \sum e_i^2 \]
\[ m_3(e) = \frac{1}{N-k} \sum e_i^3 \]

where: \( N \) is the number of observations and \( k \) is the number of regressors, the constant term included.

Then, we estimate \( \sigma_u^2 \) and \( \sigma_v^2 \) using the formulas [10]:

\[ \sigma_u^2 = \frac{[(\pi/2)(\pi/(\pi-4))m_2(e)]^{2/3}}{\pi} \]
\[ \sigma_v^2 = \frac{m_2(e) - [(\pi-2)/\pi] \sigma_u^2}{\pi} \]

Following Battese and Coelli [11], the point measure of technical efficiency \( \text{TE} \) is:

\[ \text{TE}_i = E(\exp[-u_i^2/2]) = \frac{1}{[1-F(\sigma_u^2/\sigma_v^2)]} \exp[-M_i^2 + (\sigma_u^2/2)] \]

where \( F \) denotes the distribution function of the standard normal variable. Also:

\[ M_i^2 = \frac{-\sigma_v^2}{\sigma_u^2} (\sigma_u^2 + \sigma_v^2)^{1/2} \]
\[ \sigma_u^2 = \sigma_v^2 \sigma_u^2 (\sigma_u^2 + \sigma_v^2)^{1/2} \]

4. Empirical Results and Discussion

4.1 Data and Variables

We use the Cobb-Douglas functional form to approximate production frontiers. The adopted functional form is:

\[ \ln Y = a_0 + a_1 \ln K + a_2 \ln L + a_3 \ln E + \nu - u \]

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\[ 1 \] Until the appearance of Jondrow et al. [12], only the mean technical (in)efficiency could be calculated. Later, Battese and Coelli [11] derived the point predictor of technical efficiency used in this paper.
where: $Y$ is a measure of output, $K$ a measure of capital stock, $L$ is a measure of labor, and $E$ a measure of energy spending.

The data employed are on an annual basis and cover the period 1970-1997. More specifically, output is measured through the generated electricity, labor is measured through the number of employees, and energy is measured through the fuels consumed. All the data come from corporate sources (balance sheets, etc.) except for the estimates of the capital stock that come from [6].

### 4.2 Estimates

Table 1 presents the estimate of the production function based on equation (7).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>T-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>intercept</td>
<td>-7.94</td>
<td>-6.30*</td>
</tr>
<tr>
<td>$a_1$</td>
<td>-0.05</td>
<td>-0.93</td>
</tr>
<tr>
<td>$a_2$</td>
<td>1.49</td>
<td>11.77*</td>
</tr>
<tr>
<td>$a_3$</td>
<td>0.43</td>
<td>4.90*</td>
</tr>
</tbody>
</table>

Note: * Significance at the 1% level

$R^2 = 96\%$

S.E.E. = 0.09

D.W. = 1.75

The estimated coefficients are statistically significant for all parameters, except for the capital stock. This result is expected and is related to capital’s utilization [13].

Also, the materials are not statistically significant and are, thus, not incorporated in the production function. The regression explains a very high 96% of the variability of output, and there is no evidence of autocorrelation of the residuals. Finally, it should be noted that the materials

Estimates in Table 1 imply that G.P.P.C. experienced increasing returns to scale over the period 1970-97.

The next step is, by utilizing equation (5), to estimate annual technical efficiency (T.E.) for the 1970-1997 time span, presented in Table 2. Table 3 presents corresponding estimates of technical efficiency ($W_0$) by [6] using mathematical programming techniques (D.E.A.), under the assumptions of non-constant returns to scale, of yearly activities regarded as Decision Making Units (D.M.U.) and of materials as an additional input.

### Table 2: Technical Efficiency Estimates

<table>
<thead>
<tr>
<th>Year</th>
<th>T.E.</th>
<th>Wo</th>
</tr>
</thead>
<tbody>
<tr>
<td>1970</td>
<td>0.8685</td>
<td>1.0000</td>
</tr>
<tr>
<td>1971</td>
<td>0.9468</td>
<td>1.0000</td>
</tr>
<tr>
<td>1972</td>
<td>0.9021</td>
<td>0.8659</td>
</tr>
<tr>
<td>1973</td>
<td>0.9353</td>
<td>0.8628</td>
</tr>
<tr>
<td>1974</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>1975</td>
<td>0.8950</td>
<td>1.0000</td>
</tr>
<tr>
<td>1976</td>
<td>0.8806</td>
<td>0.8754</td>
</tr>
<tr>
<td>1977</td>
<td>0.9589</td>
<td>0.9386</td>
</tr>
<tr>
<td>1978</td>
<td>0.9296</td>
<td>0.7983</td>
</tr>
<tr>
<td>1979</td>
<td>0.9773</td>
<td>0.8209</td>
</tr>
<tr>
<td>1980</td>
<td>0.8750</td>
<td>0.8100</td>
</tr>
<tr>
<td>1981</td>
<td>0.9630</td>
<td>0.8239</td>
</tr>
<tr>
<td>1982</td>
<td>0.9789</td>
<td>0.8323</td>
</tr>
<tr>
<td>1983</td>
<td>0.9838</td>
<td>0.8840</td>
</tr>
<tr>
<td>1984</td>
<td>0.9830</td>
<td>0.8477</td>
</tr>
<tr>
<td>1985</td>
<td>0.9837</td>
<td>0.8417</td>
</tr>
<tr>
<td>1986</td>
<td>0.9062</td>
<td>0.8157</td>
</tr>
<tr>
<td>1987</td>
<td>0.9125</td>
<td>0.8234</td>
</tr>
<tr>
<td>1988</td>
<td>0.9591</td>
<td>0.8706</td>
</tr>
<tr>
<td>1989</td>
<td>0.9739</td>
<td>0.8897</td>
</tr>
<tr>
<td>1990</td>
<td>0.9930</td>
<td>0.9305</td>
</tr>
<tr>
<td>1991</td>
<td>0.9932</td>
<td>0.9980</td>
</tr>
<tr>
<td>1992</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>1993</td>
<td>0.8250</td>
<td>0.9373</td>
</tr>
<tr>
<td>1994</td>
<td>0.8948</td>
<td>0.9328</td>
</tr>
<tr>
<td>1995</td>
<td>0.9167</td>
<td>0.9571</td>
</tr>
<tr>
<td>1996</td>
<td>0.9469</td>
<td>1.0000</td>
</tr>
<tr>
<td>1997</td>
<td>0.9867</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

It is evident that G.P.P.C. demonstrates technical efficiency measures ranging from 82.5% to 100%. Using a simple arithmetic average we obtain an average annual technical efficiency for the firm, higher than 90%, and equal to about 94%. Also, the firm’s technical efficiency measure reached its highest levels in 1974 and 1992.

With the aid of descriptive statistics which is used for the comparison of this sort of results [13], the estimated technical efficiency measures of the present paper are, in general terms, consistent with the findings of [6]. First, the findings of [6] show that G.P.P.C. experiences non-decreasing returns to scale
during the period 1970-1997, except for the last two years. Second, their findings demonstrate that technical efficiency ranges from 80% to 100%. Secondly, using a simple arithmetic average on the measure presented in their paper, we obtain an average annual technical efficiency higher than 90%, as in our paper, and equal to 91%. Finally, the firm’s technical efficiency measure is found to have reached its highest level in 1974 and 1992, as in this work. The comparison of the results of the two approaches can also be based on ranking of observations, etc [14].

Any differences between the two approaches’ results (e.g. in the arithmetic average technical efficiency) should not be surprising and are due to the fact that D.E.A. does not make any particular assumptions about the functional forms of the production function and mainly because D.E.A. cannot discriminate between inefficiency and noise.

5. Conclusion
The paper has measured technical efficiency in the Greek Public Power Corporation for the period 1970-1997. The theoretical framework used was the stochastic production frontier, and the functional form used was the Cobb-Douglas formulation. The estimated technical efficiency measure is obtained by using the OLS methodology. The results demonstrate a satisfactory level of technical efficiency ranging from 82.5% to 100%, with an (arithmetic) average of about 94% per year and its maximum performance in 1974 and 1992. These results are, in general terms, consistent with the findings by other researchers.

The previous analysis implies that the corporation has been experiencing increasing returns to scale and has been operating satisfactorily in terms of technical efficiency. However, there are some issues which are linked, not only to the corporation’s “internal” operation (G.P.P.C.’s privatization), but also to the privatized corporation’s position within the framework of the liberalized markets of the electricity industry, as well as of the national economies. It is evident, that if G.P.P.C. does not minimize its inefficiency in the long run, it will have problems surviving in the competitive energy market and will contribute negatively to (air) pollution.

Consequently, given the fact that the paper has not considered the concept of price efficiency and that this aspect is under current research, we believe that future research would be of great interest.

References