Investment decisions, net present value and bounded rationality

Carlo Alberto Magni

University of Modena and Reggio Emilia

August 2005

Online at http://mpra.ub.uni-muenchen.de/7451/
MPRA Paper No. 7451, posted 5. March 2008 13:06 UTC
Abstract.

The Net Present Value maximizing model has a respectable ancestry and is considered by most scholars a theoretically sound decision model. In real-life applications, decision makers use the NPV rule, but apply a subjectively determined hurdle rate, as opposed to the allegedly correct opportunity cost of capital. According to a heuristics-and-biases-program approach, this implies that the hurdle-rate rule is a biased heuristic. This paper shows that the hurdle-rate rule may be interpreted as a fruitful strategy of bounded rationality, where several important elements are integrated and condensed into an aspiration level. This paper also addresses the issue of a fruitful cooperation between bounded and unbounded rationality, of which the heuristic NPV is one significant example.

JEL codes: A11, A12, B41, C61, D46, D81, G11, G31, M21.

Keywords. Finance, methodology, investment decisions, net present value, hurdle rate, heuristics, bounded rationality, methodology.
Investment decisions, net present value and bounded rationality

Introduction

This paper deals with the Net Present Value (NPV) methodology, a widely used tool for investment decisions. It is considered theoretically sound and normatively suggested in many corporate finance textbooks (see Copeland & Weston, 1988; Bierman & Smidt, 1992; Rao, 1992; Damodaran, 1999; Copeland, Koller & Murrin, 2000; Brealey & Myers, 2000; Fernández, 2002). The concept of NPV is the brick of a normative building deep-rooted in the maximizing tradition of economics. The NPV methodology may be summarized as follows: suppose a decision maker holds a sum $X$ and is willing to invest it for one period. Suppose she has the opportunity of investing $X_0$ in a one-period project generating a payoff $X_1$ after one period. Let $r$ be the return rate of the next-best alternative available to the investor; the decision maker should invest in the project if and only if

$$X_1 - X_0 > rX.$$  \hspace{1cm} (1)

Eq. (1) may be rewritten as

$$-X_0 + \frac{X_1}{1+r} > 0.$$  \hspace{1cm} (2)

The left-hand side of (2) is called the project's Net Present Value. The NPV rule says that the decision maker should invest in the project if its NPV is positive. The discount rate $r$ is the so-called opportunity cost of capital: if a decision maker invests in the project she foregoes the opportunity of earning $r$ on her capital $X_0$. It is the rate of return of the best available alternative other than the one under consideration. The opportunity cost of capital is then taken as a norm (in the sense of Kahneman & Miller, 1986). In general, if one faces two or more mutually exclusive courses of action, one should calculate their NPVs by discounting cash flows at the respective opportunity cost of capital and choose the one with the highest NPV. The NPV model traces back to Fisher (1930), whose analysis is carried out under assumption of certainty. It may be formally derived from the following constrained optimization problem, where the investor must select the optimal consumption plan:

$$\max u(c_0, c_1) \quad \text{subject to} \quad (m_0 - X_0 - c_0)(1+r) + m_1 + X_1 = c_1$$  \hspace{1cm} (3)

where $u(\cdot)$ denotes the investor’s utility function, $r$ is the market interest rate and $c_0$, $m_t$, and $X_t$ are, respectively, the consumption, the income, and the investment flow at time $t = 1, 2$. Eq. (3) formally describes the so-called Fisher model, which provides for a separation theorem (optimal investment plan is the same regardless of the optimal consumption bundle) and for a derivation of the net present value rule as
in eq. (2) (see McMinn, 2005). Under uncertainty, we cannot say *a priori* that, for example, an *expected* return of 100 is better than an *expected* return of 90 if the former is riskier than the latter. None of the two dominates the other: one is preferable in terms of return, the other one is preferable in terms of risk (risk-aversion is assumed). So, if our alternatives are not equivalent in risk, the NPV rule may not be employed as such, because it only creates a *partial* ordering among alternatives. There is the need of finding a theoretically correct cost of capital accounting for the riskiness of the project. Unanimously, the opportunity cost of capital “should equal the return the investor could have earned on other investment opportunities with comparable risk characteristics” (Dixit & Pindyck, 1994, p. 114). Among other models, portfolio theory (Markowitz, 1952) and the Capital Asset Pricing Model (CAPM) (Sharpe, 1964; Mossin, 1966) have been developed in corporate finance for explaining the relation between risk and return. The mathematical link between these models and the NPV rule has been provided by several scholars, who consistently prove that maximization of shareholders wealth is equivalent to NPV maximization, where the cost of capital $r$ in eq. (2) is computed by making use of the CAPM’s security market line (see Tuttle & Litzenberger, 1968; Hamada, 1969; Bierman & Hass, 1973; Mossin, 1969; Rubinstein, 1973).

A more recent version of the NPV rule is the “expanded NPV” (Trigeorgis, 1986, Dixit & Pindyck, 1994), which makes use of arbitrage pricing or, alternatively, stochastic dynamic programming. It aims at taking into account possible options intrinsic in the project (option to wait, to abandon, to switch, to temporarily suspend, to enlarge scale, etc.). Proponents of the expanded NPV often call their approach “real options approach”, which is but a sophisticated version of the traditional NPV model, where the set of alternatives is inclusive of the options implicit in the project: “one can always redefine NPV by subtracting from the conventional calculation the opportunity cost of exercising the option to invest, and then say that the rule ‘invest if NPV is positive’ holds once this correction has been made” (Dixit and Pindyck p. 7). In other words, while the label ‘real options approach’ is widespread in the literature, “if others prefer to continue to use ‘positive NPV’ terminology, that is fine as long as they are careful to include all relevant option values in their definition of NPV” (p. 7).¹

Past and recent empirical evidence shows that actual decision makers turn the NPV model to a rule of thumb: they use a *subjective* discount rate as opposed to a *rational* cost of capital derived from some market model (e.g. CAPM, arbitrage theory). As a result, they employ a criterion generated in the realm of unbounded rationality but give it a distinctive bounded-rationality flavor. This paper aims at showing that this *mixed* rule may prove useful: the bounded/unbounded dichotomy peters out and the hybrid rule discloses a beneficial cooperation between bounded and unbounded rationality.

The paper is organized as follows. Section 1 surveys some empirical evidence on the use of hurdle rates by real-life decision makers so that the NPV rule is interpretable as a bounded-rationality strategy. Section 2 presents evidence that the hurdle-rate NPV rule may furnishes close-to-optimal results when compared with the expanded NPV and that it may be viewed as a comprehensive methodology that takes

---

¹ A precursory example of the use of NPV for valuing real options may be found in Merrett & Sykes (1973, p. 129)
account of several domain-specific and project-specific variables. Section 3 shows that the allegedly sound maximizing NPV may encounter some problems, among which inconsistencies with accepted standards of rationality. Section 4 presents some evidence indicating that the NPV rule is hardly interpretable as rigorously pertaining to either side of rationality. Some remarks conclude the paper.

1. The hurdle-rate heuristic

There is some awareness in the corporate finance literature that actual decision makers use the NPV rule but do not use the cost of capital suggested by corporate finance textbooks. They employ a subjective hurdle rate. Brigham (1975) surveyed 33 large, relatively sophisticated firms. Although 94% of them used DCF methodology, only 61% of the firms using DCF adopted the cost of capital as the discount rate. Summers (1987) surveyed corporations on investment decision criteria finding that 94% of reporting firms use the NPV rule employing a discount rate independent of risk, which is suggestive of the use of hurdle rates. Dixit (1992) recognizes that “firms invest in projects that they expect to yield a return in excess of a required or ‘hurdle’ rate” (p. 107). “Finance scholars have always been puzzled by the durability of … the hurdle rate rule” (Ross, 1995) and, in actual facts, “we know that hurdle rates … are used in practice” (McDonald, 2000, p. 30); “it appears common for firms to use investment criteria that do not strictly implement the NPV criterion” (ibidem, p. 13), so that their “actions do not reflect the application of current financial theory” (Gitman & Mercurio, 1982, p. 29). Graham and Harvey (2002) surveyed 392 companies and point out that “small firms are significantly less likely to use the NPV criterion or the capital asset pricing model and its variants” (p. 22). They find that sometimes the use of hurdle rates is explicitly acknowledged: “small firms were inclined to use a cost of equity determined by “what investors tell us to require” [and a] majority (in fact, nearly 60%) of the companies said that they would use a single-company wide discount rate to evaluate a new investment project, even though different projects are likely to have different risk characteristics” (p. 12). Jagannathan and Meier (2002) (henceforth J&M) observe that “managers use a … hurdle rate” (p. 3) in making decisions, instead of employing a CAPM-derived cost of capital. Relying on Poterba and Summers’ (1995) analysis, they find that “hurdle rates are not … linked to the cost of capital” (p. 22). These findings suggest that actual choice behaviors deviate from the normative NPV and decision makers interpret the rule as a satisficing strategy (Simon, 1955), letting the discount rate be a predetermined cutoff level which is considered satisfactory. Denote with \( i \) the (expected) rate of return of the project at hand, and let \( k \) be a hurdle rate subjectively prefixed by the evaluator. A simple heuristic is the following: undertake the project if \( i > k \), otherwise search for another alternative until the inequality is satisfied. It is evident that such a rule may also be rephrased in terms of NPV, with the hurdle rate \( k \) as the discount rate. That is, the decision maker will accept an investment with initial outlay \( X_0 \) and (expected) final return \( X_1 \) if (and only if) \( -X_0 + X_1 / (1 + k) > 0 \) with \( X_1 = X_0 (1 + i) \): the investment will be

---

2 All following quotations from J&M (2002) refer to the internet version.
undertaken if its NPV at the rate $k$ is positive, where $k$ represents the minimum rate of return acceptable by the investor. This rule is formally analogous to eq. (2) but is cognitively different. Replacing $r$ with $k$ means shifting from the idea of comparing equivalent-risk alternatives to the idea of accepting a satisfactory rate of return. Both $r$ and $k$ are cutoff rates, but the former refers to an objectively determined equivalent-risk alternative, the latter refers to a subjectively determined aspiration level. So doing, one leaves the realm of unbounded rationality (UR) and reaches the land of bounded rationality (BR). Yet, both equations are expression of an NPV criterion, relying as they are on discounting cash flows at a prefixed discount rate. The investor invests in the project if the value of her payoff function, calculated at the hurdle rate $k$, is positive; that is, if the rate of return of the project exceeds the aspiration level $k$. By adopting the hurdle-rate heuristic she does not confine herself to equivalent-risk alternatives (though risk and uncertainty may well be one of the factors considered in the determination of $k$). No alternative course of action is called up, no comparison is accomplished, no maximization is involved. The rate $k$ is not a rate of return of an existing alternative, and therefore it is not an opportunity cost and does not represent a foregone return. It is a subjective threshold that identifies the personal minimum required rate of return from the project. The heuristic-minded reasoner is uninterested in knowing whether there exists or not an alternative (either equivalent in risk or not) better than the one under examination, she is just interested in reaching that aspiration level, for subjective reasons.

2. Does the NPV heuristic work?

One of the studies dealing with the performance of the hurdle-rate rule is McDonald (2000). The author aims at investigating “whether the use of seemingly arbitrary investment criteria, such as hurdle rates and profitability indexes, can proxy for the use of more sophisticated real options valuation” (p. 13). He wonders whether such “simple rules are relatively robust to changes in project characteristics” and whether “a single hurdle-rate rule [can] yield approximately correct decisions for these projects” (p. 15). Focusing on an option-to-wait decision problem, and implementing variations in the cash-flow growth rate, the project volatility, the discount rate, he shows that

for a wide range of project characteristics, fixed-hurdle rate and profitability index rule can provide a good approximation to optimal investment timing decisions, in the sense that the ex ante loss from following the suboptimal rule is small; it is possible to follow the wrong investment rule without losing much of the ex ante value of the investment timing option. In fact, as the investment timing option becomes worth more and it becomes optimal to wait longer to invest, the option value becomes less sensitive to errors in investment rules.” (p. 15)

He explains that even if the rise in the discount rate lowers the project value, it also lowers the value at which investment becomes optimal, so that “a decision rule of the form ‘invest when the project has an internal rate of return of 20%’ might in fact be appropriate for a wide variety of projects” (p. 15). He underlines that “for a variety of parameters, particular hurdle-rate … rules can provide close-to-optimal

---

3 Maximization implies comparison: a comparison among mutually exclusive states.
investment decisions. Thus, it may be that firms using seemingly arbitrary ‘rules of thumb’ are approximating optimal decisions” (p. 13, italics added). J&M (2002) reach the same conclusion holding that that the use of hurdle rates should not be deemed less reliable than the use of the opportunity cost of capital as a discount rate: “managers … [take] the right decisions … because they use a hurdle rate that is higher than the cost of capital to capture the option to wait” (p. 12). Although these authors do not recommend the use of aspiration levels and rules of thumb as optimal decisions making criteria it is worthwhile noting that their conclusions implicitly (and unawarely) foster the idea that advantages can be taken from undertaking a simple heuristic:

[a] “the advantage of using a hurdle rate is that modelling all possible future options is not necessary” (J&M, 2002, p. 4) and decision makers may “find it useful to use a rule that best justifies making intuitively plausible investment decisions” (McDonald, 2000, p. 26)

[b] “managers adjust these rules … when an investment is strategic and expiring.” (McDonald, 2000, p. 30, italics added).

c] “use [of a hurdle-rate rule] in practice might stem from the success of apparently arbitrary rules that are revealed over time to be close to optimal. Managers likely observe the capital budgeting practices, in their own and other companies, and in many cases probably mimic what seems to work” (McDonald, 2000, p. 30, italics added).

It is worth noting that [a], [b] and [c] fit the three premises on which Gigerenzer (2001) bases his notion of adaptive toolbox:

[A] psychological plausibility: models of decision-making should have adequate regard “for the constraints in time, knowledge, and computational capacities that humans face” (Gigerenzer, 2001, p. 38)

[B] domain specificity: “each heuristic is specialized for certain classes of problems, which means that most of them are not applicable in a given situation” (Gigerenzer & Todd, 1999, p. 32) and “what works to make quick and accurate inferences in one domain may well not work in another. Thus, different environments can have different … heuristics” (Gigerenzer, Todd & the ABC Research Group, 1999, p. 18)

[C] ecological rationality: this consists in “the match between the structure of a heuristic and the structure of an environment” (Gigerenzer & Selten, 2001a, p. 9). Heuristics are ecologically rational in that “they are adapted to particular environments … [and] can be fast, frugal and accurate by exploiting the structure of information in natural environments” (ibidem).

Some other authors are puzzled by the “intriguing paradox” (Miller & Shapira, 2004, p. 281) of such near-optimal choice behaviors: “we have compelling evidence that managers … often do not use real option techniques … On the other hand, strategic decisions under uncertainty appear to conform to some general expectations based on real option theory … The resolution of this paradox would seem to be that, despite their biases, managers’ strategic investment decisions can loosely conform to normative real options models. Managers may employ real option reasoning, without getting all the details correct … Managers’ investment decisions may be ‘directionally correct’, even if they are not completely unbiased” (Miller & Shapira, 2004, p. 281). Actually, an ‘intriguing paradox’ is that the authors, in the concluding remarks of their papers, admit that actual choice behaviors take account of variables that are not considered by the normative models: “normative models for pricing options overlook key aspects of the behavioral and organizational contexts in which investment decisions occur” (Miller & Shapira, 2004, p. 282)

“We do not suggest that managers should use these rules of thumb” (McDonald, 2000, p. 30).
Sentences in [a] exemplify the psychological plausibility [A] of the NPV heuristic (decision makers are not able to take into account all possible future opportunities and outcomes). Sentence [b] reveals that these rules are domain-specific [B] in the sense that when the context changes (strategic investments versus nonstrategic investments, expiring versus nonexpiring projects) the rule is changed or is combined with other rules of thumb (and that the aspiration levels are adjusted too, as suggested by J&M. See above). For example, while the NPV heuristic seems to work well for a variety of industrial projects, the recognition heuristic (Borges et al., 1999; Goldstein & Gigerenzer, 1999, 2002; Borges et al., forthcoming) is inapplicable in such frameworks. Also, the hurdle-rate rule seems to work least well in case of low volatility. The errors may be avoided through a combination of rules (e.g., hurdle-rate and profitability index) to give rise to a different heuristic: “This suggests constructing a third rule as a hybrid of the two rules … Such a hybrid rule can prevent the large errors at extreme discount rates generated by either rule alone … this example … demonstrates that firms in practice might find it useful to … consider multiple rules at once” (McDonald, 2000, p. 26). Sentence [c] reveals that the use of the NPV heuristic is ecologically rational [C] due to success over time. McDonald’s words also implicitly suggest that the rules of thumb analyzed may be the result of imitation and social learning. In other terms, his words encourage the view that the satisficing NPV rule is a “do-what-others-do” (or “do-what-successful-people-do”) strategy (see Laland, 2001). J&M (2000) cites empirical researches showing that managers do not use the CAPM-derived cost of capital, contrary to the normative suggestion of most corporate finance textbooks. Decision makers apply hurdle rates which are usually higher that the cost of capital. As in McDonald (2000), the value of waiting to invest is underlined to explain this finding: the authors affirm that by “using high hurdle rates, companies in fact indirectly account for the existence of timing options” (p. 15). Resting on Poterba and Summers’ (1995) data, they show that “a relatively high hurdle rate of 12.2.% is successful in capturing most of the option value as long as the uncertainty of projects’ cash flows is high” (J&M, 2002, p. 18) and “If we take into account the option to delay the project, the financial decision is no longer crucially dependent on an exact figure of the discount rate” (p. 19). However, there are other explanations, which suggest that the use of hurdle rates is consistent with established theories in business and finance. For example, a project may absorb managerial skills and thus prevent firm from undertaking other profitable projects in the future. Thus, “If managerial time of a skilful manager is limited, she must decide when it is optimal to take a project. It may pay off to wait and not take the next best positive net present value project” (p. 21), which explains why “the use of a hurdle rate that is higher than the cost of capital … is likely to lead to near optimal decisions” (ibidem). This also explains the large variation that Poterba and Summers (1995) find in the hurdle rate of different companies: firms in the same sector face similar systematic risks and, therefore, if they complied with the CAPM-derived discount rate (based on systematic risk) they would apply similar discount rates. It is appropriate to use higher discount rates because managerial resources are

---

6 McDonald is actually concerned with three rules of thumb: hurdle-rate, profitability indexes and payback period.
7 Possibly, the reverse is true for investment in capital markets.
in limited supply. This view is consistent with the resource-based theory and the literature on Top Management Teams, according to which managerial skills is a fundamental driver of value creation (Barney, 1986, 1991, 2001; Grant, 1991; Grant & Robert, 1995; Levinthal, 1995; Bromiley, 2005). Furthermore, if a project is strategic, it may be worth undertaking it even if its NPV (computed with the “correct” cost of capital) is negative, because it promises future opportunities. In this case the use of a discount rate smaller than the cost of capital should be appropriate. Actually, J&M (2002) do affirm that “companies use low hurdle rates for strategic projects” (p. 22); and “Because it is difficult to estimate the values of all possible upcoming investment opportunities and to assign probabilities how likely these will arise, companies seem to use a low hurdle rate that takes into account that the payoffs in the future are possibly higher than the strategic investment itself suggests” (ibidem). This view is consistent with the strategy literature, according to which strategic decisions are drivers for a sustainable competitive advantage (Collis & Montgomery, 1995; Quinn & Mintzberg, 1996); “there are strategic situations where making the first move has a commitment value” (Dixit, 1992, p. 119). For this type of decisions the use of modelling is hardly helpful: “Even if highly simplified and abstracted, the associated SA [strategic assets] decision may not be solvable in closed-form equilibrium … For example, when modelled as a differential game, the problem will probably be not tractable” (Amit & Schoemaker, 1993, p. 40). Agency theory (Jensen & Meckling, 1976; Jensen, 1986) and the existence of costs of external financing may play a role in determining a hurdle rate different from normative cost of capital. J&M construct a comprehensive NPV function taking account agency costs and the costs of external financing. The authors claim that this is, according to some recent literature, the right NPV model to be maximized (pp. 23-27). That the costs of external financing may play a major role is also suggested by some empirical research about large reserves of cash retained in the firm (see Mikkelson & Parch, 1999).

As seen, the option value of waiting tend to increase the hurdle rate as well, whereas strategic considerations tend to decrease it. Brigham (1975) report that 39% of the respondents change the hurdle rate less than once in a year (p. 20, Exhibit 4), and 32% state that it “depends on conditions”: they “revise rates to reflect product and capital market condition, with revisions generally occurring less than once a year” (p. 20). Gitman and Mercurio (1982) report that 50.3% of the companies revise discount rates when environmental conditions change and 13% less frequently than annually, 11.2% when a major project is evaluated (p. 27). The ‘product and market condition’ seems to hint at Porter’s (1980) analysis, where market conditions such as rivalry, supplier power, buyer power, threat of substitutes, entry barriers are individuated as fundamental drivers for value creation. Overall, these findings evidence that hurdle rates are seen as reference levels that do not change quickly but “that may be gradually adjusted if they become too lax or binding a constraint” (Goodie et al., 1999, p. 351). Historical documents as well reveal that the hurdle-rate rule, frequently used in past centuries, were used as base levels that could be adjusted: “If 15% tended to be the rule of thumb, it was not applied slavishly by the viewers” (Brackenborough, McLean & Oldroyd, 2001, p. 144), which changed it “depending on the circumstances” (p. 143) (see also Section 4.
In other words, decision makers condense into an aspiration level several considerations related to uncertainty, decision flexibility, market conditions, future opportunities, limited managerial skills, agency costs, strategic considerations, but the BR-NPV rule is exploited in a nonrigid way so that the hurdle rate changes as the above factors change.

3. Problems in the unboundedly rational NPV

The UR-NPV rule has several shortcomings. First of all, there is no one UR-NPV, but a proliferation of UR-NPVs differing in terms of risk measure implied, equilibrium model employed, mathematical technique adopted. There is no agreement among scholars on the way the NPV technique should be implemented: “academics do not agree on an appropriate equilibrium model even for estimating firm-level discount rates, for which stock returns are observable; estimation of project-level discount rates is even more problematic” (McDonald, 2000, p. 21). Whatever the notion of risk, the equilibrium model prescribed, the mathematical tool employed, many of the assumptions required to implement it are often violated in real life. For example, the use of CAPM-derived cost of capital is often fostered in corporate finance. Resting on a considerable amount of empirical results, J&M (2002) show that “the CAPM, like all models, is only an abstraction from reality” (p. 5) and “there have been many academic challenges to the validity of the CAPM as applied in practice” (ibidem). Lander and Pincher (1998) find numerous disadvantages even in the use of the more sophisticated real-options approach (expanded NPV). They write:

“Any decision-making framework model is subject to: inappropriate assumptions … poor estimation procedures … failure to incorporate the effects of competition and the strengths and weaknesses of the firm into all aspects of the investment opportunity, ineffective information gathering of performance and measurement systems, inappropriate emphasis on short-run goals and results, or excessive conservatism or optimism … Many of the required modeling assumptions are often and consistently violated in a practical real options application. The necessary additional assumptions required for mathematical tractability limit the scope of applicability” (Lander & Pinches, pp. 542-543)

A most stringent assumption frequently used is completeness of the market. This assumption makes it possible to construct a replicating portfolio which mimics the project’s payoffs. In this case the expected rate of return of the portfolio is taken as the cost of capital and a positive NPV indicates that, regardless of preferences, the investor benefits from undertaking the project by selling short the replicating portfolio. The approach is often called the options pricing approach. It should be stressed that short sales are often not allowed and, when allowed, arbitrageurs are inevitably exposed to risk of losses (Shleifer & Vishny, 1997), contrary to what textbook arbitrage suggests. Above all, if markets are incomplete and the project is not replicable by existing assets, there is no way of computing the cost of capital, given that “without spanning, there is no theory for determining the ‘correct’ value for the discount rate $\rho$” (Dixit & Pindyck, 1994, p. 8)

8 The (unboundedly rational) fuzzy-logic version of the NPV is now another version, now commonplace in numerous disciplines (Ward, 1989; Chiu & Park, 1994; Abdel-Kader, Dugdale & Taylor, 1998; Buckley, Eslami & Feuring, 2002). The meaning of a fuzzy cost of capital and a fuzzy NPV is, admittedly, an open issue.
Smith and Nau (1995) admit that “without a market equivalent for the efficiency uncertainty, we cannot construct a perfect replicating trading strategy or identify a unique risk-neutral probability distribution and thus we cannot determine a unique option-pricing value for the project.” (p. 804). They illustrate an example where three strategies are possible and conclude that “the optimal strategy is unclear and all three strategies are potentially optimal” (p. 805).

The heuristics-and-biases-program approach may reveal itself a double-edge weapon. For example, among the various equilibrium models, “the predominant approach to estimating the cost of capital is to use the CAPM” (J&M, 2002, p. 28). The CAPM is massively endorsed in corporate finance (e.g. Copeland & Weston, 1983; Brealey & Myers, 2000; Koller, Goedhart & Wessels, 2005; Damodaran, 2006). “The CAPM provides a framework for estimating the appropriate opportunity cost to be used in evaluating an investment” (Rao, 1992, p. 33). Rubinstein (1973, footnote 10) proves that, if the CAPM assumptions are met, the project is profitable if and only if \( \frac{X_1}{X_0} - 1 > r_f + \lambda \text{cov}(\tilde{X}_1, \tilde{r}_m)/X_0 \), that is if the project’s expected return rate exceeds the CAPM-derived cost of capital. The author underlines that the cost of capital is the “appropriate discount rate for the project” (p. 174) and that “present value risk-adjusted discount rate … forms of this criterion are easily derived” (footnote 8, p. 171) as is shown by a simple manipulation of the above inequation:

\[
-X_0 + \frac{X_1}{1 + r_f + \lambda \text{cov}(\tilde{X}_1, \tilde{r}_m)/X_0} > 0.
\]

The consequent decision rule boils down to “to accepting the project with the highest net present value” (footnote 14, p. 174). It is worth noting that such a NPV is a disequilibrium (cost-based) NPV. Its use is fostered in popular finance textbooks (e.g. Copeland and Weston, 1983, 1988; Weston & Copeland, 1988; Bøssaerts and Odegaard, 2001) and it is so widespread that Ang and Lewellen (1982) consider the disequilibrium NPV “the standard discounting approach” (p. 9) in finance. Magni (2007a) shows that this unboundedly rational procedure suggested by Rubinstein deviates from an accepted standard of rationality: value additivity (warnings against the use of this NPV may be found in Ang & Lewellen, 1982; Grinblatt and Titman, 1998; Ekern, 2006). In particular, the author shows that the NPV of a portfolio of projects is different from the sum of the NPVs of the projects (Magni, 2007a, Proposition 4.1). This implies that the disequilibrium NPV furnishes different evaluations (and different decisions) depending on to the way the course of action is depicted. Loosely speaking, to receive a 100\$ banknote or to receive a 60\$ banknote and a 40\$ banknote is financially equivalent. Decision makers employing the disequilibrium NPV are trapped in a sort of mental accounting (Thaler, 1985, 1999) so that their evaluations differ depending on whether their

9 “We also have some semantic problems defining exactly what is meant by the value of a non-traded project” (Smith and Nau, 1995, p. 804, footnote 7).
10 In a private correspondence, Prof. Nau writes, referring to the assumption of complete markets: “this a strong idealization: if markets are incomplete, then there are not Arrow securities for all the states” (July 2005).
11 Rubinstein’s proposal is logically equivalent to other classical proposals presented in the late Sixties and early Seventies by such scholars as Mossin, Hamada, Tuttle and Litzenberger, Bierman and Hass (see Senbet & Thompson, 1978, for a review).
outcomes are seen as aggregate or disaggregate quantities. This amounts to saying that the disequilibrium NPV is inconsistent with the principle of description invariance, which prescribes that valuations and decisions must be invariant under changes in description of the same alternatives. Violations of this principle are known in the heuristics-and-biases tradition as framing effects (Tversky & Kahneman, 1981; Kahneman & Tversky, 1984; Soman, 2004). As opposed to the disequilibrium NPV, some scholars advocate the use of an equilibrium NPV (e.g. Bogue & Roll, 1974; Haley & Schall, 1979), which is logically deducted from the CAPM as well. It is often presented in the co-called the certainty-equivalent form:

$$-X_0 + \frac{X_1 - \lambda \text{cov}(\tilde{X}_1, \tilde{r}_m)}{1+r_f} > 0.$$ 

This equilibrium NPV is additive, but Dybvig and Ingersoll (1982) and Magni (2007a, 2007b) show that under certain (realistic) assumptions on the market rate of return, this NPV violates the no-principle arbitrage. This means that a decision maker complying with the equilibrium NPV is subject to arbitrage losses. As a result, the equilibrium NPV is not consistent with a tenet which is considered a normative benchmark for rationality (see Nau & McCardle, 1991; Nau, 1999). As a result, the CAPM-based NPVs displays deviations from accepted standards of rationality.

4. NPV rule: heuristic or unboundedly rational model?

The NPV rule (either the traditional version or the expanded one) is presented in the literature as the solution of an optimization problem (e.g. Fisher, 1930, Rubinstein, 1973, Dixit and Pyndick, 1994; McMinn, 2005). Despite this, there is evidence that it is a mixed methodology, a blending of logical and ecological rationality. As may be inferred by several historical surveys, it is the result of rigorous mathematical thinking associated to a more practical-oriented adaptive thinking of skillful decision makers. In a period where the quest for axiomatic models was cogent, Fisher (1930) just gave an optimization dressing to a technique that was practiced long since: since the times of Fibonacci’s present-value analysis (Goetzmann, 2005) in 12th century, contributions from mathematicians, engineering economists, and actuarial scientists nourished and were nourished by the non-optimizing evaluations of properties, trees, lands, collieries, coppices, buildings, leases, shops etc. made by the legal, banking, business communities (Wing, 1965; Edwards & Warman, 1981; Miller & Napier, 1993; Scorgie, 1996; Brackenborough, McLean & Oldroyd, 2001). The use of a mixed methodology consisting of a systematic discounting procedure subjectively adjusted to reflect personal judgment is exemplified in actual evaluations accomplished across centuries: Scorgie (1996), illustrating many cases, shows that since 13th century “the capitalised value or selling price of arable land and other property was determined by multiplying the annual rental by some

---

12 See also Haley & Schall (1979, pp. 182-183), for unreliability of the disequilibrium NPV in ranking projects.

13 Magni (2007b) focuses on CAPM-based capital budgeting and shows that the derivative of the NPV function may be decreasing with respect to the end-of-period cash flow in some state of nature. Dybvig and Ingersoll (1982) focuses on asset pricing and show that CAPM does not guarantee absence of arbitrage in the market.
generally accepted factor” (p. 240, italics added), which is interpretable as a discount factor. A valuation of Lord Cromwell’s property in 1469 was obtained by using a factor of 20. The latter only “provided a base valuation” (ibidem) and could be changed depending on the case at hand: “The provision for the use of another rate provided some leeway” (Scorgie, 1996, p. 240). Variations were governed by several domain-specific factors, among which “the need to use a higher interest rate where an investment project involved hazards and uncertainty” (p. 242). For example, the risk of flooding for a coal mine could suggest to decrease the factor from 20 to 4 or to 3 (i.e. raise the discount rate to 25% or 33.33%) (p. 244). Brackenborough, McLean and Oldroyd (2001), focusing on the Tyneside coal industry in the 18th century, report the use of an NPV rule where cash flows “were discounted at interest rates to reflect the viewer’s assessment of risk” (p. 141, italics added), which means that that the “responsibility for assessing the degree of risk was placed firmly in the hand of the client” (p. 144). Quoting directly from original records, they confirm that the discount rates reflected “the purchasers’ own ideas of the risk and prospects of the concern” and that the decision makers should “judge for himself, and draw his own conclusions” (p. 144). Analysts just provided the formal framework, while leaving investors to use the rule subjectively “through adjustment of the discount rate” (pp. 150-151). The use of this combined methodology is also confirmed in Wing (1965), who analyzes a paper by Van Deventer (1915) where a foreman provides the owner of a small machine shop with a rule for investment decisions: such a rule is just an unstated BR-NPV rule: “First, estimate the probable saving that the investment will make … Second, assign a probable length of life to it … Third, estimate what the investment will cost … Fourth, pick out a minimum rate of return that you will expect” (p. 475, italics added). That minimum rate of return is a “desired profit rate” (p. 475) in a satisficing sense. That the choice of the rate was meant to be subjective is confirmed by the table Van Deventer provides in his paper, which is addressed to the attention of the shop owner: it “was supplied with a range of rates of return into which he could fit his personal minimum expected rate” (p. 479, italics added). This cooperation is brought into play even in the UR realm, though it is not recognized: an outstanding example is given by Dixit and Pindyck (1994)’s use of stochastic dynamic programming in the expanded NPV, where they explicitly leave decision makers free to set their own desired discount rates: “dynamic programming can … be used to maximize the present value of the firm’s expected flow of profits, subject to an arbitrary discount rate” (p. 149, italics added); “we can … use a dynamic programming approach with an exogenously specified discount rate” (p. 185, italics added); “The dynamic programming approach started by specifying the discount rate, ρ, exogenously as a part of the objective function” (p. 120). To present a maximization problem where the objective function depends on a subjectively determined hurdle rate means to suggest a blending of an UR procedure and a BR heuristic. Interestingly, the authors themselves are puzzled by their own proposal: “One problem with this investment rule is that it is based on an arbitrary … discount rate ρ. It is not clear where this discount rate should come from” (p. 147). Yet, their book heavily relies (commendably) on this hybrid NPV rule. Even McDonald (2000), in presenting the “correct”

14 The implicit discount rate is 5% = (1/20) * 100.
maximization procedure, significantly admits: “We will be agnostic about the determination of [the discount rate] \( \rho \)” (p. 16).

One may even conjecture that the birth of the sophisticated version of the NPV rule (the expanded NPV) has been favored by the empirical findings on actual choice behaviors: some scholars may have been induced by these researches to construct a new enhanced NPV capable of explaining the empirical deviations from the well-entrenched NPV rule. This may be drawn implicitly from the concern about presenting a normative model able to explain and conform to actual choice behaviors:

Some recent developments in the theory of investment under uncertainty have offered an interesting new explanation of these phenomena … This new approach suggests that textbooks pictures of the dynamics of a competitive industry need … substantial redrawing”. (Dixit, 1992, p. 108, italics added)

The real options literature can explain why using a discount rate that is higher than the cost of capital is a good strategy when waiting opens up better opportunities. Such a discount rate is typically referred to as the hurdle rate. (J&M, 2002, p. 13, italics added)

The option insight also helps explain why the actual investment behaviour of firms differs from the received wisdom taught in business schools. Firms invest in projects that are expected to yield a return in excess of a required, “hurdle” rate … such hurdle rates are typically three or four times the cost of capital … firms do not invest until price rises substantially above long-run average cost. On the downside, firms stay in business for lengthy periods while absorbing losses, and price can fall substantially below average variable cost without disinvestment or exit. This also seems to conflict with standard theory, but … it can be explained once irreversibility and option value are accounted for. (Dixit and Pindyck, 1994, p. 7, italics added)

We found qualitative implications, and several approximate quantitative ones, that conform to experience—the high hurdle rates used by business firms when they judge investment projects, the relative inefficacy of interest rate cuts as policies to stimulate investment projects, the significant and detrimental effect of policy uncertainty on investment, and so on. (Dixit and Pindyck, 1994, p. 425)

…there is considerable anecdotal evidence that firms make investment decisions in a way that is at least roughly consistent with the theory developed in this book (for example, the use of hurdle rates that are much larger than the opportunity cost of capital as predicted by the CAPM). (Dixit & Pindyck, 1994, p. 296, italics added)

These claims may be interpreted as a regard for understanding real-life decision makers’ behaviors and providing a theory that take account of decision makers’ actual choices. If decision makers do not abide by a normative theory, they should not be tagged as irrational. Rather, one could infer that theory exhibits “failures”, “anomalies” (Dixit & Pindyck, p. 4), “abstraction from reality (J&M; p. 5).”\(^{15}\) There is a need of new theories that explain decision maker’s heuristics, which may give insights for construction of new theories and models that take account of information, context, environment and personal evaluation. The two sides of rationality should (at least attempt to) go along a common road:

Any decisions-making framework, however, can only improve a manager’s understanding of the problem at hand and help him/her to make a more informed and consistent decision. No decision-making framework can guarantee a “good” outcome and there is no substitute for managerial effort.

\(^{15}\) Dixit and Pindyck, as well as J&M’s, use empirical evidence to dismiss theories (traditional NPV, CAPM) that do not comply with actual behaviors, not to tag actual choice behaviors as irrational.

The quest for a fruitful cooperation of bounded and unbounded rationality is now compelling.

Conclusions

The heuristics-and-biases program draws attention on logical fallacies of human reasoning which cloud human minds (Tversky & Kahneman, 1974; Kahneman, Slovic & Tversky, 1982; Kahneman & Tversky, 1996). Choice behaviors are biased because they deviate from accepted standards of rationality; conversely, advocates of bounded rationality are concerned with showing that heuristics may often prove useful in decision-making (Gigerenzer, Todd & the ABC Research Group, 1999). The latter aver that cognitive illusions disappear once heuristics are seen as adaptive tools rather than logical devices for solving decision-making processes (Gigerenzer, 1996; Gigerenzer, 2000). Gigerenzer and the ABC group endorse the idea that human beings’ rationality is ecological rather than logical, and their fast-and-frugal-heuristics program aims at presenting a number of heuristics that are successfully applied in real-life in specific environments. This work draws attention on the NPV maximization model, which is a keystone in economics and finance. The main results of this work may be summarized as follows:

(1) Real-life decision makers often use the NPV methodology, but discount cash flows with a hurdle rate which differs from the cost of capital normatively suggested (e.g. CAPM, arbitrage pricing)

(2) The hurdle-rate rule may be interpreted as a boundedly rational approach to investment decisions: decision makers rest on aspiration levels subjectively determined

(3) Some empirical evidence show that the bounded-rationality approach is ecologically rational, domain-specific, psychological plausible. The hurdle rate rule leads to close-to-optimal solutions when confronted with the expanded NPV

(4) Several factors affect the hurdle rate: uncertainty, future opportunities, rationing of managerial skills, strategic considerations, agency costs, costs of external financing. This suggests that this approach is consistent with several theories and models: real options approach, resource-based theory, Top Management Team literature, agency theory, strategy literature. The hurdle rate seems to be set at a base level but may fluctuate around it depending on the drivers above cited

(5) The NPV maximizing model is not univocal: it may take several different forms. The use a particular version depends on the circumstances: for example, the arbitrage-pricing NPV may not be used if the actual market is not complete. Some of the versions are often inapplicable and/or reveal deviations from accepted standards of rationality (additivity, no-arbitrage, description invariance)

(6) Both unboundedly rational NPV and boundedly rational NPV have their own shortcomings, if taken in isolation, but an interaction between the two system of reasoning may improve performance. Actually, the very distinction between the two is rather artificial and only useful as a metaphor (see Gigerenzer & Regier, 1996): as attested by recent historical researches, the origins of the NPV lies in the common efforts of various categories of people facing the need of both logical and eco-logical rationality.
In such a way, the paper enters the debate as to whether unbounded or bounded rationality should be judged more apt to help decision makers. The latter rests primarily on the idea of heuristics as fruitful tools to be used in specific environments, the former focuses on the equation heuristics=biases, in the sense that heuristics are often considered internally inconsistent and for this reason inappropriate for decision-making. This paper is consistent with a bounded/ecological rationality approach (and, indeed, it recommends bounded and ecological rationality as a fruitful and indispensable vision of rationality): “the heuristics themselves are sensible estimation procedures that are by no measure “irrational”.” (Gilovich and Griffin, 2002, p.  However, this paper may suggest a direction for research and act as a stimulus for further inquiries. The scientific niche that might be disclosed could offer unexpected views about bounded and unbounded rationality and their interrelations. A possible payoff of such a view is not solely that internal consistency is a double-edged weapon (and therefore one should not entirely base decision-making on it) but that bounded rationality and unbounded rationality are not necessarily rivals. This view is then consistent with a nondichotomous dual-process theory of reasoning, according to which individuals are furnished with two cognitive systems: a heuristic-based (intuitive) system and rule-based (analytic) systems that cooperate in producing answers to decision problems. Bounded and unbounded rationality may be viewed as two sides of the same medal. This seems to be the new direction of the heuristics-and-biases program. But the latter seems also interested to stress that System 1 (rule-based) and System 2 (heuristic-based) do cooperate even though “the accuracy of (System 2) decision rules rests on the validity of a System 1 computation” (Gilovich & Griffin, 2002, p. 17). That System 1 is always superior to System 2 is here disconfirmed, since the rule-based model has its own biases as well. Also, to rigidly split our reasoning processes in two Systems may prove a useful metaphor for scientific research but often results in a simplistic explanation. One never knows where either System begins and where it ends, and the distinction between the two is rather fuzzy: it is not simple to ascertain what an association is and what a rule is, and to find clear-cut differences between them (Gigerenzer & Regier, 1996). Unbounded rationality itself, as derived from logic and mathematics, may not abstain to consider itself as a derivation of ecological rationality: logic and mathematics should be intended as the most advanced step of human simulation, and the impressive degree we have achieved in such an ability may be seen as the result of an evolutionary process, in which its surviving value has been adaptively tested. Simulation is a tool for anticipating and as such is indispensable for discovering and creating. Therefore logic and mathematics, as symbolic tools, assemble the experience of our ancestors (Monod, 1970, pp. 171-172), or, to put it in a nutshell, even logic is ecological.

If a narrow interpretation of the two-process theory means that the two systems always give contradictory answers, then this paper aims at showing the opposite; if a less rigid interpretation of it is followed, then a dual-process theory may actually be of some help to understand (decision-making and) decision-making models. Evidently, such a theory cannot substitute a meticulous analysis of cognitive processes where focus is not placed on the opposition of the two systems but rather on the interaction
between the two and on the mechanisms that combine them, as well as on the theoretical question concerning how much of each reposes in the other. Certainly, “the two-systems view … helps to clarify the differences and similarities between the … “heuristics and biases” program and … the “fast and frugal heuristic” program” (Gilovich & Griffin, 2002, p. 16) and its “potential usefulness of analyses of this sort” (Gigerenzer & Regier, 1996) is undeniable. But collecting more and more binary oppositions—and labeling these “two-process theories”—does not necessarily enhance clarity. Dichotomies can be an important step, but they cannot substitute for theories of cognitive processes. (Gigerenzer, 2000, p. 293).

More “fluid theories” and more blending will perhaps help us understand decisions makers and help decision makers cope with complex decision problems (with the pleasant by-product of conciliating the two rival parties).

References


