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This paper analyses the strength of the financial system of Tunisia through the construction of an Index of Financial Safety (IFS). Over the period 2000Q1 – 2014Q3, the IFS is built using a wide range of financial and macroeconomic indicators. The empirical results show that it can capture the disturbances in Tunisian financial system with sufficient accuracy. The nonlinear autoregressive with exogenous input (NARX) model with Levenberg-Marquardt algorithm of training was selected to forecast changes in IFS, and provides significant results.

JEL classification: G10, E00, C43, C45, C53

Keywords: index of financial safety of a country (IFS), forecast, nonlinear autoregressive with exogenous input (NARX) model, neural networks.

I. Introduction

The underlying root causes of financial and macroeconomic imbalances lie not only in disequilibrium in current account balances, such as the huge current account deficit in the USA or the large current account surplus in China, but also in financial recession that has shaken some Arab world countries as a result to political events. Following the revolution of January 14, 2011, the economic and financial situation in Tunisia worsened. The average economic growth rate has decreased from 4% during the period 2007-2010 to 1.5% over 2011-2014, whereas the inflation rate has reached 5% during the last three years against 3.5% from 2007 to 2010[1]. The sovereign credit rating of this country has experienced an unexpected wave of downgrades since the beginning of the revolution in 2011.The rating agency, Standard and Poor’s (S&P) lowered Tunisia from “BBB/stable” in 2011 to “B/-“ in August 2013[2]. These downgrades are due to several factors. Indeed, the financial performance of Tunisian banking sector and asset quality indicators suffered from the combination of post-revolution instability and Europe's slowing economies. Thus, political risk has increased; growth prospects have deteriorated; external liquidity has declined with the drop in foreign exchange reserves; fiscal performance weakened compared to historical orthodoxy of Tunisia in terms of public finance management; and the monetary policy became inefficient with the exhaustion of all the ways available to the central bank to help to remedy the situation. Moreover, the local capital market is small and banks' access to external funding is restricted and concentrated primarily on Tunisian expatriate deposits or long-term loans from multilateral lending institutions. This recession has had negative impacts on the financial strength of the country as well as the access’s conditions to the international financial market.

[1] Source: Central Bank of Tunisia.
[2] Source: Central Bank of Tunisia.

To avoid financial recession and instability, policymakers and researchers highlighted the need for developing some indices or indicators, such as Early Warning Systems (Caggiano *et al.*, 2014; Catullo *et al.*, 2015; Li *et al.*, 2015), financial soundness indicators (Cihak and Schaeck, 2010; Kasselaki and Tagkalakis, 2013), monetary conditions index (Batini and Turnbull, 2002; Osborne-Kinch and Holton , 2010) and financial conditions index (Matheson, 2012; Guihuan and Yu, 2014; Angelopoulou *et al.*, 2014). Appraising the financial health of a country refers to know whether its financial structure is balanced or not. Financial safety indices enable to assess the ability of the country to face a recession or to seize opportunities for development. The aim of this paper is to investigate the financial safety of Tunisia by constructing a financial safety index through the use of factor analysis and nonlinear autoregressive with exogenous inputs (NARX) modelling. There is a wide range of studies on this topic, especially on Early Warning Systems (EWS). Recently, Matkovskyy (2012, 2013, 2014) developed a new index, namely index of financial safety (IFS), which is based mainly on financial and macroeconomic variables. Our contribution in this paper is to continue in the same line of Matkovskyy (2012, 2013, 2014) while taking into account some specificities of Tunisia. In particular, as this country has witnessed a revolution, we include new variables as proxies for political stability conditions in modelling financial safety.

The remainder of the paper proceeds as follows. Section 2 provides an overview on the existing literature related to the estimation of financial indices. Section 3 presents the data and the methodology. In section 4, we discuss the empirical results while section 5 concludes.

II. Literature review

Financial indicators have been the subject of several researches in the wake of the global financial crisis. Early Warning Systems (EWS) are among the models the most commonly used in preventing financial crises. The EWS enable the prediction of the occurrence of a financial turmoil over a given time period. Caggiano *et al.* (2014) developed an EWS for predicting systemic banking crisis in 35 Sub-Saharan African countries over the period 1980–2008. They used a multinomial logit model with 8 explanatory variables. Their results show that crises in these countries are associated with low economic growth, banking system illiquidity and large foreign exchange net open positions. Similarly, Hmili and Bouraoui (2015) applied an EWS on 6 Asian emerging countries over the period 1973-2012, but they enlarged the explanatory variables and employed 13 measures ranged into macroeconomic, financial and external variables. Based on multivariate panel logit approach, they pointed out that only 6 measures are solid predictors of banking crises, in particular inflation which demonstrates the most significant power. The selection of independent variables is so-over arbitrary and differs from one author to another. In line with this assumption, Drehmann and Juselius (2014) tested the effectiveness of early warning indicators of banking crises in 26 economies. Among 10 macroeconomic variables, the authors indicated that credit-to-GDP gap and debt service ratio are the best performing early warning indicators. The authors emphasize the importance of focusing only on the most relevant indicators, rather than selecting a wide range of potential variables.

To continue supporting macro-prudential analysis and evaluate strengths and vulnerabilities of financial systems, the International Monetary Fund (IMF) created at the end of the 90s the Financial Soundness Indicators (FSI). FSIs are considered as synthetic indicators related to the stability level of a financial system. Cihak and Schaeck (2010) and Babihuga (2007) investigated whether FSIs provide an accurate signal for the identification of banking crises. By using multivariate logit regression on a panel of 100 countries during the period 1994-2007, Cihak and Schaeck (2010) report that the probability of occurring banking crises is lowered with high capital to risk-weighted assets and high return on equity. Non-performing loans to total loans and capital adequacy ratio are also found to be useful indicators for the build up of banking turmoil. Whereas Babihuga (2007), by using a panel dataset covering 96 countries, finds evidence that inflation rate, real effective exchange rate, and real interest rates emerge as important determinants of FSIs. The author adds that the relationship between macroeconomic indicators and FSIs changes significantly across the sample of countries. Kasselaki and Tagkalakis (2013) analysed also the behaviour of FSIs in time of crisis. They considered 20 OECD economies between 1997 and 2009, and demonstrated that both regulatory capital to risk-weighted assets and non-performing loans to total loans have risen, while loan loss provisions as well as profitability have dropped dramatically.

Some other papers have focused on other financial and economic indicators, namely monetary conditions index and financial conditions index. Monetary conditions index (MCI) constitutes a measure of the stance of monetary policy through the combination of both interest rates and exchange rates. In practice, MCIs are used as operational short-run targets for monetary policy. Batini and Turnbull (2002) estimated a macro-econometric model over the period 1984Q4-1993Q3 to build a dynamic MCI for the United Kingdom. Their findings show that dynamic MCI is a useful indicator for both stance and inflation. However, Osborne-Kinch and Holton (2010) argue that although that MCI is straightforward and easy to calculate, it cannot be considered as reliable stand-alone element in assessing the monetary policy, given the uncertainty in its estimation and interpretation.

To overcome the shortcomings of MCIs, a financial conditions index (FCI) was developed. Compared to MCI, the FCI incorporates additional variables relevant to the financial side of a country in attempt to give exhaustive information about the state of the economy. Guihuan and Yu (2014) selected 5 variables, namely money supply, exchange rate, interest rate, stock price and real estate price to construct a FCI for China over the period 1998-2013. Based on principal component analysis methodology, they found that FCI is pertinent to reflect China’s financial conditions, and can serve as a valuable indicator to forecast the overall economy trend. Angelopoulou *et al.* (2014) extended the set of variables to 21 in modelling FCIs for the euro countries during the period 2003-2011. Broadly, the indices appeared to provide a significant picture on financial conditions. The analysis of FCIs for individual countries showed divergences in financial conditions across them, and thus, the authors conclude that comparison between the euro area countries is not easy in interpreting their FCIs.

Finally, Matkovskyy (2012, 2013, 2014) proposed an approach to explore the strength of the financial system of South Africa and Turkey, respectively, by developing a new financial indicator, called the index of financial safety (IFS). He defined financial safety of a country as a state in which all the components of the financial system are protected against internal and external threats. The index is constructed using a set of 16 macroeconomic variables over the period 1992Q1-2011Q1 for South Africa and between 2001Q4 and 2011Q2 for Turkey. The results of both studies are similar, and highlighted that IFS constitute a good indicator for capturing distresses in the financial system. In addition, it was shown for South Africa that the use of BVAR and NARX models are relevant for forecasting the future dynamics of IFS with sufficient accuracy.

Overall, most of variables used in constructing financial indices are common in the above studies. These variables are mainly ranged into financial and economic categories. To contribute to this literature, we will extend the set of variables by including new variables linked to the political situation of a country. This extension is in line with the theories of financial safety and political instability.

III. Methodological framework

The Index of financial safety of a country is constructed as a function of partial features of the synthetic categories escribing conditions of a financial system and has a conditional meaning. The number and contents of the specific macro-level indicators being synthesized dependent on a series of objective and subjective time varying factors over specified period of time.

A central issue is to choose the correct combination of variables which can offer reasonably consistent signals of changing conditions in financial safety for a country. The estimation of an index of financial safety of Tunisia, therefore, is based on the identification of key indicators, following Matkovskyy (2013, 2014). The initial list of the indicators includes 15 variables, namely: money in circulation/M2*100, money in circulation/GDP*100, M1/M2*100%, M2/GDP*100, M2/M0, PPI / WPI, money market interest rates, GDP/M2, M2/money in circulation, monetary base/reserves, coverage of import by reserves, total domestic credit/GDP, M2/ market capitalization, changes of share price index to a previous quarter and real effective exchange rate.

The studies of Borensztein *et al.* (1998) and Xu (2000), among others, show that generally a measure of FDI flows is positively related with per capita GDP growth or productivity, as well as with income and employment prospects. But positive effects of inward FDIs are not automatic for host countries, and depend on policies in place *e.g*. macro-economic policies, political situation. According to Cohen and Levinthal (1989), Blomstrom *et al.* (2000) and Borensztein *et al.* (1998), there is evidence that the development of local capabilities is crucial in benefiting from FDI.

The empirical findings of Busse and Hefeker (2007) proof that government stability, religious tensions, and democratic accountability are highly associated with foreign investment inflows, and negative changes in government and political institutions which are resulted in increased political risk, could undesirably influence the inward FDIs activity. In this context, less-developed countries are much sensitive and the empirical results of Demekas *et al.* (2007) confirm the predominance of gravity factors such as host market size and geographical and cultural proximity between source and host country.

On the other hand, financial safety of a country also depends not only on inner conditions of a country, but also on cross-border influences in the form of liabilities and assets transfer, particularly in terms of control and influence on the management of enterprises. Therefore, we extended the list of variables used in the Index of Financial Safety construction with inward FDI observations as the proxy variable for political risks capturing.

Another variable that might serves the similar role, namely the proxy for political instability, democratisation and a country's creditworthiness is net claims on government (Lee, 1993; Block, 2001). This variable represents a fiscal policy in the country and includes loans to central government institutions net of deposits, and consists of two counterparts: claims on central government that includes government promissory notes, government bridging finance programs, other claims; and liabilities to central government, that comprise of current account, import deposits, aid counterpart account. Therefore, to increase the index of financial safety sensitivity we included net claims on government to the factor model of Tunisia.

To increase the performance of the index, two other new variables are included, which are banking sector assets panelised by inflation and portfolio equity flows, which is in line with the theory of financial safety. The motivation to include banking sector assets is the fact, that this indicator of physical and financial "property" of a bank describes size and significance within the banking sector. If the value of the banking sector assets falls, it might mean decreasing in solvency and ability to repay their debts.

Portfolio equity flows consist of the sum of country funds, depository receipts, and direct purchases of shares by foreign investors. There is also the evidence that international portfolio flows have a positive influence on stock returns (Tesar and Werner, 1994; Bohn and Tesar, 1996; and Brennan and Cao, 1997; etc.).

Table 1 represents the whole indicators chosen to build the index of financial safety of Tunisia. This set of indicators provides a reasonably complete picture of the financial system conditions in both short and long term perspective.

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Based on the indicators described in Table 1, the construction of the index includes the following steps: unifying scales of measurement, weight coefficients calculation and finally the aggregate index deriving.

We use the following rages of values for the purpose of normalization: ±5% of variation in the chosen variables derives the optimum values, ±15% of variation in the chosen variables forms cordon values and ±25% of variation in the chosen variables create the range for extreme values. These values were chosen based on previous research (Matkovskyy, 2013, 2014)

Next, to unify scales of measurement of raw data, we apply the following equations:

$$Z_{ij} = \begin{cases} 1, x_{optym}^{min} \le x_{ij} \le x_{optym}^{max}, x - \frac{stimulant}{no} stimulant; \\ \frac{x_{ij} - x_{cordon}^{min}}{x_{cordon}^{max} - x_{cordon}^{min}}, x_{cordon}^{min} \le x_{ij} \le x_{cordon}^{max}, x - stimulant; \\ \frac{x_{cordon}^{max} - x_{ij}}{x_{cordon}^{max} - x_{cordon}^{min}}, x_{cordon}^{min} \le x_{ij} \le x_{cordon}^{max}, x - no\ stimulant; \\ \frac{x_{ij} - x_{extreme}^{min}}{x_{extreme}^{max} - x_{extreme}^{min}}, x_{extreme}^{min} \le x_{ij} \le x_{extreme}^{max}, x - stimulant; \\ \frac{x_{extreme}^{max} - x_{ij}}{x_{extreme}^{max} - x_{extreme}^{min}}, x_{extreme}^{min} \le x_{ij} \le x_{extreme}^{max}, x - no\ stimulant. \end{cases} \quad (1)$$

where, Z_{ij} is the normalized value of raw data; x_{ij}, x_{optym}^{min} and x_{optym}^{max} are the minimum and maximum optimum values; x_{cordon}^{min} and x_{cordon}^{max} are the minimum and maximum of the cordon values; and $x_{extreme}^{min}$ and $x_{extreme}^{max}$ are the minimum and maximum of the extreme values, respectively.

The next milestone includes the calculation of the weighted coefficients (a_{ij}) by applying factor analysis and principal components. Therefore, the aggregation of raw signals from the chosen variables and further index construction is based on the logic of the superposition principle and principal component methodology with VARIMAX rotation to fit the final index in the rage from 0 to 1. Superposition principle means that the net response of the financial safety index caused by two or more stimuli is the sum of the responses which would have been caused by each stimulus individually[3].

The weight coefficients a_{ij} are calculated as follows:

$$a_{ij} = \frac{c_{ij} \cdot | d_{ij} |}{\sum c_{ij} \, | d_{ij} |} \tag{2}$$

where d_{ij} is the weight of the factor, and c_{ij} is a deposit of component j in the summarized dispersion of the collection of indicators of i-th element of the financial safety (% total of variance). Weights are normalized to sum to unity.

The integral index of financial safety (*IFS*) of a country is then as follows:

[3] Every solution, (X_t), to the first order nonhomogeneous linear difference equation $X_t = \varphi X_{t-1} + Z_t$, $\varphi \neq 0$ can be represented as the sum of the general solution to homogenous equation $X_t = \varphi X_{t-1}$., ($X^{(g)}{}_t$), and a particular solution to the nonhomogeneous equation, ($X^{(p)}{}_t$): $X_t = X^{(g)}{}_t + X^{(p)}{}_t$.

$$IFS_j = \sum_i a_{ij} \cdot z_{ij} \tag{3}$$

where IFS_j is a j-th counterpart of the integral index and z_{ij} is the normalized indicator of the factor derived by applying Equation 1.

IV. Empirical results and discussion

The Index estimation results

We acquired quarterly data of Tunisia in the time frame from 2000Q1 – 2014Q3[4] (source of data: International Financial Statistics database and www.thomsonone.com).

After applying Equation 1, we performed Horn's (1965) “parallel" analysis to the normalised data of Tunisia. This analysis suggests that the rational number of factors and principal components equals 2-3. The resulting weights show that the following variables have the highest weights: FDI- inward, real effective exchange rate, M2/Money in circulation (Credit Multiplicator), Money Market Rate, Portfolio Equity Flows, Total Domestic credit/GDP, Money in circulation/GDP*100, M2/M0, PPI / WPI, M2/GDP*100 and velocity of money circulation.

The final Index of financial safety and its dynamics are shown in Fig.1 below.

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[4] During the period 1997-2007, the governance problems in Tunisia affected reporting quality of the financial information provided by the companies (Klai and Omri, 2011). Therefore, this might affect further empirical results.

According to Fig.1, the Index of financial safety of Tunisia is able to capture the main distresses in the Tunisian financial system. From 2000Q1 till 2007Q4, Tunisian IFS grows, indicating that the level of security in Tunisia is increasing. The positive dynamics of IFS of Tunisia during 2000Q1-2007Q4 was ensured by the high contribution of M2/M0 (29% of contribution to IFS), FDIs-inward (19% of contribution to IFS), credit multiplicator (17% of contribution to IFS), M2/market capitalization (11% of contribution to IFS) and M1/M2*100% (9% of contribution to IFS) (Fig.2). We can also assume, that money in circulation, velocity of money, M2/GDP, PPI/WPI, REER, share market, domestic credits, bank sector assets/inflation, monetary rate and coverage of import by reserves had not a dominant role in financial safety of Tunisia. Indeed, it corresponds to the real conditions in the economy of Tunisia. Foreign Direct Investment (FDI) inflows were relatively significant, but mainly focused in the energy sector. According to World Bank, approximately 60 percent on average during 2006-2012 of attracted FDI mainly targeted natural resources. The country's banking system was a sort of a tool for privileged access and was heavily used by the governor and his circle during the pre-revolution period to build up their wealth. As a result, banking sector imposed a significant cost on the economy as they have accumulated significant losses, demonstrating inefficiency and low profitability. Despite the stable level of domestic credits, it remains below the potential for Tunisia because of ineffectiveness at channelling resources in banking system (Source: World Development Indicators).

In terms of currency, Tunisia tightly manages its currency through strict control of capital account. Also, there are many restrictions on capital mobility. Tax incentives are quite ineffective as the main policy instrument to attract investment (IFC, 2009).

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Tunisia safety was tangibly affected in 2007 by the world financial crisis despite the relatively loose ties of Tunisian financial system with the international capital markets. Next, in 2009 Tunisia financial safety was dampened by the international financial crisis and recession in Tunisian's major export destinations countries, namely France and Italy. Although the Tunisia financial system is not highly integrated into world finance, the international crisis influenced Tunisia and the Index of financial safety captures these disturbances. Banking sector continues to demonstrate inefficiency. According to the statistics, the average return on assets (ROA) in Tunisia in 2010 was 0.9%, which is the lowest level among Turkey, Morocco, Jordan and Egypt (Source: Bankscope).

The negative tendency continued in 2011 was caused by domestic events (the revolution) as well as the conflict in neighbouring Libya. The average return on equity (ROE) in Tunisia was 9.9% in 2012, which is low compared to the same indicators in comparator countries, such as Turkey, Morocco, Jordan or Egypt. Tunisia domestic capital markets demonstrate weakness that is caused by weak domestic demand, lack of yield curve, and lax enforcement of prudential banking regulation (according to IMF and World Bank 2012).

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Post-revolution instability in Tunisia has led to a decline in investment, illiquidity and perturbation of prices. The smallest contribution to the integrated IFS of Tunisia over 2007Q2-2011Q3 is observed in the following variables (see Fig.3): inward FDIs, banking sector assets/inflation, claims on government, monetary rate, Tunisian share price index, market capitalization, and coverage of import by reserves. This provides the base for identification of the weakest sides of the Tunisian financial system during the analysed period of time. During 2007Q2-2011Q3 period of time, the contribution of inward FDIs shrunk from 20% to 0% and, indeed, was replaced by remittances as one of the principal sources of foreign exchange. As we can see in Fig.3, the counterparts which dominated over 2000-2007 period of time decreased and were replaced by the minor indicators of financial safety that is caused by the structural reforms. The Central Bank of Tunisia performed a sound control of inflation that caused the increasing of its contribution to the aggregated index of financial safety (the Central Bank has adopted the policy of inflation targeting; it was the twinning project with the Bank of France). Due to gradually tightening monetary policy, the key rate was doubled (to 4%) and provided flexibility to the interbank interest rates as well as ensured better functioning of market mechanisms. The policy also included the steps towards modernization by improvement of market practices and increase the solidity and competitiveness of banks.

Prediction of IFS of Tunisia

Prediction is performed by means of the nonlinear autoregressive with exogenous inputs (NARX) model that represents the neural networks and can emulate any nonlinear, dynamic state space model.

The NARX is a recurrent dynamic network, with feedback connections enclosing several layers (incl. hidden layers) of the network. This model is characterized by non-linear relations between the past inputs, past outputs and the predicted process output, and can be delineated by a high order differential equation. The defining equation for the NARX model is the following (Leontaritis & Billings, 1985; Siegelmann *et al*., 1997; Haykin, 1999; Menezes and Barreto, 2006 *etc*.):

$$y_t = f\left(y_{t-1}, y_{t-2}, \dots, y_{t-n_y}, u_{t-1}, u_{t-2}, \dots, u_{t-n_u}\right) + e_t \tag{4}$$

where the next value of the dependent process y_t is regressed on previous values of the output signal (y) and previous values of an independent input signal (u); n_u and n_y are the corresponding maximum lags for input and output; e_t explains the uncertainties and possible noise. The function f is a nonlinear function. The system input vector with a known dimension $n = n_y + n_u$ is

$$\bar{X} = [y_{t-1}\ y_{t-2} \dots\ y_{t-n_y}\ u_{t-1}\ u_{t-2} \dots\ u_{t-n_u}]' \tag{5}$$

The function f is unknown. Thus, it is approximated by the regression model of the form:

$$y_t = \sum_{i=0}^{n_u} a_i . u_{t-i} + \sum_{j=0}^{n_y} b_j . y_{t-j} + \sum_{i=0}^{n_u} \sum_{j=1}^{n_u} a_{i,j} . u_{t-i} . u_{t-j} + \sum_{i=1}^{n_y} \sum_{j=1}^{n_y} b_{i,j} . y_{t-i} . y_{t-j} + \sum_{i=0}^{n_u} \sum_{j=1}^{n_y} c_{i,j} . u_{t-i} . y_{t-j} + e_t \tag{6}$$

where a_i and $a_{i,j}$ are the linear and nonlinear coefficients of the originating exogenous term respectively; b_j and $b_{i,j}$ are the linear and nonlinear coefficients of the autoregressive term respectively; $c_{i,j}$ are the coefficients of the nonlinear terms.

The implementation of the approximation of the function f also allows for a vector NARX model to be estimated (i.e.: the input and output can be multidimensional). The detailed model is provided in Appendix 1.

Although, the Tunisian raw time series include only 59 observations of 19 exogenous variables, we trained the network to generate the response (IFS of Tunisia) with adequate quality.

For this purpose, we applied Levenberg-Marquardt algorithm for network training. It is an iterative and adaptive technique that finds the minimum values of a multivariate function. This minimum is derived as the sum of squares of non-linear real-valued functions (Marquardt, 1963). In the estimation process, when model parameters are not close to their optimal values, the Levenberg-Marquardt algorithm acts in a highly convergent gradient-descent way by updating parameter values in the direction opposite to the gradient of the objective function. In other cases, when parameters are close to their optimal values, the Levenberg-Marquardt algorithm starts acting as the Gauss-Newton method, assuming that the objective multivariate function is quadratic in parameters near their optimal solutions.

The basis of Levenberg-Marquardt algorithm is the linear approximation to f . When the performance function is in the form of a sum of squares, the Hessian matrix is approximated in the way:

$$H = J'J \tag{7}$$

and the gradient is calculated as:

$$g = J'\epsilon \tag{8}$$

where J is the Jacobian matrix[5] with the first derivatives of the network errors with respect to the weight and biases; ϵ – is a vector of network errors. Next, the algorithm applies the received approximation to the Hessian matrix:

[5] The Jacobian matrix max be calculated with the application of a standard backpropagation technics

$$x_{k+1} = x_k - [J'J + \mu I]^{-1} J'\epsilon. \quad (9)$$

In Equation 9, if a scalar $\mu = 0$, the algorithm becomes the Newton's method. In other cases, this method becomes gradient descent with a small step size. The detailed analysis of Levenberg-Marquardt algorithm is beyond the scope of this research (for more comprehensive treatments please refer, among others, to Nielsen, 1999; Nocedal & Wright, 1999; Kelley, 1999 *etc*.).

We randomly divided input vectors and targets vectors into three sets as follows: 75% are used for training; 15% are used to validate that the network is generalizing and to stop training before overfitting; the last 15% are used as a completely independent test of network generalization.

The performance of the system models is compared using a mean square error normalized with respect to the variance of the target signal. Mean Squared Error (MSE) is defined as:

$$MSE = \frac{1}{N}\sum_{i=1}^{N}(e_i)^2 = \frac{1}{N}\sum_{i=1}^{N}(t_i - a_i)^2 \quad (10)$$

where t_i – are target outputs, a_i – are network outputs.

According to our results (Table 2), the small values of *MSE* and the high values of *R* give us the base to consider the obtained NARX model as adequate.

-------------------- insert Table 2 about here --------------------

Fig. 4 shows that the process of network's performance is improved during training the network for IFS of Tunisian prediction model. This kind of performance is measured in terms of mean square error and it is shown in log scale. It is rapidly decreasing while the network is trained. In our case, the results are reasonable, because the following:

- The final mean-square error is very small (less than 10^{-2});

- The test set error and the validation set error have similar characteristics.

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The training data indicates a reasonably good fitting a function of the IFS of Tunisia to a set of raw data (Fig. 5). The validation and the test results also show that R values that are high. Not perfect fitting might be explained that we have not long time-series (59 observations).

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Fig. 6 shows how the error sizes are distributed. Typically, most errors are nearby zero that also illustrates a reasonably good fit.

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We also tested correlation of errors with the input sequence. The perfect prediction model means that all the correlations should be zero. In our case, all of the correlations are within the confidence bounds around zero.

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Fig. 8 describes the function of autocorrelations of errors to validate the network performance. Autocorrelation describes how the prediction errors are related in time. For the perfect model, there should be only one nonzero value of the autocorrelation at zero lag (this is the mean square error). It means that there is no correlation in prediction errors with each other and there is white noise. In our case the autocorrelation of errors are within the 95% confidence limits around zero.

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Fig. 9 below shows the results of Tunisian IFS simulation and based on it, we can assume that NARX prediction model is adequate, as it shows small quantity of errors.

V. Concluding remarks

We reinvestigated the findings of Matkovskyy (2012, 2013, 2014) and improved the performance of this Index of financial safety. For this purpose we extended the model for Index construction with inward FDIs, banking sector assets/inflation, portfolio equity flows and net claims on government. Based on the empirical results, there is evidence that the Index of Financial Safety is able to capture the main distresses in financial system of Tunisia.

An analysis of the constructed IFS can identify the main counterparts with the most negative dynamics during the disturbances in the financial system of Tunisia. The smallest contribution to the integrated IFS of Tunisia during crisis is observed in inward FDIs, banking sector assets/inflation, net claims on government, monetary rate, Tunisian share price index, market capitalization and coverage of import by reserves. Therefore, these indicators are critical in ensuring a sufficient level of the financial safety of Tunisia during crisis. The magnitudes M2, inflation, money market rate, FDI- inward, REER, portfolio equity flows and ratio of total domestic credit to GDP confirm their importance for the financial system conditions monitoring.

The results also show that the nonlinear autoregressive with exogenous input (NARX) model with Levenberg-Marquardt algorithm for network training can ensure adequate quality of the forecast of short-time series. The NARX model with nu=4 and ny= 4 and the number of hidden neurons=48 was chosen since it gave the best performance according to MSE.

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Appendix 1

Equation (3) may be re-written in the matrix form:

$$\begin{bmatrix} y_t \\ y_{t+1} \\ \vdots \\ y_{t+n_y} \end{bmatrix} = a.u' + b.y' + A.[U]' + B.[Y]' + C.[X]' \tag{A1}$$

where

$$a = [a_0 \; a_1 \; \dots \; a_{n_u}]' \tag{A2}$$

$$b = [b_1 \; b_2 \; \dots \; b_{n_y}]' \tag{A3}$$

$$A = [a_{0,0} \; a_{0,1} \; \dots \; a_{0,n_u} a_{1,1} \; \dots \; a_{n_u,n_u}]' \tag{A4}$$

$$B = [b_{1,1} \; b_{1,2} \; \dots \; b_{1,n_y} b_{2,2} \; \dots \; b_{n_y,n_y}]' \tag{A5}$$

$$C = [c_{0,1} \; c_{0,2} \; \dots \; c_{0,n_y} c_{1,1} \; \dots \; c_{n_u,n_y}]' \tag{A6}$$

$$u = [u_{t-1} \; u_{t-2} \; \dots \; u_{n_y}] \tag{A7}$$

$$y = [y_{t-1} \; y_{t-2} \; \dots \; y_{n_y}] \tag{A8}$$

$$U = [u_t.u_t u_t.u_{t-1}.u_{t-2} \dots u_t.u_{t-n_u}.u_{t-1}.u_{t-1} \dots u_{t-n_u} u_{t-n_u}] \tag{A9}$$

$$Y = [u_t.y_{t-1} u.y_{t-2} \dots u_t.y_{t-n_y} u_{t-1}.y_{t-1} \dots u_{t-n_u}.u_{t-n_y}] \tag{A10}$$

Alternatively, Equation 4 may be re-written as:

$$y_t = [u'y'U'Y'X'] \begin{bmatrix} a \\ b \\ A \\ B \\ C \end{bmatrix} \tag{A11}$$

To simplify we denote:

$$\bar{Y} = y_t \tag{A12}$$

$$\bar{U} = [u'y'U'Y'X'] \tag{A13}$$

and

$$\bar{C} = [a\ b\ A\ B\ C]' \quad \text{(A14)}$$

Which means that equation (A11) now is simplified as:

$$\bar{Y} = \bar{U}.\bar{C} \quad \text{(A15)}$$

while the solution to the problem can be presented by

$$\bar{C} = \bar{U} \backslash \bar{Y} \quad \text{(A16)}$$

Table 1. The financial safety variables

Indicators	non-stimulant/stimulant
Money in circulation/M2*100	non-stimulant
Money in circulation/GDP*100	non-stimulant
M1/M2*100%	stimulant
M2/GDP*100	stimulant
M2/M0	stimulant
PPI / WPI	non-stimulant
Money market interest rates %	non-stimulant
GDP/M2	stimulant
M2/Money in circulation	non-stimulant
Monetary base/reserves	stimulant
Coverage of import by reserves	stimulant
Total domestic credit/GDP	non-stimulant
M2/ market capitalization	non-stimulant
Changes of share price index % to a previous quarter	stimulant
Real effective exchange rate	non-stimulant
Inward FDIs	stimulant
Banking Sector Assets/Inflation	stimulant
Portfolio Equity Flows	stimulant
Claims on Government (net)	non-stimulant

Source: Authors' analysis; Data source: http://www.thomsonone.com/

Table 2. The result of the NARX model training for Tunisian IFS prediction (Levenberg-Marquardt algorithm)

	Target value	*MSE*	*R*
Training	41	1.04352e-17	1.0000
Validation	9	1.41096e-3	9.1299e-1
Testing	9	4.51787e-4	8.04725e-1

Source: Computed by the authors
Notes: The number of the hidden neurons equals 48, delay equals 4

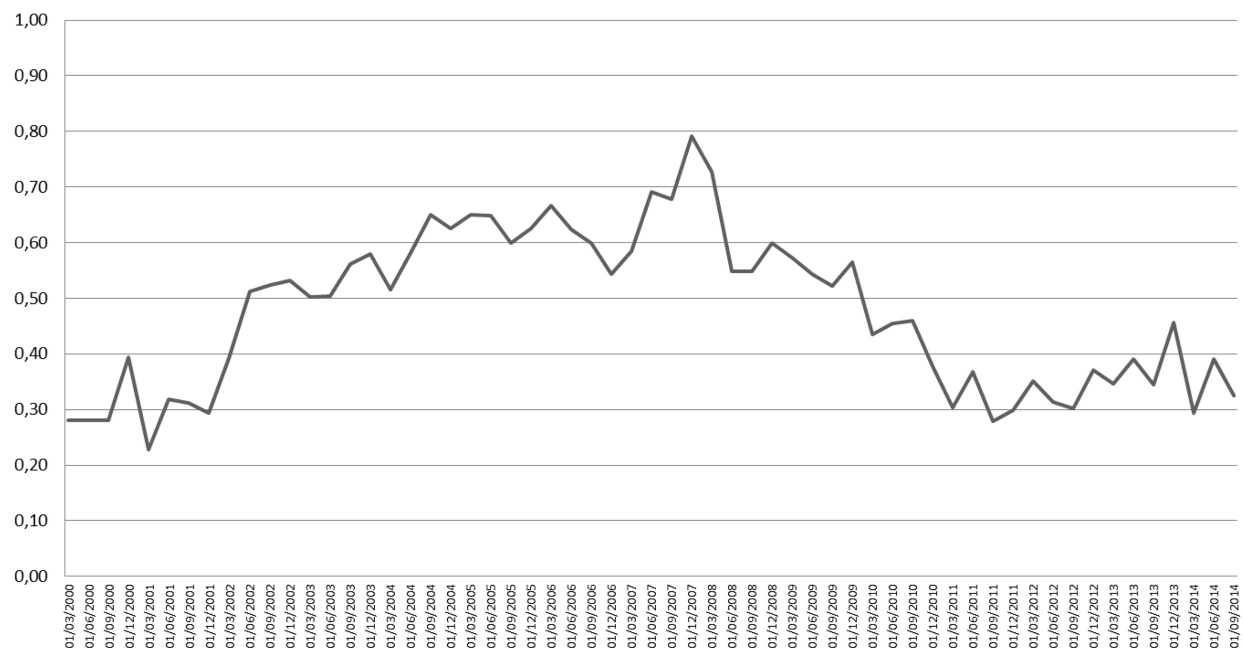

Fig.1. The index of financial safety of Tunisia over 2000Q1-2014Q3 period of time

Source: Computed by the authors

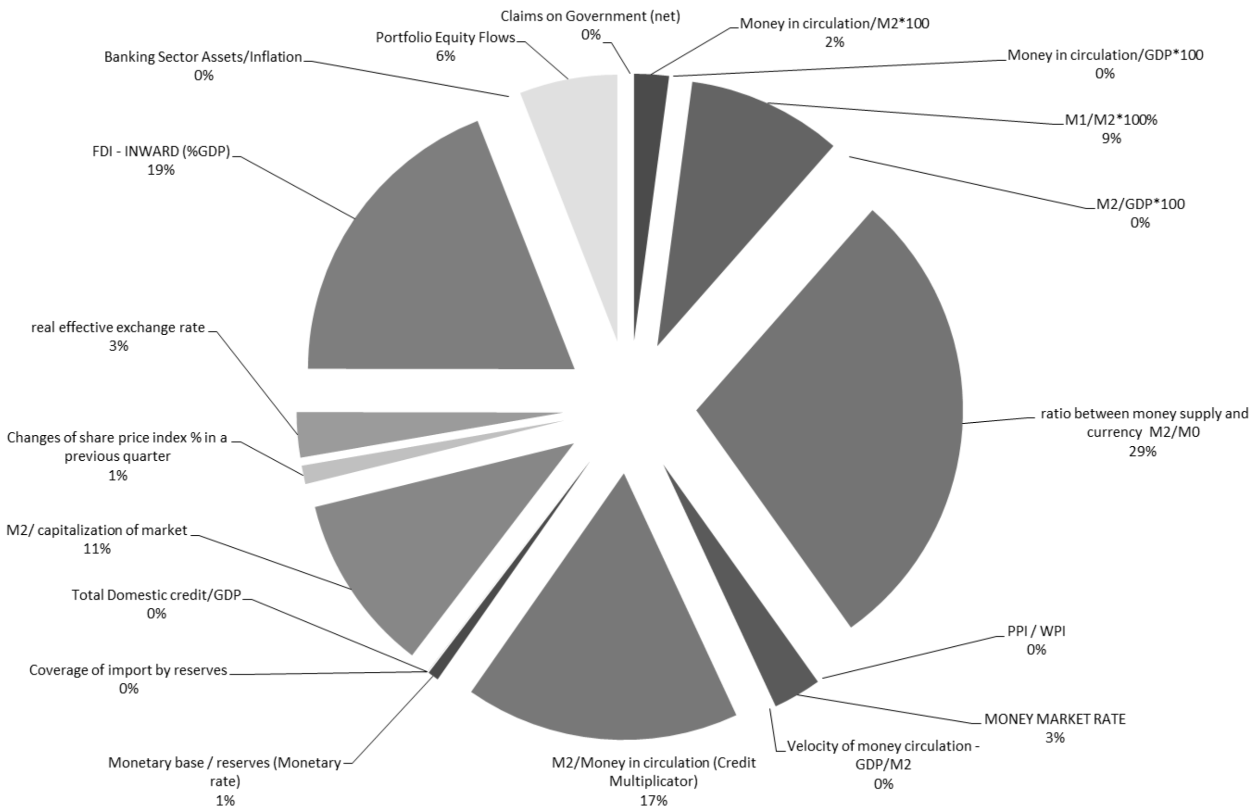

Fig.2. The counterparts of the index of financial safety of Tunisia over 2000Q1-2007Q4 period of time

Source: Computed by the authors

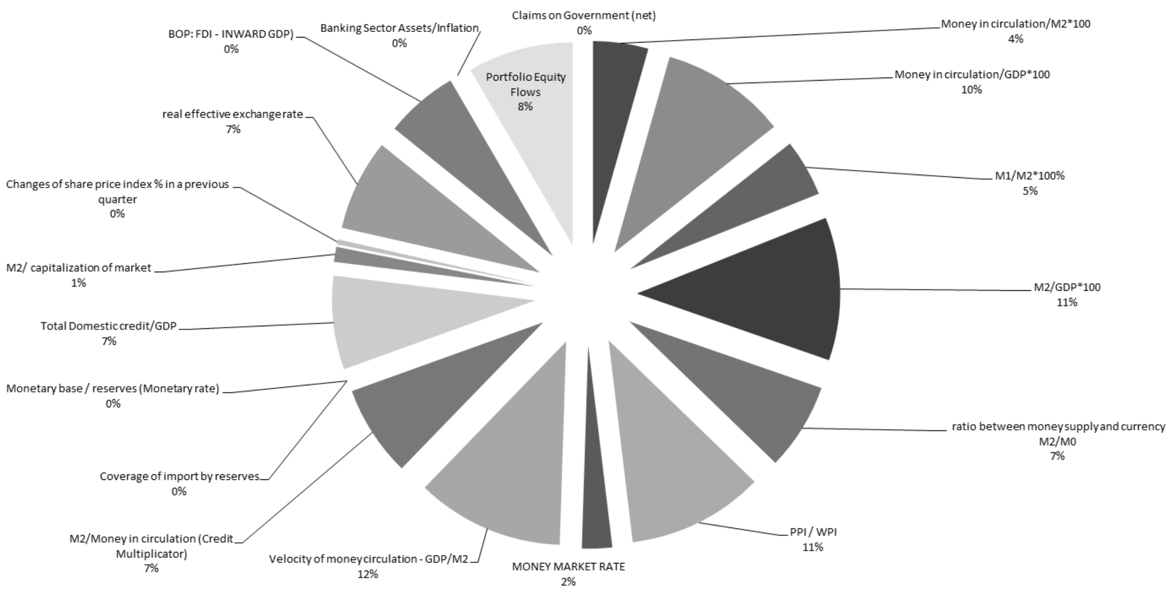

Fig. 3. The counterparts of the index of financial safety of Tunisia over 2007Q4-2011Q3 period of time

Source: Computed by the authors

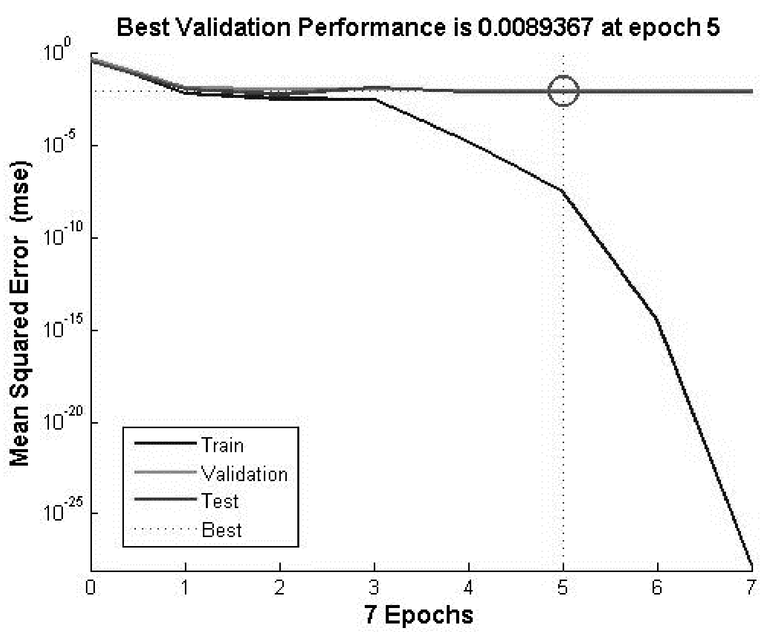

Fig. 4. The process of network's performance improved during training

Source: Computed by the authors

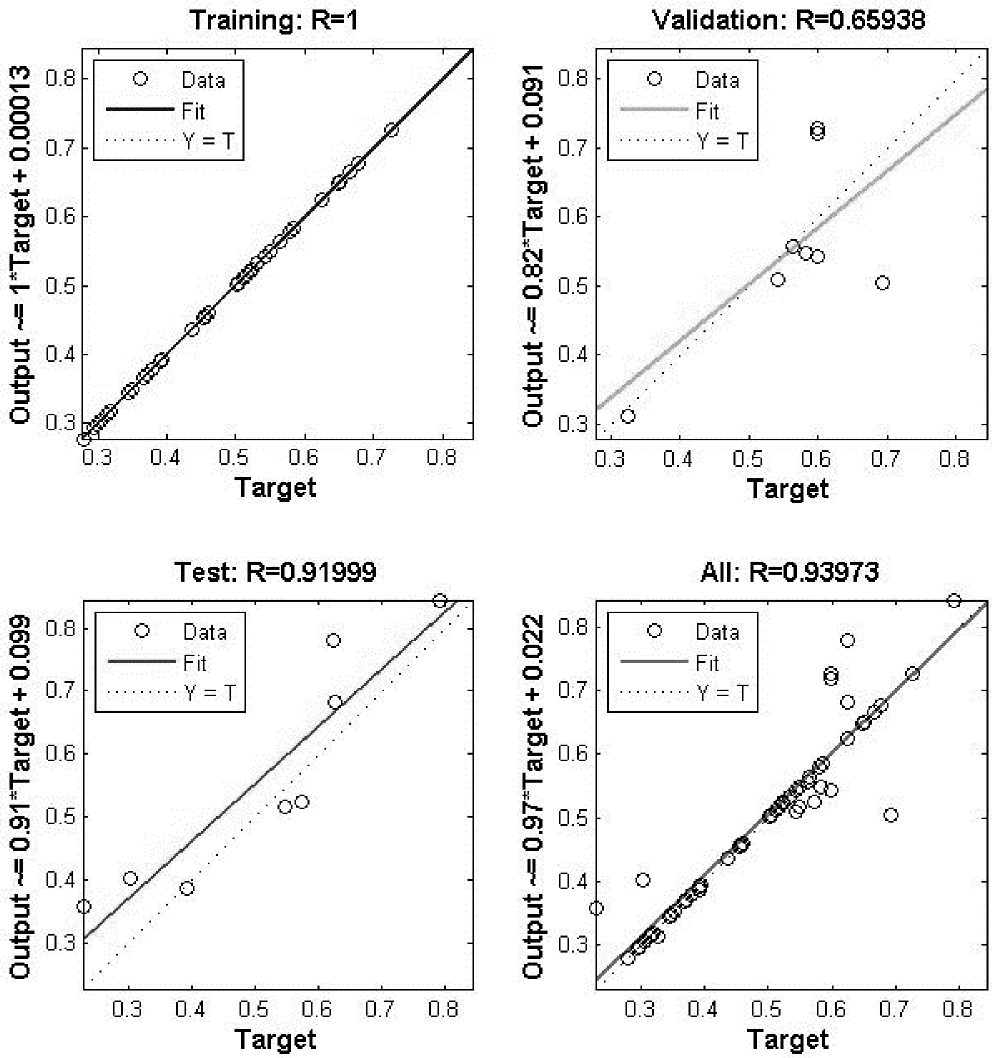

Fig. 5. Regression plot for the NARX to predict IFS of Tunisia

Source: Computed by the authors

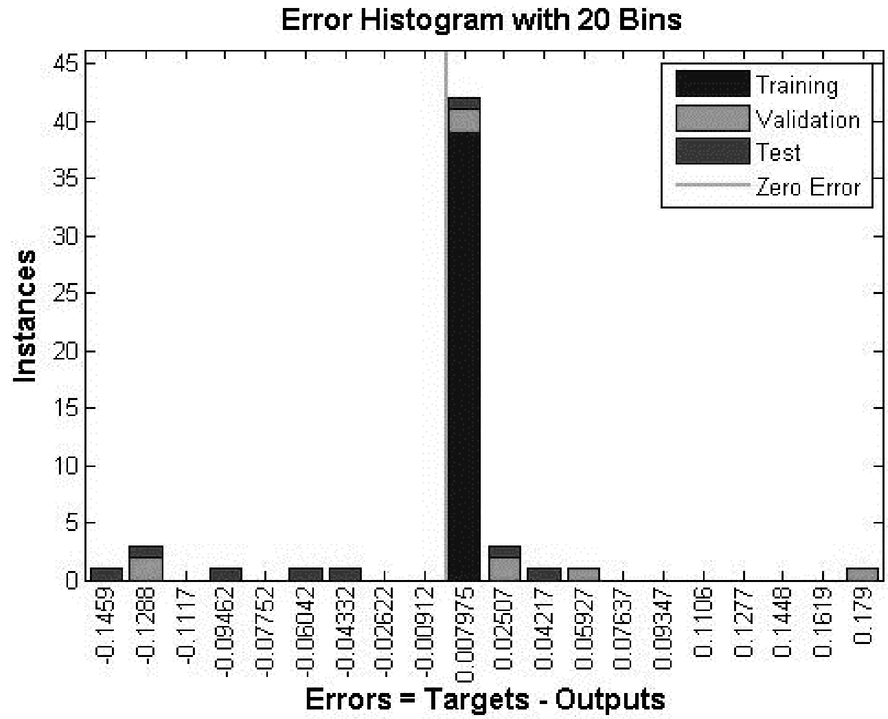

Fig. 6. Error histogram of the NARX prediction model for IFS of Tunisia

Source: Computed by the authors

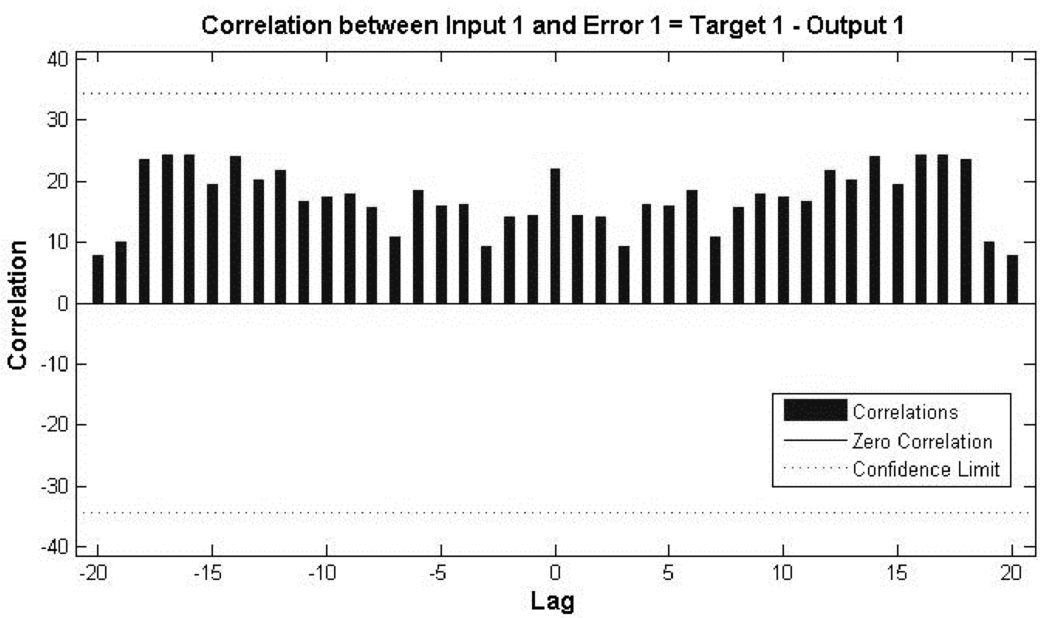

Fig. 7. The input-output cross-correlation function

Source: Computed by the authors

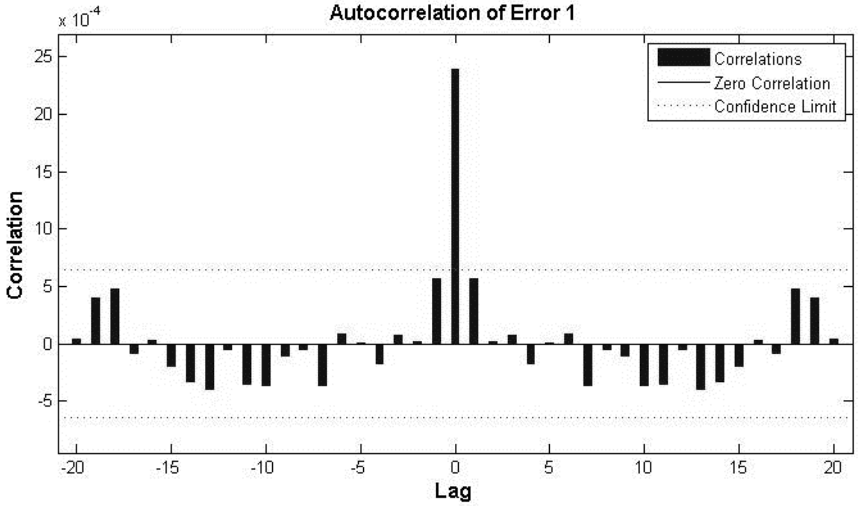

Fig. 8. Autocorrelation of errors of NARX prediction model for IFS of Tunisia

Source: Computed by the authors

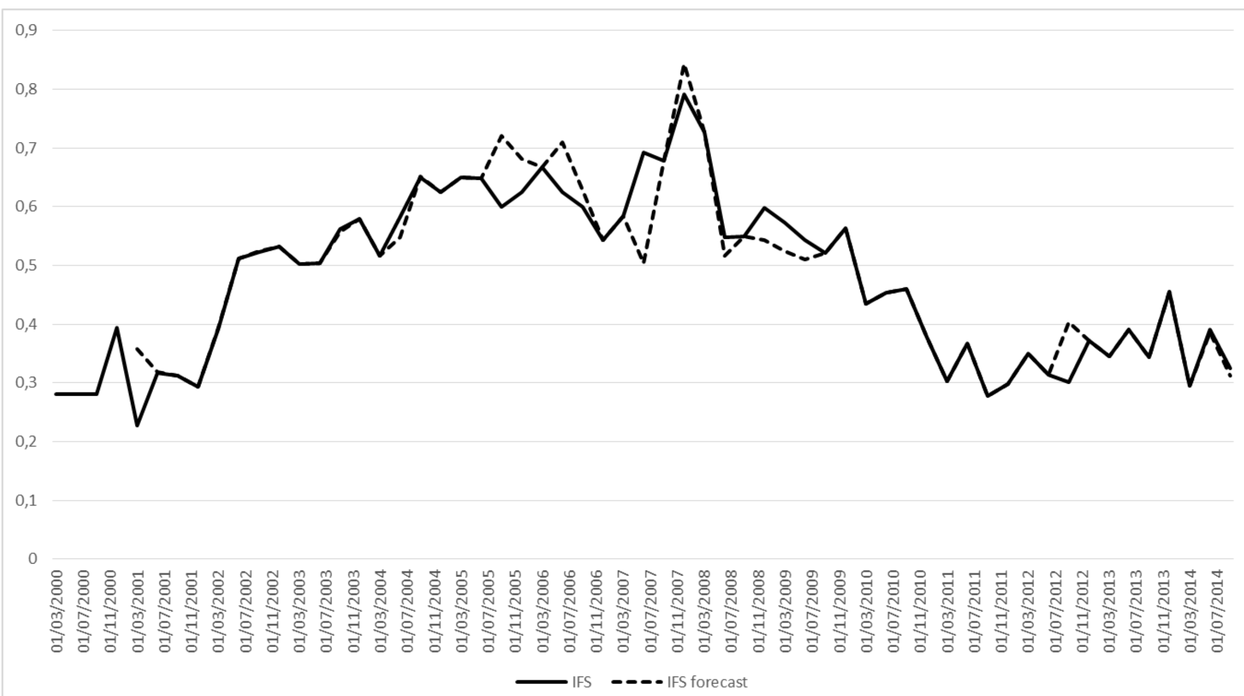

Fig. 9 Response of NARX prediction model for IFS of Tunisia (trained by the Levenberg-Marquardt algorithm)

Source: Computed by the authors