Specialization, Matching Intensity and Income Inequality of Sellers

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Abstract

We develop a simple model with heterogeneous agents and search frictions to study how increases in matching intensity between buyers and sellers determine the level of income inequality among sellers. Our findings indicate that a reduction in search frictions leads to higher inequality and induces buyers to purchase goods and services only from specialized sellers.

JEL classification: C78; O30
Keywords: game theory; income inequality; matching; technology; value functions

1 Introduction

The impact of technological change on income inequality is well-documented in the literature (Acemoglu, 2002). Technological advances increase the demand for skilled labor leading to a widening of the wage gap between skilled and unskilled workers (Acemoglu, 1998; Card and Dinardo, 2002). Moreover, the reduction of the power of low-skilled workers due to closer monitoring of their effort, triggered by adoption of new technologies, (Skott and Guy, 2007) and

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the so-called 'general' purpose technology (Aghion et al., 2002) -as expressed by skill transferability and vintage capital compatibility- can lead to higher inequality. In the current paper, the effect of technology on income inequality is examined within a different context; we utilize a simple matching model to study how increases in matching intensity between buyers and sellers[1] affects income inequality among sellers.

Our findings indicate that a reduction in frictions due to technological advancements leads to equilibrium outcomes where buyers purchase goods and services only from specialized sellers. In this case, income inequality is high. In contrast, when frictions are high, finding a specialized seller is more difficult leading to matches with non-specialized sellers and hence a less polarized distribution of sellers' earnings. Our results are consistent with the findings of Brynjolfsson et al. (2016). In this study, a positive trend in income inequality among professional sellers in eBay is identified. Furthermore, Brynjolfsson et al. (2016) conclude that growth is experienced most likely by sellers providing specialized offerings. The rest of the paper is structured as follows. The next Section presents the model while Section 3 discusses market equilibria. Our main results are presented in Section 4. Finally, Section 5 concludes the paper.

2 The Model

2.1 Environment

We consider a market for a service. The market comprises of a large number (normalized to one) of buyers and the same number of sellers who live in

[1] These increases can be translated into reductions in search frictions enabling buyers to find the seller meeting their needs best. Decreases in frictions may be caused by developments in information technology (e.g., development of e-commerce).

continuous time. Buyers and sellers are of two types; h and w.[2] The proportion of h-type buyers and h-type sellers is ϕ and θ, respectively (where $0 \leq \theta, \phi \leq 1$). Sellers and buyers meet each other at a Poisson arrival rate β (see Mortensen, 1986). The utility that buyers get from the service they receive from sellers is u. Seller i has cost (disutility of work) zero for i job, $i = h, w$, where seller i has cost $c \in (0, u]$ for not i job. As soon as they match, agents bargain over the price through a Nash bargaining process (we assume 50/50 bargaining power). When agents of different type meet, then the match will form if the value of c is small enough and more precisely if it is below a threshold value (reservation value). We assume that all participants are risk neutral discounting the future at the same rate r. Buyers leave the market after trading and they are replaced by identical ones (clones). Sellers are infinite lived and stay in the market forever.

2.2 Exchange

The price that sellers charge buyers will be

$$p_{ij}^{*} = \arg\max_{p_{ij}} [u - p_{ij} - V_j^b]^{\frac{1}{2}} [p_{ij} - c]^{\frac{1}{2}} \tag{1}$$

where $i, j = h, w$. The first subscript denotes the type of seller and the second the type of buyer. Hence p_{ij} is the price charged by i seller to j buyer. V_j^b represents the value of being a j type buyer. The outside options are the continuation values. Solving (1) we get

$$p_{ij}^{*} = \frac{1}{2}[u + c - V_j^b] \tag{2}$$

[2] h (w) type buyers demand sellers for h (w) type jobs. h (w) type sellers are specialized in h (w) type jobs.

where $c = 0$ if $i = j$, and $c \in (0, u]$ if $i \neq j$.

3 Market Equilibria

Equilibria are patterns of trading from which no one wants to deviate. Hence our objective, is to identify candidate equilibrium patterns of trade and find the portions of the parameter space for which they are self-sustaining.

In type 1 equilibrium pattern all agents trade with each other. Let V_i^{tn} be the value[3] of being a group t individual of type i when everyone behaves according to type n trading pattern.[4]

In the case of type 1 equilibrium pattern we have that

$$rV_h^{b1} = \beta[\theta(u - p_{hh}) + (1-\theta)(u - p_{wh}) - V_h^{b1}] \tag{3a}$$

$$rV_w^{b1} = \beta[\theta(u - p_{hw}) + (1-\theta)(u - p_{ww}) - V_w^{b1}] \tag{3b}$$

$$rV_h^{s1} = \beta[\phi p_{hh} + (1-\phi)(p_{hw} - c)] \tag{3c}$$

$$rV_w^{s1} = \beta[\phi(p_{wh} - c) + (1-\phi)p_{ww}] \tag{3d}$$

By substituting p_{ij}^*'s and doing the algebra we get

$$(r + \frac{\beta}{2})V_h^{b1} = \beta[(\theta u/2) + (1-\theta)(u-c)/2] \tag{4a}$$

$$(r + \frac{\beta}{2})V_w^{b1} = \beta[\theta(u-c)/2 + (1-\theta)u/2] \tag{4b}$$

[3] where t = buyer (b), seller (s).

[4] Alternatively, V_i^{tn} is the discounted expected value of a group t individual of type i under n equilibrium trading pattern.

$$rV_h^{s1} = \beta[\frac{\phi}{2}(u - V_h^{b1}) + \frac{(1-\phi)}{2}(u - c - V_w^{b1})] \tag{4c}$$

$$rV_w^{s1} = \beta[\frac{\phi}{2}(u - c - V_h^{b1}) + \frac{(1-\phi)}{2}(u - V_w^{b1})] \tag{4d}$$

In order to find the portions of the parameter space which ensure the existence of type 1 equilibrium, we have to consider whether an individual of any type and group may want to deviate from the above defined trading pattern. Firstly, we ensure that participation in the market is worthwhile. Hence, we get the following participation (rationality) constraints; $V_h^{b1}, V_w^{b1}, V_h^{s1}$ and $V_w^{s1} \geq 0$ (solution of (3a)-(3d) should satisfy the rationality constraints). Therefore, trading with agents of different type is worthwhile only if the value of trading (and re-entering the market[5]) exceeds that of keep searching for a trading partner. In other words, we get the following

$$u - p_{wh} \geq V_h^{b1},\ u - p_{hw} \geq V_w^{b1},\ p_{wh} + V_h^{s1} - c \geq V_h^{s1},\ p_{hw} + V_w^{s1} - c \geq V_w^{s1}$$

Hence the relevant incentive compatible constraints are

$$u - p_{wh} \geq V_h^{b1},\ u - p_{hw} \geq V_w^{b1},\ p_{wh} - c \geq 0,\ p_{hw} - c \geq 0 \tag{5}$$

By substituting the relevant values for p_{ij}^*'s from (2) and doing some algebra, we get that the above constraints are satisfied if the following conditions hold.

$$\frac{1}{2}[u - c - V_h^{b1}] \geq 0 \tag{6}$$

and

[5] This holds for sellers.

$$\frac{1}{2}[u - c - V_w^{b1}] \geq 0 \tag{7}$$

It can be easily shown that if (6) and (7) are satisfied then the trade with agents of the same type is worthwhile as well. By substituting (4a) and (4b) into (6) and (7), respectively and solving with respect to c we get the following reservation values:

$$\tilde{c}_h(\theta) = u[1/(1+\gamma)] \tag{8}$$

where $\gamma = \beta\theta/2r$ and

$$\tilde{c}_w(\theta) = u[1/(1+\zeta)] \tag{9}$$

where $\zeta = \beta(1-\theta)/2r$.

If c exceeds the above reservation values then type 1 trading pattern is no longer an equilibrium since there is an incentive for deviation. If the proportion of h-type sellers is greater than 1/2 then $\tilde{c}_w > \tilde{c}_h$ and therefore type 1 trading pattern is an equilibrium if $c \leq \tilde{c}_h$. Doing the same for $\theta = 1/2$ and $\theta < 1/2$ and summarizing all the above results we get the following proposition.

Proposition 1. *Type 1 trading pattern is an equilibrium if $c \leq \tilde{c}_h(\theta)$ for $\theta > 1/2$, $c \leq \tilde{c}_w(\theta)$ for $\theta < 1/2$, $c \leq \tilde{c} = \tilde{c}_w(\theta) = \tilde{c}_h(\theta)$ for $\theta = 1/2$, given β, r.*

In type 2 equilibrium pattern h-type buyers do not trade with w-type sellers (but w-type buyers trade with h-type sellers). The relevant equations for buyers and sellers will be

$$(r + \beta\theta/2)V_h^{b2} = \beta(\theta u/2) \tag{10a}$$

$$(r+\frac{\beta}{2})V_w^{b2} \ = \beta[\theta(u-c)/2+(1-\theta)u/2] \tag{10b}$$

$$rV_h^{s2} = \beta\frac{\phi}{2}(u-V_h^{b2}) \tag{10c}$$

$$rV_w^{s2} = \beta[\frac{\phi}{2}(u-c-V_h^{b2})+\frac{(1-\phi)}{2}(u-V_w^{b2})] \tag{10d}$$

Given the above equations and Proposition 1, we get

Proposition 2. *Type 2 trading pattern is an equilibrium if* $\tilde{c}_w \geq c > \tilde{c}_h$ *and* $\theta > 1/2$, *given* β, r.

In type 3 trading pattern w-type buyers do not trade with h-type sellers (but h-type buyers trade with w-sellers). The relevant equation for each individual is

$$(r+\frac{\beta}{2})V_h^{b3} \ = \beta[(\theta u/2)+(1-\theta)(u-c)/2] \tag{11a}$$

$$[r+\beta(1-\theta)/2]V_w^{b3} \ = \beta(1-\theta)u/2 \tag{11b}$$

$$rV_h^{s3} = \beta[\frac{\phi}{2}(u-V_h^{b3})+\frac{(1-\phi)}{2}(u-c-V_w^{b3})] \tag{11c}$$

$$rV_w^{s3} = \beta\frac{(1-\phi)}{2}(u-V_w^{b3}) \tag{11d}$$

Following the same logic, we get

Proposition 3. *Type 3 trading pattern is an equilibrium if* $\tilde{c}_h \geq c > \tilde{c}_w$ *and* $\theta < 1/2$, *given* β, r.

In type 4 equilibrium buyers of type i do not trade with sellers of type j. The relevant equations are

$$(r+\beta\theta/2)V_h^{b4} \ = \beta(\theta u/2) \tag{12a}$$

$$[r+\beta(1-\theta)/2]V_w^{b4} \ = \beta(1-\theta)u/2 \tag{12b}$$

$$rV_h^{s4} = \beta\frac{\phi}{2}(u - V_h^{b4}) \tag{12c}$$

$$rV_w^{s4} = \beta\frac{(1-\phi)}{2}(u - V_w^{b4}) \tag{12d}$$

Proposition 4. *Type 4 trading pattern is an equilibrium if $c > \tilde{c}_w$ for $\theta > 1/2$, $c > \tilde{c}_h$ for $\theta < 1/2$, $c > \tilde{c} = \tilde{c}_w = \tilde{c}_h$ for $\theta = 1/2$, given β, r.*

Finally, when $c = \tilde{c}_h$ or $c = \tilde{c}_w$ there is a continuum of mixed strategy equilibria. In such cases, we assume that all individuals (weakly) prefer to stick to the trading pattern specified each time.

4 Matching rate and income differentials

Figure 1 graphs $\tilde{c}_w$ and $\tilde{c}_h$ against θ. The regions marked 1, 2, 3 and 4 correspond to the values of c and θ for which the unique equilibrium trading patterns are respectively Type 1 (every meeting generates transactions), Type 2 ('h-type equilibrium'),Type 3 ('w-type equilibrium') and Type 4 (i type buyers do not trade with j type sellers). Moreover $\tilde{c}_h(\theta)$ is decreasing and convex, $\tilde{c}_w(\theta)$ is increasing and convex, $\tilde{c}_h(1) > 0$ and $\tilde{c}_w(1) = u$.

By differentiating $\tilde{c}_w$ and $\tilde{c}_h$ with respect to matching rate β, we get

$$\frac{\partial \tilde{c}_h}{\partial \beta} = -\frac{2r\theta u}{(2r + \beta\theta)^2} \leq 0 \tag{13}$$

$$\frac{\partial \tilde{c}_w}{\partial \beta} = -\frac{2r(1-\theta)u}{[2r + \beta(1-\theta)]^2} \leq 0 \tag{14}$$

The above derivatives indicate the amount by which $\tilde{c}_w$ and $\tilde{c}_h$ will decrease if the matching rate will increase by one per cent. Since both derivatives are negative, an increase in β will rotate (rightward for $\tilde{c}_w$ and leftward for $\tilde{c}_h$) the graph of each reservation value around u. The new reservation curves will be $\tilde{c}_w'$ and $\tilde{c}_h'$ as illustrated in figure 2.

[Figure 1 about here]

[Figure 2 about here]

As we can see from figure 2 an increase in matching rate decreases area 1 and increases area 4. In other words, an increase in β increases the range of c's leading to a type 4 equilibrium (buyers of type i do not trade with sellers of type j) and decreases the range of c's leading to a type 1 equilibrium (every meeting generates transactions).

Let assume now that we are initially in type 1 equilibrium trading pattern and $\theta = 1/2$ (i.e., the proportion of h and w type sellers is the same). Moreover, we assume that $\tilde{c} - \frac{ru}{(2r+\beta/2)^2} < c \leq \tilde{c} = \frac{4ru}{4r+\beta}$, where $\frac{4ru}{4r+\beta}$ is the value of reservation cost for $\theta = 1/2$ and $\frac{ru}{(2r+\beta/2)^2}$ is the value by which reservation cost decreases when β increases by one per cent. We set $\tilde{c} - \frac{ru}{(2r+\beta/2)^2} < c$, in order to ensure that an increase in β by one unit will shift us to equilibrium trading pattern 4 (look at figure 2). In type 1 equilibrium the absolute difference between expected lifetime income of sellers is

$$\begin{aligned}\Delta_1 &= \left|V_h^{s1} - V_w^{s1}\right| = \left|\begin{array}{c}\frac{\beta\phi u}{2r} - \frac{\beta\phi}{2r}V_h^{b1} + \frac{\beta u}{2r} - \frac{\beta c}{2r} - \frac{\beta}{2r}V_w^{b1} - \frac{\beta\phi u}{2r} + \frac{\beta\phi c}{2r} + \frac{\beta\phi}{2r}V_w^{b1} - \frac{\beta\phi u}{2r} \\ +\frac{\beta\phi c}{2r} + \frac{\beta\phi}{2r}V_h^{b1} - \frac{\beta u}{2r} + \frac{\beta}{2r}V_w^{b1} + \frac{\beta\phi u}{2r} - \frac{\beta\phi}{2r}V_w^{b1}\end{array}\right| \\ &= \left|\frac{2\beta\phi c}{2r} - \frac{\beta c}{2r}\right| \Longrightarrow \Delta_1 = \frac{\beta\left|2\phi - 1\right|}{2r}c \qquad (15)\end{aligned}$$

For $\phi > 1/2$ (i.e., the proportion of h-type buyers is greater than the proportion of w-type ones) the difference is positive and $\frac{\beta|2\phi-1|}{2r}c = \frac{\beta(2\phi-1)}{2r}c$. If

$\phi < 1/2$ then the difference is negative and $\frac{\beta|2\phi-1|}{2r}c = -\frac{\beta(2\phi-1)}{2r}c$. the greater value which Δ_1 can take is $\frac{2\beta|2\phi-1|u}{4r+\beta}$ (i.e., when $c = \tilde{c}$). Under the assumption that $c > \tilde{c} - \frac{ru}{(2r+\beta/2)^2}$, an increase in β will lead to type 4 equilibrium. The absolute difference of sellers' income will be

$$\Delta_4 = \left|V_h^{s4} - V_w^{s4}\right| \left|\frac{\beta\phi u}{2r} - \frac{\beta\phi}{2r}V_h^{b4} - \frac{\beta u}{2r} + \frac{\beta}{2r}V_w^{b4} + \frac{\beta\phi u}{2r} - \frac{\beta\phi}{2r}V_w^{b4}\right| \qquad (16)$$

But for $\theta = 1/2$, $V_w^{b4} = V_h^{b4}$ and hence $\Delta_4 = \left|\frac{2\beta\phi u}{2r} - \frac{2\beta\phi}{2r}V_h^{b4} - \frac{\beta u}{2r} + \frac{\beta}{2r}V_h^{b4}\right| = \left|\frac{\beta(2\phi-1)(u-V_h^{b4})}{2r}\right|$. By substituting V_h^{b4}, we get that $\Delta_4 = \frac{2\beta_4|2\phi-1|u}{4r+\beta_4}$. The subscript in β indicates that β in Δ_4 is different from β in Δ_1 (actually $\beta_4 > \beta$, where β is the matching rate in type 1 equilibrium trading pattern). When β increases then $\frac{\beta}{4r+\beta}$ increases as well. Hence $\frac{2\beta_4|2\phi-1|u}{4r+\beta_4} > \frac{2\beta|2\phi-1|u}{4r+\beta}$. Since Δ_4 is greater than the greater value that Δ_1 gets, then Δ_4 is always greater than Δ_1. It can be easily proven that shifts from equilibrium 1 to equilibria 2, 3 or 4 (due to β), lead to the same result. Therefore, we conclude that an increase in β leads to an increase in income differentials. Hence, factors which can increase matching rate (such as developments in information technology) intensify the inequality of sellers' income.

5 Conclusion

Utilizing a simple model with heterogeneous buyers and sellers, where informational frictions are present, we show that factors reducing their level, such as the introduction and development of information technology, will aggravate income inequality among sellers. This paper calls for further research on this topic either by a theoretical or an empirical perspective.

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Figures

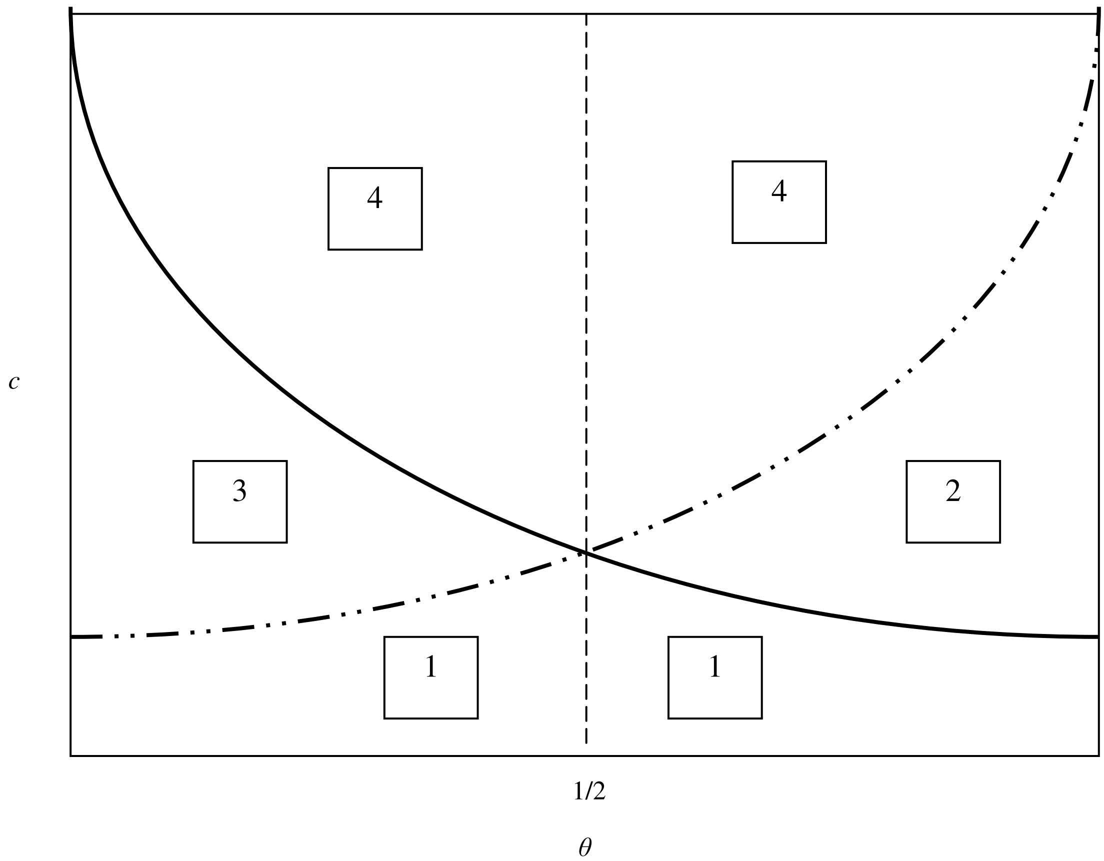

Figure 1: Equilibrium trading patterns

Notes:

denotes $\tilde{c}_w(\theta)$

denotes $\tilde{c}_h(\theta)$

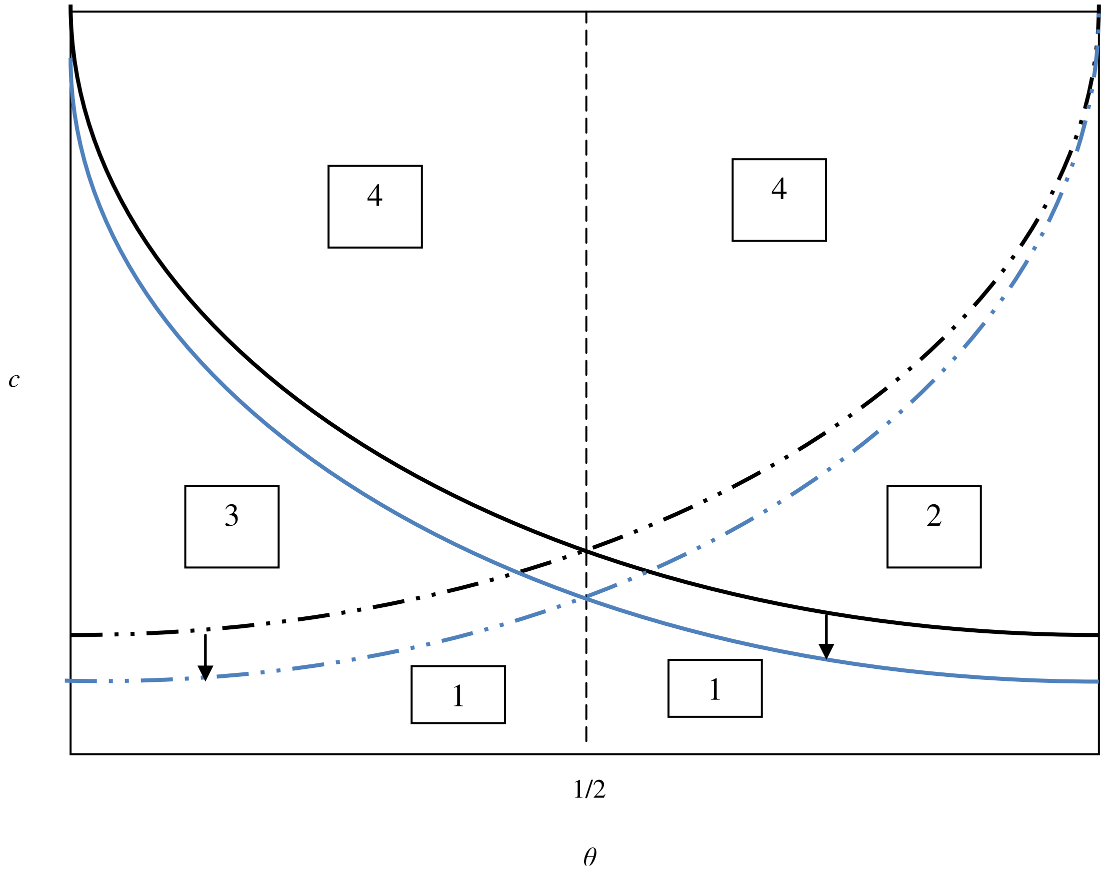

Figure 2: The impact of an increase in β on reservation values

Notes:

denotes $\tilde{c}_w(\theta)$

denotes $\tilde{c}_h(\theta)$

denotes $\tilde{c}_w{}'(\theta)$

denotes $\tilde{c}_h{}'(\theta)$