The financial consequences of the ’Women Men 40’ pension scheme concept in pay-as-you-go pension systems

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THE FINANCIAL CONSEQUENCES OF THE “WOMEN & MEN 40” PENSION SCHEME CONCEPT IN PAY-AS-YOU-GO PENSION SYSTEMS

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ABSTRACT

With the help of five models, this paper analyses pension systems in general and the direct financial effects of the retirement-age-reducing concept mentioned in the title. The first part assumes a financially unchanged environment, when earnings are permanent, there is no interest and everyone lives for a predetermined length of time (*deterministic models*). Taking into consideration actual mortality rates (*stochastic model*) we give an idealised description of the current pay-as-you-go pension system, which, however, does not significantly differ from real life. The most important consequence of taking mortality into consideration is the *life insurance effect*. The contribution of individuals who deceased prior to reaching a pensionable age, will increase the sources for the pension of survivors. Every forint the survivors paid themselves is worth 1.5–2 times as much on account of this life-insurance effect.

In the second part of the paper incomes are assumed to increase and interest also accounted for. Here we highlight the advantage of funded pension schemes in that for the same amount of pension they require a third to half of the pension insurance contributions of the pay-as-you-go pension system, because half to two-thirds of pension funding comes from the returns on invested payments. Analysis of this reveals the hidden state deficit inherent in the pay-as-you-go pension scheme, and the fact that every active employee pays the interests on it on a monthly basis when paying two to three times more in pension insurance contribution than would be necessary.

The third part demonstrates that if both women and men were to retire after 40 years of employment, it would entail pensions cuts of 9–12% for females and males respectively or a general pension cut of 19% for both sexes on average, if the present balance of contributions and pension payments are to be kept in the future.

JEL codes: C88, H55, G22

Keywords: pension system, early retirement at 40, modelling the pay-as-you-go pension system

INTRODUCTION

It is well known that the amendment of the Pension Act[1] that entered into force in January 2011 is called "Women 40 programme" for short. What it stands for is that regardless of age, a woman who has acquired 40 years of "eligibility," as defined by Hungarian legislation, is entitled to retire with a full pension even if she is still below the statutory retirement age. According to the provisions of the Pension Act, the main difference between the calculation of *"service period"* and *"eligibility period"* is that the latter can include neither the time spent in tertiary education nor any period of time where the individual paid all her contributions but did not draw a salary. As a consequence of these specific regulations, the "Women 40 programme" chiefly benefits women under the age of 63 (the present number for retirement age for women) who have worked from the age 18, never obtained a college/university degree and were never (or were rarely) unemployed. The demand for the only early retirement possibility granted by law exceeded the government's expectations, and to date some 123 thousand women retired on these grounds.[2] In this paper "Men 40 programme" will refer to the idea of granting men a similar benefit. It is obvious without proof or analysis that these programmes (would) increase pension costs while at the same time reducing pension contribution revenues.

These changes are clearly at odds with the European Union's general strategy and the re-orientation strategies decided on by numerous Members States, which focuses on delaying the retirement age.[3] On the other hand, however, it should be noted that the programme(s) in question are "exceptional" in Hungary's case, since Hungary complied with EU legislation by introducing in 2012, as a main principle, the prohibition of retirement before reaching the pensionable age.[4] The purpose of this paper is to quantify the combined effects of these two "special" programmes with respect to the replacement rates (to be defined later).

The models to be used in the present paper are simplified models, but even these models allow for the presentation of the salient features of the Hungarian pay-as-you-go pension system and for the quantification of a number of correlations that often escape the attention of the economists with a basic knowledge of macroeconomics.

The following conditions were considered to be generally valid:

1 Act 81(18)(2a) – (2d) of 1997

2 Farkas (2015).

3 Ebbinghaus – Hofäcker (2015); Gomez-Leon et al. (2015); Bönke et al. (2015)

4 For the first controversies of the Hungarian solution, see Simonovits (2015).

1. It is assumed that everybody starts work at the age of $L = 21$. That approximates the average 18–25 age of entering work; also, the choice of L corresponds to the age of entering work following the obtaining of a bachelor-level degree.[5]
2. Wages (W) will be considered to be the super-gross amount (the sum of an employee's gross wage plus social and health insurance) which an active person costs his or her employer. All tax and contribution rates will apply to this amount, thus excluding the complicating fact that a portion of pension contributions is paid by the employer.[6]
3. All amounts remaining after taxes, other contributions and pension savings and/or contributions, are assumed to be spent, that is, there are no other savings.
4. The only pension considered in this paper is old-age pension. Like the experts of the Round Table of Pension Old Age[7], this paper shall not be concerned with disability, orphans' and widowers' pension either.
5. The financial balance of the pension sector is taken for granted, that is, it is assumed that if women had not been allowed to retire after 40 years of employment and the same is not granted to men, the pension contributions equal the pension paid by the state.[8]
6. Our models do not account for the absolute size of wage; consequently, the question does not come up whether an earnings-related pension will provide a sufficient income in a beneficiary's old age in comparison with subsistence indices.
7. This paper does not account for the operative (administrative) costs of the pension system, which constitute a mere 0.8% of pension payments.

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5 In these parameters we have adopted the system employed András Simonovits (2002; 2015).

6 Put this way is a simplification of the legal situation because the 24% pension contribution of an employer is now part of the social contribution tax which, Hungarian legislation provides, has to be considered to be a "contribution to the costs of common social needs" and no longer needs to be spent exclusively on pension payments. These feature in the tables of the Budget Act jointly, under the name of "portion of the social contribution tax belonging to the Pension Fund and employers' pension insurance contribution". Exactly how much of it "*belongs to*" to Pension Fund is decided on an annual basis by Parliament. Calculated in contribution points, the pension payments by the Fund amounted to less than a third of employers' pension contributions. Quantified, this means that 10/(10+24) = 29.4%. The proportions are similar on the basis of the 2015 budget.

7 Holtzer (2010)

8 While that might not be true, it will help shed light on the effect of the two programmes on the pension system, *ceteris paribus*.

1. A FINANCIALLY UNCHANGED ENVIRONMENT: PERMANENT EARNINGS, NO INTEREST AND NO RETURNS

Part 1 will simplistically assume that there is no inflation; wages, consumption and interests are permanent each year, and the average profit rate characteristic of the entire economy is zero.

1.1 Deterministic models

These models are called deterministic because they assume everyone lives to a given age. In the first model it will be assumed that a person

a) lives to the age of $D = 80$;
b) year R will be the last full year that person will have worked;
c) this person was never unemployed during his/her active period;
d) he/she paid taxes and contributions on his/her real wages;
e) he/she never undertook grey or black labour.[9]

The choice of the $D = 80$ age allows for a simple, easy-to-interpret life path. If a person worked to the age of 60 ($R = 60$) the proportion of active and inactive years will be exactly $A/N = 2{:}1$.

1.1.1 Individual precautionary savings – self-financed pension

First, the effect of the pensionable age (R) will be examined, being as it is, the only independent variable. In this case, *R is a pre-planned* value, where the rate of savings is adjusted to entire active period of the individual's life. The individual can only count on his/herself by saving up during their active years for the time when he or she no longer wishes to work. Their private pension will obviously be earnings related, and will serve to maintain the same consumption rate in their pension (N) years as during their active (A) years, that is to achieve a replacement rate (r) of 100%.[10] The individual will put aside s per cent of W wage, will pay a taxes

9 One of the greatest problems of the current Hungarian pension system is that conditions [(III, IV, V3., 4., 5.] are not even close to being met (see Augusztinovics 2005; Augusztinovics – Köllő 2007.

10 The replacement rate is the rate of self-financed precautionary pension or the "normal" pension and the net incomes of active years. Its significance is that it shows the extent of pension services compared to last year's net incomes. In Hungary this rate – as calculated by the standard OECD methodology – was 95% in 2012, that is, not much below 100%. The mandatory state system in Japan and the United Kingdom provides a 41% and 42% replacement rate, respectively (see https://data.oecd.org/pension/net-pension-replacement-rates.htm, downloaded: 22.11.2015). That is possible because in these two countries – and in developed market economies in general – the elderly population have considerable financial savings, while the same cannot be said for Hungary where, as it is well known, old-age savings are more the exception.

in the form of income tax, health insurance, etc. and consume the remaining $W(1 - a - s)$ amount. Consumption during the inactive years ($N = D - R$) equals the savings of the active years ($A = R + 1 - L$), so it can be established that

$$(D - R) \times W \times (1 - a - s) = (R + 1 - L) \times W \times s \qquad (1)$$

Rearrangement of equation (1) yields

$$s = \frac{1-a}{A/N+1} \qquad (2)$$

Assuming taxation charges on personal incomes to be a = 25%[11], the values of A/N and s can be seen as a function of R in Figure 1. The basic case is this: L = 21, R = 60, D = 80, length of active age: 40; inactive age: 20 years.

Figure 1
Necessary rate of savings (s) and A/N as a function of *planned* R, with a certain life expectancy of 80

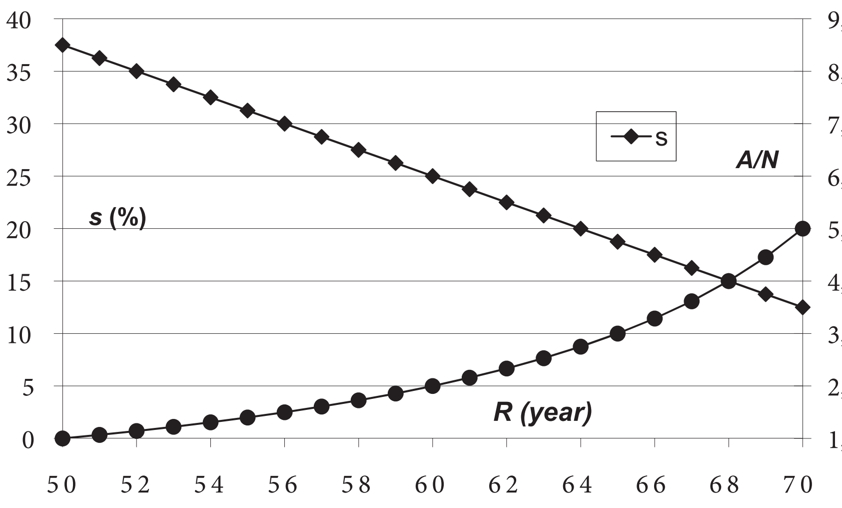


Figure 1 shows the rate of savings (s) for given periods of active years. When the active period is shorter, a larger rate of savings is required to maintain the same level of consumption in a person's inactive age as in their active period. However, the larger rate of savings merely reflects the main disadvantage: where the active period is shorter, due to lower life earnings, consumption will be lower throughout that person's whole life than would be possible with a longer *planned* active

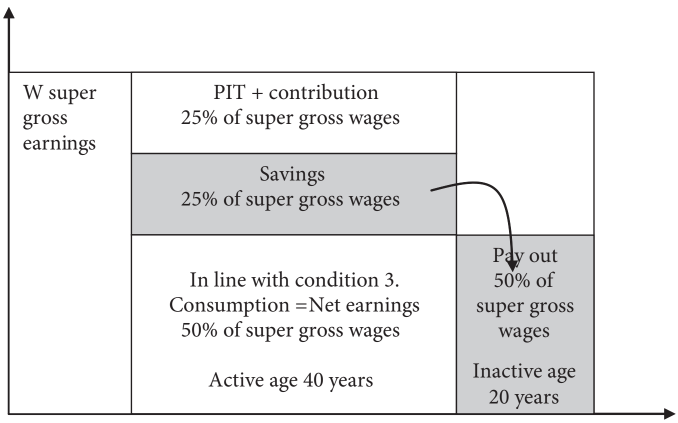
11 Average data for Hungary – according to Hungarian Central Statistical Office – correspond to this in 2015.

age. Taking this into consideration, a specific goal can be determined, namely if a person wishes their consumption to reach at least 50 per cent of their previous working income, they need to work at least to the age of 60 and the earliest point of retirement is age 61. This will correspond to the selected basic case where *A*/*N* = 2.0. *Figure 2* shows how, during a person's 40 active years, half of the earnings go to consumption and a quarter to savings, which will ultimately cover an unchanged level of consumption in the following 20 inactive years.

In other words, with the given parameters, a person's consumption will be stationary regardless of whether he/she is pensioned or working. The very simplicity of this model makes it strikingly clear how a person can consume as much as their life earnings after tax. **Precautionary savings and using up those savings only means regrouping in time.**

Since changing the retirement age alters both the length of active and inactive years, the rate of necessary savings is sensitive to the *planned* length of a person's active age. Figure 1 shows that a 5% reduction in the active age (*R* reduces from 60 to 58, making the active age 38 instead of 40) entails a 2.5 percentage point increase in necessary savings (from 25.0% to 27.5%) which corresponds to a 10% increase in savings.

Figure 2
Rates of taxes, savings, consumption and payments in inactive age



Note: Basic case *L* = 21, *R* = 60, *D* = 80, active age 40 years, inactive age 20 years.

The model demonstrates the consequence of the *unplanned* shortening of the active period, the smaller pension. If a person spent 50% of the active-age income on consumption (that is 25% of earnings was put aside for pension), but later changes his/her mind or falls ill and decides to retire prior to the age of 61, the replacement rate will inevitably fall. Equation (1) therefore needs to be complemented, because the equality of old-age consumption and active-age savings is guaranteed by alteration of the replacement rate for the given pension contribution (s) corresponding to the planned length of active age:

$$h \times N \times W \times (1 - a - s) = A \times W \times s \tag{3}$$

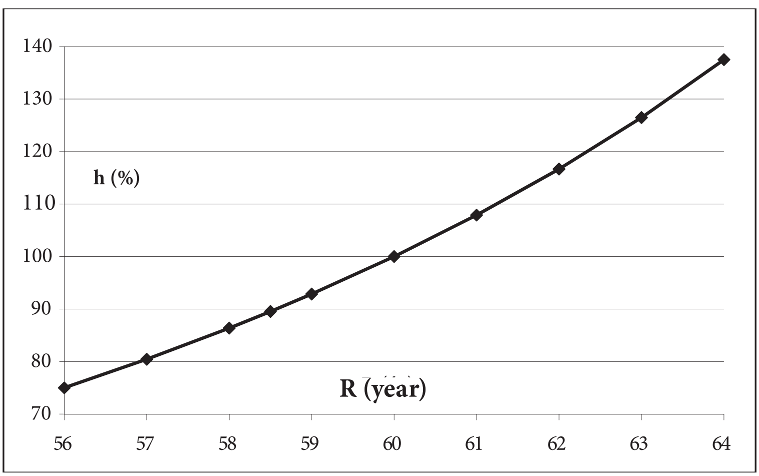
Expressing the replacement rate:

$$h = \frac{s \times A}{(1 - a - s) \times N} \tag{4}$$

The results are summed up in Figure 3. If an individual works until the age of 58.5 instead of the planned 60, the replacement rate will be 89.5%, that is, the old-age consumption will reduce by 10.5%. I.e. the individual will have that much less money to spend compared to their active-age net income.

The precautionary self-retirement model might at first seem unrealistic. However, this model can be also interpreted as a voluntary, supplementary insurance scheme. This already exists in most countries in the world, including Hungary (personal pensions or contributory/voluntary savings constitute the so-called 3rd Pillar in the World Bank Pension Conceptual Framework), and it will almost certainly be necessary in the future. At the same time, the figures above reveal what **large amount of savings a person would have to make during their active period, if the aim is to provide for the old-age pension exclusively or almost entirely from voluntary savings such as these** (as opposed to a mandatory, pooled pension system, where a 10–20% rate of savings would suffice).

Figure 3
Replacement rate (h) in the case of an *unplanned* change of R



1.2 Precautionary savings at the population level – a pension system model with life expectancies

The model used in this section already constitutes a real-life pension system, because individual savings are required to be paid into a pension fund; the rate of savings is determined by law; and by definition it covers a larger populace. The savings here correspond to a pension contribution (hereinafter contribution) which, in the majority of developed countries, are levied by the state and distributed by the state as pension.[12] The size of the pool allows for consideration of the fact that members of various generations retire at different ages. Equations (2) and (4) remain valid, but the meaning of *A* and *N* will be different. Rather than signifying the number of active and pension years of an individual, they will denote the number of active and pensioned individuals. Consequently, the model can reflect the effect of other factors, such as members of various generations starting work and retiring at different times.

In the second version of the model, we select data that closer approach real values in Hungary. The data come partly from statistical reports, and are partly estimations. Because the purpose of the inquiry is to calculate, *ceteris paribus*, the effect of introducing the Women 40 and the Men 40 programmes on the replacement rate, a good approximation will suffice.

This model is also deterministic. At the age of 62 everybody has the same average life expectancy, that is, 77 for men and 82 for women.[13] $R + 1 = 63$, an age that people born in 1952 will have reached in 2015, and is the retirement age for people born in 1953. The replacement rate will be assumed to be the statistical 95% with women's earnings being 86% of men's.[14]

First, we calculate the contribution rate corresponding to the financial equilibrium separately for men and women, and then jointly for both.

1.2.1 Contribution rate (s) for men only

Equation (3) with male data is as follows:

$$W_M \times s \times A = h \times W_M \times (1 - a - s) \times \mathrm{N}, \qquad (5)$$

where $A = (R - L + 1) \times l$ and $N = (D - R) \times l$. The number of living members of the cohort in its t^{th} year is $l(t)$, and in this model it is considered to be the same in every year, that is, it can be considered to be l. Consequently,

12 The fact that being part of the pension system is mandatory has no role in this model.

13 Mária Hablicsekné Richter (2011).

14 Farkas (2015).

$$s \times A = h \times (1 - a) \times N - h \times s \times N \tag{6}$$

Rearranged for *s*:

$$s = \frac{h \times (1-a)}{h + A/N} \tag{7}$$

Using the data ($A = 42 \times l$, $N = 14 \times l$) yields $s_M = 18.0\%$.

1.2.2 Contribution rate (s) for women only

Using the corresponding figure ($A = 42 \times l$, $N = 19 \times l$) for equation (7) yields $s_W = 22.5\%$.

1.2.3. Contribution rate (s) for the entire population

Altering equation (5) for both sexes:

$$W_M \times s \times A \times (1 + 0.86) = h \times W_M \times (1 - a - s) \times (N_M + 0.86 \times N_W) \tag{8}$$

After simplification and rearrangement:

$$s \times [h \times (N_M + 0.86 \times N_W) + 1.86 \times A] = h \times (1 - a) \times (N_M + 0.86 \times N_W) \tag{9}$$

From equation (9), the contribution rate can be calculated by means of an equation not unlike (7):

$$s = \frac{h \times (1-a)}{h + 1{,}86 \times \dfrac{A}{N_M + 0{,}86 \cdot N_W}} \tag{10}$$

Replacing the adequate data in equation (10) yields ***s*****=20.2%** for the entire population.

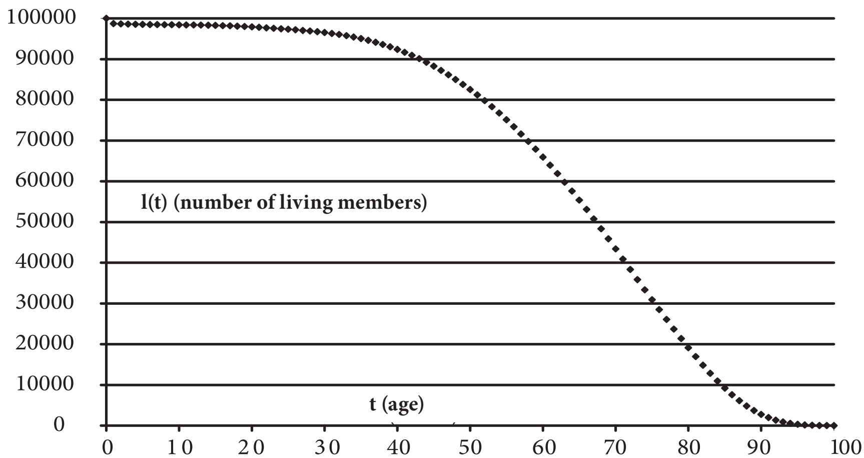
It can be established from the above that if every Hungarian working man and woman were to work continuously throughout their careers, and were to achieve the average life expectancy, women and men would have to pay a contribution rate of about 22 and 18 per cent, respectively.

1.3 The stochastic model, gradual mortality

1.3.1 Life insurance effect

In contrast with sections 1.1 and 1.2 where death was assumed to occur at specific times (the age of 80, 77 and 82), this model will account for the gradual occurrence of death. Figure 4 shows the mortality curve – which one might call, more optimistically, survival curve – which reveals for every year how many of a hundred thousand people born in the same year are still alive.

Figure 4
Mortality curve



The above figure clearly reveals the following logical consequences:

- The number of people in each cohort is not constant, but decreases from one year to the other;
- Not every member of a population born in the same year pays contributions; consequently total contributions at a given rate will be somewhat lower than as if everyone were to live to retirement age. The contribution rate is just 89% for the 21–60 age group.
- Only a portion – 40–70% – of a given cohort will live up to retirement age and take out pension payments.[15] Due to the age groups of diminishing numbers, the sum total of pension payments is, for 61–100 years, only 50% of the total pension of the 20 age groups (aged 61–80) consisting of full numbers.

15 It might be low by EU comparison, it is an important fact that positive changes can be observed in Hungary for life expectancy not only at birth, but also at 60 (Monostori, 2015).

There are, however, further non-trivial consequences of the above simplistic assumptions:

- The necessary pension contribution cannot be calculated for a determined life expectancy;
- Deciding what to do with the contributions of people who do not live to take out pension payments depends on the choice of model. The solution used by systems built on the logic of social security is that the contributions of deceased people go to the survivors. This makes pension insurance **an entitlement-type life insurance scheme** and accordingly, it provides extra services compared to private pension in that every forint of survivors' contributions will be worth considerably more than the contributions of people who decease prior to their life expectancy.[16]

Calculating the actual values is a more complex task than the previous ones. Equation (2) works here too, but the *A/N* parameter can only be calculated by means of the following definite integral formula (11):

$$\frac{A(R)}{N(R)} = \frac{\int_{L}^{R} l(t)dt}{\int_{R+1}^{\infty} l(t)dt} \tag{11}$$

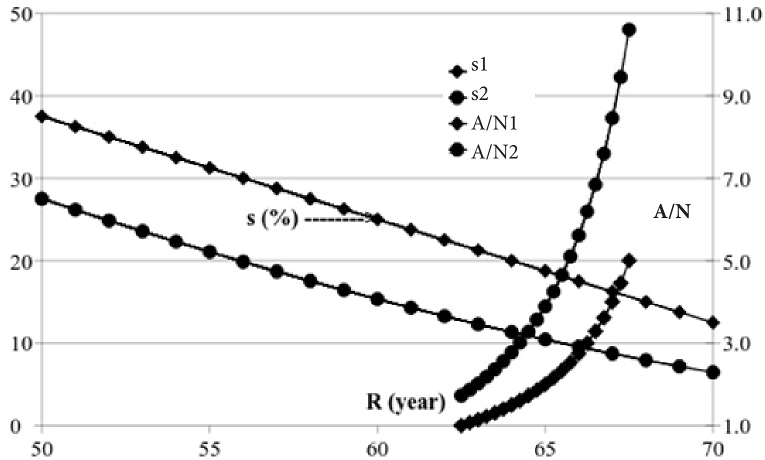
Because demographical data are usually provided annually, the integrals are replaced by the sum of annual data, which makes the formula used for the calculations the following:

$$\frac{A(R)}{N(R)} = \frac{\sum_{t=L}^{R} l(t)}{\sum_{t=R+1}^{100} l(t)} \tag{12}$$

Due to gradual mortality the difference is that due to more favourable *A/N* values the necessary contribution rate is smaller for each *R* than in the deterministic and individual model in I/1. The results are summed up and compared in Figure 5 below.

16 The general public are completely unaware of this similarity, even though the relatively familiar Hungarian personal income tax rules treat pension insurance payments as such (Banyár et al., 2014).

Figure 5
Proportion of active/pensioned years (A/N) and the necessary rate of savings (s) as a function of R for the deterministic (1) and stochastic (2) model



In the stochastic model, the contribution rate is lower and its value more sensitive to the length of a person's active age than in the deterministic model. Figure 5 reveals that a reduction of a person's active age by 5% (R reduces from 60 to 58 years, and the active years will be 38 instead of 40) entails a 2.2 percentage point increase in savings (15.3% to 17.5%) and means a 14.3% increase in savings. Naturally that is only true for people who live to retirement age. At the same time, the survivors' contributions are increased by the contributions of the deceased, which means that their every forint in contributions will be worth 1.5–2 forints[17] when they receive pension. How much more the insured person will receive also depends on how much longer they live than the average life expectancy typical of their sex.

However, if a person dies before retirement age, the majority of their contributions will be lost to their spouse and heirs. If a person dies a few years after retiring, the person will take out a smaller amount of pension than the present value of their contributions. This difference cannot be inherited either.[18]

In sum, the current pension system is a form of insurance that pools pension contributors into a risk community. Under this system, only persons living to retirement and achieving their statistical life expectancy will break even. Persons

17 Naturally instead of HUF we might have written USD or EUR.

18 In actual fact in most countries the spouse or partner of the deceased can be granted a widower's pension, whose value, however, is a mere 20–30% of the old-age pension the deceased would have received. This is not dealt with in this paper

living longer profit from the system, while people dying before the retirement age lose out. This is the price of having the security of a pension till death. It should be noted, however, that this is still far removed from reality. This model

- assumes that an equal number of people are born in every year, meaning that it is irrelevant whether the pension system is pay-as-you-go or funded; and
- does not account for inflation, interest, or profits; the present value of contributions and pension payments is independent from the time/date and corresponds to the nominal value.

1.4 Calculations based on real demographical data

First we establish the actual numbers for each age group [$l(t)$]. Figure 6 shows demographical data for 2015. The number of active and pensioned individuals is given in the numerator and denominator of equation (12).

Unemployment and the fact that part of active individuals work in the grey or black economy (and consequently pay less or no contribution) will be taken into account. Unemployment rate will be taken to be 10%[19] and unpaid contributions (due to grey and black-market work) will be accounted for by further reducing the rate of contributors by 15%. Because the volume of the grey and black economy is an estimated HUF 5,000 billion, that is, one sixth of the GDP, 15% is a good approximation: $A^* = (1 - 0.10 - 0.15) \times A = 0.75 \times A$.

Because the numbers of women and men are never the same, not even during the active age, equation (10) will be altered as follows:

$$s = \frac{h \times (1-a)}{h + \frac{A_M + 0{,}86 \times A_W}{N_M + 0{,}86 \times N_W}} \qquad (13)$$

The resulting contribution rate for the entire population (18.6%) does not significantly differ from the 20.2% in model I/2. However, **after considering the existence of unemployment and the contribution shortfall due to the grey and black economies, the necessary contribution rate** further increases from 18.6% to **22.9%**.

19 If public workers are considered to be unemployed (as many theoretical economists will argue), this largely corresponds to the current situation.

Figure 6
Population data by single-year age for 2015

Source: Website of the Hungarian Central Statistical Office (2015), dates in table; the above figure is by the authors.

2. FINANCIALLY CHANGING ENVIRONMENT THAT ACCOUNTS FOR INCOME RISES AND RETURNS

In Part 2 we introduce two new conditions: (1) saved and invested capital has returns (r) larger than zero; (2) incomes can increase. This inquiry will be limited to the effect of returns and gradual mortality.

2.1. Deterministic model, self-financed pension

2.1.1 Permanent income and changing returns

Due to the presence of returns, the present value of both savings and pension payments (i.e. the monetary value of pension in retirement age) can be calculated as an annuity. The present value of savings at the time of birth[20] is as follows:

20 This was chosen because it is a well-defined point of reference that does not depend on anything else.

$$JÉ(L,R)=\frac{W\times s}{r}\times\left(\frac{1}{(1+r)^{L}}-\frac{1}{(1+r)^{R}}\right)\tag{14}$$

The present value of pension payments at the time of birth:

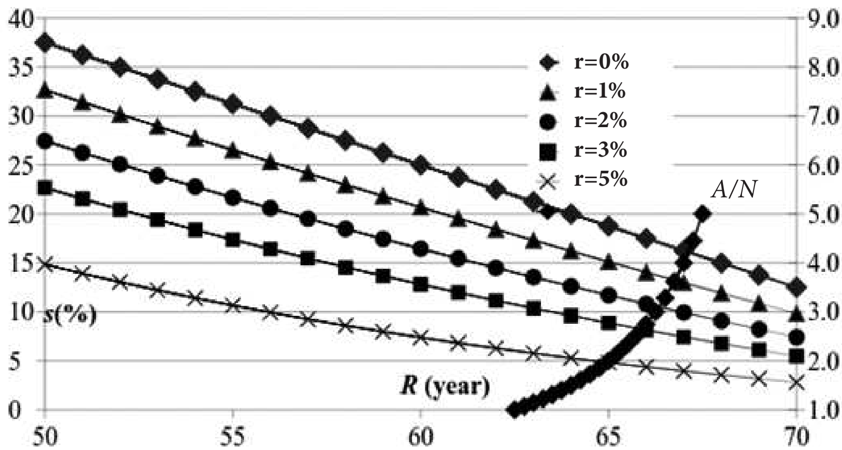
$$JÉ(R,D)=\frac{W\times(1-a-s)}{r}\times\left(\frac{1}{(1+r)^{R}}-\frac{1}{(1+r)^{D}}\right)\tag{15}$$

Expressing *s* from the equality of the two present-value equations yields

$$s=(1-a)\times\frac{\dfrac{1}{(1+r)^{R}}-\dfrac{1}{(1+r)^{D}}}{\dfrac{1}{(1+r)^{L}}-\dfrac{1}{(1+r)^{D}}}\tag{16}$$

The results as a function of *R* and *r* are summed up in Figure 7.

Figure 7
Pension contribution (s) and A/N as a function of R
with a life span of 80 years



Leading to an increase in capital, the reinvestment of returns considerably reduces the volume of required pension contributions: for returns of 3%, contributions reduce by half; and for returns of 5% to a third or a quarter. This also increases the amount of money that can be spent on consumption in both active and pension years. In the case of self-financed pension after the age of 60, consumption will be not be 50% of one's income, but 62% for returns of 3%, and 68% for returns of 5%; meaning a 24% and 36% increase in living standards respectively.

Naturally, to achieve this result, it is necessary to make investments, that is, to deposit contributions. At returns of 3%, the total amount of capital financing the pension contribution will be 9.66 times the annual income and at 5% 8.93 times. The larger the returns the smaller amount of capital is necessary for the payment of the same pension. However, returns have a greater effect than obvious at first sight. The capital stock financing pensions in year R will be one's own savings; if there are no returns, it will be 10 times one's annual income (100% of the capital), at returns of 3% 5.12 times (53% of the capital), and at 5% just 2.96 times (33% of the capital). In other words, following the reinvestment of returns compound interests double or more than triple the original capital. Consequently, pension savings are not only "processed" through the system, but also, thanks to capital market investments and bank loans, they contribute capital to the economy. A share of the profits of financed enterprises and the interests on bank-deposit loans create a source of growth of the capital that constitutes the basis of pensions. It is based on the growth of the economy and returns are the results therein.

2.1.2 Growing income and varying returns

Let returns continue to be *r*, let the annual rate of growth of incomes be *g* and let $r > g$. The present value of savings and pension payments can still be calculated as an annuity, but contributions are not permanent but are growing. The present value of growing savings – like growing annuities – at the time of birth is as follows[21]:

$$JÉ(L,R) = \frac{W_1 \times s}{r-g} \times \left(\frac{1}{(1+r)^L} - \frac{1}{(1+r)^R} \right) \qquad (17)$$

The present value of pension payments at the time of birth:

$$JÉ(R,D) = \frac{W_1 \times (1-a-s)}{r-g} \times \left(\frac{1}{(1+r)^R} - \frac{1}{(1+r)^D} \right) \qquad (18)$$

Equations (17) and (18) differ from (14) and (15) only in that they contain $W_1/(r-g)$ instead of W/r. Due to the fact that in the course of setting up and arranging the resultant equation these factors drop out, the dependency of pension contributions on *R* and *r* is not affected by whether the income is permanent or growing, assuming that the returns exceed the rate of growth of incomes. For this case too the dependency of pension contributions on *R* and *r* is shown in equation (16) and Figure 7.

21 Illés (1998)

2.2 Stochastic model

2.2.1 Funded pension

This model differs from 1.3 in that it is funded: savings are deposited on private accounts where the returns are reinvested until retirement and this accumulated capital (which continues to make returns) covers a person's pension. It has been shown how (for fortunate survivors, at least) contribution rates are reduced by actual mortality (1.3) and interests in the determined model (2.1). Actual mortality and returns together will further reduce necessary contributions.

This model cannot be described by a closed formula, because there are too many parameters changing from year to year, including the number of people in an age group, pension contributions and pension payments, as well as the volume of the pension fund that generates returns. The average income of the age groups also changes (albeit it has not been accounted for here) partly because of inflation matching and partly because of career progression.

In calculating this model it will only be assumed that the financial balance is correct for the individual age groups only. In other words, consisting of contributions of the members of the examined age group and aggregated with returns, the pension fund will be exhausted by pension payments when the last member of the age group deceases statistically. We are looking for the *s* contribution rate for which this holds true.

If every member of an age group begins work at the age of *L* and retires at the age of *R* + 1, the accumulation of *T* pension fund of the age group for *R* years can be expressed by means of the following series of recursive formulae:

$$T(L)=W(L)\times s\times l(L) \tag{19}$$

$$T(L+1) = T(L)\times(1+r) + W(L+1)\times s\times l(L+1) \tag{20}$$

$$T(t)=T(t-1)\times(1+r)+W(t)\times s\times l(t) \tag{21}$$

$$T(R)=T(R-1)\times(1+r)+W(R)\times s\times l(R) \tag{22}$$

T(*R*) can be expressed by means of a closed formulae, as follows:

$$T(R)=s\times\sum_{t=L}^{R}W(t)\times l(t)\times(1+r)^{R-t} \tag{23}$$

but it is more appropriate to use a series of recursive formulae, because it works better in an Excel chart and the use of the pension fund can subsequently calculated by means of a recursive formula:

$$T(R+1) = T(R) \times (1+r) - W(R+1) \times (1-a-s) \times l(R+1) \quad (24)$$

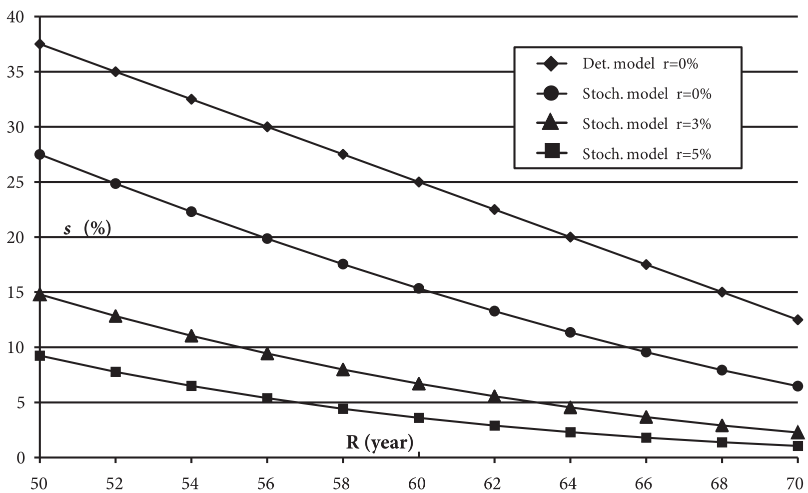
$$T(R+2) = T(R+1) \times (1+r) - W(R+2) \times (1-a-s) \times l(R+2) \quad (25)$$

$$T(100) = T(99) \times (1+r) - W(100) \times (1-a-s) \times l(100) = 0 \quad (26)$$

Next, we need to find the *s* rate of savings for which the value of formula (26) will be zero; when the age group dies out – that is, *l*(*t*) reduces to zero – and *T*(*t*), i.e. its pension fund capital equals zero. Owing to the use of the series of recursive formulae, this model works for both permanent and varying incomes.

The results are summed up in Figure 8. The necessary pension contribution rate reduces significantly, and returns of 3% produce similar results as 5% returns in the deterministic model (cf. Table 1.).[22] In financial terms, this two percentage point difference means that the internal rate of return of the demographical effect of gradual mortality is 2%.

Figure 8
Necessary pension contribution (s) as a function of R in the deterministic and stochastic model



22 Another thought: in the pay-as-you-go system as we know it there is an implicit public debt element which, if the state decided overnight to swift to a funded pension system, would instantly become explicit. Just because a debt is implicit it still exists as a debt, and every active earner is paying back that debt by paying at least twice as much in pension contribution in the pay-as-you-go system as they would have to in the funded system with a real returns of 3%. The curves in Figure 8 for r = 0% and r = 3%, and column 3 in Table 4 clearly highlight that fact.

The models and their results have been summed up in Table 1. When there is no interest and everyone lives for the same length of time, the pension contribution rate will be 25% because everyone can only consume as much as they earned during their active years. When actual mortality and/or interests are taken into account, the contribution rate can go down, because actual savings are boosted from external sources. These additional sources of money comprise the contributions of individuals who died before reaching retirement age, which are, in turn, contribute to the financing of the survivors’ pensions, and the compound interests of savings. Both resources are considerable, and even individually they can reduce pension contribution rates by almost a half, and they mutually reinforce each other.

Table 1
Pension contribution rate (s) in the various models necessary to ensure a 100% replacement rate

R = 60 years	Deterministic model	Stochastic model
No interest	25.0%	15.3%
Interest, $r = 3\%$	12.8%	6.7%

3. THE EFFECT OF THE “WOMEN 40” AND THE “MEN 40” PROGRAMMES ON THE EQUILIBRIUM REPLACEMENT RATE

It is assumed that a person will have reached the 40-year eligibility period at the age of 58, if that person started work at 18, his/her employment was never interrupted, with the exception of maternity leave for women. The “Women 40” and “Men 40” programmes mean that in addition to 63-year-olds, anyone in the five age groups of 58–62-year-olds who meets these criteria is allowed to retire. People meeting the criteria constitute some 30% of the five age groups, which corresponds to current actual data for women. Let us assume furthermore that none of the people applying for retirement are unemployed or working in the grey or black markets.[23]
The differences caused by the “Women 40” and “Men 40” programmes are marked with the variables Δ_M and Δ_W, for which the contribution rate is as follows:

$$B = W_M \times s \times A_M - W_M \times s \times \Delta_M + W_W \times s \times A_W - W_W \times s \times \Delta_W \tag{27}$$

23 In Hungary it is unlikely that they will have acquired 40 years of eligibility before the age of 63.

The volume of pension payments:

$$K = h \times (1 - a - s) \times (W_M \times N_M + W_M \times \Delta_M + W_W \times N_W + W_W \times \Delta_W) \quad (28)$$

The two being equal, following quantification and considering that $W_N = 0.86 \times W_W$, it can be established that:

$$s \times (A_M + 0.86 \times A_W - \Delta_M - 0.86 \times \Delta_W) = \\ = h \times (1 - a - s) \times (P_M + 0.86 \times P_W + \Delta_M + 0.86 \times \Delta_W) \quad (29)$$

In the next step, rearranging equation (29) produces the following *r* replacement rate:

$$h = \frac{s \times (A_M + 0,86 \times A_W - \Delta_M - \Delta_W \times 0,86)}{(1 - a - s) \times (N_M + 0,86 \times N_W + \Delta_M + \Delta_W \times 0,86)} \quad (30)$$

The replacement rates and their comparison are summed up in Tables 2 and 3 below.

Table 2
Replacement rates depending on the "Women 40" and "Men 40" programmes, with actual demographic data*

Women 40	**Men 40**	A_W	A_M	N_W	N_M	h	***h* reduction**
no	no	2,901,081	2,856,755	1,251,707	776,413	95.0%	0.0%
yes	no	2,778,755	2,856,755	1,374,033	776,413	88.1%	8.2%
no	yes	2,901,081	2,715,151	1,251,707	918,017	85.9%	10.8%
yes	yes	2,778,755	2,715,151	1,374,033	918,017	**80.0%**	**18.0%**

Note: *The reduction of *h* is expressed in % (s = 18.6%)

Table 3
Replacement rates depending on the "Women 40" and "Men 40" programmes, with actual demographic data*

Women 40	Men 40	A_W	A_M	N_W	N_M	h	h reduction
no	no	2,175,811	2,142,566	1,251,707	776,413	95.0%	0.0%
yes	no	2,053,485	2,142,566	1,374,033	776,413	87.5%	8.9%
no	yes	2,175,811	2,000,963	1,251,707	918,017	85.1%	11.7%
yes	yes	2,053,485	2,000,963	1,374,033	918,017	**78.7%**	**19.4%**

Note: *taking into consideration unemployment and the grey/black economy (s = 22.9%)

When considering the demographic facts behind our finding, it is easy to see that the considerable drop in replacement rates can be attributed to the fact that the beneficiaries of these programmes are the largest male and female age groups of 1953–1957.

As regards a prognosis for the "Women 40" and "Men 40" programmes, it can be said that unfavourable demographic processes, unemployment and the grey and black economy have deteriorated the financial situation of Hungary's pension system, which has been further impaired by the introduction of the "Women 40" programme in 2011. It is not very far-fetched to assume that introduction of the "Men 40" programme would worsen this situation to the extent that – *ceteris paribus* – it would be necessary to cut pensions by over 19%, that is, taking the 95% replacement rate in out models down below 79%. That would mean the answer to *András Farkas*'s question ("Should we help a hundred thousand by hurting two million?") would be "yes", and it should be noted furthermore that the a fifth of the pension given to the hundred thousand would have to be soon taken away.

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