Competitiveness and subsidy or tax policy for new technology adoption in duopoly

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We consider a problem of subsidy or tax policy for new technology adoption by duopolistic firms. The technology is developed in and transferred by a foreign country to the domestic country. It is free but each firm must expend some fixed set-up cost for education of its staff to adopt and use it. Assuming that each firm maximizes the weighted average of absolute and relative profits, we examine the relationship between competitiveness and subsidy or tax policies for technology adoption, and show that when firm behavior is not competitive (the weight on the relative profit is small), the optimal policy of the government may be taxation; when firm behavior is competitive (the weight on the relative profit is large), the optimal policy is subsidization or inaction and not taxation. However, if firm behavior is extremely competitive (close to perfect competition), taxation case re-emerges.

**Keywords:** new technology adoption, duopoly, subsidy, tax

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1 Introduction

As an example of situations to which the results of this paper can be applied we may consider the following story. There is a duopolistic industry in a developing country. The firms in the industry produce a homogeneous good. They can use a common new production technology which is more efficient than the present technology. The production cost with the new technology is lower than that with the present technology. This new technology is developed by a laboratory or a firm in a foreign developed country, and the government of the developed country wants to transfer the new technology to the developing country as a foreign aid free of charge. However, each firm in the developing country must expend some fixed set-up cost for education of its staff to adopt and use the new technology.

This story is one example. The approach of this paper may be applied to other situations such as technology transfer between firms in one country, or between firms located in two developed countries.

We consider a problem of subsidy or tax policy for new technology adoption of duopolistic firms under the assumption that each firm maximizes the weighted average of absolute and relative profits. The weight on the relative profit indicates the severity of competition. We examine the relationship between competitiveness and subsidy or tax policies for technology adoption.

This paper is based on our two-fold interests in analyses of duopolistic or oligopolistic industries.

1) The first is an interest in the relation between technology adoption and competitiveness under oligopoly. The important reference is Brander and Spencer (1983). They analyze strategic use of research and development, which takes place before the associated output is produced, by imperfectly competitive firms. They showed that such strategic use of R&D will increase the total amount of R&D undertaken, increase total output, and lower industry profit. Although the strategic use of R&D introduces inefficiency which means that total costs are not minimized for the output chosen, net welfare rises if products are homogeneous, marginal cost is non-decreasing, and demand is convex or linear.

There are references about licensor's strategic behavior, under duopoly or oligopoly such as Katz and Shapiro (1985), Kamien and Tauman (1986), Sen and Tauman (2007), La Manna (1993), and under Stackelberg competition such as Filippini (2005), Kabiraj (2004), Wang and Yang (2004). Using cooperative game theory, Watanabe and Muto (2008) studies the equilibrium among licensor and licensees. Also, we introduce two papers concerning competitiveness. Boone (2001) analyzes the relation between licensor’s incentive to innovate and licensee’s competition, and has found the U-shaped relation. In Matsumura et. al. (2013), the relation between competitiveness, which is represented by the weight on the relative profit in maximization of the weighted average of absolute and relative profits as this paper, and endogenous R&D investments is analyzed. They showed that the R&D investment is U-shaped with respect to competitiveness like Boone (2001).
About social welfare and optimal policy, it has shown that optimal R&D investment is S-shaped with respect to competitiveness, and enhancement of competitiveness changes the optimal policy from taxation to subsidization. Their results and the results of this paper are similar, but the model in Matsumura et. al. (2013) is restricted to a symmetric equilibrium, and do not refer to fixed cost and the difference of cost functions. The model about technology adoption behavior where new technology is exogenously given as this paper is discussed in Zhang et. al. (2014) which focuses on the uncertainty of R&D investment, and Hattori and Tanaka (2014) which focuses on the relation between technology adoption and competitiveness using the relative profit maximization, but these do not analyze the social welfare. About social welfare, Pal (2010) shows that technology adoption may change the market outcome, and the social welfare is larger in Cournot competition than in Bertrand competition. The model in Pal (2010) and the model of this paper are similar. But in Pal (2010) government’s policies are not analyzed. Moreover, Elberfeld and Nti (2004) focuses on the spillover of new technology, and Hattori and Y. Tanaka (2015) focuses on the difference of cost functions, and these claim the over or under investment for the society in some situation. This paper extends Hattori and Y. Tanaka (2015) to a case of various competitiveness.

Concerning empirical studies H. Tanaka (H. Tanaka (2004), (2013)) examined success factors of technology transfer from Japan to Taiwan (Republic of China) in post Word War 2 period. He stressed the importance of social capability which reduces the technology transfer cost such as human capacity of bureaucracy and managers, and the role of the government for improvement of infrastructure, education and training of workers and engineers. Also, he claims that the government should choose the industry where foreign company can receive the tax benefit. And he concludes that these factors which promote technology transfer lead Taiwan to economic growth.

(2) The second is an interest in the property of relative profit maximization, or maximization of weighted average of absolute and relative profits of firms. Theoretical justification of relative profit maximization is mainly based on evolutionary game theoretic point of view. Schaffer (1989) demonstrates with a Darwinian model of economic natural selection that if firms have market power, profit-maximizers are not necessarily the best survivors. A unilateral deviation from Cournot equilibrium decreases the profit of the deviator, but decreases the other firm’s profit even more. On the condition of being better than other competitors, firms that deviate from Cournot equilibrium achieve higher payoffs than the payoffs they receive under Cournot equilibrium. He defines the finite population evolutionarily stable strategy (FPES). It is a strategy of a player that maximizes his relative payoff. And in Vega-Redondo(1997) it was shown that FPES is a strategy that survives in the long run equilibrium or stochastically stable state of a dynamic stochastically evolutionary game developed by Kandori, Mailath and Rob (1993). Also Vega-Redondo(1997) showed that in a homogeneous good case if firms in an oligopoly maximize relative profits, a Walrasian equilibrium can be induced.

We consider the following three-stage game in a duopoly with a homogeneous good.

1. The first stage: The government determines the level of lump-sum subsidies to or lump-sum taxes on the firms.
2. The second stage: The firms decide whether they adopt new technology or not.
3. The third stage: The firms determine their outputs.

The social welfare is defined to be consumers’ surplus plus firms’ profits. Lump-sum subsidies to the firms are financed by lump-sum taxes on the consumers, and revenues from taxes on the firms are transferred to the consumers in a lump-sum manner. These lump-sum taxes and transfers are not related to the good of this industry. Excluding income effects they do not affect demand for the good, and they are canceled out in the social welfare.

We will show that when firm behavior is not competitive (the weight on the relative profit is small), the optimal policy of the government may be taxation; when firm behavior is competitive (the weight on the relative profit is large) but not extremely competitive (close to perfect competition), the optimal policy is subsidization or inaction and not taxation. The optimal policy may be taxation if the effect of new technology adoption by one firm on the social welfare is smaller than the effect on the objective function of a firm (the weighted average of absolute and relative profits). The reason why the optimal policy may be taxation when firms’ behavior is less competitive seems to be the fact that the social welfare includes the consumers’ surplus, and when the competitiveness is small, the volume of consumption is small. However, if firms’ behavior is extremely competitive (close to perfect competition), taxation case re-emerges.

2 The model

Two firms, Firm A and B, produce a homogeneous good, and consider adoption of new technology. Technology itself is free, but each firm must expend a fixed set-up cost. Denote the outputs of Firm A and B by \( x_A \) and \( x_B \), the price of the good by \( p \). The utility function of consumers is

\[
\begin{align*}
\quad u &= a(x_A + x_B) - \frac{1}{2}(x_A + x_B)^2,
\end{align*}
\]

where \( a \) is a positive constant. The inverse demand function is derived as follows.

\[
\begin{align*}
\quad p &= a - x_A - x_B.
\end{align*}
\]

The cost functions of the firms before adoption of new technology are \( cx_i, i = A, B \), and the cost of each firm after adoption of new technology is zero. A fixed set-up cost is \( e \). \( c \) and \( e \) are positive constants and common to both firms. There exists no fixed cost other than the set-up cost.

The social welfare \( W \) is defined to be the sum of consumers’ surplus and firms’ profits as follows;

\[
\begin{align*}
\quad W &= a(x_A + x_B) - \frac{1}{2}(x_A + x_B)^2 - p(x_A + x_B) + [p(x_A + x_B) - c_A(x_A) - c_B(x_B)]
\end{align*}
\]

\[
\begin{align*}
\quad &= a(x_A + x_B) - \frac{1}{2}(x_A + x_B)^2 - c_A(x_A) - c_B(x_B).
\end{align*}
\]

\( c_A(x_A) \) and \( c_B(x_B) \) generally denote the cost functions of the firms. They may include set-up costs. We analyze the optimal subsidization or taxation policies of the government for adoption of new technology by firms when the firms seek to maximize the weighted
average of their absolute profits and relative profits.

If adoption of new technology and non-adoption are indifferent for a firm, then it adopts new technology, and if adoption and non-adoption are indifferent for the society, the government chooses adoption.

3 Competitiveness of firm behavior in imperfectly competitive market

According to Matsumura and Matsushima (2012), Matsumura et. al. (2013), Shibata (2014) and Matsumura and Okamura (2015), we assume that each firm maximizes the weighted average of its absolute profit and relative profit. The relative profit of each firm is the difference between its absolute profit and the absolute profit of the rival firm. Let \( \pi_A \) and \( \pi_B \) be the absolute profits of Firm A and B. Then, the objective functions of Firm A and B are defined to be

\[
\Pi_A = (1 - \alpha)\pi_A + \alpha(\pi_A - \pi_B) = \pi_A - \alpha\pi_B,
\]

and

\[
\Pi_B = (1 - \alpha)\pi_B + \alpha(\pi_B - \pi_A) = \pi_B - \alpha\pi_A.
\]

\( \alpha \) is the weight on the relative profit. We assume \(-1 < \alpha < 1\).

There are some literature which justifies maximization of relative profit or the weighted average of absolute and relative profits by firms, and as stated in the introduction there is justification of relative profit maximization based on evolutionary game theory. However, similarly to Matsumura and Matsushima (2012), Matsumura et. al. (2013) and Matsumura and Okamura (2015), we do not necessarily assume that firms really maximize the weighted average of their absolute and relative profits. \( \alpha \) is interpreted as a parameter indicating the severity of competition. In a duopoly with a homogeneous good when \( \alpha \) is near to 1, the equilibrium outcome is near to the equilibrium in a perfectly competitive market. When \( \alpha = 0 \), the model is reduced to the standard Cournot case. On the other hand, if \( \alpha \) is near to \(-1\), each firm’s behavior is near to joint profit maximization, and thus, the outcome corresponds to that of collusion. Hence, \( \alpha \in (-1, 0) \) implies an intermediate competitiveness between monopoly and Cournot duopoly, and \( \alpha \in (0, 1) \) implies an intermediate competitiveness between Cournot duopoly and perfect competition. We assume that \( a \) is sufficiently larger than \( c \) and \( \alpha > -1 + \frac{2c}{a} \) so that the equilibrium outputs of both firms when one firm adopts new technology are positive.

4 Firm Behavior

The values of the objective functions of the firms are as follows. Before adoption of new technology,

\[
\Pi_A = (a - x_{A} - x_{B})x_{A} - cx_{A} - \alpha[(a - x_{A} - x_{B})x_{B} - cx_{B}],
\]
\[ \Pi_b = (a - x_A - x_B)x_B - cx_B - \alpha [(a - x_A - x_B)x_A - cx_A]. \]

After adoption of new technology by both firms,
\[ \Pi_A = (a - x_A - x_B)x_A - e - \alpha (a - x_A - x_B)x_B + \alpha e, \]
\[ \Pi_B = (a - x_A - x_B)x_B - e - \alpha (a - x_A - x_B)x_A + \alpha e. \]

After adoption of new technology by only Firm A,
\[ \Pi_A = (a - x_A - x_B)x_A - e - \alpha [(a - x_A - x_B)x_B - cx_B], \]
\[ \Pi_B = (a - x_A - x_B)x_B - cx_B - \alpha (a - x_A - x_B)x_A + \alpha e. \]

After adoption of new technology by only Firm B,
\[ \Pi_A = (a - x_A - x_B)x_A - cx_A - \alpha (a - x_A - x_B)x_B + \alpha e, \]
\[ \Pi_B = (a - x_A - x_B)x_B - e - \alpha [(a - x_A - x_B)x_A - cx_A]. \]

We assume Cournot type behavior of firms. There are four cases.

1. The conditions for maximization of the objective functions of the firms when no firm adopts new technology are
\[ a - 2x_A - x_B + \alpha x_B = 0, \quad a - x_A - 2x_B - c + \alpha x_A = 0. \]
The equilibrium outputs, price and the equilibrium values of the objective functions of the firms are written as
\[ x_A^0 = x_B^0 = \frac{a - c}{3 - \alpha}, \quad p^0 = \frac{(1 - \alpha)a + 2c}{3 - \alpha}, \quad \Pi_A^0 = \Pi_B^0 = \frac{(1 - \alpha)^2(a - c)^2}{(3 - \alpha)^3}. \]

2. The conditions for maximization of the objective functions of the firms when both firms adopt new technology are
\[ a - 2x_A - x_B + \alpha x_B = 0, \quad a - x_A - 2x_B + \alpha x_A = 0. \]
The equilibrium outputs, price and the equilibrium values of the objective functions of the firms are written as
\[ \bar{x}_A = \bar{x}_B = \frac{a}{3 - \alpha}, \quad \bar{p} = \frac{(1 - \alpha)a}{3 - \alpha}, \quad \bar{\Pi}_A = \bar{\Pi}_B = \frac{(1 - \alpha)^2a^2}{(3 - \alpha)^2} - (1 - \alpha)e. \]

3. If only Firm A adopts new technology, the conditions for maximization of the objective functions of the firms are
\[ a - 2x_A - x_B + \alpha x_B = 0, \quad a - x_A - 2x_B + \alpha x_A - c = 0. \]
The equilibrium outputs, price and the equilibrium values of the objective functions of the firms are written as
\[ x_A^4 = \frac{(1 + \alpha)a + (1 - \alpha)c}{(3 - \alpha)(1 + \alpha)}, \quad x_B^4 = \frac{(1 + \alpha)a - 2c}{(3 - \alpha)(1 + \alpha)}, \quad p^4 = \frac{(1 - \alpha)a + c}{3 - \alpha}, \]
\[ \Pi_A^e = \frac{(1 + \alpha)(1 - \alpha)^2 a^2 + (2 + 3\alpha - \alpha^3)ac + (1 - 5\alpha + 2\alpha^2)c^2}{(1 + \alpha)(3 - \alpha)^2} - e, \]
\[ \Pi_B^e = \frac{(1 + \alpha)(1 - \alpha)^2 a^2 - (1 + \alpha)(4 - 3\alpha + \alpha^3)ac + (4 - 3\alpha + \alpha^2)c^2}{(1 + \alpha)(3 - \alpha)^2} + \alpha e. \]

4. If only Firm B adopts new technology, the equilibrium outputs, price and the equilibrium values of the objective functions of the firms are written as
\[ x_A^B = \frac{(1 + \alpha)a - 2c}{(3 - \alpha)(1 + \alpha)}, \quad x_B^B = \frac{(1 + \alpha)a + (1 - \alpha)c}{(3 - \alpha)(1 + \alpha)}, \quad p_B = \frac{(1 - \alpha)a + c}{3 - \alpha}, \]
\[ \Pi_A^B = \frac{(1 + \alpha)(1 - \alpha)^2 a^2 - (1 + \alpha)(4 - 3\alpha + \alpha^3)ac + (4 - 3\alpha + \alpha^2)c^2}{(1 + \alpha)(3 - \alpha)^2} + \alpha e, \]
\[ \Pi_B^B = \frac{(1 + \alpha)(1 - \alpha)^2 a^2 + (2 + 3\alpha - \alpha^3)ac + (1 - 5\alpha + 2\alpha^2)c^2}{(1 + \alpha)(3 - \alpha)^2} - e. \]

Let define \( e^0 \) be the value of \( e \) such that \( \Pi_A^e = \Pi_A^0 \) or \( \Pi_B^e = \Pi_B^0 \), that is, adopting and non-adopting the new technology are indifferent for each firm when the rival firm does not adopt it. Then,
\[ e^0 = \frac{[(1 + \alpha)a - \alpha c](4 - 3\alpha + \alpha^2)c}{(1 + \alpha)(3 - \alpha)^2}. \]
Define \( e^1 \) be the value of \( e \) such that \( \tilde{\Pi}_A = \Pi_A^0 \) or \( \tilde{\Pi}_B = \Pi_B^0 \), that is, adopting and non-adopting the new technology are indifferent for each firm when the rival firm adopts it. Then,
\[ e^1 = \frac{[(1 + \alpha)a - c](4 - 3\alpha + \alpha^2)c}{(1 + \alpha)(3 - \alpha)^2}. \]
Clearly, since \(|\alpha| < 1\), we have \( e^0 > e^1 \). If \( e > e^0 \) \( (e \leq e^0) \), the best response of a firm is non-adopt (adoption) of new technology when the other firm does not adopt. If \( e > e^1 \) \( (e \leq e^1) \), the best response of a firm is non-adopt (adoption) of new technology when the other firm adopts. Thus, the sub-game perfect equilibria of the game after the second stage are as follows.

**Lemma 1**

1. If \( e \leq e^1 \), the sub-game perfect equilibrium is a state such that both firms adopt new technology. In this case \( e \leq e^1 \) and \( e \leq e^0 \), so adoption of new technology is the dominant strategy for both firms.
2. If \( e^1 < e \leq e^0 \), the sub-game perfect equilibrium is a state such that one firm, Firm A
or B, adopts new technology. In this case $e \leq e^0$ and $e > e^1$, so adoption of new technology is the best response to non-adoption, and non-adoption is the best response to adoption.

3. If $e > e^0$, the sub-game perfect equilibrium is a state such that no firm adopts new technology. In this case $e > e^0$ and $e > e^1$, so non-adoption is the dominant strategy for both firms.

5 Social welfare

In this section we consider the social welfare. When both firms adopt new technology, we write

$$W^2 = \frac{2(2-\alpha)a^2}{(3-\alpha)^2} - 2e,$$

when one firm adopts new technology, we write

$$W^1 = \frac{4a(a-c)(2-\alpha)(1+\alpha) + (11-5\alpha)c^2}{2(1+\alpha)(3-\alpha)^2} - e,$$

when no firm adopts new technology, we write

$$W^0 = \frac{2(2-\alpha)(a-c)^2}{(3-\alpha)^2}.$$

Let define $e^0_w$ be the value of $e$ such that $W^1 = W^0$, that is, adopting and non-adopting the new technology by one firm are indifferent for the society when no firm adopts it. Then,

$$e^0_w = \frac{[4a(2-\alpha)(1+\alpha) + (4\alpha^2 - 9\alpha + 3)c]}{2(1+\alpha)(3-\alpha)^2}.$$

Define $e^1_w$ be the value of $e$ such that $W^2 = W^1$, that is, adopting and non-adopting the new technology by one more firm are indifferent for the society when one firm adopts it. Then,

$$e^1_w = \frac{[4a(2-\alpha)(1+\alpha) + (5\alpha - 11)c]}{2(1+\alpha)(3-\alpha)^2}.$$

If $e \leq e^0_w$ ($e > e^0_w$), $W^1 \geq W^0$ ($W^1 < W^0$); and if $e \leq e^1_w$ ($e > e^1_w$), $W^2 \geq W^1$ ($W^2 < W^1$).

Comparing $e^0_w$ and $e^1_w$,

$$e^0_w - e^1_w = \frac{(2\alpha^2 - 7\alpha + 7)c^2}{(1+\alpha)(3-\alpha)^2} > 0.$$

Thus, we get the following lemma.

Lemma 2

1. If $e \leq e^1_w$, $W^2$ is the maximum, and adoption of new technology by both firms is optimal.

2. If $e^1_w < e \leq e^0_w$, $W^1$ is the maximum, and adoption of new technology by one firm
is optimal.

3. If \( e > e_w^0 \), \( W^0 \) is the maximum, and adoption of new technology by no firm is optimal.

6 Subsidization or taxation policy

6.1 Main results

We know \( e^0_w > e^1_w \) and \( e^0 > e^1 \). Comparing \( e^0_w \), \( e^1_w \) with \( e^0 \) and \( e^1 \) yields

\[
e^1_w - e^1 = \frac{[2a\alpha(1-\alpha)+(2\alpha-3)c]c}{2(3-\alpha)^3},
\]

\[
e^0_w - e^1 = \frac{[2a\alpha(1-\alpha)(1+\alpha)+(6\alpha^2-15\alpha+11)c]c}{2(1+\alpha)(3-\alpha)^3},
\]

\[
e^1_w - e^0 = \frac{[2a\alpha(1-\alpha)(1+\alpha)+(2\alpha^3-6\alpha^2+13\alpha-11)c]c}{2(1+\alpha)(3-\alpha)^3},
\]

\[
e^0_w - e^0 = \frac{[2a\alpha(1-\alpha)+2\alpha^2-4\alpha+3)c]c}{2(3-\alpha)^5}.
\]

They may be positive or negative. However, \( e^0_w - e^1 \) is likely to be positive, and \( e^1_w - e^0 \) is likely to be negative for reasonable values of variables. Suppose \( a = 10, c = 2 \). Then,

1. If \( \alpha = 0.9 \),
   \( e^1_w - e^1 = -0.14, e^0_w - e^1 = 0.97, e^1_w - e^0 = -0.24, e^0_w - e^0 = 0.87. \)

2. If \( \alpha = 0.5 \),
   \( e^1_w - e^1 = 0.16, e^0_w - e^1 = 1.87, e^1_w - e^0 = -0.43, e^0_w - e^0 = 1.28. \)

3. If \( \alpha = -0.1 \),
   \( e^1_w - e^1 = -0.89, e^0_w - e^1 = 2.68, e^1_w - e^0 = -3.09, e^0_w - e^0 = 0.48. \)

4. If \( \alpha = -0.5(> -1 + \frac{2c}{a}) \),
   \( e^1_w - e^1 = -1.88, e^0_w - e^1 = 5.31, e^1_w - e^0 = -7.51, e^0_w - e^0 = -0.32. \)

Thus, the larger the value of \( \alpha \), that is, the larger the competitiveness is, the larger the
values of \( e^1 - e^1 \) and \( e^0 - e^0 \) are. However, if \( \alpha \) is extremely large (close to perfect
competition), \( e^1 - e^1 \) is small again. We consider the following three cases.

1. Case A (\( \alpha \) is large): \( e^0 > e^0 > e^1 > e^1 \).
2. Case B (\( \alpha \) is a bit small, or \( \alpha \) is extremely large): \( e^0 \geq e^0 > e^1 > e^1 \).
3. Case C (\( \alpha \) is very small): \( e^0 > e^0 > e^1 > e^1 \).

Then, we obtain the following results.

Theorem 1 The optimal policies should be as follows;

1. Case A:
   (a) If \( e > e^0 \), the government should do nothing.
   (b) If \( e^0 < e \leq e^0 \), the government should give subsidies to the firms. The level of the
       subsidy to each firm must not be smaller than \( e - e^0 \) and must be smaller than \( e - e^1 \).
       Only one firm actually receives a subsidy and adopts new technology.
   (c) If \( e^1 < e \leq e^0 \), the government should do nothing.
   (d) If \( e^1 < e \leq e^1 \), the government should give subsidies to both firms. The level of the
       subsidy to each firm must not be smaller than \( e - e^1 \).
   (e) If \( e \leq e^1 \), the government should do nothing.

2. Case B:
   (a) If \( e > e^0 \), the government should do nothing. The same result as (1) of 1.
   (b) If \( e^0 < e \leq e^0 \), the government should give subsidies to the firms. The same result as
       (2) of 1.
   (c) If \( e^1 < e \leq e^0 \), the government should do nothing. The same result as (3) of 1.
   (d) If \( e^1 < e \leq e^1 \), the government should impose taxes on the firms. The level of the tax
       on each firm must be larger than \( e^1 - e \), and must not be larger than \( e^0 - e \). With this tax
       scheme only one firm pays the tax and adopts new technology.
   (e) If \( e \leq e^1 \), the government should do nothing. The same result as (5) of 1.

3. Case C:
   (a) If \( e > e^0 \), the government should do nothing. The same result as (1) of 1 and 2.
   (b) If \( e^1 < e \leq e^0 \), the government should impose taxes on the firms. The level of the tax
       on each firm must be larger than \( e^0 - e \).
   (c) If \( e^1 < e \leq e^0 \), the government should do nothing. The same result as (3) of 1 and 2.
   (d) If \( e^1 < e \leq e^1 \), the government should impose taxes on the firms. The same result as
       (4) of 2.
   (e) If \( e \leq e^1 \), the government should do nothing. The same result as (5) of 1 and 2.
Proof. 

1. Case A:
   (a) In this case \( W^0 \) is optimal and no firm adopts new technology without subsidy nor tax. Thus, the government should do nothing.
   (b) In this case \( W^1 \) is optimal but no firm adopts new technology without subsidy nor tax. The government should give subsidies to the firms. The level of the subsidy to each firm must not be smaller than \( e - e^0 \) and must be smaller than \( e - e^1 \). With this subsidy scheme only one firm actually receives a subsidy and adopts new technology at the equilibrium.
   (c) In this case \( W^1 \) is optimal and only one firm adopts new technology without subsidy nor tax. The government should do nothing.
   (d) In this case \( W^2 \) is optimal but only one firm adopts new technology without subsidy nor tax. The government should give subsidies to both firms. The level of the subsidy to each firm must not be smaller than \( e - e^1 \). If a subsidy is given to only one firm, the other firm does not adopt.
   (e) In this case \( W^2 \) is optimal and both firms adopt new technology without subsidy nor tax. Thus, the government should do nothing.

2. Case B:
   (a) In this case \( W^1 \) is optimal but both firms adopt new technology. Thus, the government should impose taxes on the firms. The level of the tax on each firm must be larger than \( e^1 - e \), and must not be larger than \( e^0 - e \). With this tax scheme only one firm pays the tax and adopts new technology.

3. Case C:
   (a) In this case \( W^0 \) is optimal, but one firm adopts new technology. Thus, the government should impose taxes on the firms. The level of the tax on each firm must be larger than \( e^0 - e \).
6.2 Example

Let us consider an example. Assume $a = 10$ and $c = 2$. Then, the values of $e^0$, $e^1$, $e_w^0$ and $e_w^1$ are

$$e^0 = \frac{4(4\alpha + 5)(\alpha^2 - 3\alpha + 4)}{(3 - \alpha)^2(\alpha + 1)}, \quad e^1 = \frac{4(5\alpha + 4)(\alpha^2 - 3\alpha + 4)}{(3 - \alpha)^2(\alpha + 1)},$$

$$e_w^0 = \frac{86 - 32\alpha^2 + 22\alpha}{(3 - \alpha)^2(\alpha + 1)}, \quad e_w^1 = \frac{58 - 40\alpha^2 + 50\alpha}{(3 - \alpha)^2(\alpha + 1)}.$$

Case A, B and C are obtained as follows.

1. Case A: If $\frac{6 - \sqrt{6}}{10} (\approx 0.355) < \alpha < \frac{6 + \sqrt{6}}{10} (\approx 0.845)$, $e^0 > e^1 > e_w^0 > e_w^1$.

2. Case B: If $\frac{3 - \sqrt{33}}{8} (\approx -0.343) \leq \alpha \leq \frac{6 - \sqrt{6}}{10}$ or $\alpha \geq \frac{6 + \sqrt{6}}{10}$, $e^0 \geq e^1 > e_w^0 \geq e_w^1$. 
3. Case C: If \( \alpha < \frac{3 - \sqrt{33}}{8} \), \( e^0 > e^0_w > e^1 > e^1_w \).

The relationships among the competitiveness (the value of \( \alpha \)), the value of \( e \) and the types of optimal policies, subsidization, taxation or inaction (doing nothing) are depicted in Figure 1. \( S \), \( T \) and \( N \) denotes the domains of the values of \( \alpha \) and \( e \) in which the optimal policies are, respectively, subsidization, taxation and inaction. In Figure 2 we depict the relation between the competitiveness and the (minimum) level of subsidy or tax in the case where \( e = 8.6 \). The positive value means a subsidy, and the negative value means a tax. In Figure 3 we depict the case where \( e = 6.5 \). In this case the government never gives a subsidy. In this figure there is a discontinuity of the curve because the optimal number of new technology adopting firm changes from one to two, but both firms adopt new technology without subsidy nor tax across \( \alpha \approx 0.01646 \).

6.3 Some discussions

![Figure 2: Relation between competitiveness and the level of subsidy or tax: \( e = 8.6 \)]
The effects of the competitiveness expressed by the value of $\alpha$ on the government’s subsidy or tax policy are decomposed as follows.

1. A subsidy encourages technology adoption, which tends to increase social welfare because of the production increase and cost reduction.
2. While social welfare cares about the profits of two firms, each firm maximizes its own profit minus a fraction $\alpha$ of its competitor’s profit. Therefore, each firm’s decision making has a negative externality on its competitor, and profits can be too small relative to the social optimal. Hence, the government may charge a tax so that profits may go up.

We discuss about our results in some detail. In Case A at the equilibria the government give subsidies to the firms or does nothing. There is no equilibrium with taxes. In Case B there are equilibria with taxes and equilibria with subsidies. In Case C at the equilibria the government impose taxes on the firms or does nothing. There is no equilibrium with subsidies. The difference between $e^0_w$ and $e^0$ and the difference between $e^0_i$ and $e^i$ are important for the pattern of the optimal policy, subsidization or taxation. $e^0_w$ ($e^0_i$) represents the effect of new technology adoption by one firm on the social welfare when no firm (one firm) adopts, and $e^0$ ($e^i$) represents the effect on the weighted average of absolute and relative profits of the adopting firm when the other firm
does not adopt (adopts). The optimal policy may be taxation if \( e^0_w < e^0 \) or \( e^1_w < e^1 \). When the value of \( \alpha \) is large (not extremely large), \( e^0_w > e^0 \) \( (e^1_w > e^1) \), and the smaller the value of \( \alpha \) is, the smaller the value of \( e^0_w - e^0 \) \( (e^1_w - e^1) \) is. The reason why the optimal policy may be taxation when \( \alpha \) is small seems to be the following fact.

The social welfare is the sum of the consumers’ surplus and the profits of the firms. When the competitiveness is small \((\alpha \text{ is small})\), the outputs of the firms and the volume of consumption are small, and so the effect of new technology adoption by one firm on the social welfare is small relatively to the effect on the objective functions of the firms.

However, if firm behavior is extremely competitive (close to perfect competition), taxation case re-emerges because \( e^1 > e^1_w \). The reason may be the following fact.

If the competitiveness is very large, the objective function of a non-adopting firm (a firm which does not adopt new technology) when the other firms adopts is negative with large absolute value, and so the effect of new technology adoption by that firm \((e^1)\) is large$^4$.

7 Ongoing and future researches

We are proceeding research in two directions. The first is the extension of the results in this paper to a case of oligopoly. The second is the studies of strategies by firms and public policies around licensing problem. In this paper we consider free technology transfer and fixed set-up cost. New technology developed by an innovating firm spreads to other firms in a domestic or a foreign country by licensing as well as free transfer. We are studying choice of strategies by an innovating firm, to license its new technology to other firms without entering the market, or to license its technology at the same time enter the market, or to enter the market without license.

Further, there exist some valuable future research themes, for example, spillover effect (d’Aspremont and Jacquemin (1988), Shibata (2014)), initial asymmetries between firms (Lahiri and Ono (1999), Kitahara and Matsumura (2006)), uncertainty (Matsumura (2003), Kitahara and Matsumura (2006)) and patent race (O’Donoghue (1998), Ishibashi and Matsumura (2006)).

References


$^4$ For example, when \( a = 10, c = 2 \), \( \Pi^a_d = -8.3 \) if \( \alpha = 0.9 \), \( \Pi^a_d = -3.6 \) if \( \alpha = 0.5 \).


