Learning by doing, low level equilibrium trap, and effect of domestic policies on child labour

Chakraborty, Kamalika and Chakraborty, Bidisha

Jadavpur University

22 October 2016

Learning by doing, low level equilibrium trap, and effect of domestic policies on child labour

Kamalika Chakraborty

Jadavpur University, India

Bidisha Chakraborty

Jadavpur University, India

Abstract

This paper builds an overlapping generations household economy model with learning by doing effect in unskilled work. We study the short run equilibrium of schooling, relationship between child schooling and parental schooling, long run dynamics of schooling and human capital and relative effectiveness of two domestic policies- child labour ban and education subsidy on schooling. We find some interesting results. If parents working in unskilled sector do not experience any schooling at their childhood, they will never send their children for schooling. But the relationship between parental schooling and child schooling may not be monotonic. This relationship depends on other factors like subsistence consumption expenditure, learning by doing effect, responsiveness of wage to human capital in skilled sector, efficiency of education technology. Existence of low level equilibrium trap for unskilled parent depends on the specific form of human capital accumulation function. For a certain range of parental schooling time path of child schooling will be oscillating in nature. Decrease in child wage increases steady state schooling only if the maximum possible adult unskilled wage exceeds the sum of the schooling cost and subsistence expenditure of the household. If unskilled adult wage is sufficiently small, education subsidy is more effective in enhancing schooling than banning child labour.

Keywords: child labour, schooling, human capital, low level equilibrium trap, oscillation, child labour ban, education subsidy

JEL Classification Numbers: J22, J24, J820, I210

1. Introduction

Child labour is a persistent problem across the globe, especially in the developing countries. A number of rules and conventions have been laid down all over the world to fight child labour. This paper builds a theoretical model to examine the relative effectiveness of two types of domestic policies to combat child labour-a child labour ban and an education subsidy. In spite of steady decline in the incidence of child labour over the last decade, ILO estimates show that the number of child workers across the globe is still quite high. According to the ILO estimates, in 2012, there were about 168 million child labourers in the world, of whom more than two thirds (120 million) were in the age group 5 to 14 years old. In 2012, the largest number of child labourers was in Asia and the Pacific (77.7 million), followed by Sub-Saharan Africa with 59.0 million, Latin America and the Caribbean with 12.5 million and Middle East and North Africa with 9.2 million. In relative terms, Sub-Saharan Africa ranks highest. About 1 in 5 children was in child labour in the region.

These figures point out that tackling the problem of child labour still remains a challenging issue for the developing countries across the world. Both domestic as well as international policies may be undertaken to reduce the incidence of child labour. However in this paper we restrict our analysis only to domestic policies. A number of theoretical papers deal with the effectiveness of domestic policies to reduce child labour. The pioneer work on child labour by Basu and Van (1998) shows that in case of multiple equilibria in the labour market, a total ban on child labour can take the economy from bad equilibrium to good equilibrium. All working class households will be better off. But if there is only one equilibrium, a total ban could harm worker households and also benefit them. A partial ban may not always reduce child labour but may reduce only child wage. However utility of the worker household may or may not increase. According to Baland and Robinson (2000) small ban on child labour can be Pareto improving. A ban on child labour reduces the supply of child labour while increasing the supply of adult labour in the future. As a result, current wages of both adults and children are likely to rise and future wages are likely to fall. Thus while children's utility is likely to rise in most cases, parental welfare will increase only when the effect on current wages dominates. The paper by Dessy and Pallage (2001) states that compulsory bans on child labour help sending signals to investors that investment in human capital will be made in the near future and thus skilled labour is likely to be available. Ban or compulsory education will be counterproductive if the cost of investment is very high. Instead a policy that subsidizes technology and imposes compulsory education can help to move the economy from bad equilibrium to a good one. Dinopoulos and Zhao (2007), in their paper on globalisation, show that a ban on child labours benefits adult unskilled workers but hurts adult skilled workers. According to Emerson and Knabb (2006), P.Ranjan (1999, 2001) banning child labour can reduce dynastic welfare, increase poverty and further accentuate income inequality within society. Few papers deal with the policy issues on harmful forms of child labour. According to Rogers and Swinnerton (2002), a ban on exploitative child labour has ambiguous effects i.e. child employment and aggregate output may rise or fall. Dessy and Pallage (2005) states that a ban on worst forms of child labour in poor countries is likely to be welfare reducing as these forms of labour have important economic role to play. The wages earned by the children by working in these jobs help in human capital accumulation. So by denying them work, they are being relegated to an even worse situation.

There is a small set of literature that deals with the effect of education policy on child labour. Emerson and Knabb (2006) show that compulsory education policy may actually reduce welfare. According to Chaudhuri and Mukhopadhyay (2003), a rise in the education subsidy may produce counterproductive results on the supply of child labour in the urban area. Moreover it may raise the level of urban unemployment of adults even when adult labour and child labour are not substitutes to each other. The average income of the urban poor families may also decrease as a consequence. Chaudhuri (2004) states that the effects of increase in education subsidy on child labour depends on relative strength of two effects-namely labour re-allocation effect and the contradictory effect which exerts a downward pressure on the incidence of child labour. Mukherjee and Sinha (2006) and Estevez (2011) argue in favour of education subsidy in improving school attendance. According to Estevez (2011), an education subsidy will reduce the incidence of child labour, increase the household income and will also indirectly increase the unskilled wage.

Some empirical papers on domestic policies on child labour deserve mention as well. Fabre and Pallage (2011) works within a dynamic, general equilibrium model calibrated to South Africa in the 1990s. It shows that in an economy with idiosyncratic shocks to adult employment, child labour ban deprives the households of an important way of smoothing consumption. Schultz (2004) evaluates the performance of Progresa (provides education grants to poor mothers) program in rural Mexico and has concluded that there has been significant reduction in child work for those families who have been induced by the program to enrol their child in school. However the magnitude of the response cannot offset more than a fifth of the total consumption gains associated with the program grants. The paper by Ravallion and Wodon (2000) studies the effects of a targeted enrolment subsidy on children's labour force participation and school enrolment in rural Bangladesh. Results suggest that the enrolment subsidy reduced the incidence of child labour but this effect accounts for a small proportion of the increase in school enrolment. So reduction of child labour not necessarily implies increase in schooling. Krueger and Donohue (2005) calibrate their model to USA data around 1880 and conclude that introducing free education results in substantial welfare gains, whereas a child labour ban induces small welfare losses.

None of the papers mentioned so far have theoretically examined the effects of ban and education subsidy on steady state schooling and steady state human capital of the child labour in the presence of learning by doing effect in unskilled sector. Learning by doing effect is included in unskilled wages in our paper. Dessy and Pallage (2005), in their paper on worst forms of child labour, consider the learning by doing effect in the human capital accumulation function. There are many other papers which emphasize on the learning by doing effect. However these papers do not deal with the issue of child labour. Arrow (1962), Mao (2012), Parente (1994), Hippel and Tyre (1993) deal with learning by doing effect but in different context. In our paper individuals earn an extra income as adult in the unskilled sector if they have work experience as child labour in their young age. This is how learning by doing effect enters into our model. Learning by doing often occurs through apprenticeship and real life apprenticeship is found mostly in informal or unskilled sector e.g in fishing, poultry, farming etc. According to World Employment Report 1998-99-"In Kenya, with its relatively well developed formal training system, there are more apprentices enrolled in the informal sector than trainees in the formal sector", while "in Egypt, over 80% of craftsmen in the construction sector acquire their skills through traditional apprenticeship." According to the report, child labour is common in the field of apprenticeship. According to ILO's report on Employment Sector (2008), apprenticeship has been providing the traditional solution for developing and financing vocational skills of young people in poor societies. Estimations suggest that 80% of the skills imparted in the informal economy in West Africa are transferred through apprenticeship. In Benin, in 2005, approximately 2000,000 young apprentices were trained, which represents ten times as many apprentices than students in vocational and technical education. The present paper includes learning by doing effect in wages of unskilled labour and makes a comparison between the effect of child labour ban and education subsidy on child labour.

The present paper builds an overlapping generations model of household economy consisting of a skilled sector and an unskilled sector. If one individual is employed in skilled sector she gets wage proportional to human capital whereas unskilled sector gives a fixed return and a positive learning by doing effect generated from working in her childhood. Human capital formation of child is included in the parental utility function and parental choice of schooling vis-a-vis child work is considered. More educated parent gives more stress on human capital accumulation of the child in his utility function. This paper attempts to understand the effects of child labour ban and education subsidy on steady state schooling and steady state human capital of child labour. Moreover this paper studies the relative effectiveness of child labour ban and education subsidy in improving schooling of the child.

In case of unskilled parent we find in this model that if parents are completely uneducated, they will not send their children for schooling. We also find that there exist two steady state equilibria of schooling in the presence of intercept term in human capital accumulation function. The low level equilibrium represents a trap[1]. Once the trap is crossed schooling keeps on increasing and the time path of schooling is convergent towards higher of the equilibrium. Once the higher of the steady state level of parental human capital is crossed time path of schooling becomes convergent but oscillating in nature[2]. However if the intercept term is absent in human capital accumulation function the low level equilibrium trap does not exist anymore. There is only one equilibrium. The time path of schooling of the child is steadily convergent in nature when approached from below steady state level of parental schooling and beyond the steady state value of schooling the time path of schooling is convergent but oscillating in nature.

In the presence of intercept term in human capital accumulation function we find that for skilled parent, one steady state equilibrium prevails which is unstable in nature. Below the steady state, schooling keeps on falling till it converges to zero and beyond the steady state; schooling keeps on rising till it converges to full schooling. In case where the intercept term is absent in human capital accumulation function, if skilled parents also send their children for partial schooling then there exists a critical level of parental schooling beyond which steady growth of schooling of child takes place and eventually it converges towards full schooling. However below that critical level, schooling of child keeps on falling till it converges to the unskilled level steady state schooling in an oscillating manner. But if schooling required to be engaged as skilled worker is higher than the critical level of parental schooling beyond which they send their children for full schooling then there exists only one steady state equilibrium in case of skilled parent and that is full schooling.

---

[1] Papers dealing with child labour trap include Basu and Van (1998), Emerson and Knabb (2007), Gupta (2001), Emerson and Souza (2003), Basu (1999), Sarkar and Sarkar (2012), Sasmal and Guillen (2011) and others.

[2] Oscillations in human capital accumulations are found in Zhang (2015), Zhang (2014), Croix and Licandro (1999), Croix (2001) but in different context.

We also find that if human capital accumulation depends only on schooling of the child, then in case of unskilled parent, as child wage falls, steady state schooling increases only if the maximum unskilled adult wage exceeds the sum of subsistence consumption expenditure and schooling cost of the household. A fall in schooling cost increases the steady state schooling and steady state human capital. Comparing the effects of ban and education subsidy on steady state human capital in case of unskilled parent, we get the result that if unskilled adult wage is sufficiently small compared to subsistence consumption expenditure of the household the effect of giving education subsidy is more effective in enhancing schooling than banning child labour. We try to capture the real life problem of child labour as much as we can in this model. In reality a poor parent of developing country often ponders over the issue that whether my child would be benefitted from learning by doing effect earned because of work experience as child labour or they would be benefitted more if they go to school. Moreover we also observe that educated parents derive more satisfaction by sending their children to school.

The rest of this paper is organized as follows. Section 2 describes the basic model. Section 3 describes the short run equilibrium. Section 4 discusses the long run dynamics. In section 5 we consider the case where human capital formation of child depends entirely on her schooling and the child does not accumulate any human capital in absence of schooling. Concluding remarks are made in section 6.

2. The Model

We consider an economy that consists of identical households in overlapping generations framework[3]. Each household consists of one adult and one child. We consider two parents as one adult and two children as one child. The economy consists of two sectors- a skilled sector and an unskilled sector. In first period agents are children. They may either work in unskilled sector or go to school. In second period, the agent on reaching adulthood may either work in unskilled sector or in the skilled sector. If one individual is employed in skilled sector she gets wage proportional to human capital whereas unskilled sector gives a fixed return and learning by doing effect. The adult or the parent decides the time allocation of her child between work and schooling. Utility function of the adult depends on family consumption and human capital formation of the child. More educated parents have more preference towards child's human capital.

Following Glomm (1997), we assume parental choice of human capital investment. The adult decides how much time her child would devote to work in the unskilled sector and how much time for schooling by maximizing utility subject to the budget constraint. In first period, time devoted to schooling is denoted by 's_{t-1}' and that to work is '1- s_{t-1}'. Working generates positive learning by doing effects denoted by '$(1- s_{t-1})\underline{h}$'which helps him to earn a higher wage in future on joining the unskilled sector. In second period the adult sends her child to school for 's_t' units of time and for the remaining '$(1-s_t)$' units of time, the child is employed in the unskilled sector. Wages earned by the adult and by the child constitute the total income of the household. If the child joins the skilled sector, on becoming adult, she gets a wage in the skilled sector which is a fixed proportion of the human capital possessed by her (δh_t)[4]. In

[3] Overlapping generations framework has been adopted by Becker and Tomes (1979), Acemoglu and Pischke (2000), Glomm (1997), Glomm and Ravikumar (1998) and many more.

[4] Hare and Ulph (1979) assume that wage rate depends on ability and amount of education received by an individual.

unskilled sector the adult gets a return 'A+ $(1-s_{t-1})\underline{h}$' where'$(1-s_{t-1})\underline{h}$' denotes positive learning by doing effect generated from working as a child and 'A' denotes the fixed return.

A child, by working in the unskilled sector gets a fixed return which is less than the return obtained by the adults from unskilled sector.

Like Moav (2005), this paper assumes that human capital evolution is independent of physical capital. Human capital accumulation function of a child is assumed to take the following form[5]:

$$h_{t+1} = bs_t + \underset{\smile}{h} \quad (1)$$

where 's_t' is the time devoted to studies by the child, b>0 is a positive constant representing education technology and $\underset{\smile}{h}$ represents minimum level of human capital possessed by the child even if she does not attend school.

In case of unskilled parent household income is given by:

$$Y_t = [A+ (1- s_{t-1})\underline{h}\ ] + A\varphi\ (1-s_t)\ , \quad (2)$$

where Y_t is total income of the household, A is wage earned by the adult in unskilled sector, $(1-s_{t-1})\underline{h}$ is the positive learning by doing effect that an adult receives if she has work experience as child labour, φ is the fraction of adult wage that a child labour receives. Here $0<\varphi<1$ is a positive constant.

The household spends its income on purchasing consumption good and schooling of the child. So, the budget constraint of the household is given by:

$$[A+ (1- s_{t-1})\underline{h}\ ]+ A\varphi\ (1-s_t) = p_c c_t + \rho\ s_t\ , \quad (3)$$

where p_c is the price of the consumption good, $p_c c_t$ represents the total consumption expenditure and $\rho\ s_t$ denotes the expenditure on schooling of the child. When adults work in skilled sector, household income is given by:

$$Y_t = w_t + A\varphi\ (1-s_t)\ ,$$

where w_t is the wage earned by the adult in the skilled sector. We assume wage earned in skilled sector (w_t) is proportional to the human capital acquired by that individual i.e. $w_t = \delta h_t$.

Utility function of an adult of the representative household is defined as follows:

$$U_t = \ln(c_t - \underline{c}) + s_{t-1} \ln(bs_t + \underset{\smile}{h}) \quad \text{if } c_t \geq \underline{c}$$

$$= -\infty \text{ otherwise} \quad (4)$$

where c_t represents consumption, $\underline{c}$ represents subsistence consumption. The utility function is defined on the range $c_t \geq \underline{c}$. Utility depends on consumption of the adult and human capital formation of the child. Higher is the education level of the parent, more is the importance they give to human capital accumulation of the child.

Let us first apply the model in the short run equilibrium context and understand the relationship between parental human capital and schooling of the child.

[5] Inclusion of parental human capital in human capital accumulation of child yields nonlinear equations and makes the model very complicated. So, for the sake of simplicity the human capital accumulation of child is assumed to take this form.

3. Short-run Equilibrium

3.1 Parents working in unskilled sector

Utility maximization problem of an adult of the representative household working in unskilled sector is to maximize the utility, given by equation (4) subject to the budget constraint given by equation (3) with respect to the decision variables of the household viz, c_t and s_t

From the first order conditions[6] of the optimization problem, if there exists an interior solution of s_t , we obtain:

$$s_t = \frac{bs_{t-1}[A+(1-s_{t-1})\underline{h}+\varphi A-p_c\underline{c}]-\underset{\smile}{h}\,(A\varphi+\rho)}{b(A\varphi+\rho)(1+s_{t-1})} \tag{5}$$

$s_t > 0$ if $-b\underline{h}s_{t-1}^2 + bs_{t-1}\{A(1+\varphi) + \underline{h} - p_c\underline{c}\} - \underset{\smile}{h}\,(A\varphi+\rho) > 0$.

This implies that if $A(1+\varphi)+\underline{h}-p_c\underline{c} > 0$, then only we get (2 positive values of s_{t-1} for which) $s_t > 0$. So it is a necessary condition for s_t being positive.

Note that if $s_{t-1}=0$ then $s_t=0$ because $dz/ds_t<0$. So we arrive at the following proposition:

Proposition 1: If parents do not experience any schooling at their childhood they will never send their children for schooling.

There exists a range of s_{t-1} for which $s_t > 0$.
$s_t = 1$ if $-b\underline{h}s_{t-1}^2 + bs_{t-1}[A+\underline{h}-p_c\underline{c}-\rho] - (A\varphi+\rho)(b+\underset{\smile}{h}) \geq 0$

This implies that if $A+\underline{h}-p_c\underline{c}-\rho>0$, then only we get (2 positive values of s_{t-1} for which) $s_t = 1$. There also exists a range of s_{t-1} for which $s_t > 0$.

Proposition 2: There exists a particular range of parental schooling for which child schooling is positive and a particular range of parental schooling for which child experiences full schooling.

Differentiating s_t with respect to s_{t-1} we get

$$\frac{ds_t}{ds_{t-1}} = \frac{1}{b\,(A\varphi+\rho)(1+s_{t-1})^2}[-b\underline{h}s_{t-1}^2 - 2bs_{t-1}\underline{h} + b\,\{A\,(1+\varphi) + \underline{h}-p_c\underline{c}\} + \underset{\smile}{h}\,(A\varphi+\rho)]$$

$$\frac{ds_t}{ds_{t-1}} > 0 \text{ if } [-b\underline{h}s_{t-1}^2 - 2bs_{t-1}\underline{h} + b\,\{A\,(1+\varphi) + \underline{h}-p_c\underline{c}\} + \underset{\smile}{h}\,(A\varphi+\rho)] > 0$$

$$\text{or } s_{t-1} < \pm\sqrt{\frac{A(1+\varphi)}{\underline{h}} + \frac{\underset{\smile}{h}(A\varphi+\rho)}{b\underline{h}} + 2 - \frac{p_c\underline{c}}{\underline{h}}}\ -1=N$$

We ignore the negative term since s_{t-1} cannot be negative.

$$\frac{d^2s_t}{ds_{t-1}^2} = \frac{-2[b\underline{h}(1+s_{t-1})^2 + \{-b\underline{h}s_{t-1}^2 - 2bs_{t-1}\underline{h} + b\,\{A\,(1+\varphi)+\underline{h}-p_c\underline{c}\}+\underset{\smile}{h}\,(A\varphi+\rho)\}]}{b(A\varphi+\rho)(1+s_{t-1})^3}$$

[6] For detailed derivation please see equations (A.1), (A.2) and (A.3) of Appendix A.

$\frac{d^2 s_t}{ds_{t-1}^2}$ <0 if $-b\underline{h}s_{t-1}^2 - 2bs_{t-1}\underline{h} + b\{A(1+\varphi) + \underline{h} - p_c\underline{c}\} + \underset{\smile}{h}(A\varphi + \rho)\} + b\underline{h}(1+s_{t-1})^2 > 0$

or $b\{A(1+\varphi) + 2\underline{h} - p_c\underline{c}\} + \underset{\smile}{h}(A\varphi + \rho) > 0$

$\frac{d^2 s_t}{ds_{t-1}^2}$ >0 if $b\{A(1+\varphi) + 2\underline{h} - p_c\underline{c}\} + \underset{\smile}{h}(A\varphi + \rho) < 0$

We assume $b\{A(1+\varphi) + 2\underline{h} - p_c\underline{c}\} + \underset{\smile}{h}(A\varphi + \rho) > 0$ otherwise $\sqrt{\frac{A(1+\varphi)}{\underline{h}} + \frac{\underset{\smile}{h}(A\varphi+\rho)}{b\underline{h}} + 2 - \frac{p_c\underline{c}}{\underline{h}}}$ becomes an imaginary number.

If N≥1, s_{t-1} is always less than N. This implies that when N≥1 $\frac{ds_t}{ds_{t-1}}$ >0 always.

Now N≥1 implies $\frac{A(1+\varphi) - p_c\underline{c}}{\underline{h}} + \frac{\underset{\smile}{h}(A\varphi+\rho)}{b\underline{h}} \geq 2$.

Sufficient condition for this to hold is that $A(1+\varphi) - p_c\underline{c} > 0$, A (1+φ) must be high and $\underline{h}$ must be low.

Proposition 3: Parents being employed in unskilled sector, if total earnings of the household exceed the subsistence consumption expenditure of the household and are high and learning by doing effect in unskilled sector is low then schooling of child always increases with increase in schooling of the parent.

If N<0, then this condition is never satisfied. Equilibrium does not exist.
Therefore if N is a fraction, then $\frac{ds_t}{ds_{t-1}}$ >0 till N is reached and thereafter $\frac{ds_t}{ds_{t-1}}$ <0.

N is a fraction when 0<N<1. This implies $-1 < \frac{\{A(1+\varphi) - p_c\underline{\underline{c}}\}}{\underline{h}} + \frac{\underset{\smile}{h}(A\varphi+\rho)}{b\underline{h}} < 2$.

If $A(1+\varphi) > p_c\underline{c}$ but A (1+φ) is low and $\underline{h}$ is high then this inequality is likely to be satisfied. However this is only the sufficient condition for the above inequality to hold true.

$\frac{d^2 s_t}{ds_{t-1}^2}$ <0 throughout.

Proposition 4: Parents being employed as an unskilled labour, if total earnings of the household exceed subsistence consumption expenditure but are low and learning by doing effect is high, then below a particular level of parental schooling there is positive relationship between parental schooling and schooling of the child. But beyond a certain level of parental schooling, schooling of the child decreases with increase in parental level of schooling.

The reason behind obtaining such result is when in spite of going to a school for quite long time parents are still working in unskilled sector they lack motivation to send their children to school. Moreover as unskilled parents went to school themselves they are losing a part of income that they would have earned had they have not gone to school in their childhood Below a particular level of parental schooling, parental schooling and child schooling are positively related because it is assumed that more educated parents derive more satisfaction from sending their children to school. But given low levels of earnings of the household and high learning by doing effect in unskilled sector, beyond a particular level of parental schooling there is a negative relationship between parental level of schooling and child schooling.

3.2 Parents working in skilled sector

When parents work in the skilled sector, the incentive compatibility condition requires that wage earned in skilled sector is higher than the wage earned in unskilled sector. This implies that

$w_{t+1} >$ A+ (1- s_t)$\underline{h}$ which implies that $\delta(bs_t+\underset{\smile}{h}) >$ A+ (1- s_t)$\underline{h}$ i.e. $s_t > \frac{A+\underline{h}-\delta\underset{\smile}{h}}{\delta b+\underline{h}} = \underline{s}$. This implies that only if $s_t > \underline{s}$, then only individuals join the skilled sector.

When adults work in skilled sector the budget constraint of the household is given by:
$\delta\, bs_{t-1} + \delta\underset{\smile}{h} + A\varphi\,(1-s_t) = p_c c_t + \rho\, s_t$ where $\delta\, bs_{t-1} + \delta\underset{\smile}{h}$ denotes income of the adult working in the skilled sector.

In this case schooling of the child is given by

$$s_t = \frac{bs_{t-1}\left[\delta bs_{t-1}+\delta\underset{\smile}{h}+\varphi A-p_c\underline{c}\right]-\underset{\smile}{h}(A\varphi+\rho)}{b(A\varphi+\rho)(1+s_{t-1})}$$

$s_t > 0$ if $\delta b^2 s_{t-1}^2 + bs_{t-1}(\delta\underset{\smile}{h} + \varphi A - p_c\underline{c}) - \underset{\smile}{h}(A\varphi+\rho) > 0$. This implies that we get one positive value of s_{t-1} above which $s_t > 0$ whatever be the sign of $\delta\underset{\smile}{h} + \varphi A - p_c\underline{c}$.

$s_t = 1$ if $\delta b^2 s_{t-1}^2 + b[\delta\underset{\smile}{h} - p_c\underline{c} - \rho]\, s_{t-1} - (A\varphi+\rho)(b+\underset{\smile}{h}) \geq 0$

This implies that there exists a positive value of s_{t-1} say $\hat{s}$ for which $s_t = 1$ whatever be the sign of $\delta\underset{\smile}{h} - p_c\underline{c} - \rho$.

If $\underline{s} > \hat{s}$ then all parents who are employed in skilled sector send their children for full schooling.

Differentiating s_t with respect to s_{t-1} we get

$$\frac{ds_t}{ds_{t-1}} = \frac{1}{b\,(A\varphi+\rho)(1+s_{t-1})^2}[\delta b^2 s_{t-1}^2 + 2\delta b^2 s_{t-1} + \delta b\underset{\smile}{h} + bA\varphi - bp_c\underline{c} + \underset{\smile}{h}(A\varphi+\rho)]$$

$\frac{ds_t}{ds_{t-1}} > 0$ if $\delta b^2 s_{t-1}^2 + 2\delta b^2 s_{t-1} + \delta b\underset{\smile}{h} + bA\varphi - bp_c\underline{c} + \underset{\smile}{h}(A\varphi+\rho) > 0$

or $s_{t-1} > \pm\sqrt{\frac{p_c\underline{c}}{\delta b} - \frac{\underset{\smile}{h}}{b} - \frac{A\varphi}{\delta b} - \frac{\underset{\smile}{h}(A\varphi+\rho)}{\delta b^2} + 1 -}\; -1 = R$

If $R \leq 0$, this condition is always satisfied. So for $R \leq 0$, $\frac{ds_t}{ds_{t-1}} > 0$ always.

Now $R \leq 0$ implies $\frac{p_c\underline{c}}{\delta b} - \{\frac{\delta\underset{\smile}{h}+bA\varphi+\underset{\smile}{h}(A\varphi+\rho)}{\delta b^2}\} \leq 0$.

This implies that if $p_c\underline{c}$ is low and δ and b are high, then in case of skilled parent, schooling of child will increase with increase in schooling of parent.

Proposition 5: Parents being employed in skilled sector, if subsistence consumption expenditure of the household is low and responsiveness of wage to human capital in skilled sector is high and education technology is highly efficient then schooling of child always increases with increase in schooling of the parent.

If $R \geq 1$, this condition is never satisfied. So for $R \geq 1$, $\frac{ds_t}{ds_{t-1}} \leq 0$ always. Hence there does not exist any equilibrium (see Figure 2).

If $0<R<1$, then s_t is falling till R is reached and beyond R, s_t is rising.

R is a fraction when $0<R<1$. This implies $0<\frac{p_c\underline{c}}{\delta b}-\frac{\underset{\smile}{h}}{b}-\frac{A\varphi}{\delta b}-\frac{\underset{\smile}{h}(A\varphi+\rho)}{\delta b^2}<3$.

If $p_c\underline{c}$ is sufficiently high and δ and b are low then this condition is likely to be satisfied.

Proposition 6: Parents being employed as a skilled labour, if subsistence consumption expenditure is sufficiently high and responsiveness of wage to human capital is low and education technology is less efficient, then below a particular level of parental schooling there is negative relationship between parental schooling and schooling of the child. But beyond a certain level of parental schooling, schooling of the child increases with increase in parental level of schooling.

The reason for this is that since subsistence expenditure of the household is quite high and responsiveness of wage to human capital for skilled parent is low and also education technology is not very efficient, till a particular level of parental schooling is reached schooling of child does not increase with increase in schooling of the parent. More educated parents give more importance to human capital formation of the child and hence to schooling of the child. So beyond a particular level of human capital only schooling of child increases with increase in schooling of the parent. Beyond this level high subsistence expenditure, low responsiveness of wage to human capital and low level of education technology no longer play a role in determining schooling of the child.

4. Long run Dynamics

4.1 Dynamics of schooling when parents work in the unskilled sector

Putting $s_t = s_{t-1} = s^*$ in the expression of s_t we get

$[b\,(A\varphi+\rho)+ b\underline{h}\,]\, s^{*2} + b\,[(A\varphi+\rho) - \{A(1+\varphi)+ \underline{h}-p_c\underline{c}\}]s^* + \underset{\smile}{h}\,(A\varphi+\rho) = 0$

This implies that if $b[(A\varphi+\rho) - \{A\,(1+\varphi) + \underline{h}-p_c\underline{c}\}] <0$ i.e. if $A + \underline{h} > \rho + p_c\underline{c}$, then only we get two positive values of s^*, otherwise we do not get any positive value of s^*.

The dynamics of s_t is shown in the following diagram:

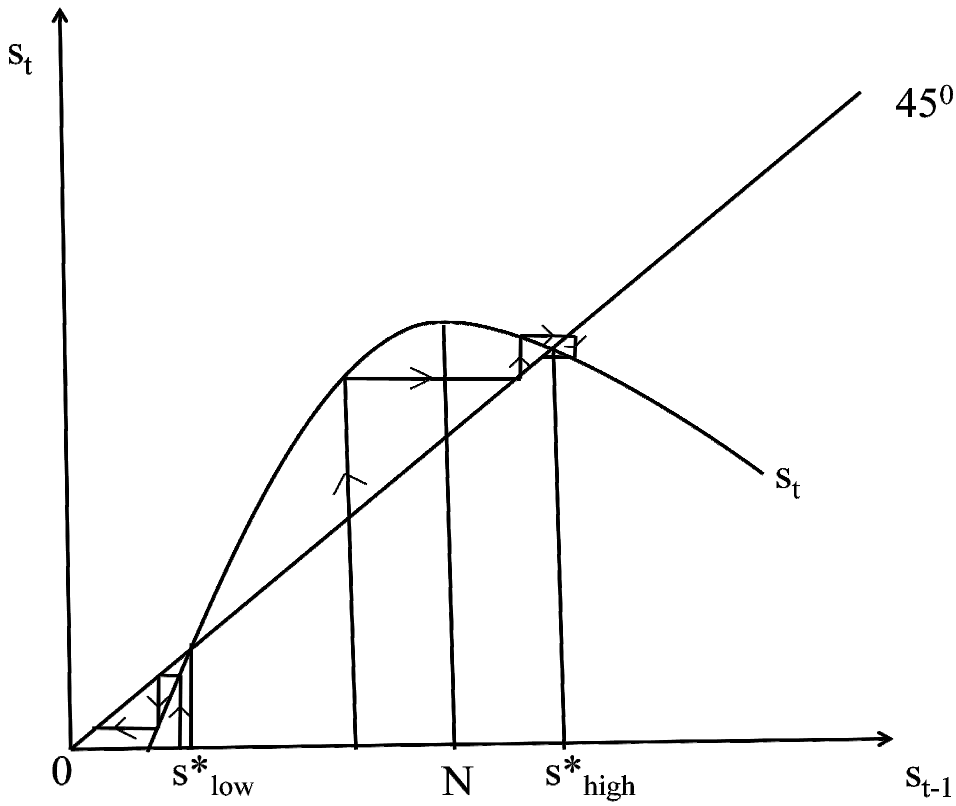

Figure 1: Dynamics of schooling for unskilled sector when intercept term exists in human capital accumulation

We demonstrate here the case where N is a fraction. So s_t is rising till N is reached and beyond N, s_t is falling. There are two steady state equilibria. The low level steady state s^*_{low} is unstable and represents a low level equilibrium trap. If s_{t-1} is below s^*_{low} schooling keeps on falling till becomes zero. Beyond s^*_{low} schooling keeps on increasing. Once N is crossed, the time path becomes convergent and oscillating. s^*_{high} represents the high level equilibrium.

Proposition 7: In case of unskilled parent, steady state of schooling exists if unskilled wage is sufficiently high to cover education cost and subsistence expenditure. Steady state equilibrium, if it exists, is not unique. There exist two steady states of schooling. When parental level of schooling is below a critical level(s^*_{low}), the economy gets trapped in a low level equilibrium trap. Beyond that critical level the time path of schooling of the child is convergent towards high level equilibrium (s^*_{high}).Beyond s^*_{high} the time path of schooling is convergent but oscillating in nature.

In this paper we are obtaining fluctuations in schooling because of the assumptions of higher altruism by more educated parents and learning by doing effect in unskilled wage.

4.2 Dynamics of human capital when parents work in the unskilled sector

In this section we discuss dynamics of human capital for the dynasties of which parents work in unskilled sector. Since human capital accumulation function is given by $h_t = bs_{t-1} + \underset{\smile}{h}$. $s_t = s_{t-1}$ implies that $h_t = h_{t+1}$ i.e. when schooling is at steady state ($s_t = s_{t-1} = s^*$) human capital will also be in steady state ($h_t = h_{t+1} = h^*$). The time path of human capital will be similar to that of schooling- first convergent and then oscillating and convergent. Here growth rate of human capital is constant. The comparative static results which hold true for steady state schooling will hold true for steady state human capital as well. Therefore $\frac{dh^*}{d\varphi}, \frac{dh^*}{d\rho}$ and $\frac{dh^*}{dA}$ will have same signs as $\frac{ds^*}{d\varphi}, \frac{ds^*}{d\rho}$ and $\frac{ds^*}{dA}$. We have made the comparative static analysis for the case where $\underset{\smile}{h} = 0$ in the above model.

4.3 Dynamics of schooling when parents work in the skilled sector

Putting $s_t = s_{t-1} = s^*$ in the expression of s_t we get

$[b(\delta - A\varphi - \rho)]\, s^{*2} + b\,[\delta\underset{\smile}{h} - p_c\underline{c} - \rho]s^* - \underset{\smile}{h}\,(A\varphi + \rho) = 0$

We assume $\delta > A\varphi + \rho$ and $\delta\underset{\smile}{h} > p_c\underline{c} + \rho$ to get one positive value of s^*

The dynamics of s_t is shown in the following diagram:

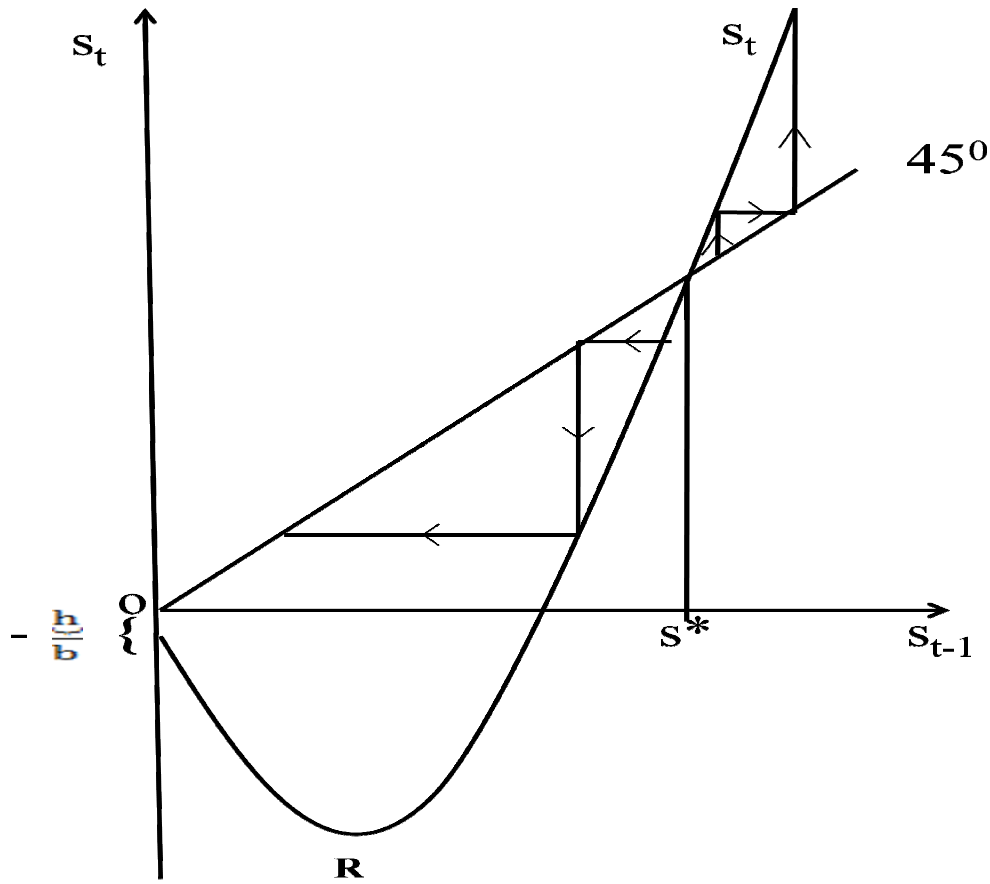

Figure 2: Dynamics of schooling for skilled sector when intercept term exists in human capital accumulation

In Figure 2 we illustrate the case where R is a fraction. So s_t is falling till R is reached and beyond R, s_t is rising. There is one steady state equilibrium given by s^*. If s_{t-1} is below s^*

schooling keeps on falling till becomes zero. Above s* schooling keeps on increasing till it converges to full schooling.

Proposition 8: In case of skilled parent, one steady state of schooling exists, which is unstable in nature. When parental level of schooling is below a critical level(s*), schooling keeps on falling till it becomes zero. Beyond that critical level schooling keeps on increasing till it converges to full schooling.

If it is assumed that parental schooling must be at least s* for being employed as skilled labour then for skilled parent child schooling always converges to unity.

4.4 Dynamics of human capital when parents work in the skilled sector

In this section we discuss dynamics of human capital for the skilled parent. Since human capital accumulation function is given by $h_t = bs_{t-1} + \underset{\smile}{h}$. $s_t = s_{t-1}$ implies that $h_t = h_{t+1}$ i.e. when schooling is at steady state ($s_t = s_{t-1} = s^*$) human capital will also be in steady state ($h_t = h_{t+1} = h^*$). The time path of human capital will be similar to that of schooling- divergent in nature. Here growth rate of human capital is constant.

5. Case when $\underset{\smile}{h} = 0$ in the above model-No intercept term in human capital accumulation function

In this case we assume that human capital formation of child depends only on schooling of the child and not on minimum level of human capital possessed by the child even if she does not attend school. Under this special case, human capital accumulation function is given as follows:

$h_{t+1} = bs_t$

5.1 Parents working in unskilled sector

From the optimization problem of unskilled parent we get[7]

$$s_t = \frac{s_{t-1}[A+(1-s_{t-1})\underline{h}+\varphi A-p_c\underline{c}]}{(A\varphi+\rho)(1+s_{t-1})}$$

Differentiating s_t with respect to s_{t-1} we get

$$\frac{ds_t}{ds_{t-1}} = \frac{A+(1-s_{t-1})\underline{h}+A\varphi-p_c\underline{c}-\underline{h}\, s_{t-1}(1+s_{t-1})}{(A\varphi+\rho)(1+s_{t-1})^2}$$

$$\frac{ds_t}{ds_{t-1}} > 0 \text{ if } [-\underline{h}s_{t-1}^2 - 2s_{t-1}\underline{h} + \{A\,(1+\varphi) + \underline{h} - p_c\underline{c}\} > 0$$

$$\text{or } s_{t-1} < \pm\sqrt{\frac{A(1+\varphi)}{\underline{h}} + 2 - \frac{p_c\underline{c}}{\underline{h}}} - 1 = M$$

[7] For detailed derivations please see Appendix (B.2) and (B.4) of Appendix B.

We ignore the negative term since s_{t-1} cannot be negative.

$$\frac{d^2 s_t}{ds_{t-1}^2} = \frac{-2[\underline{h}(1+s_{t-1})^2+\{A\,(1+\varphi)+(1-s_{t-1})\underline{h}-p_c\underline{c}-\underline{h}\,s_{t-1}(1+s_{t-1})\}]}{(A\varphi+\rho)(1+s_{t-1})^3}$$

$\frac{d^2 s_t}{ds_{t-1}^2}$ <0 if $\underline{h}(1+s_{t-1})^2 + A\,(1+\varphi) + (1-s_{t-1})\underline{h} - p_c\underline{c} - \underline{h}\,s_{t-1}(1+s_{t-1})$ >0

or $A(1+\varphi) + 2\underline{h} - p_c\underline{c} > 0$

$\frac{d^2 s_t}{ds_{t-1}^2}$ >0 if $A\,(1+\varphi) + 2\underline{h} - p_c\underline{c} < 0$

We assume $A(1+\varphi) + 2\underline{h} - p_c\underline{c} > 0$ otherwise $\sqrt{\frac{A(1+\varphi)}{\underline{h}} + 2 - \frac{p_c\underline{c}}{\underline{h}}}$ becomes an imaginary number.

If M≥1, s_{t-1} is always less than M. This implies that when M≥1 $\frac{ds_t}{ds_{t-1}}$ >0 always.

Now M≥1 implies $\frac{A(1+\varphi)-p_c\underline{c}}{\underline{h}} \geq 2$.

The above condition will hold true if $A(1+\varphi) - p_c\underline{c}$ >0, A (1+φ) is high and $\underline{h}$ is low.

Thus Proposition 3 holds true here as well.

If M<0, then this condition is never satisfied. Equilibrium does not exist.

Therefore if M is a fraction, then $\frac{ds_t}{ds_{t-1}}$ >0 till M is reached and thereafter $\frac{ds_t}{ds_{t-1}}$ <0.

Now M is a fraction when 0<M<1. This implies $-1 < \frac{A(1+\varphi)-p_c\underline{c}}{\underline{h}} < 2$.

If $A(1+\varphi) > p_c\underline{c}$ but A (1+φ) is low and $\underline{h}$ is high then this inequality is likely to be satisfied.

$\frac{d^2 s_t}{ds_{t-1}^2}$ <0 throughout.

When human capital formation of child depends only on time devoted for her own schooling then also if total earnings of the household exceed subsistence consumption expenditure but are low and learning by doing effect is high, then below a particular level of parental schooling there is positive relationship between parental schooling and schooling of the child. But beyond a certain level of parental schooling, schooling of the child decreases with increase in parental level of schooling. So even in this special case we observe that proposition 4 holds good.

5.2 Parents working in skilled sector

From the optimization problem of skilled parent we get

$s_t = \frac{s_{t-1}[\delta b s_{t-1} + \varphi A - p_c \underline{c}]}{(A\varphi+\rho)(1+s_{t-1})}$

Differentiating s_t with respect to s_{t-1} we get

$\frac{ds_t}{ds_{t-1}} = \frac{\delta b s_{t-1}^2 + 2\delta b s_{t-1} + A\varphi - p_c \underline{c}}{(A\varphi+\rho)(1+ s_{t-1})^2}$

$\frac{ds_t}{ds_{t-1}} > 0$ if $\delta b s_{t-1}^2 + 2\delta b s_{t-1} + A\varphi - p_c \underline{c} > 0$.

or $s_{t-1} > \pm\sqrt{\frac{p_c \underline{c}}{\delta b} - \frac{A\varphi}{\delta b} + 1} - 1 = K$

If $K \leq 0$ this condition is always satisfied. So for $K \leq 0$ $\frac{ds_t}{ds_{t-1}} > 0$ always.

If $K \geq 1$, this condition is never satisfied.

If $0<K<1$, then $\frac{ds_t}{ds_{t-1}} < 0$ till K is reached and beyond K $\frac{ds_t}{ds_{t-1}} > 0$.

5.3 Dynamics of schooling and human capital

Let us denote the steady state schooling in unskilled sector as

$s_u^* = \frac{A + \underline{h} - p_c \underline{c} - \rho}{A\varphi + \rho + \underline{h}}$

$s_u^* = 0$ if $A + \underline{h} \leq p_c \underline{c} + \rho$

$s_u^* = 1$ if $A - A\varphi - p_c \underline{c} - \underline{h} \geq 2\rho$

The steady state schooling in skilled sector is given as

$s_s^* = \frac{p_c \underline{c} + \rho}{\delta b - A\varphi - \rho}$

$s_s^* > 0$ if $\delta b - A\varphi - \rho > 0$

The dynamics of s_t in both unskilled and skilled sectors are shown in the following diagram

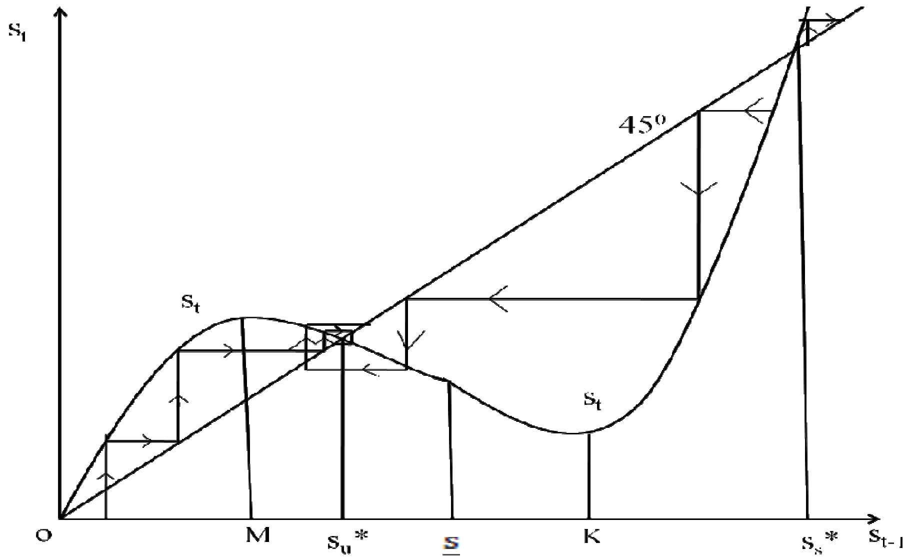

Figure 3: Dynamics of schooling for both skilled and unskilled sectors when there is no intercept term in human capital accumulation

In Figure 3 we demonstrate that case of unskilled parent where M is a fraction. In case of unskilled parent, till M is reached, time path of s_t is convergent but once M is crossed, the time path becomes convergent and oscillating. The low level equilibrium trap vanishes and there is only one steady state level of schooling in the case of unskilled parent. s_u^* is a stable equilibrium.

Proposition 9: In case of unskilled parent (parental skill below $\underline{s}$), when minimum level of human capital possessed by the child if she does not attend school is zero or the intercept term in human capital accumulation function is absent, the time path of schooling of the child is steadily convergent in nature when approached from below steady state level of parental schooling s_u* and beyond that s_u*, the time path of schooling is convergent but oscillating in nature. The low level equilibrium trap does not exist anymore.

So, for unskilled parent we notice that the intercept term in human capital accumulation function is responsible for creating a trap in schooling.

s_t for skilled parent - s_{t-1} for skilled parent > 0 if $s_{t-1} > \frac{p_c\underline{c}+\rho}{\delta b - A\varphi - \rho} = s_s^*$. This implies that once s_t crosses s_s^*, s_t will be greater than s_{t-1} i.e. s_t curve will lie above the 45^0 line in case of skilled sector.

The incentive compatibility condition implies $w_{t+1} > A + (1 - s_t)\underline{h}$ i.e. $s_t > \underline{s} = \frac{A + \underline{h}}{\delta b + \underline{h}}$. Only when schooling exceeds this particular level individuals join the skilled sector.

Now s_t for skilled parent at $\underline{s}$ = s_t in case of unskilled parent at $\underline{s}$.This implies that the s_t curve will be continuous at $s_t = \underline{s}$.

If $\frac{p_c\underline{c}+\rho}{\delta b-A\varphi-\rho} > \frac{A+\underline{h}}{\delta b+\underline{h}}$ then $s_s^* > \underline{s} > s_u^*$[8]

In our model we assume $\frac{p_c\underline{c}+\rho}{\delta b-A\varphi-\rho} > \frac{A+\underline{h}}{\delta b+\underline{h}}$. Therefore $s_s^* > \underline{s} > s_u^*$

In skilled sector, s_t (at $s_{t-1} = \underline{s}$) - $\underline{s} = \frac{\underline{s}[\underline{s}\,(\delta b-A\varphi-\rho)-(p_c\underline{c}+\rho)]}{(A\varphi+\rho)(1+\underline{s})}$

Since we assume in our paper $\frac{p_c\underline{c}+\rho}{\delta b-A\varphi-\rho} > \frac{A+\underline{h}}{\delta b+\underline{h}}$, therefore [$s_t$ in skilled sector (at $s_{t-1} = \underline{s}$)-$\underline{s}$] <0. This implies that s_t at $\underline{s}$ lies below the 45^0 line in skilled sector.

When the parental level of schooling lies between s_u^* and $\underline{s}$, schooling of child keeps on falling till it converges to the unskilled level steady state schooling s_u* in an oscillating manner.

In Figure 3 we consider that case of skilled parent where K is fraction and $\underline{s}$ < K. In this case we consider the situation when even skilled parent may send their child for partial schooling. Here, for skilled sector s_t curve with respect to s_{t-1} is falling till K is reached and beyond K, s_t curve in skilled sector is rising. Note that s_s^* denotes an unstable equilibrium for the parents working in skilled sector. Below s_s^*, s_t in skilled sector keeps on falling and eventually converges to steady state equilibrium in unskilled sector in an oscillating manner. Beyond s_s^* schooling of child keeps on increasing and will eventually converge to s_t=1.Hence the dynasties having parental skill level between $\underline{s}$ and s_s *may end up in the situation where next generations will be working as unskilled labour. Lower is s_s^*, lower is the parental level of human capital required to launch the economy on the path of steady growth of schooling. Lower s_s^* is thus good for the economy. Increase in education cost (ρ), child wage (Aφ) and subsistence consumption expenditure ($p_c\underline{c}$) thus leads to higher s_s^* which is not good for the economy. Increase in responsiveness of wage to human capital (δ) and improvement in education technology (rise in b) lead to lower s_s^* which is good for the economy.

If $\underline{s}$ > K then s_t in skilled sector will be rising throughout.

Proposition 10: In case of skilled parent there exists a critical level of parental schooling beyond which steady growth of schooling of child takes place. However there exists a certain range of parental human capital for which schooling of child keeps on falling till it converges to the unskilled level steady state schooling in an oscillating manner.

Now in skilled sector $s_t = \frac{s_{t-1}[\delta b s_{t-1}+\varphi A-p_c\underline{c}]}{(A\varphi+\rho)(1+s_{t-1})}$

s_t >0 if $\delta b s_{t-1}^2 + s_{t-1}(\varphi A - p_c\underline{c}) > 0$. Thus $(\varphi A - p_c\underline{c}) > 0$ is sufficient condition for s_t >0.

$s_t = 1$ if $\delta b s_{t-1}^2 - (p_c\underline{c}+\rho)\, s_{t-1} - (A\varphi+\rho) \geq 0$

This implies that there exists one positive value of s_{t-1} say $\hat{s}$ for which $s_t = 1$.

If $\underline{s} > \hat{s}$ then all parents who are employed in skilled sector send their children for full schooling.

[8] For detailed derivation please see Appendix B

The dynamics of schooling for this case is shown in the following diagram:

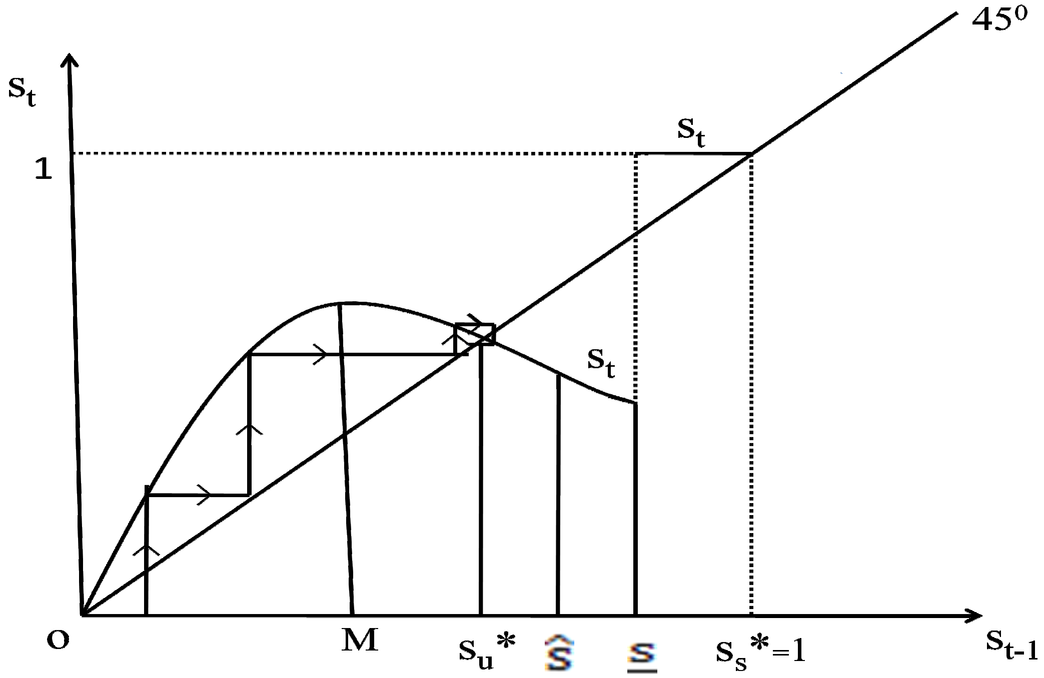

Figure 4: Dynamics of schooling for skilled sector in absence of intercept term in human capital accumulation and when $\underline{s} > \hat{s}$

In case of unskilled parent, till M is reached, time path of s_t is convergent but once M is crossed, the time path becomes convergent and oscillating. There is only one steady state level of schooling in the case of unskilled parent s_u^* which is a stable equilibrium. Parents who work in skilled sector always send their children for full schooling. Steady state schooling in skilled sector i.e. $s_s^* = 1$.

So in this paper we are getting low level equilibrium trap for unskilled parent when level of human capital acquired by their children is positive even if they do not attend school at all or in other words if there exists an intercept term in human capital accumulation function. On the other hand, if there does not exist any intercept term in human capital accumulation function, for skilled parent if $\underline{s} < \hat{s}$ this implies that even if being skilled parents they send their children for partial schooling then in this case there exists a critical level of parental schooling beyond which steady growth of schooling of child takes place and eventually it converges towards full schooling. However below that critical level, schooling of child keeps on falling till it converges to the unskilled level steady state schooling in an oscillating manner. In the long run after few dynasties their grandchildren may end up working as unskilled labour.

Note that in the case where parents work in unskilled sector the following hold true[9]:

[9] For detailed derivation please see equations (B.6), (B.7) and (B.8) of Appendix B.

i) $\frac{ds_u^*}{d\varphi} < 0$ if $A+\underline{h}>\rho + p_c\underline{c}$. This implies that if the maximum possible adult unskilled wage exceeds the sum of the schooling cost and subsistence expenditure of the household, steady state schooling of child of unskilled parent increases with fall in child wage. Thus due to child labour ban, when child wage falls, steady state schooling increases when parents work in unskilled sector.

iii) $\frac{ds_u^*}{d\rho} < 0$. This implies that with fall in schooling cost steady state schooling in unskilled sector increases. So an education subsidy is going to increase steady state schooling of the child.

iv) $\frac{ds_u^*}{dA} > 0$ .This implies that with increase in unskilled adult wage, steady state schooling will increase.

Proposition 11: Steady state schooling of child of unskilled parent increases with increase in adult unskilled wage but decreases with increase in education cost. It increases with fall in child wage only if the maximum possible adult unskilled wage exceeds the sum of the schooling cost and subsistence expenditure of the household.

When schooling will be in steady state, human capital will also be in steady state and dynamics of human capital is same as dynamics of schooling in this case too.

5.4 Comparison between the effects of ban and subsidy

In this section we compare the effects of ban and education subsidy on steady state schooling in the case where parents work in the unskilled sector.

$$\left|\frac{ds^*}{d\rho}\right| - \left|\frac{ds^*}{d\varphi}\right|$$

$$= \frac{A(\varphi+\rho)+(1-A)\left(A-p_c\underline{c}\right)+\underline{h}\,(2-A)}{\left(A\varphi+\rho+\underline{h}\right)^2}$$

If $1<A<p_c\underline{c}<2$, then the above expression is positive.

Again if $p_c\underline{c}<A<1$ then also the above expression is positive.

This implies that if adult unskilled wage is less than subsistence consumption expenditure or even if adult unskilled wage exceeds subsistence expenditure but is less than one, the effect of giving education subsidy is higher than child labour ban in enhancing schooling.

Proposition 12: If adult unskilled wage is less than subsistence consumption expenditure or even if adult unskilled wage exceeds subsistence expenditure but is sufficiently small, the effect of giving education subsidy is higher than child labour ban in enhancing schooling.

6. Conclusion

Child labour continues to remain a social evil in developing countries. The present paper addresses the issue of child labour in the presence of learning by doing effect on unskilled wage.

This paper builds an overlapping generations model of household economy consisting of a skilled sector and an unskilled sector. If one individual is employed in skilled sector she gets wage proportional to human capital whereas unskilled sector wage includes a positive learning by doing effect generated from working in her childhood along with a fixed return. Human capital formation of the child is included in the parental utility function and parental choice of schooling vis-a-vis child work is considered. We consider two cases-firstly we consider the case where human capital accumulation of the child depends on time devoted to schooling by the child and minimum level of human capital possessed by the child even if she does not attend school and next we consider the case where human capital accumulation of the child depends only on minimum level of human capital possessed by the child even if she does not attend school.

When human capital accumulation of the child depends on time devoted to schooling by the child and minimum level of human capital possessed by the child even if she does not attend school we find that there exists two steady state equilibria of schooling for unskilled parent. The low level equilibrium represents a trap. Once the trap is crossed schooling keeps on increasing and the time path of schooling is convergent towards the higher of the equilibrium. Once higher equilibrium is crossed time path of schooling becomes convergent but oscillating in nature. When human capital of child depends only on schooling of child and there is no intercept term in human capital accumulation function, the trap ceases to exist.

When human capital accumulation of the child depends on time devoted to schooling by the child and minimum level of human capital possessed by the child even if she does not attend school we find that one steady state equilibrium exists which is unstable in nature. Below the steady state schooling keeps on falling till it becomes zero. Beyond the steady state, schooling keeps on increasing till it converges to full schooling. When human capital of child depends only on schooling of child and there is no intercept term in human capital accumulation function, if schooling required to be engaged as skilled worker is higher than the critical level of parental schooling beyond which they send their children for full schooling then there exists only one steady state equilibrium in case of skilled parent and that is full schooling. But if skilled parents also send their children for partial schooling then there exists a critical level of parental schooling beyond which steady growth of schooling of child takes place and eventually it converges towards full schooling. However below that critical level, schooling of child keeps on falling till it converges to the unskilled level steady state schooling in an oscillating manner.

This paper attempts to understand the effects of child labour ban and education subsidy on steady state schooling of child labour in such a situation. Moreover this paper studies the relative effectiveness of child labour ban and education subsidy in improving schooling of the child in such a situation. When human capital formation of child depends only on schooling of the child, we find that in case of unskilled parent, as child wage falls, steady state schooling and steady state human capital increase only if the unskilled adult wage exceeds the sum of subsistence consumption expenditure and schooling cost of the household. A fall in schooling cost increases the steady state schooling and steady state human capital for unskilled parent. Comparing the effects of ban and education subsidy on steady state human capital in case of unskilled parent, we get the result that if unskilled adult is sufficiently small, the effect of giving education subsidy is more effective in enhancing schooling than banning child labour.

References

Acemoglu, D. and J.S. Pischke (2000) "Changes in the wage structure, family income, and children's education" *NBER Working Paper Number* **7986.**

Arrow, K.J. (1962) "The Economic Implications of Learning by Doing" *The Review of Economic Studies*, **29(3)**, 155-173.

Baland, J. and J.Robinson (2000) "Is Child Labour Inefficient?" *Journal of Political Economy*, **108**, 663-679.

Basu, K. (1999) "Child Labor: Causes, Consequence and Cure with Remarks on International Labor Standards" *Journal of Economic Literature*, **37(3)**, 1083-1119.

Basu, K. and P. Van (1998) "The Economics of Child Labour" *The American Economic Review,* **88,** 412-27.

Becker, G.S. and N. Tomes (1979) "An Equilibrium Theory of the Distribution of Income and Intergenerational Mobility" *Journal of Political Economy*, **87 (6)**.

Chaudhuri, S.(2004) "Incidence of Child Labour, Free Education policy, and Economic Liberalisation in a Developing Economy" *The Pakistan Development Review*, **43(1)**, 1-25.

Chaudhuri, S. And U. Mukhopadhyay (2003) "Free Education Policy and Trade Liberalization: Consequences on Child and Adult Labour Markets in a Small Open Economy" *Journal of Economic Integration*, **18(2)**, 336-359.

Croix, D. (2001) "Growth dynamics and education spending: The role of inherited tastes and abilities" *European Economic Review*, **45**, 1415-1438.

Croix, D. And O. Licandro (1999) "Life expectancy and endogenous growth" *Economic Letters*, **65**, 255-263.

Dessy, S. and S.Pallage (2001) "Child labor and Coordination failures" *Journal of Development Economics*, **65(2)**, 469-476.

Dessy, S. and S.Pallage (2005) "A Theory of the Worst Forms of Child Labour" *The Economic Journal*, **115(500)**, 68-87.

Dinopoulos, E. And L. Zhao (2007) "Child Labor and Globalization" *Journal of Labor Economics*, **25(3)**, 553-579.

Emerson, P.M. and A.P. Souza (2003) "Is there a Child Labor Trap? Intergenerational Persistence of Child Labor in Brazil" *Economic Development and Cultural Change.*

Emerson, P.M. and S.D. Knabb (2006) "Opportunity, Inequality and the Intergenerational Transmission of Child Labour" *Economica, New Series,* **73(291)**, 413-434.

Emerson, P.M. and S.D. Knabb (2007) "Fiscal Policy, Expectation Traps and Child Labor" *Economic Inquiry* **45 (3)** 453-469.

Estevez, K. (2011) "Nutritional Efficiency Wages and Child Labour" *Economic Modelling*, **28**, 1793-1801.

Fabre, A. and S. Pallage (2011) "Child Labor, Idiosyncratic Shocks, and Social Policy" *CIRPEE Working Paper Number* **11-15**.

Glomm, G.(1997) "Parental choice of human capital investment" *Journal of Development Economics*, **53,** 99-114.

Glomm G. and B. Ravikumar (1998) "Increasing returns, human capital, and the Kuznets curve" *Journal of Development Economics*, **55**, 353-367.

Gupta, M.R. (2001) "Child labour, skill formation and capital accumulation: a theoretical analysis" *Keio Economic Studies*, **38(2)**, 23-40.

Hare, P.G. and D.T. Ulph (1979) "On Education and Distribution" *Journal of Political Economy*, **87(5)**.

Hippel, E.V. and M. Tyre (1993) "How 〝Learning by Doing〞 is Done: Problem Identification in Novel Process Equipment" SSM WP# BPS 3521-3593.

Krueger, D. and J.T. Donohue (2005) "On the distributional consequences of child labor legislation" *International Economic Review*, **46(3)**.

Mao, Z.Y. (2012) "Learning-by Doing and its implications for Economic Growth and International Trade".

Moav, O. (2005) "Cheap children and the Persistence of Poverty" *Economic Journal, Royal Economic Society*, **115(500)**, 88-110.

Mukherjee, D. And U. Sinha (2006) "Schooling, Job prospect and Child Labour in a Developing Economy" *mimeo: Indian Statistical Institute, Kolkata*.

Parente, S.L. (1994) "Technology Adoption, Learning-by-Doing and Economic Growth" *Journal of Economic Theory*, **63**, 346-369.

Ranjan, P. (1999) "An economic analysis of child labour" *Economic Letters*, **64**, 99-105.

Ranjan, P. (2001) "Credit constraints and the phenomenon of child labor" *Journal of Development Economics*, **64**, 81-102.

Ravallion, M. and Q.T. Wodon (2000) "Does Child Labour Displace Schooling? Evidence on Behavioural responses to an Enrollment Subsidy" *The Economic Journal*, **110(462)**.

Rogers, C.A. and K.A. Swinnerton (2002) "A Theory of Exploitative Child Labor"

Sarkar, J. And D. Sarkar (2012) "Why does child labour persist with declining poverty?" *NCER Working Paper Number* **84**.

Sasmal, J. and J. Guillen (2015) "Poverty, educational Failure and the Child Labour Trap: The Indian Experience" *Global Business Review*, **16**, 270-280.

Schultz, T, P. (2004) "School subsidies for the poor: evaluating the Mexican Progresa poverty program" *Journal of Development Economics*, **74**, 199-250.

Zhang, W.B. (2014) "Endogenous population with human and physical capital accumulation" *International Review of Economics, Springer*, **61(3)**, 231-252.

Zhang, W.B. (2015) "Economic Oscillations with Endogenous Population, Human Capital and Wealth".

Appendix A- General Model

In case of unskilled parent, the optimization problem is

Max $U_t = \ln (c_t - \underline{c}) + s_{t-1} \ln (bs_t + \underset{\smile}{h})$

subject to $[A + (1 - s_{t-1})\underline{h}] + A\varphi (1 - s_t) = p_c c_t + \rho s_t$,

$c_t \geq \underline{c}$

and $0 \leq s_t, s_{t-1} \leq 1$

with respect to the decision variables of the household, viz, c_t and s_t.

In case of unskilled parent, the Lagrangian function is

$Z = \ln(c_t - \underline{c}) + s_{t-1} \ln (bs_t + \underset{\smile}{h}) + \lambda \; [\{A + (1 - s_{t-1})\underline{h}\} + A\varphi (1 - s_t) - p_c c_t - \rho s_t] + \theta(c_t - \underline{c})$

where λ is the Lagrange multiplier. The first order conditions for maximization of utility are given by:

$$\frac{\delta Z}{\delta c_t} = \frac{1}{c_t - \underline{c}} - \lambda p_c + \theta = 0 \tag{A.1}$$

$$\frac{\delta Z}{\delta s_t} = \frac{b s_{t-1}}{b s_t + \underset{\smile}{h}} - \lambda\,(A\varphi + \rho) = 0 \tag{A.2}$$

$$\theta \geq 0,\ \theta\,(c_t - \underline{c}) = 0 \tag{A.3}$$

From (A.1) and budget constraint $[A + (1 - s_{t-1})\underline{h}\,] + A\varphi\,(1 - s_t) = p_c c_t + \rho\, s_t$, we get

$$\frac{1}{A + (1 - s_{t-1})\underline{h} + A\varphi\,(1 - s_t) - p_c\underline{c} - \rho\, s_t} = \lambda \tag{A.4}$$

From (A.2) and (A.4) we get,

$$s_t = \frac{b s_{t-1}\left[A + (1 - s_{t-1})\underline{h} + \varphi A - p_c\underline{c}\right] - \underset{\smile}{h}\,(A\varphi + \rho)}{b(A\varphi + \rho)(1 + s_{t-1})} \tag{A.5}$$

In case of skilled parent, the optimization problem is

Max $U_t = \ln(c_t - \underline{c}) + s_{t-1} \ln(b s_t + \underset{\smile}{h}\,)$

subject to $[(\delta b s_{t-1} + \delta \underset{\smile}{h}\,] + A\varphi\,(1 - s_t) = p_c\, c_t + \rho\, s_t$,

$c_t \geq \underline{c}$

and $0 \leq s_t,\ s_{t-1} \leq 1$

with respect to the decision variables of the household, viz, c_t and s_t.

In case of skilled parent, the Lagrangian function is

$$Z = \ln(c_t - \underline{c}) + s_{t-1} \ln(b s_t + \underset{\smile}{h}) + \lambda\,[\delta\, b s_{t-1} + \delta \underset{\smile}{h} + A\varphi\,(1 - s_t) - p_c c_t - \rho\, s_t\,] + \theta(c_t - \underline{c})$$

where λ is the Lagrange multiplier. The first order conditions for maximization of utility are given by:

$$\frac{\delta Z}{\delta c_t} = \frac{1}{c_t - \underline{c}} - \lambda p_c + \theta = 0 \tag{A.6}$$

$$\frac{\delta Z}{\delta s_t} = \frac{b s_{t-1}}{b s_t + \underset{\smile}{h}} - \lambda\,(A\varphi + \rho) = 0 \tag{A.7}$$

$$\theta \geq 0,\ \theta\,(c_t - \underline{c}) = 0 \tag{A.8}$$

From (A.6) and budget constraint $\delta\, b s_{t-1} + \delta \underset{\smile}{h} + A\varphi\,(1 - s_t) + A\varphi\,(1 - s_t) = p_c c_t + \rho\, s_t$, we get

$$\frac{1}{\delta\, b s_{t-1} + \delta \underset{\smile}{h} + A\varphi\,(1 - s_t) - p_c\underline{c} - \rho\, s_t} = \lambda \tag{A.9}$$

From (A.7) and (A.9) we get,

$$s_t = \frac{b s_{t-1}\left[\delta\, b s_{t-1} + \delta \underbrace{h} + \varphi A - p_c \underline{c}\right] - \underbrace{h}\,(A\varphi+\rho)}{b(A\varphi+\rho)(1+s_{t-1})} \qquad \text{(A.10)}$$

Appendix B: Case where $\underbrace{h}$ =0

In case of unskilled parent the Lagrangian function is

$Z= \ln(c_t - \underline{c}) + s_{t-1} \ln(b s_t) + \lambda\ [\{A + (1 - s_{t-1})\underline{h}\} + A\varphi\,(1-s_t) - p_c c_t - \rho\, s_t] + \theta(c_t - \underline{c})$

where λ is the Lagrange multiplier. The decision variables of the household are c_t and s_t. The first order conditions for maximization of utility are given by:

$$\frac{\delta Z}{\delta c_t} = \frac{1}{c_t - \underline{c}} - \lambda p_c + \theta = 0 \qquad \text{(B.1)}$$

$$\frac{\delta Z}{\delta s_t} = \frac{s_{t-1}}{s_t} - \lambda\,(A\varphi+\rho) = 0 \qquad \text{(B.2)}$$

$$\theta \geq 0,\ \theta\,(c_t - \underline{c}) = 0 \qquad \text{(B.3)}$$

From (B.1) and budget constraint $[A + (1 - s_{t-1})\underline{h}] + A\varphi\,(1-s_t) = p_c c_t + \rho\, s_t$, we get

$$\frac{1}{A + (1 - s_{t-1})\underline{h} + A\varphi\,(1 - s_t) - p_c \underline{c} - \rho s_t} = \lambda \qquad \text{(B.4)}$$

From (B.2) and (B.4) we get,

$$s_t = \frac{s_{t-1}\left[A + (1 - s_{t-1})\underline{h} + \varphi A - p_c \underline{c}\right]}{(A\varphi+\rho)(1+s_{t-1})} \qquad \text{(B.5)}$$

$$\frac{ds_u^*}{d\varphi} = \frac{-A[A + \underline{h} - p_c \underline{c} - \rho]}{\left(A\varphi + \rho + \underline{h}\right)^2} \qquad \text{(B.6)}$$

$$\frac{ds_u^*}{d\rho} = \frac{-[A(1+\varphi) + 2\underline{h} - p_c \underline{c}]}{\left(A\varphi + \rho + \underline{h}\right)^2} \qquad \text{(B.7)}$$

$$\frac{ds_u^*}{dA} = \frac{\rho + p_c \underline{c}\varphi + \rho\varphi + \underline{h}(1-\varphi)}{\left(A\varphi + \rho + \underline{h}\right)^2} \qquad \text{(B.8)}$$

In case of skilled parent the Lagrangian function is $Z= \ln(c_t - \underline{c}) + s_{t-1} \ln(b s_t) + \lambda\ [\delta b s_{t-1} + A\varphi\,(1-s_t) - p_c c_t - \rho\, s_t] + \theta(c_t - \underline{c})$

where λ is the Lagrange multiplier. The decision variables of the household are c_t and s_t. The first order conditions for maximization of utility are given by:

$$\frac{\delta Z}{\delta c_t} = \frac{1}{c_t - \underline{c}} - \lambda p_c + \theta = 0 \qquad \text{(B.9)}$$

$$\frac{\delta Z}{\delta s_t} = \frac{s_{t-1}}{s_t} - \lambda\,(A\varphi+\rho) = 0 \qquad \text{(B.10)}$$

$\theta \geq 0, \theta\,(c_t - \underline{c}) = 0$ (B.11)

From (B.9) and budget constraint $\delta b s_{t-1} + A\varphi\,(1-s_t) = p_c c_t + \rho\, s_t$,we get

$$\frac{1}{\delta\, bs_{t-1} + A\varphi\,(1-s_t) - p_c\underline{c} - \rho\, s_t} = \lambda \quad \text{(B.12)}$$

From (B.10) and (B.12) we get,

$$s_t = \frac{s_{t-1}[\delta bs_{t-1} + \varphi A - p_c\underline{c}]}{(A\varphi+\rho)(1+s_{t-1})} \quad \text{(B.13)}$$

Relation between s_u^*, $\underline{s}$ and s_s^*

$s_u^* = \frac{A + \underline{h} - p_c\underline{c} - \rho}{A\varphi + \rho + \underline{h}}$

$\underline{s} = \frac{A + \underline{h}}{\delta b + \underline{h}}$

$s_s^* = \frac{p_c\underline{c} + \rho}{\delta b - A\varphi - \rho}$

$s_s^* > \underline{s}$ if $\frac{p_c\underline{c} + \rho}{\delta b - A\varphi - \rho} > \frac{A + \underline{h}}{\delta b + \underline{h}}$

$s_u^* < \underline{s}$ if $\frac{(A + \underline{h})(\delta b - A\varphi - \rho) - (\delta b + \underline{h})(p_c\underline{c} + \rho)}{(\delta b + \underline{h})(A\varphi + \rho + \underline{h})} < 0$

Now if $\frac{p_c\underline{c} + \rho}{\delta b - A\varphi - \rho} > \frac{A + \underline{h}}{\delta b + \underline{h}}$, then $s_u^* < \underline{s}$

Therefore if $\frac{p_c\underline{c} + \rho}{\delta b - A\varphi - \rho} > \frac{A + \underline{h}}{\delta b + \underline{h}}$ then $s_s^* > \underline{s} > s_u^*$