Optimal Monetary Policy in Behavioral New Keynesian Model

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Abstract

This paper conducts the first assessment of the optimal monetary policy in the case of behavioral New Keynesian model proposed by Gabaix (2016). Consistent with the previous studies, I find that monetary policy under commitment continues to be important for optimal policy, but the optimal policy is found to be more history-dependent than in the traditional New Keynesian model. Importantly, I find that monetary policy under discretion may be optimal under some constraints on the parameters of the model which seems to correspond better to the reality of the conduct of monetary policy in central banks of developing, emerging and transitional economies. This finding is considered as filling the

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gap that always has been between the practice and the theory of the optimal monetary policy.

JEL Codes: D84, E52.

1 Introduction

While the literature on optimal monetary policy has been explored widely in the case of the traditional New Keynesian model under different complications[1] , there is little research in this area in the case of New Keynesian models with bounded rationality. In fact, the behavioral New Keynesian model as proposed by Gabaix (2016) departs from the traditional model by relaxing the assumption of fully rational agents. Within the new framework, agents are assumed to be partially myopic and do not anticipate the future perfectly. The features incorporated in the behavioral New Keynesian model resolve lot of unsettled questions that has been left unresolved by the traditional New Keynesian model. First, the zero lower bound when it is hit implies large depressions in the traditional model. In contrast, depressions are contained and moderated when bounded rationality is assumed. Second, when the economy is in the zero lower bound forever the Taylor rule

[1] e.g. Clarida et al. (1999), Gali (2008, 2015), Woodford (2010).

is violated and the equilibrium is indeterminate (from one period to another economy jump randomly to different equilibrium). Such theoretical result has not been observed in the aftermath of the crisis of 2008. In this regard, the new model contributes to avoid this strong result of multiple equilibria. Third, Cochrane (2016) has pointed out that in the traditional New Keynesian model a rise of interest rate may lead to a rise in inflation (an equilibrium among others, due to the multiple equilibria problem). However, the behavioral model helps to overturn this striking result[2].

Moreover, the rationale for adopting this behavioral New Keynesian model to study the optimal monetary policy lies in the critiques that have been addressed to the traditional model, its underlying assumptions and its policy implications. As has been pointed out by Stiglitz (2010), one important underlying assumption of the traditional model is the rational behavior of the economy, but the economy seems inconsistent with any model of rationality. Such criticism has been carried out by the behavioral New Keynesian model which is assuming partially myopic agents which is leading to relax the assumption of rationality. In addition, Stiglitz criticized even more the policy prescriptions that arise from the traditional model. Such criticism will be

[2]For more problems that has been resolved by this behavioral approach see Gabaix (2016).

carried out through this paper.

Looking at the number of questions that has been addressed with the behavioral New Keynesian model and the results that have been found, I expect to be interesting to study the optimality of monetary policy within this new framework, in order to come out with some results that reconcile between the theory of optimal monetary policy and the practice of the central banks. Indeed, the literature on the optimal monetary policy largely gives the credit to commitment[3] in setting an optimal policy while the discretion is seen as yielding to undesirable rule for the conduct of monetary policy[4] . However, the practice in most central banks in developing, emerging and transitional economies is different from what the theory suggests (except in some exceptional circumstances like the forward guidance). In practice, in every period (month or trimester) the committee of monetary policy decides on interest rate without committing to future plan due to the structural changes that face those economies. Such a policy can be qualified as discretion, not in the literal sense, but to the extent that the central bank reoptimizes every period without preannouncing any trajectory for future policies. Taking into account these observations, my aim in this paper is to fill this gap between

[3]See Orphanides (2007)

[4]see Clarida et al. (1999)

the practice and the theory of optimal monetary policy.

The present paper proposes to study the optimality of monetary policy within Behavioral New Keynesian model with bounded rational agents proposed in Gabaix (2016). Within a wide range of central banking literature, two methods have been used. The first one is the linear quadratic problem used by Clarida et al. (1999)and Gali (2008, 2015). In this approach the central bank seeks to choose a path for inflation and output gap that minimize a quadratic loss function. The second approach is that of welfare-based utility maximization by Rotemberg and Woodford (1999). However, connection can be established between utility maximization and linear-quadratic policy problems of the sort[5] . One possible limitation of this approach is that, while the widely used representative agent approach may be a micro founded method to study the behavior of the economy, it could be highly misleading methodology for studying welfare. As illustrated in Clarida et al. (1999), if some groups suffer more in recessions than others and there are incomplete insurance and credit markets, then the utility of a hypothetical representative agent might not provide an accurate barometer of cyclical fluctuations in welfare. That's why literature takes a pragmatic approach to this issue

[5]I refer here to the Chapter of Woodford (2010) for a formal presentation of the link between the two methods.

by simply assuming that the objective of monetary policy is to minimize the squared deviations of output and inflation from their respective target levels.

In this paper, I work with the first approach of linear quadratic deviations to study the optimal monetary policy within the behavioral New Keynesian model of Gabaix (2016). The remainder of the paper is the following. The second section will be dedicated to a brief presentation of the behavioral New Keynesian model and a comparison with the basic one. Section three will study the optimal monetary policy under discretion and the forth section will take up the same question under commitment. A final section will discuss the main results.

2 Behavioral New Keynesian Model: Remainder and Comparison

The model that will be used is based on Gabaix (2016a and 2016b). This framework is based on the psychological foundation of "Bounded Rationality", so the assumption of rational expectations hypothesis is relaxed. In this approach, agents build a simplified model of the world. The representation crated will be sparse, in the sense that agents will pay attention to just a few parameters in order to make an economic decision. This will lead to a modified New Keynesian model.

The behavioral New Keynesian model is designed to overcome the unrealism of the infinitely forward-looking agent who computes the whole equilibrium in her own head. Moreover, it can be viewed as an alternative to the overstatement of expectations in the traditional New Keynesian models as raised by Blachard (2008):

> "*That anticipations matter a lot is obviously true. That people and firms look into the future, directly or by relying on the forecasts of others, in forming anticipations is also obviously true. Whether the basic model does not overstate the role of anticipations is however open to question.*"

General Framework: Sparse Max Operator

Gabaix (2014) can be viewed as the starting point of his work on bounded rationality. The sparse max operator is a generalization of the traditional max operator under constraint.

Under the rational version, agent faces a maximization problem, $max_a v(a, x)$, where a is an action and x is a state variable. There is an attention vector,

m, and an attention-dependent function $v(a, x, m)$. It can be represented as:

$$v(a, x, m) \equiv v(a, m_1 x_1, ..., m_n x_n)$$

This function can be perceived as utility function when the consumer is partially inattentive to the vector x. When$m_i = 1$, it means that the agent is fully attentive to the variable x_i when making her decision. In contrast, when $m_i = 0$ the agent is completely inattentive to x_i which means that agent think this variable is not relevant for her decision.

Solving the problem of maximizing the attention-dependent function will lead to a solution of the form:

$$a(x, m) := argmax_a v(a, x, m)$$

The sparse max operator will be the same as the traditional max operator, but including some vector of inattention[6] which will lead to a solution that contains the agents' inattention parameters.

Behavioral New Keynesian Model

Following Gabaix (2016b), the behavioral IS relation is the following:

$$\tilde{y}_t = M E_t \tilde{y}_{t+1} - \sigma(i_t - E_t \pi_{t+1} - r_t^n) \tag{1}$$

[6]More developments and cases can be found in Gabaix (2014, 2016a).

With $M \in [0,1]$ representing the inattention of the household. One can notice that the weight given to expectation is becoming inferior of that in the traditional NK model. Moreover, in this framework the traditional NK model can be obtained as a particular case.

For the behavioral New Keynesian Phillips Curve, it is represented by the equation:

$$\pi_t = \beta M^f E_t \pi_{t+1} + \kappa \tilde{y}_t \tag{2}$$

When $M^f = 1$ we recover the traditional model. The behavioral model changes simply β to βM^f in order to account for some myopia about the future evolutions.

Notation and Definition

y_t the (log) output, y_t^e the efficient output and y_t^n is the natural output. $x_t \equiv y_t - y_t^e$ is the welfare relevant output gap. The relationship that links those variables: $\tilde{y}_t \equiv x_t + (y_t^e - y_t^n)$.

By using this relation, one can easily transform the IS and Phillips curve equations in term of welfare-relevant output gap as:

$$x_t = M E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1} - r_t^e) \tag{3}$$

With $r_t^e = r_t^n + \frac{1-M\rho_u}{\sigma}$ is the efficient interest rate perceived by households.

$$\pi_t = \beta M^f E_t \pi_{t+1} + \kappa x_t + u_t \tag{4}$$

With $u_t \equiv \kappa(y_t^e - y_t^n)$ is a cost-push shock, that evolves as an $AR(1)$: $u_t = \rho_u u_{t-1} + \epsilon_t$.

3 Optimal Monetary Policy Under Discretion

The central bank makes whatever decision is optimal each period without committing itself to any future actions. That case is often referred in the literature as optimal policy under discretion. One can observe that this scenario accords best with reality. In practice, no central bank makes any kind of commitment about its future monetary policy (except in some exceptional circumstances like the forward guidance operated after the crisis). In this respect, it seems important to study the optimal policy in this context.

Under discretion, the central bank solves the problem in sequential way. Each period the central bank seeks to minimize:

$$\pi_t^2 + \alpha_x x_t^2$$

Subject to:

$$\pi_t = \beta M^f E_t \pi_{t+1} + \kappa x_t + u_t$$

coefficient α_x represents the weight of output gap fluctuations (relative to inflation) in the loss function, and is given by $\alpha_x = \frac{\kappa}{\epsilon}$ where κ is the coefficient on x_t in the New Keynesian Phillips curve, and ϵ is the elasticity of substitution between goods. One can interpretα_x as the weight attached by the central bank to deviations of output from its efficient level in its own loss function, which does not necessarily have to coincide with the household's.

The optimality condition for the problem above is given by:

$$x_t = -\frac{\kappa}{\alpha_x}\pi_t \tag{5}$$

The relation (5) must be satisfied forever in order to minimize the loss function; it can be called the "targeting rule" for the central bank. In the face of inflationary pressures (due to cost-push shock), the central bank must act by driving output below its efficient level. Such a policy is widely called by "leaning against the wind" policy, which must be the response of monetary authority until the condition is satisfied. Though this principle may seem obvious, it provides very simple criteria for evaluating monetary policy.

Using the condition (5) to replace for x_t in (4), I obtain:

$$\pi_t = \frac{\alpha_x \beta M^f}{\alpha_x + \kappa^2} E_t \pi_{t+1} + \frac{\alpha_x}{\alpha_x + \kappa^2} u_t \tag{6}$$

Solving forward leads to:

$$\pi_t = \frac{\alpha_x}{\alpha_x + \kappa^2 - \alpha_x \beta M^f \rho_u} u_t \tag{7}$$

And for the output gap I find:

$$x_t = \frac{-\kappa}{\alpha_x + \kappa^2 - \alpha_x \beta M^f \rho_u} u_t \tag{8}$$

If we take the following notation: $\psi_M = \frac{1}{\alpha_x + \kappa^2 - \alpha_x \beta M^f \rho_u}$, then I find the compressed expressions: $\pi_t = \alpha_x \psi_M u_t$ and $x_t = -\kappa \psi_M u_t$.

The expressions for inflation and output gap, simply, states that central bank lets the output gap and inflation deviates from their targets by a value that is proportional to the cost-push shock. However, the central bank cannot choose the values of those variables. One possible method is to set an interest rate rule that will lead to the desired values of inflation and output gap. By writing the IS equation in term of those expressions, I find:

$$-\kappa \psi_M u_t = M(-\kappa \psi_M u_{t+1}) - \sigma(i_t - \alpha_x \psi_M u_{t+1} - r_t^e)$$

By simplifying, I find the optimal interest rate under discretion is:

$$i_t = r_t^e + \Psi_{M,i} u_t \tag{9}$$

With: $\Psi_{M,i} = \alpha_x \psi_M \rho_u + \frac{\kappa}{\sigma} \psi_M - \frac{\kappa}{\sigma} M \psi_M \rho_u$.

Uniqueness of the Equilibrium

In order to assess the optimality of the policy rule (9), we have to check if the equilibrium that will be obtained is unique. Otherwise, this policy rule will be undesirable in case it leads to multiple equilibria.

If the previous rule is used to eliminate the nominal interest rate in (3), I will have the following equations:

$$x_t = M E_t x_{t+1} - \sigma(\Psi_{M,i} u_t - E_t \pi_{t+1})$$

$$\pi_t = \beta M^f E_t \pi_{t+1} + \kappa x_t + u_t$$

We can write this system in matrix form as:

$$\begin{pmatrix} x_t \\ \pi_t \end{pmatrix} = \begin{pmatrix} M & \sigma \\ \kappa M & \beta M^f + \kappa\sigma \end{pmatrix} \begin{pmatrix} E_t x_{t+1} \\ E_t \pi_{t+1} \end{pmatrix} + \begin{pmatrix} -\sigma\Psi_{M,i} \\ 1 - \sigma\kappa\Psi_{M,i} \end{pmatrix} u_t$$

Which can be represented as:

$$z_t = A E_t z_{t+1} + B u_t \tag{10}$$

With: $z_t = (x_t, \pi_t)'$ and $A = \begin{pmatrix} M & \sigma \\ \kappa M & \beta M^f + \kappa\sigma \end{pmatrix}$ and $B = \begin{pmatrix} -\sigma\Psi_{M,i} \\ 1 - \sigma\kappa\Psi_{M,i} \end{pmatrix}$.

The equilibrium will be unique if and only if the eigenvalues of A are inside the unit circle.

Proposition:(*Condition of uniqueness of the equilibrium*)

The rule derived in the case of monetary policy under discretion yields to unique equilibrium if and only if the following condition is satisfied:

$$M + \beta M^f + \kappa\sigma - \beta M M^f < 1.$$

Under the rational expectation hypothesis in the traditional New Keynesian model: $M = M^f = 1$*, this condition will be :*$1+\kappa\sigma < 1$ *which is not satisfied.*

Proof. See Appendix 1. □

Within the framework that takes into account the inattention of agents, the monetary policy that seeks to act every period in order to minimize the welfare losses will be optimal under the condition derived in proposition 1. In contrast, in the traditional model this rule of monetary policy is seen as undesirable rule because it leads to multiple equilibria phenomena (see King et al. (2003) and Gali (2008)). So, the behavioral model contributes to overcome the problem of multiple equilibrium outcomes in the case of discretion.

Under the traditional model, the discretionary monetary policy is undesirable for the simple fact that agents have an infinite horizon and today's decisions are formulated based on expectations of the future. Within this framework, agents form their expectations taking into account how the central bank adjusts policy, given that the central bank is free to reoptimize every period. The rational expectations equilibrium resulting have the property that agents are uncertain about the future conduct of monetary policy, hence the equilibrium obtained is not optimal and the discretion is not a suited strategy for optimal policy conduct in this framework. However, this conduct of monetary policy is the closest to reality. Unlike the traditional model, monetary policy under discretion appears to be optimal if the model parameters satisfy the proposition 1.

By comparing our finding to the traditional case, borrowing solutions from Clarida et al. (1999) for the NK model and preserving my notation:

$$\pi_t = \alpha_x q u_t \tag{11}$$

$$x_t = -\kappa q u_t \tag{12}$$

with: $q = \frac{1}{\alpha_x + \kappa^2 - \alpha_x \beta \rho_u}$ Some simple arithmetic shows that:

$$q > \psi_M \tag{13}$$

The conclusion that arises from this result is that the central bank adjusts aggressively inflation and output gap in the traditional case (by the parameter q), while in the behavioral New Keynesian model the response is much smoother. In my belief, in the bounded rationality case inflation and output gap depend less on the expectations, respectively by M^f and M. To the extent that expectations no more influence the actual variables, the policy that act to stabilize inflation and output every period may perceived as optimal.

In order to be analytically tractable and to visualize our results in a simple way, we have to turn to some simulation results.

Figure 1: Response in the case of Permanent cost-push shock in the New Keynesian Model

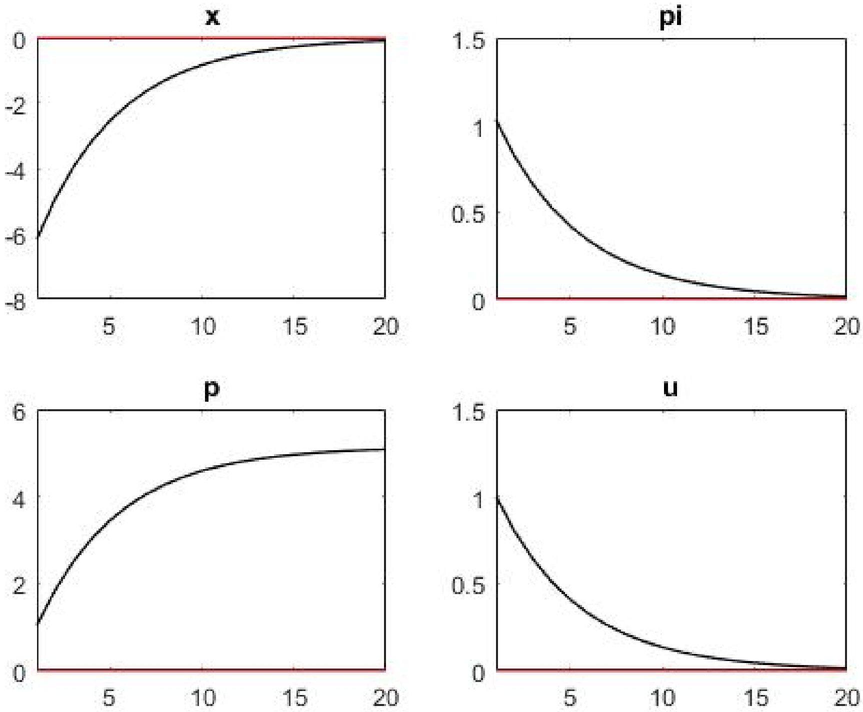
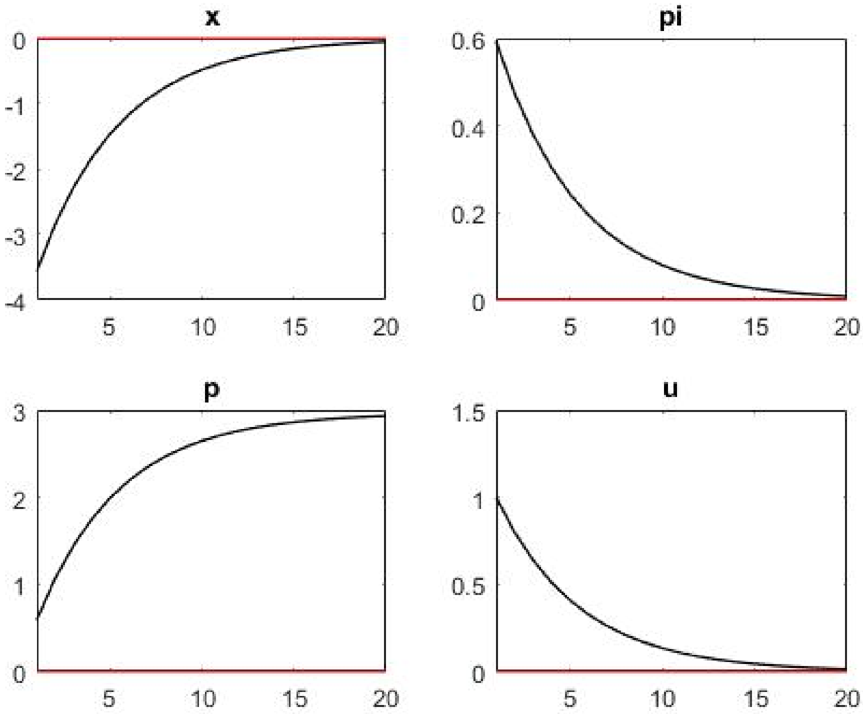


Figure 1 shows the reactions of output gap, inflation and price level in response to one percent permanent cost-push shock in the case of the traditional New Keynesian model [7].It is clear that this shock implies an increase one by one in inflation and hence in the price level. In the front of this negative shock, the central bank seeks to stabilize the economy by reducing the output gap by six percent regarding the relationship (12). However, as shown by the inequality (13) the response of the central bank will be less aggressive if the agents' inattention is incorporated in the model.

[7]the model used is that of Gali (2008) and the code for the simulation was provided by Johannes Pfeifer and it is available at : www.github.com

Figure 2: Response in the case of Permanent cost-push shock in the Behavioral New Keynesian Model



In the figure 2, I reported the simulation results [8] in the case of the Behavioral New Keynesian model with the following values: $M = 0.4$ for the degree of myopia for households, and $M^f = 0.7$ for the myopia of firms. The reason why I suppose that households are more myopic than firms is that those laters have the financial means to buy forecasting services in order to form more accurate expectations. As a result, I find that inflation does not increase one by one in response to cost-push shock instead it rises only by 0.6 percent rather than 1 percent as in the traditional model. In response

[8] The code used is modified and adapted from Johannes Pfeifer and it is available by demand

to this shock the central bank reduces the output gap only by around three percent which is represented by the relation (13).

To summarize, the behavioral New Keynesian model has two novelties with respect to the literature in the case of optimal monetary policy under discretion. First and unlike what the traditional New Keynesian model suggests, the interest rate rule derived is optimal under certain condition on the parameters of the model. Second, the response of the central bank to cost-push shocks is less aggressive as in the traditional model. Myopia of agents about the future plays a key role as long as the monetary policy actions are a synonym of expectations management as stated in King et al. (2008).

4 Optimal Monetary Policy Under Commitment

The unconstrained solution

In this case, the central bank is assumed to be credible firstly, and to be able to commit to a policy plan. Monetary authority must be able to choose a path for output gap and inflation $(x_t, \pi_t)_{t=0}^{\infty}$ over the infinitely-lived horizon in order to minimize the intertemporal loss function:

$$\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t(\pi_t^2+\alpha_x x_t^2)$$

Subject to the sequence of constraints:

$$\pi_t = \beta M^f E_t \pi_{t+1} + \kappa x_t + u_t$$

By writing the Lagrangian and differentiating with respect to x_t and π_t, it yields the optimality conditions:

$$\alpha_x x_t - \kappa \gamma_t = 0$$

$$\pi_t + \gamma_t - \gamma_{t-1} M^f = 0$$

With γ_t's are the Lagrangian multipliers.

The first order conditions yields to the condition:

$$x_t - M^f x_{t-1} = -\frac{\kappa}{\alpha_x}\pi_t$$

By iterating this condition, we obtain:

$$x_t = -\frac{\kappa}{\alpha_x}\sum_{i=0}^{\infty}(M^f)^i \pi_{t-i} + (M^f)^t x_0 \tag{14}$$

The relation (14) constitutes the targeting rule under commitment. Central bank sets the size of output gap taking into account the actual and past inflation weighted by the terms of myopia. When the central bank looks at past inflation, it will be done with some sort of inattention implying that just more recent observations that will be given more importance in setting the value of output gap.

Two remarks can be drawn at this stage. Firstly and by comparing the targeting rule (14) under commitment with (3.2) the rule under discretion, I find that under commitment the targeting rule is more history-dependent than in the previous section. The central bank commits to stabilize inflation by lowering the actual output under its efficient level, but also by lowering the future output if it is needed in order to smooth the central bank's actions. This result is consistent with the study of Woodford (2009) in a New Keynesian framework with Near Rational agents. It is appealing in this regard to notice that in the present paper as in Woodford (2009) the rational expectations hypothesis is relaxed, and both models yields to the same result. Secondly and by comparing (4.1) with the targeting rule resulting from the traditional model stating that central bank needs to lower output below its efficient level in proportion to the deviation of the price level from an implicit target (See Gali 2008), such a theoretical result may not be suited for the central banks conduct of monetary policy in reality. In the case of the behavioral model, the targeting rule (4.1) is linking the output gap with inflation and the reactions for the central bank will be to set values for output gap given the past path for inflation.

By combining the optimality condition with the New Keynesian Phillips curve (2.4), I obtain:

$$\pi_t = -\delta \frac{\kappa^2}{\alpha_x} \sum_{i=0}^{t} (M^f)^i \pi_{t-i} + \delta \beta M^f E_t \pi_{t+1} + \delta u_t$$

With $\delta \equiv \frac{\alpha_x}{\alpha_x + \kappa^2}$ and I assume that $x_0 = 0$ in the starting period the economy was on equilibrium.

The stationary solution for this equation is:

$$\pi_t = -\eta \frac{\kappa^2}{\alpha_x} \sum_{i=0}^{t} (M^f)^i \pi_{t-i} + \frac{\eta}{1 - \eta \beta M^f \rho_u} u_t \tag{15}$$

With: $\eta \equiv \frac{1 - \sqrt{1 - 4\beta\delta^2}}{2\beta\delta}$.

Solving for the output gap, I find:

$$x_t = (1 + \frac{\kappa \eta}{M^f}) x_{t-1} - \frac{\eta \kappa}{\alpha_x (1 - \eta \beta M^f \rho_u)} u_t \tag{16}$$

Relations (15) and (16) represent the reactions of the central bank to a cost-push shock. It appears that both equations are history-dependent more than the traditional model. The central bank responds to a cost-push shock in the current and future periods until inflation and output gap return to their original targets. This slow adjustment process of targeted variables induces some persistence in the behavior of the output gap and the rate of inflation.

Once again and to visualize more clearly the results, it is necessarily to illustrate by some simulations.

Figure 3: Response in the case of Permanent cost-push shock in the New Keynesian Model

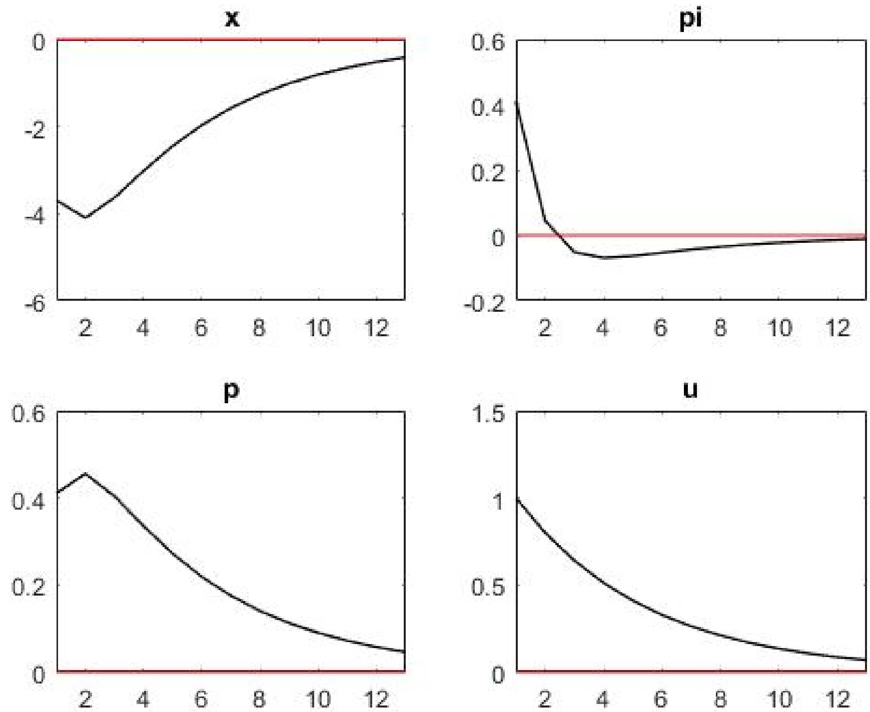
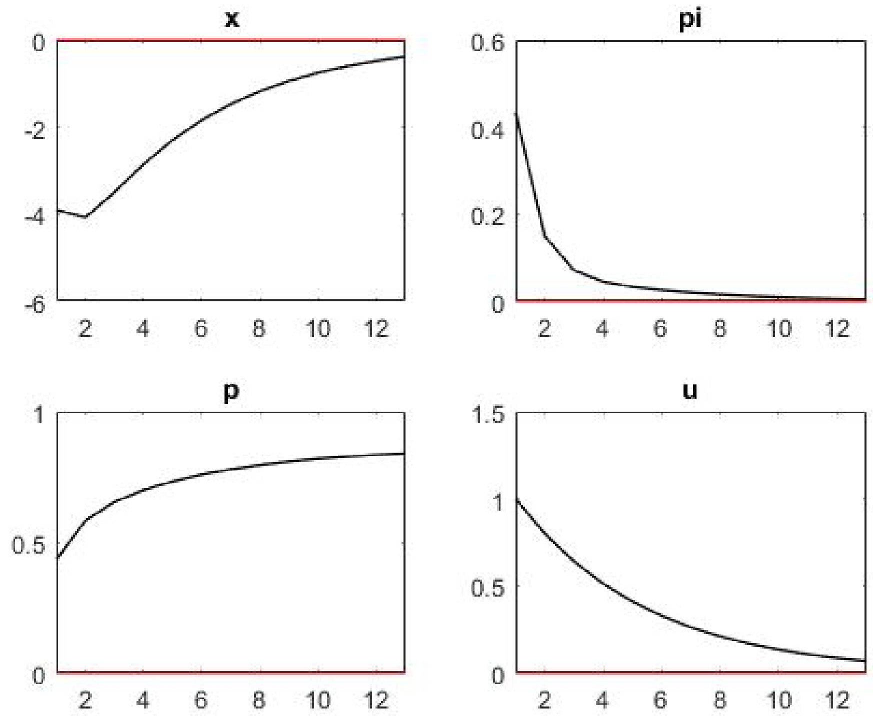


Figure 3 extends the reaction of target variables following a cost-push shock in the case of traditional New Keynesian model. It is thus clear that inflation rose 0.6 percent while the output gap decreases by 4 percent.Comparing this results with those of the figure 4, in the case of behavioral New Keynesian model, I find the same outcomes in terms of changes in the output gap and inflation. Though the two models are facing the same shock, one can remark that the price level in figure 3 returns to its initial level, due to the decrease in inflation below its target in the medium term, unlike the figure 4 in which the price level presents some persitent behavior. This difference can be explained by the finding that in the behavioral model the variables are more history-dependent than in the traditional model.

Figure 4: Response in the case of Permanent cost-push shock in the Behavioral New Keynesian Model



As the unconstrained solution seems difficult either to implement or to completely interpret, I now turn to characterize the optimal policy in some constrained family of solutions.

The constrained solution

In this case, I accordingly consider a rule for the output gap x_t that is contingent on the fundamental shock u_t, in the following way:

$$x_t = -\omega u_t \tag{17}$$

for all t, where $\omega > 0$ is the coefficient of the feedback rule.

Combining equation (*) with the Phillips curve (2.2), in turn, implies that inflation under the rule,π_t, is also a linear function of the cost push shock:

$$\pi_t = \beta M^f E_t \pi_{t+1} - \kappa\omega u_t + u_t$$

Solving forward, it yields to:

$$\pi_t = \frac{1-\kappa\omega}{1-\beta M^f \rho_u} u_t$$

This can be rewritten as:

$$\pi_t = \frac{\kappa}{1-\beta M^f \rho_u} x_t + \frac{1}{1-\beta M^f \rho_u} u_t$$

The problem of the central bank will be to find the optimal solution parameter ω by minimizing the welfare loss function subject to the condition

(*). It follows that:

$$x_t = \frac{-\kappa}{\alpha_x(1-\beta M^f \rho_u)}\pi_t$$

The relationship (*) represents the central bank targeting rule under commitment in the case of the behavioral New Keynesian model. While the unconstrained solution seems to be too complex to completely interpret, the targeting rule in the family of solutions of type (4.4) provides a tractable analytical solution describing the way the central bank act in response to cost-push shock. It looks very close to the targeting rule under discretion; they differ just by the term: $\frac{1}{1-\beta M^f \rho_u}$.

Comparing the targeting rule (*) with the one obtained in the case of the traditional model as in Clarida et al. (1999):

$$x_t = \frac{-\kappa}{\alpha_x(1-\beta\rho_u)}\pi_t$$

One can notice that, in absolute term, the response of the central bank in the case of the traditional model will be much harder than in the behavioral model in lowering the output gap.

To sum up, in this section two families of solutions under commitment were presented relaying to the behavioral New Keynesian model. The first one is the unconstrained solution of the optimal monetary policy, which concludes that the targeting rule for the central bank is more history dependent than in the traditional case but also it includes the terms of firms myopia. Such feature in the targeting rule highlights the importance of private expectations in terms of the conducts in monetary policy. The policy recommendation that arises from this fact is to take into account the private expectations form while conducting monetary policy.The second family of solutions is the constrained one, and it sheds light about the differences between the traditional case and the behavioral case in some simple analytical and tractable way.

5 Discussion

The present paper constitutes an assessment of the optimal monetary policy under discretion and commitment in a behavioral New Keynesian framework. With the comparison of the traditional New Keynesian model, we have provided a condition on the parameters of the model when the discretionary moenaty policy can be optimal which constitutes a theoritical foundation of the practice of the central banks in the sense of deciding period-by-period the optimal response to economic development. Moreover, in the case of the policy under commitment we showed that the response of the central bank is very smooth due to persistence that arises in the targeted variables.

In one hand, our results in the case of discretion can be supported by numerous studies that take up this question under different angles. As Sauer (2010) noted:"Discretion gains relative to the timeless-perspective rule—i.e., the short-run losses become relatively more important—if the private sector behaves less forward looking", which is satisfied in the behavioral New Keynesian model. Another study that supports our finding is that of Dennis (2010), in which he shows that discretion is found to dominate timeless perspective policymaking when the price/wage Phillips curves are relatively flat due to firm-specific capital (or labor).

In the other hand, commitment continues to be important for the optimality of the monetary policy as in the benchmark model. However, the targeted variables are shown to be more history-dependent in the behavioral model and since the authority commits to a history-dependent policy in the future, it is able to optimally spread the effects of shocks over several periods. This result is in line with the most of the optimal monetary policy literature but the most important finding is that the actions of the central bank in the behavioral model are less aggressive than the traditional one.

As pointed out by Dotsey (2008), the rationale behind preferring the commitment over discretion lies in the fact that policymakers who can commit have the ability to follow through on promised actions that they can influence expectations in a desirable way. The discretionary planner makes no promises and, as a result, does not have a similar ability to influence expectations. A planner who can commit to future actions in various situations can affect what people expect will happen in these situations, and these expectations influence current behavior. But as in the behavioral New Keynesian model, the assumption of rational agents is relaxed and then those policymakers prefrences are to nuance.

Turning to a comparison of the values of the policy loss function, the simulations conducted in the previous two sections show the following results reported in the table below:

•	New Keynesian Model	B. New Keynesian Model
Under Discretion	503	325
Under Commitment	146	145

One can figure out that under commitment both models deliver approximately the same value of the loss function. Although, the behavioral model performs well when it comes to the case of discretionary policy. In addition, when comparing the discretion versus commitment under the same model it appears that commitment is superior to discretion in both models. However, the difference of loss between the discretion and commitment in the behavioral model is much smaller than in the traditional New Keynesian model.

What policy recommendations can be drawn ? Firstly, the monetary policy under discretion can be optimal for developing, emerging and transitional economies where the policy making process is much closer to discretion rather than commitment. Such a result constitutes a theoretical foundation of the practice of lot of central banks around the world and my respond to the crticisms raised by Stiglitz (2010). Secondly, the improvement of the monetary policy framework for these less developed central banks comes out with an important increase in welfare as had been shown by the simulations result.

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Appendix 1. Proof of Proposition 1

In order to have a unique equilibrium, matrix A must have eigenvalues inside the unit circle.

$$z_t = AE_t z_{t+1} + Bu_t$$

First, the characteristic function of A is defined as follows: $f(\lambda) = det(A - \lambda I)$. With I is the unity matrix.

By a simple calculation of the determinant, I find:

$$f(\lambda) = \lambda^2 - (M + \beta M^f + \kappa\sigma)\lambda + \beta M M^f$$

The eigenvalues of A are the solutions of the equation: $f(\lambda) = 0$, by solving this second order equation I find the following solutions:

$$\lambda_1 = \frac{M + \beta M^f + \kappa\sigma + \sqrt{\Delta}}{2}$$

$$\lambda_2 = \frac{M + \beta M^f + \kappa\sigma - \sqrt{\Delta}}{2}$$

With: $\Delta = (M+\beta M^f+\kappa\sigma)^2 - 4\beta M M^f > 0$. Which assure the existence for the real solutions λ_1 and λ_2.

Since I have $\lambda_2 < \lambda_1$, in order to find a condition on the uniqueness of the equilibrium I have to verify just that λ_1 is inside the unist cercle and the other condition will follow.

$$\lambda_1 < 1 \Rightarrow M + \beta M^f + \kappa\sigma - \beta M M^f < 1$$

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