Towards a Democratization of Knowledge with Topological Emphasis in Economics

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Abstract We formulate and prove a theorem which consists in how the natural endogenous antagonist interaction of agents who look for understanding a generalizable phenomenon, results in a tendency towards chaos. This takes us to the final absolution of implementing the majority rule as the only instrument that generates socially acceptable knowledge, escaping from the chaos tendency. Finally, we extend our analysis to consider the arise of multiple simultaneous antagonist postures on the explanation of a phenomenon, and through an application of the Pythagoras theorem, we prove that it takes less effort or sacrifice for an agent to learn strategically to get an explanation, than if she was the creator of the concerning knowledge, which implies different consequences of possible topological private and public tendencies.

Keywords Antagonist Endogenous Knowledge · Social Entropy · Chaos Theorem · Social Choice

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1 Introduction

> Pure mathematics is, in its way, the poetry of logical ideas.
> Albert Einstein

This work is meant to capture a broad representation of the process which takes place on the formation of knowledge. It is based on qualitative observations more than on quantitative ones. Although this generalization can be understood as explanatory for different sciences, we shall introduce it with examples of economics that result highly illustrative, not only because we are more familiarized with them, but also because we consider that are more popular among readers.

The formation of knowledge is always preceded by complicated eternal and antagonist debates which are leaded by thinkers that truly believe they are right, as if their position was a sort of religion. For example, we can look at the well known Keynesians vs Neoclassical debate. Another more recent but not less essential in the understanding of economics which starts from questioning the very foundations of microeconomics, is the Sraffian capital debate vs the Neoclassical praxis, where according to each group of prominent thinkers, the truth lies in their position[1].

Most of these debates are developed upon wether some assumptions or properties should be treated as truth or not[2], and the deeper the reader gets, the more likely will be for her to join a partial position.

Among other debates we can highlight the different opinions on the future of the economic growth of the countries, where authors like Kaldor argued that the regions show convergence in real per capita GDP levels, while for example Accinelli et al. (2010) and Howitt and Mayer-Foulkes (2005) explain how the nations diverge depending on initial levels.

Although there are specific theoretical positions based on the statistic inference that look for answering which of the previously mentioned views is right, such as the approach to assess robustness proposed by Sala-i-Martin (1997), deep thinkers do not seem to be satisfied, and keep arising with new points of view or proposed variables concerning the debated topics, diverging continuously.

Considering the antagonistic nature of these phenomena, we shall develop a representation and a theorem which allows the reader to understand the divergent dynamic of positions over facts, also enhancing the visualization of a chaotic future in the absence of a stopping mechanism such as a knowledge democracy. Posteriorly we prove a sacrifice theorem which taking into account the existence of knowledge prerequisites, stands that it is easier to follow than innovating, and we do this through an application of the Pythagoras theorem. Furthermore, this work can also be found as an exposition of foundations behind why it is important not only for economists to study Social Choice.

2 The model

All the agents are located in a convex closed set of R_+^n which contains the vector zero. The model is based on the intuitive assumption of that one of the dimensions captures the level of antagonism and capability, such that if the agents are located in a closure it means that are the creators of the deepest explanation which justifies an antagonist position. This means that the agents in this dimension's closures go deeper in the debated points (considering more aspects), which pushes them to such position, and that they propose the more complex and ruling explanations, where the closures of such dimension thus represent a current boundary of human understanding[3]. Furthermore this means that the intermediate point in this dimension is occupied by the agents who are indifferent in terms of the discussed topic.

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[1] e.g. Petri (2009; 2013), Garegnani (2003; 2005), Mandler (2002; 2005), Lazzarini (2011).

[2] For example, the treatment of the capital as an homogeneous good of the models of economic growth, discussed in the mentioned Sraffian capital debate, which leads to wondering about the existence of the correct properties of an aggregate production.

[3] A unidimensional representation of the location of agents can be found in Hotelling (1929).

The rest of the dimensions capture a location of the agents in terms of other aspects, like space or a sport taste. Each agent $i \in N$ has a personal scope to share theoretical findings representing her flexibility to interact with the individuals who are different. We accept that the scope in tolerating other positions is equal for all the agents, and that it is equally annoying and possible for them to do a personal sacrifice in any direction despite their personal characteristics and the income distribution. *Therefore* we can represent the individual's scope with a constant distance k, such that the k radio ball surrounding an agent denotes her effective interaction area[4].

The first entry of the vector of coordinates of the location of an agent $i \in N$ at the time t is given by the real number $l_i \in [A_t, D_t]$, where A_t and D_t represent the borders of human knowledge.

Although looking for understanding the intertemporal metric behavior of the new knowledge happenings could takes us to fruitful findings, in this work we rather consider the following properties, which are usually present in the process that we are dealing with.

Constant presence of leadership: There are at least two agents $i, j \in N$ located such that $l_i = A_t$ and $l_j = D_t \ \forall t \in R_+$. Moreover, the individuals in this closure are called knowledge leaders.

This property considers how it is important to notice that the closures exist because there is at least one agent in the already mentioned frontier of human knowledge. Moreover, by common sense we know that all the inventions or new theories are built constantly using previous ones as a base or influence[5], and to represent this we define the following intertemporal property.

Constant scientific progress: $A_t \leq A_{t-x} - \frac{1}{\int_{t-x}^{t} 1 du}$ and $D_t \geq D_{t-x} + \frac{1}{\int_{t-x}^{t} 1 du} \ \forall x > 0$.

2.1 Local consensus

As we have previously mentioned, the agents are able to come up with an explanation about the discussed topic, however, the communication among agents can take them to understand and/or support deeper and *thus* ruling theories which are proposed by others. This means that if an agent i is located inside of the closed k radio ball surrounding j, the deeper explanation which is accessed by these two agents can be supported by both as a kind of local consensus.

As we have previously mentioned, a location l_i does not only capture a level of capability, but a posture as well, such that an individual in the middle of the interval $[A_t, D_t]$, is located there because is not capable of taking a posture on her own. Furthermore, the possible local consensus also imply that this "indifferent" individual could support a partial position under certain conditions, which we will later explain.

A set of interaction is a coalition $S \in 2^N$ which satisfies $|S| > 1$, formed by the individuals who are located such that, for any agent $i \in S$ there is at least another individual $j \in S$ with the k radio ball surrounding her having a non empty intersection with the k radio ball surrounding i.

To represent how the agents have defined a posture on the theoretical debate, after interacting, the individuals will always support the deepest thesis which they can access and agree with. In other words, they will support the position of the agent who belongs to their set of interaction, and that is closer to the closure in the dimension of positions to which they are closer. Moreover, an "indifferent" agent will support the position of the individual within her interaction set, that is more distant from hers[6].

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[4] We could reject the assumption about the interdimensional equally annoying and possible sacrifices, and represent the maximum sacrifice of the *jth* dimension by k_j, which *means* that the effective interaction area of an individual would be given by an n dimensional ellipse. Moreover, depending on different hypothesis of behavior, we could represent an effective interaction area with a not necessarily convex closed set, surrounding an agent without altering our qualitative results.

[5] This means that the leaders are always able to pass to more capable individuals, the interest for continuing the search for deeper and better explanations.

[6] The deepest explanation that she has received, where being part of a local consensus does not necesarily mean that the explanation has been truly understood.

As we can see, this means that among the agents forming a coalition $S \in 2^N$ there are one or two consensus to which we shall refer as local consensus.

2.2 Debate and Social Problems

As it can be verified, these debates can originate problems which affect negatively the life style of the population showing violent consequences. This is, although it may be "normal" for many regions to presence such kind of "generalizable" social happenings, the produced effects can reach devastating outcomes.

Considering how the local consensus are already in favor of one of the theoretical positions, and that the individuals look for "approaching" and sharing the "true" explanation, imposing their views over the others, we shall define a measure of social stability likelihood based on how divergent the positions are.

The antagonism which derives naturally from the knowledge formation is as we have mentioned "normally" problematic, and both postures or thoughts have usually followers, which is why independently of the formed intermediate local consensus and the number of followers, we shall base our measure on how difficult it becomes to reach a global consensus that avoids social problems.

In this way, for a given period t we can just take the distance $I_t = D_t - A_t$ as a global indicator of social instability, representing the impossibility to reach agreements, such that a higher I_t indicates a more chaotic social situation.

H is the set of coalitions of elements of the real numbers with cardinality two. The function $f_t : H \rightarrow H$ takes the pair of leading positions $\{A_{t-1}, D_{t-1}\}$ and gives back the knowledge frontiers of the next period $\{A_t, D_t\}$, satisfying the "constant scientific progress" property. This means that f_t is a function that focuses only on the evolution of the frontiers of knowledge.

Theorem 1 (Chaos) $\lim_{t \rightarrow \infty} I_t = \infty$.

Proof From the "constant scientific progress" property we get that $t > x \Leftrightarrow A_t < A_x$ and $D_t > D_x$. From this we deduce that $D_t - A_t > D_x - A_x \Leftrightarrow t > x$. Considering the strict increase of the distances in the middle we get that $t - x \rightarrow \infty \iff (D_t - A_t) - (D_x - A_x) \rightarrow \infty$. Finally we can just consider the particular case of $x = 0$, and any posterior t tending to infinite illustrates the divergence and chaos.

As we can see this theorem means that with the pass of time, the understanding of the agents on an issue gets deeper, and that it also becomes more difficult for them to reach a global consensus or agreement, thus tending to more chaotic social dynamics at least due to the need for adopting a posture when general decisions are taken[7]. Furthermore, for some reason, as a sort of established rivalry of novelties, due to a continuous application of the mean value theorem, we know that the indifferent agent(s) shall constantly occupy the mean point between the closures.

2.3 Escaping from Chaos

Now that we have identified the tendency towards chaos, what could be done in order to avoid such a cruel destiny for the human kind?

Although dictatorship of knowledge seems to be the easiest alternative in terms of implementation, because it does not require the approval of each individual, we recognize the right of the agents to participate in the formation of the information which shall be considered as true in the functioning of their world. Therefore, based on this we propose the majority rule as the mechanism through which a final more desired consensus and thus, a socially acceptable knowledge could be adopted, also highlighting how the individuals may increase their participation and promotion of local consensus to impact their daily living.

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[7] It can be verified that in the equality case of the "constant scientific progress" property $\frac{\triangle D_t}{\triangle D_{t+1}}, \frac{\triangle A_t}{\triangle A_{t+1}} = \frac{t+1}{t} > 1$.

Raza et. al (2007) demonstrate that the social democratization of knowledge for e-learning (without confusing it with the free access concept) can lead to the establishment of viable global civil society, helping millions in Asia, Africa and South America to contribute and share the fruits of knowledge explosion in a just, equitable and honorable fashion. On the other hand, some works that study problems which can arise when the majority rule is used are Condorcet (1785), Plata (1999), and the **manipulation possibility** of strategic voting studied by Salvador Barbera (Jackson and Sonnenschein, 2011).

Assuming the democracy mechanism to start being implemented in the period t' does not mean that the evaluations of the function $f_{t'}$ start showing convergence[8], but instead it means that the index $I_{t'}$ becomes useless due to the intellectual exercise subordination to the will of the majority. Therefore, the chosen postures on the matters would depend on the distribution of individuals over the different local consensus[9], which implies avoiding social problems at a possible cost of exercising a "minor" to the frontier local consensus[10].

2.4 Social Stability and Multiple Antagonist Postures

As we have previously mentioned, because of the antagonist nature of knowledge there are polar disagreements between thinkers, however, in the advance of a phenomenon understanding we can find the formation of new debates and antagonist positions within a single posture. Considering how this could take place, we get that the local consensus could now be located in different points.

The interpretation of this is given by the arise of an extra dimension which indicates the position of the agents in terms of the new debate, where a new direction can be taken at certain point of the previous postures. Moreover, notice that considering more than one debate allows the possibility of the agents occupying more than one position at the same time, because a leader in a position could also be a leader in another debate within that position.

The formation of multiple debates is of our concern because of the social instability that derives from it. In order to understand the social instability it is fundamental to understand when there are knowledge requirements to be able to get a posture in a new debate. To illustrate this we consider the following case:

Case 1 Strict requirements debate: this happens when in order to get a posture in a new debate, the agents need to be able to form part of a local consensus which is at least in the depth level where the new position arises.

In this way if there were not requirements in a debate, the agents could be located in any coordinate within the newly considered closed intervals. In order to represent how the requirements to get a position in a debate, can increase depending on how deep into the new posture an agent can be, we assume that the agents can only be located in terms of a new debate as it follows.

Denoting the indifferent location of the *jth* debate by r_j, the individuals can only be located in terms of their posture on a phenomenon, within the triangle that has the level of the arise of a new debate j as base, and height $D_t - r_j$ or $r_j - A_t$ depending on the adopted posture, where D_t and A_t are the frontiers of human knowledge in terms of the *jth* debate. Moreover, this means that when the debate from which the new one arose gets deeper, then the agents could also be located in the area of the rectangle[11] that has the new boundary minus the level of the *jth* arise as a base, and $D_t - r_j$ or $r_j - A_t$ as height.

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[8] Which could be true if the mechanism included the dictation of the prohibition to study the bound of the loosing position.

[9] Santerre (2008) focuses on how the scientific and technical culture has become an interface, stimulating exchanges between scientists and other social actors, resulting in research being more attuned to community needs.

[10] This is because as we have previously mentioned, the agents who are part of different local consensus are not necessarily able to effectively interact with some one who explains them at least convincingly enough a deeper consensus, or the frontier of their position.

[11] Or square.

This also means that if more than one debate arises at a single level of knowledge, then the individuals could be located in terms of position within the triangle in the l dimensions, where there were $l-1$ arising debates.

The interpretation of this visualization is that an agent located in the hypotenuse of the triangle or "below" does not really get the new debate, but instead it has some of the required knowledge to get it.

Considering how each of these dimensions keeps being associated with the capability and effort of the agents, we formulate the following theorem.

Theorem 2 (Sacrifice) *It takes less effort, sacrifice or capability to learn or copy requirements than to innovate, propose, or discover to get to the knowledge of the frontiers of an arisen debate.*

Proof For $l-1$ arising debates we can write this argument as $h < a+b$, where h is the distance between the previous indifference point and the new frontiers' coordinates, a is the base of the triangle in the l dimensions, and b is its height.

Since $h^2 = a^2 + b^2$ and $(a+b)^2 > a^2 + b^2$, we have proved the theorem.

An example of this theorem is given by how it takes less effort to learn math and economic intuition simultaneously, than to first understand math[12] and then economics to be able to get the explanation of an economic phenomenon like monopoly pricing. In other words, this theorem means that given the rational agents, it is easier to get somewhere when the final destination is known, or that it is easier to get something which requires knowledge that is already possessed by the individual.

In terms of stability this means that although the **topologically** defined debates within a debate can bring problematic social dynamics, the added instability is even higher because there can be more **distinct** local consensus[13]. Therefore, to consider this in a simple way we redefine the instability indicator for l related debates, as if there were not knowledge requisites in the following

$$I_t = \sum \prod_{i=1} \left(D_{i,t} - A_{i,t} \right)$$

for which it is obvious how for different of these **very well topologically** defined arising debates that are in the same dimension[14], only the more advanced boundaries should be considered. Moreover, based on the satisfaction of the "constant scientific progress" property, we can verify and prove how the previous chaos theorem is true for all the newly defined I_t!

3 Conclusions

From the representation we get a clear explanation and intuition behind the formation of local consensus that look for explaining certain phenomenon.

We introduced a measure of social instability which is based on how difficult it becomes for the individuals to agree and form a global consensus. Moreover, our theorem contributed in showing how the evolution of the boundaries of knowledge leads a population to increasingly chaotic social dynamics, which took us to the unavoidable proposal of a mechanism, to reach social agreements on which knowledge shall be considered as true or mainstream, in this way facilitating the taking of general decisions.

As we can see, our work contributed not only to address questions about tendencies over time in terms of social problems and stability, but also to establish a solution to the incoming chaos named the democratization of knowledge, which has the advantage of allowing the individuals to participate in the formation

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[12] Or Phisics.

[13] The set of possible local positions including consensus is $2^N \backslash \phi$, and for enough existent knowledge, the possible knowledge tendencies are **at least** as many as possible set of disjoint non empty coalitions.

[14] i.e. that require the same kind of knowledge requisites.

of scientific information. In this way, the winning postures on the matters would depend on the distribution of individuals among the different local consensus, which implies avoiding social problems at a possible cost of exercising an inferior to the frontier local consensus. Furthermore, we could remark how it may already be "normal" in many regions to observe the employment of this kind of mechanisms, which can be interpreted as a justification not only for economists to get deeper in the study of Social Choice.

Finally, the sacrifice theorem allows us to get the importance of knowledge requirements for the understanding of a scientific posture, and to remark the efforts of the individuals who although chaotically, build information at the frontiers of human knowledge, implying different consequences of possible private and public tendencies.

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