Wage and employment determination in a dynamic insider-outsider model

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Wage and Employment Determination in a Dynamic Insider-Outsider Model*

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Abstract

In this paper, I develop a dynamic insider-outsider model in which a union of incumbent workers is called in to choose the wage of its members by taking into account the optimal employment policy of firms that, in turn, are assumed to decide the firing rate of insiders and the number of outsiders to hire. Under the assumption that incumbents are able to observe their own productivity, the one of outsiders and the amount of labour turnover costs paid by firms, I analytically show that the initial stock of insiders may pin down an equilibrium in which the trajectories of incumbents, their own wage and the firing rate are jointly determined and the proposition according to which insiders have an incentive to keep their numbers small holds on a static and a dynamic perspective. Moreover, I numerically show that in this setting insiders adjust their wage in order to retain their positions and such a behaviour leads to asymmetric employment fluctuations and a certain degree of wage stickiness.

JEL Classification: E24; E32; J31; J64.

Keywords: Insider-outsider theory; Union modeling; Wage and employment dynamics; Optimal control.

1 Introduction

The insider-outsider theory of employment and unemployment rests on the assumption that there is a fundamental asymmetry in wage setting process among incumbent workers (insiders) and unemployed workers who are looking for a job (outsiders). On the one hand, relying on labour turnover costs and/or firm-specific skills that create a productivity premium, insiders are

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assumed to have a strong bargaining power in the wage setting process - sometimes strengthened via formal or informal unions - and to exploit this power in order to maximize their pay and foster their employment opportunities. On the other hand, outsiders are assumed to have no market power and when they have the chance to find a job their wage is usually quite close to the reservation level and they have no say over their employment prospects.

Undoubtedly, the main backers of the insider-outsider theory are Lindbeck and Snower (1984, 1989) who developed a long array of works that uncover a wide range of consequences in terms of wages and (un)employment triggered by the interest conflict between insiders and outsiders in the labour market such as the existence and persistence of involuntary unemployment, the inflexibility of insiders' wage, trade union corporatism and the asymmetry of wage-employment movements during recessions and expansions. Moreover, Carruth and Oswald (1987) provide a model of union behaviour grounded on the distinction between insider and outsider workers and discuss some related macroeconomic implications.[1] Furthermore, Blanchard and Summers (1986) as well as Gottfries and Horn (1987) rely on insider-outsider relations to explain the strong persistence of unemployment observed in Europe during the 80's.

In the latest survey article dedicated to the theoretical approach that they contributed to start, Lindbeck and Snower (2002) point out that one of the most complex and open question of the insider-outsider theory is the way in which employment - and wages - moves through time in response to labour market shocks. Specifically, while there is a number of insider-outsider contributions that show how to fix the static levels of employment and wages at any point in time, dynamic insider-outsider models - despite some fair exceptions - lag somehow behind. Similarly, Sanfey (1995) argues that modifying traditional union models to take into account the distinction between insiders and outsiders is straightforward, but that task is actually much more complicated in dynamic models.

In confirmation of this argument, the dynamic insider-outsider literature counts a limited set of contributions. For instance, Solow (1985) outlines a two-period insider-outsider model but he is mainly concerned about what happens in the first and he does not derive any explicit dynamic law for employment and wages.[2] Drazen and Gottfries (1990) set forth a dynamic optimizing insider-outsider framework developed over an infinite horizon, but they model the evolution of wages and employment by means of few discrete realizations of the two variables.[3] Huizinga and Schiantarelli (1992) develop a discrete-time model in which a firm and a union of insiders efficiently bargain over the wage and employment but the two authors focus only on

[1]For instance, the possibility that productivity improvements can feed into pure wage increases for insiders with no, or minor, effects on employment.

[2]Recognizing the inherent dynamic of the insider-outsider hypothesis, a similar exercise is carried out by Vetter and Andersen (1994).

[3]Namely, high or low wages vis-à-vis one or two slots of workers which are taken as proxies of low and high employment.

the path of employment adjustments driven by productivity shocks. Fukuda and Owen (2008) build a series of overlapping-generations (OLG) models with human capital accumulation in which the insider-outsider dichotomy is conveyed in terms of firm-specific versus general skills.[4] More recently, Galì (2016) sets forth a New Keynesian DSGE model with an insider-outsider labour market and explores the implications of that environment for the design of monetary policy.

To the best of my knowledge, the present contribution is the first attempt to provide a stochastic dynamic insider-outsider model developed within an optimal control framework with continuous time and infinite horizon. Specifically, I build a theoretical setting in which a union of insider workers is called in to choose the wage of its members in an inter-temporal setting by taking into account the optimal employment policy of firms that, in turn, are assumed to decide the firing rate of insiders as well as the number of outsiders - or entrants - to hire in each instant. In a partial equilibrium perspective, the dynamic interaction between the decisions of firms and the ones of the union describes how wages and employment are continuously determined and move through time in a typical insider-outsider economy subject to productivity shocks.

The results of this theoretical exploration can be summarized as follows. First, maintaining the informational assumptions of textbook insider-outsider models, i.e., assuming that insiders are actually able to observe their own productivity, the one of outsiders and the amount of labour turnover costs borne by firms (cf. Lindbeck and Snower, 1989), the initial stock of incumbent workers may pin down a well-defined equilibrium of the model economy in which the dynamics of the insider labour force, its wage and the firing rate of incumbent workers are jointly determined. Second, the proposition according to which insider workers have an incentive to keep their numbers small holds on a static as well as on a dynamic perspective. Moreover, relying on the outcome of some numerical simulations, it becomes possible to show that in this model economy insiders adjust their wage in order to retain their positions and such behaviour leads to asymmetric employment fluctuations and a certain degree of wage stickiness.

The paper is arranged as follows. Section 2 develops the theoretical model. Section 3 provides some numerical simulations. Finally, section 4 concludes.

2 The Model

I consider a model economy in which time is continuous and the time horizon is infinite. Within this economy, in each instant, say t, a representative firm starts with a pool of $L_I(t)$ insider workers. Those fully-fledged employees may randomly loose their job at the exogenously given rate b. Moreover, the firm is assumed to be in the position to accelerate the outflow of seasoned workers. Consequently, denoting newly hired workers - or entrants - by $L_E(t)$ and assuming that

[4] Different OLG models in which the insider-outsider distinction is concerned are given by McCausland (1998) and Begg (1988).

outsiders becomes insiders just immediately after their appointment, the dynamic evolution of the stock of incumbent workers can be written as

$$\dot{L}_I(t) = L_E(t) - (\Phi(t) + b) L_I(t) \qquad b > 0 \tag{1}$$

where $\Phi(t)$ is the instantaneous firing rate decided by the firm.

The expression in (1) implicitly defines the way in which outsiders turn into insiders and it reveals that when $\Phi(t)$ is higher than b, the firing decisions of the firm exacerbate the 'normal' labour market turnover. Moreover, when $\Phi(t)$ is equal to zero, the dynamics of incumbent workers is completely determined by the decision to hire outsiders put forward by the firm.[5]

Similarly to Guerrazzi (2011), I assume that produced output is a stochastic quadratic combination of the employed labour force. Therefore, under the hypothesis that the flow of entrants is less productive than the stock of insiders, the production function of the representative firm is given by

$$Y(t) = A(t)(L_I(t) + \phi L_E(t)) - \frac{\alpha}{2}\left(L_I^2(t) + L_E^2(t)\right) \qquad 0 < \phi < 1,\ \alpha > 0 \tag{2}$$

where $Y(t)$ is the level of output, $A(t)$ is a productivity shock, ϕ measures the productivity differential between insiders and outsiders while α is a parameter that conveys the slope of labour demand.

The quadratic specification in eq. (2) has the virtue to deliver linear marginal productivity schedules both for insiders and outsiders whose distance is proportional to the parameter ϕ.[6] As argued by Lindbeck and Snower (1987), such a wedge between the productivity of the two different categories of workers - together with labour turnover costs - provides a measure of the degree of insiderness (cf. Manzini and Snower, 2002).

Denoting by w_E the exogenously given reservation wage that has to be paid to outsider workers, the picture of the marginal productivity of the two kind of workers implied by the production function in eq. (2) is illustrated in figure 1.

[5]The existence of a positive job destruction rate for incumbent workers is a prerequisite for (1) to have a stationary solution in which both $L_I(t)$ and $L_E(t)$ are positive and constant over time.

[6]This can be straightforwardly verified by differencing $Y(t)$, respectively, with respect to $L_I(t)$ and $L_E(t)$.

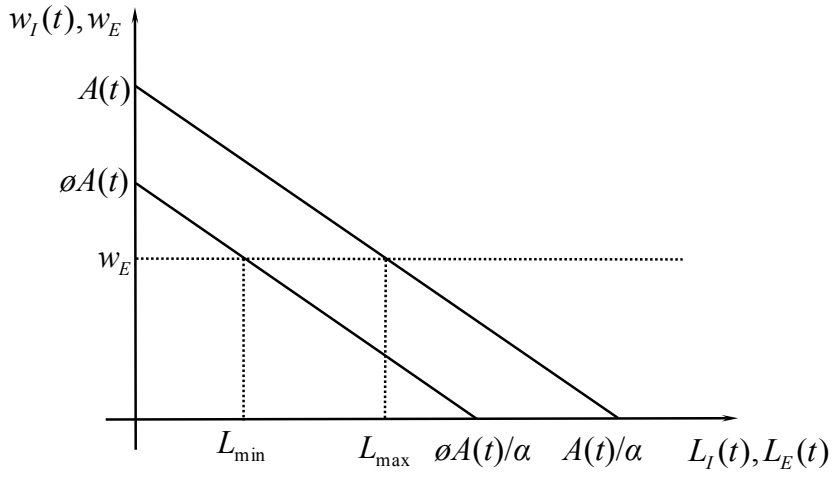

Figure 1: Marginal productivity of insiders and outsiders

The diagram in figure 1 illustrates, respectively, the marginal productivity of insiders and outsiders. In details, the former crosses w_E in $L_{\max}$, the latter in $L_{\min}$. The exogeneity of the reservation wage is the feature that identifies this setting as a partial equilibrium model (e.g. Huizinga and Schiantarelli, 1992). In the remainder of the paper, I will assume that the information conveyed by the two linear schedules is common knowledge both for workers and the firm.

Productivity disturbances evolve over time in a way that is known both by the firm and the workers. Formally speaking, supply shocks are assumed to follow an Ornstein-Uhlnebeck process. Consequently, the dynamics of A is given by

$$\dot{A}(t) = \kappa(\mu_A - A(t)) + \sigma_A \dot{x} \qquad \kappa > 0,\ \mu_A > 0,\ \sigma_A > 0 \tag{3}$$

where κ is the mean reversion rate, μ_A is the long-run mean of the process, σ_A is its instantaneous standard deviation while $\dot{x}$ is a standard Brownian motion with zero drift and unit variance (cf. Cox and Miller, 1967).

Insiders workers are assumed to be organized in an internal labour union whose task is to set the real wage of its members by taking into account the hiring and firing decisions of the firm.[7] Similarly with Dertouzos and Pencavel (1981), I will assume that the preferences of the union are given by a Stone-Geary utility function in which the reference values for the insider labour force and the insider wage are both set to zero. Therefore, the union of insider workers will have the following Cobb-Douglas objective function:

$$U(L_I(t), w_I(t)) \equiv (L_I(t))^{\beta} (w_I(t))^{1-\beta} \qquad 0 < \beta < 1 \tag{4}$$

where β $(1-\beta)$ measures the relative weight of employment (wage) in the union preferences.

[7] As argued by Drazen and Gottfries (1990), one may think of a union which organizes an industry and that is large relative to firms in that industry.

Eq. (4) implies that the union suffers a loss (experiences a gain) when the firm decides to reduce (increase) the stock of incumbent workers. Furthermore, the expression in (4) straightforwardly reveals that the welfare of outsiders is given no weight in the preferences of the wage-setting union.

In what follows, I develop the dynamic problem of the firm and the one of the union. Moreover, I discuss the existence of a stationary solution for both problems and I explore the dynamics of the model economy in the neighbourhood of that stable allocation.

2.1 The problem of the firm

Consistently with Solow (1985), the representative risk-neutral firm is assumed to maximize the discounted flow of its expected profit by taking the trajectory of the insider wages as given but - at the same time - considering that its hiring and firing decisions have an impact on the dynamics of incumbent workers, i.e., on the evolution of union's membership. Consequently, omitting the functional dependence of variables on time, the problem of the firm can be written as

$$\begin{gathered}\max_{\Phi, L_E} \int_{t=0}^{+\infty} \exp\left(-\rho t\right)\left(Y - w_I L_I - w_E L_E - \Phi L_I F\right) dt \qquad \rho > 0 \\ \text{s.to} \\ \dot{L}_I = L_E - \left(\Phi + b\right) L_I \\ L_I\left(0\right) = \overline{L}_I\end{gathered} \tag{5}$$

where ρ is the discount rate, F are firing costs while $\overline{L}_I$ is the initial number of insider workers.

In the subsequent analysis, I will make two assumption about firing cost. First, F is quadratic with respect to the outflow of incumbent workers decided by the firm. The underlying idea behind this hypothesis is that firing costs - associated with strikes, work-to-rule and union litigations - increase more than proportionally with respect to the outflow of insiders (cf. Cooper and Willis, 2009). Moreover, F is assumed to be directly proportional to the insider wage. Consequently, I will consider the following expression:

$$F = \frac{1}{2}\Phi L_I w_I \tag{6}$$

As far as the production function in (2) and firing costs in (6) are concerned, the first-order conditions (FOCs) for the problem in (5) are given by

$$\Phi L_I w_I + \Lambda = 0 \tag{7}$$

$$A\phi - \alpha L_E - w_E + \Lambda = 0 \tag{8}$$

$$\dot{\Lambda} = \alpha L_I - A + w_I + \Phi^2 w_I L_I + \Lambda\left(\Phi + b + \rho\right) \tag{9}$$

$$\lim_{t \to +\infty} \exp\left(-\rho t\right) \Lambda L_I = 0 \tag{10}$$

where Λ is the costate variable on the dynamic constraint describing the evolution of insider workers.

Eq.s (7) and (8) are, respectively, the FOC with respect to Φ and L_E. Moreover, the differential equation in (9) describes the optimal path of Λ, whereas (10) is the required transversality condition.

The results in eq.s (7) and (8) straightforwardly lead to

$$w_I = \frac{A\phi - \alpha L_E - w_E}{\Phi L_I} \tag{11}$$

Eq. (11) is a trivial manipulation of the FOCS with respect to Φ and L_E and it has two important implications. First, from an economic point of view it implies that, everything else being equal, the firm is willing to pay an insider wage that increases (decreases) as the size of incumbent workers decreases (increases). Therefore, according to the optimal hiring and firing decisions of the firm, insider workers gain from keeping their numbers small; indeed; as argued by Solow (1985), a large number of insiders increases the union utility loss driven by an increase of the firing rate and decreases the magnitude of w_I.

Moreover, from an analytical point of view, eq. (11) implies that for given values of w_I, Φ, w_E, A and L_I is also given the value of L_E. Consequently, the problem of the firm can be developed by focousing on one control only. Specifically, in the remainder of the paper, I will describe the solution of the firm problem in terms of the implied trajectories for the firing rate and the stock of insider workers.

Exploiting the results in $(7) - (9)$ and assuming that the firm actually knows the process that drives productivity shocks, the rates of growth of L_I and Φ can be written, respectively, as

$$\frac{\dot{L}_I}{L_I} = \frac{A\phi - w_E - \Phi\left(\left(\alpha + w_I\right) - \alpha b\right) L_I}{\alpha L_I} \tag{12}$$

$$\frac{\dot{\Phi}}{\Phi} = \frac{A - \alpha L_I - w_I + \Phi L_I w_I \left(b + \rho\right)}{\Phi L_I w_I} - \frac{\dot{L}_I}{L_I} \tag{13}$$

Given the initial value of L_I and the trajectory of w_I, the solution of the firm problem conveyed by eq.s (12) and (13) pins down the evolution of the insider labour force and the path of the firing rate that convey the optimal employment strategy of the representative firm. Therefore, in order to close the model, we have to find a way to link the trajectory of w_I to the endogenous - and exogenous - variables entering the two expressions above.

2.2 The problem of the union

Continuing the analogy with Solow (1985), the union of insider workers is assumed to set the wage of its members by taking into account that its membership dynamics is influenced by firm's hiring and firing decisions described above. Consequently, under the assumptions that (i) the union discounts utility at the same rate that the firm uses to discount profit and (ii), the union knows how the firm's makes its decision about the hiring of outsiders, the problem of the union is given by

$$\begin{gathered} \max_{w_I} \int_{t=0}^{+\infty} \exp\left(-\rho t\right) L_I^{\beta} w_I^{1-\beta} dt \\ \text{s.to} \\ \dot{L}_I = \tfrac{A\phi - w_E}{\alpha} - \left(\tfrac{\Phi(\alpha + w_I) + \alpha b}{\alpha}\right) L_I \\ L_I\left(0\right) = \overline{L}_I \end{gathered} \tag{14}$$

One may argue that the union of insider workers should set the wage of its members by taking into account also the dynamics of the firing rate. However, such an addition in the union problem would lead to two distinct inconsistencies. First, from a theoretical point of view, the assumption that the union set the insider wage by considering the effects of its choices on the firing rate is at odds with observation that in bad times outflows from employment are exacerbated and a substantial number of insiders may actually loose their positions.[8]

Moreover, from an analytical point of view, the inclusion of $\dot{\Phi}$ among the dynamic constraints of the union problem requires the existence of an initial value for the firing rate. However, according to the expressions in (5), such an initial value is a jump variable of the firm problem whose existence would call for the definition of a trajectory for the insider wage. Therefore, I will assume that union set the wage by considering how the decision of the firm to hire outsiders is affect by its wage policy, the level of the reservation wage and labour turnover costs.[9]

The FOCs for the problem in (14) are given by

$$\left(1 - \beta\right)\left(\frac{L_I}{w_I}\right)^{\beta} - \Gamma \frac{\Phi L_I}{\alpha} = 0 \tag{15}$$

$$\dot{\Gamma} = \Gamma\left(\Phi\left(\frac{\alpha + w_I}{\alpha}\right) + b + \rho\right) - \beta\left(\frac{w_I}{L_I}\right)^{1-\beta} \tag{16}$$

[8] The underlying assumption is that insiders cannot adopt a wage strategy that guarantees a firing rate equal to zero.

[9] In other words, the only piece of information required by the union to solve its problem is how the insider wage affects the evolution of the state variable of the firm problem. As conveyed by eq. (11), this amounts to assume that the union is able to observe the marginal productivity for insiders and outsiders as well as the amount of marginal firing costs paid by the firm.

$$\lim_{t\to+\infty} \exp(-\rho t)\,\Gamma L_I = 0 \tag{17}$$

where Γ is the costate variable on the dynamic constraint describing the evolution of union membership.

Eq. (15) is the FOC with respect to w_I. Moreover, the differential equation in (16) defines the optimal path of Γ, whereas (17) is the required transversality condition.

Exploiting the results in (15) and (16), the growth rate of the insider wage can be written as

$$\frac{\dot{w}_I}{w_I} = \frac{\Phi w_I - (1-\beta)\left(\Phi\left(\alpha + w_I\right) + \alpha\left(b+\rho\right)\right)}{\alpha\left(1-\beta\right)} - \frac{1-\beta}{\beta}\frac{\dot{L}_I}{L_I} \tag{18}$$

The expression in (18) reveals that rate of growth of the insider labour force spills over with a negative sign into the dynamics of w_I. Consequently, the incentive of insider workers of keeping their numbers small holds also on a dynamic perspective. In other words, the dynamic concern about the evolution of the insider labour force motivates the union to set a wage trajectory that limits the expansion of its members.[10]

Given the initial value of L_I, the solution of the union problem pins down the optimal trajectory of the insider wage by taking into account the way in which its membership dynamics is influenced by the optimal employment policy of the firm. Therefore, for any given value of the initial insider workforce, the expressions in (3), (12), (13) and (18) describe the whole dynamics of the model economy.

2.3 Steady-states

Within the model under investigation steady-state equilibria are defined as the set of terns $\{w_I^*, \Phi^*, L_I^*\}$ such that $\dot{w}_I = \dot{\Phi} = \dot{L}_I = \dot{A} = 0$. The elements of this set collect allocations in which the insider wage is stable over time and in each instant the number of hired outsiders is equal to the number of fired insiders. From a formal point of view, their derivation can be made in three simple steps. First, setting $\dot{L}_I = 0$ and $\dot{A} = 0$, respectively, in (3) and (12), the steady-state equilibrium of the insider labour force can be conveyed as

$$L_I^* = \frac{\mu_A \phi - w_E}{\Phi^*\left(\alpha + w_I^*\right) + \alpha b} \tag{19}$$

The expression in (19) reveals that when the long-run marginal productivity of outsiders is equal to the reservation wage, the steady-state insider workforce becomes equal to zero. Conse-

[10] From a formal point of view, this result is related to the fact that the union does not have a reference value for the wage entering its preferences. Whenever a wage reference is taken into account, the sign of the spillover of union membership into the insider wage dynamics also depends on the degree of wage orientation of the union, i.e., on the point size of the parameter β.

quently, the existence of a positive value of L_I^* requires that he long-run marginal productivity of outsiders is higher than their wage.[11]

Second, setting $\dot{\Phi} = 0$ and $\dot{A} = 0$, respectively, in (3) and (13), and plugging the result in (19) allows to write the steady-state firing rate as

$$\Phi^* = \frac{\alpha\left(\mu_A\phi - w_E - b\left(\mu_A - w_I^*\right)\right)}{\left(\alpha + w_I^*\right)\left(\mu_A - w_I^*\right) + \left(\mu_A\phi - w_E\right)w_I^*\left(b + \rho\right)} \tag{20}$$

Eq. (20) states that when the long-run marginal productivity of outsiders and insiders is equal, respectively, to the reservation wage and the equilibrium insider wage, the steady-state firing rate becomes indeterminate.

Furthermore, setting $\dot{w}_I = 0$ in (18) and plugging the results in (19) and (20) allows to derive an equation in which the only unknown is the steady-state value of the insider wage. Specifically,

$$\frac{\alpha\mu_A + \gamma_0 w_I^* - \left(w_I^*\right)^2}{\mu_A\left(\phi - b\right) - w_E + bw_I^*} = -\frac{\alpha\left(1-\beta\right) + w_I^*\left(1 - 2\beta\right)}{\left(1-\beta\right)\left(b+\rho\right)} \tag{21}$$

where $\gamma_0 \equiv \left(\mu_A\phi - w_E\right)\left(b + \rho\right) + \mu_A - \alpha$.

On the one hand, the expression on the LHS of (21) is a reverse-u-shaped function of w_I^* whose roots are, respectively, $P_0 \equiv 1/2\left(\gamma_0 - \sqrt{\gamma_0^2 + 4\alpha\mu_A}\right)$ and $P_1 \equiv 1/2\left(\gamma_0 + \sqrt{\gamma_0^2 + 4\alpha\mu_A}\right)$, whereas its vertical intercept is equal to $P_2 \equiv \alpha\mu_A\left(\mu_A\left(\phi - b\right) - w_E\right)^{-1}$. On the other hand, the expression on RHS is a linear function of w_I^* whose root (slope) is equal to $Q_0 \equiv -\alpha\left(1-\beta\right)\left(1-2\beta\right)^{-1}$ $\left(\delta \equiv -\left(1-2\beta\right)\left(\left(1-\beta\right)\left(b+\rho\right)\right)^{-1}\right)$.

Since unions usually assumed to be more employment-oriented than wage oriented (e.g. Sanfey, 1995), i.e., $\beta > 1/2$, the linear function on the RHS of (21) should have a positive slope and a positive (negative) horizontal (vertical) intercept equal to Q_0 ($Q_1 \equiv -\alpha/\left(b+\rho\right)$). Consequently, selecting a suitable values for μ_A, ϕ, b and w_E, eq. (21) admits a positive solution for the steady-state insider wage. Thereafter, once w_I^* is given, eq.s (19) and (20) allow to retrieve, respectively, the steady-state values for the insider workforce and the firing rate. The steady-state solution for w_I is illustrated in figure 2.

[11] This is a sign of the inefficiency of the steady-state equilibrium.

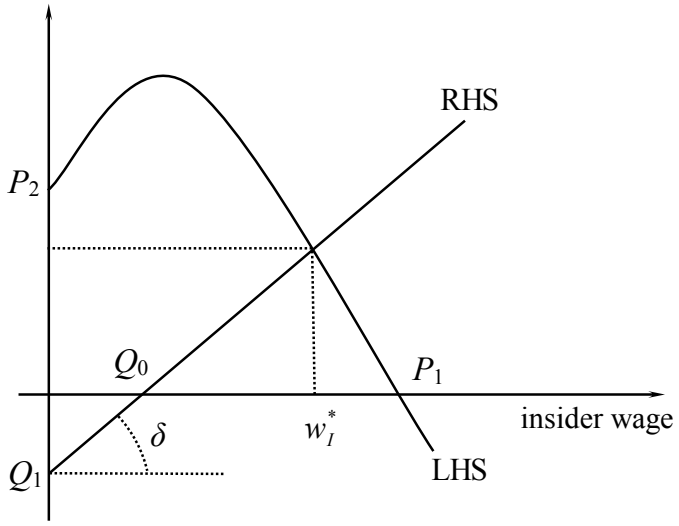

Figure 2: Steady-state

The diagram in figure 2 - together with eq.s (20) and (21) - shows that, everything else being equal, the higher (lower) the value of long-run productivity, i.e., the higher (lower) μ_A, the higher (lower) the steady-state values of the insider wage and the lower (higher) the firing rate. Moreover, since the steady-state values for the insider wage and the firing rate tend to move in opposite direction, eq. (19) reveals that the effects of long-run productivity movements on the steady-state value of union membership remain analytically uncertain.

2.4 Local dynamics

Taking into account the tern $\{w_I^*, \Phi^*, L_I^*\}$ derived above, the local dynamics of the model economy is described by the following linear 3×3 system:

$$\begin{pmatrix} \dot{w}_I \\ \dot{\Phi} \\ \dot{L}_I \end{pmatrix} = \begin{bmatrix} j_{1,1} & j_{1,2} & \frac{w_I^*(1-\beta)(\mu_A\phi - w_E)}{\alpha\beta\left(L_I^*\right)^2} \\ \frac{\alpha\left(\alpha L_I^* - \mu_A\right) + \left(\Phi^* w_I^*\right)^2 L_I^*}{\alpha L_I^* \left(w_I^*\right)^2} & \frac{\alpha(b+\rho)+\Phi(\alpha+w_I)}{\alpha} & j_{2,3} \\ -\frac{\Phi^* L_I^*}{\alpha} & -\frac{L_I^*\left(\alpha+w_I^*\right)}{\alpha} & -\frac{\Phi^*\left(\alpha+w_I^*\right)+\alpha b}{\alpha} \end{bmatrix} \begin{pmatrix} w_I - w_I^* \\ \Phi - \Phi^* \\ L_I - L_I^* \end{pmatrix} \tag{22}$$

where the two unspecified elements of the Jacobian matrix (J) are given, respectively, by

$$j_{1,1} \equiv \frac{\beta\Phi^* w_I^* (1+\beta) - (1-\beta)\left(\Phi^*\left(\alpha + w_I^*\right) + \alpha(b+\rho)\right)}{\alpha\beta(1-\beta)} \tag{23}$$

$$j_{1,2} \equiv \frac{w_I^*\left(w_I^* - (\alpha + w_I^*)(1-\beta)\right)}{\alpha(1-\beta)} \tag{24}$$

$$j_{2,3} \equiv \frac{\alpha\left(w_I^* - \mu_A\right) + w_I^*\Phi^*\left(\mu_A\phi - w_E\right)}{\alpha w_I^* \left(L_I^*\right)^2} \tag{25}$$

Considering a situation in which all the endogenous variables of the model economy have a positive stationary solution and taking appropriate values for μ_A, ϕ and w_E, it become possible to state that the steady-state allocation $\{w_I^*, \Phi^*, L_I^*\}$ is a saddle-point.[12] In this case, there is only one trajectory that satisfies (12), (13) and (18) which converges to the steady state while all the others diverge. In other words, in the proposed dynamic insider-outsider model the equilibrium path is locally determinate, i.e., taking a given initial value of the insider labour force $\left(\overline{L}_I\right)$, there is a unique vector $\left[\begin{array}{cc} \Phi(0) & w_I(0) \end{array}\right]$ in the neighbourhood of $\{\Phi^*, w_I^*\}$ that generates a trajectory converging to $\{w_I^*, \Phi^*, L_I^*\}$. Specifically, the values of $\Phi(0)$ and $w_I(0)$ should be selected to satisfy the transversality conditions in (10) and (17) by placing the system of (12), (13) and (18) exactly on the stable branch of the saddle point $\{w_I^*, \Phi^*, L_I^*\}$.

In the remainder of the paper, the stable saddle path followed by w_I, Φ, L_I and, implicitly, by L_E, is taken as the perfect-foresight - or rational expectations - path of the model economy.

3 Quantitative implications of the model

In order to offer a quantitative assessment of the dynamic insider-outsider model developed in the previous section, now I resort to some numerical simulations. Consequently, I provide a suitable calibration of the model economy. Thereafter, I derive the optimal trajectories for w_I, Φ, L_I and L_E generated by the linear dynamic system in (22) when productivity disturbances evolve according to the stochastic process in (3).[13]

3.1 Calibration

Following Eckstein et al. (2006), I map the period of the model economy into monthly figures over a typical 50 years horizon, i.e., the recurrent time span explored in business cycles analysis (e.g. Shimer, 2005). Thereafter, for reasons of data availability, the model is calibrated by taking as reference the US economy.[14]

Specifically, the slope of labour demand is calibrated by means of the value of the labour share provided by Kydland and Prescott (1982).[15] The value of the discount rate is taken from Giammarioli (2003). The separation rate is set by adjusting on a monthly base the JOLT-based quarterly estimations retrieved by Shimer (2005). The productivity differential between insiders and outsiders is fixed according to the US union wage premium estimated by Blanchflower and

[12] It would be possible to show that μ_A, ϕ and w_E are bifurcation parameters for the linear system in (22); indeed, the sign of $j_{1,3}$ and $j_{2,3}$ is strictly related to the magnitude of those parameters. Technical details are available from the authors upon request.

[13] MATLAB codes are available from the author upon request.

[14] The applicability of the insider-outsider theory to the US labour market is explicitly acknowledged by Solow (1985) and Lindbeck and Snower (2002).

[15] The hypothesis underlying the calibration of α is that the production function in (2) can be seen as the integration outcome of log-linear demand schedules for insiders and outsiders.

Bryson (2004). The figure of the reservation wage is obtained by combining the mentioned value of the labour share with the point estimation of the US replacement rate provided by van Vliet and Caminada (2012). Union's employment weight is fixed according to the estimates of Pencavel (1985). The value of long-run productivity is calibrated in order to replicate the long-run US employment rate. The productivity reversion rate is fixed at the value of the negative root of the system in (22) that generates the saddle path converging towards $\{w_I^*, \Phi^*, L_I^*\}$. Finally, the productivity variance is set in order to mimic the out-of-trend fluctuations of TFP retrieved by Fernald (2012).

The whole set of parameter values and their respective description is given in table 1.

PARAMETER	DESCRIPTION	VALUE
b	*Separation rate*	0.0250
ρ	*Discount rate*	0.0300
ϕ	*Productivity differential*	0.8300
α	*Labour demand slope*	0.6400
w_E	*Reservation wage*	0.3648
μ_A	*Long-run productivity*	1.1610
β	*Union's employment weight*	0.6490
κ	*Productivity reversion rate*	-1.1775
σ_A	*Productivity variance*	0.0085

Table 1: Calibration

The parameter values in table 1 lead to the following steady-state references: $w_I^* = 0.8395$, $\Phi^* = 0.7512$, $L_I^* = 0.6816$ and $L_E^* = 0.2640$. Those figures implies that the suggested calibration delivers a the picture of a labour market in which more than 70% of workers are unionized. Interestingly, this membership ratio is fairly close to the one actually prevailing in the US postal services over the last 30 years (cf. Hirsh and Macpherson, 2016).

3.2 The dynamics of w_I, Φ, L_I and L_E

Exploiting the parameters values in table 1, the implied trajectories of w_I, Φ, L_I and L_E are plotted in the four panels of figure 3 while some descriptive statistics are given in table 2.[16]

[16]Initial conditions are set at the values implied by deterministic steady-state values for A and L_I. Moreover, in order to mitigate the effect of initial conditions on the first stage of the simulation process, similarly to Shimer (2005), artificial series are derived by throwing away the first $1,000$ replications.

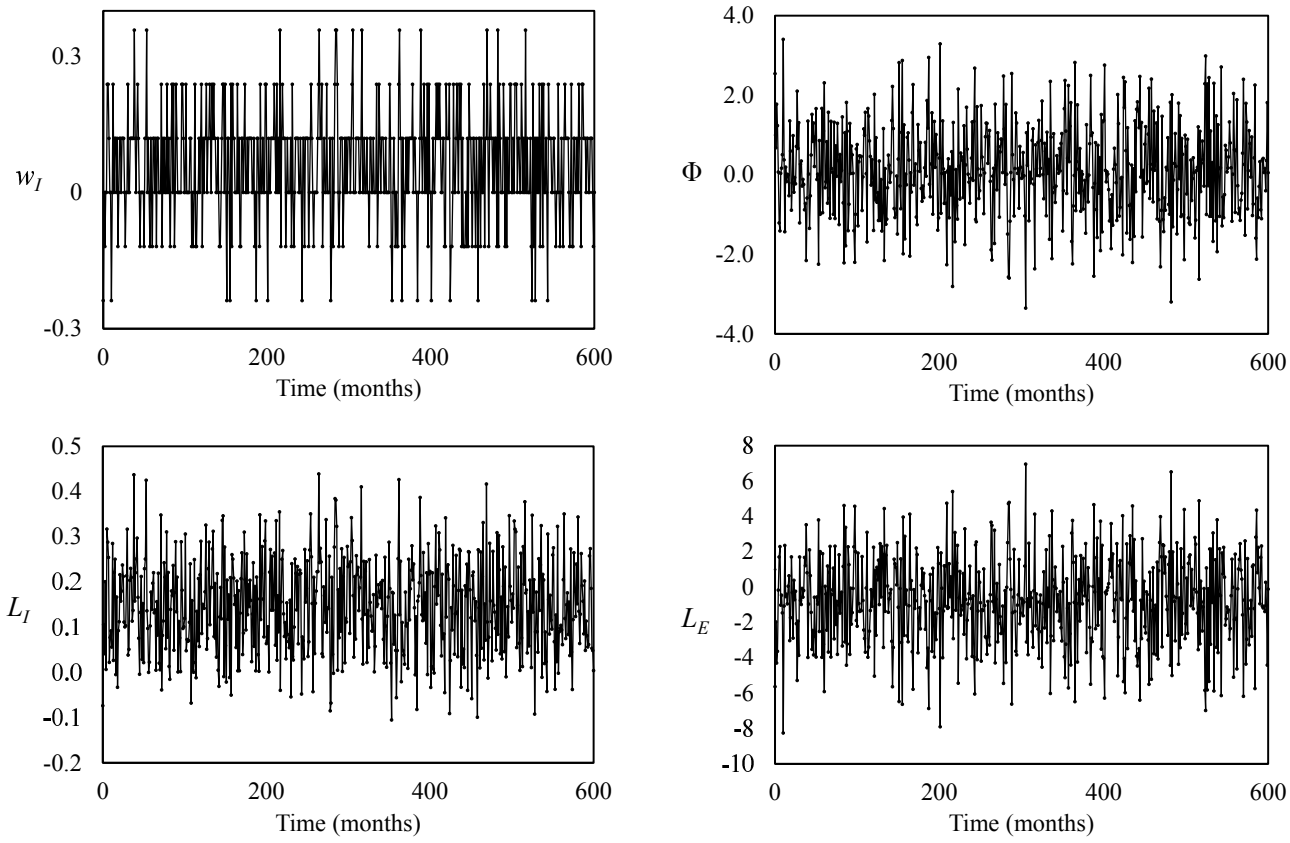

Figure 3: Fluctuations driven by productivity shocks
(% deviations from steady-state values)

		A	w_I	Φ	L_I	L_E
Standard deviation		0.0074	0.0013	0.0118	0.0011	0.0259
Autocorrelation		0.1017	0.0803	0.1032	0.0144	0.1041
Correlation matrix	A	1	0.9633	−0.9984	0.6444	0.9824
	w_I	–	1	−0.9473	0.8219	0.8992
	Φ	–	–	1	−0.6005	−0.9914
	L_I	–	–	–	1	0.4919
	L_E	–	–	–	–	1

Table 2: Descriptive statistics
(Logarithmic deviations from steady-state values)

The diagrams in figure 3 and the estimates in table 2 allow to draw the following conclusions. First, a positive (negative) productivity shock leads to an increase (decrease) in the insider wage, in the number of incumbent workers and in the number of entrants. By contrast, a positive (negative) productivity shock leads to a reduction (increase) of the firing rate of insiders.

Second, the theoretical framework developed in section 2 conveys a certain degree of wage stickiness; indeed, with the exception of the stock of incumbents, the amplitude of the insider wage fluctuations is definitely smaller with respect to the one of other concerned variables.[17] By contrast, the more volatile variable is the number of hired outsiders. This latter finding suggests that in this setting employment adjustments in expansions and recessions tend mainly to involve non-unionized workers (cf. Kugler and Saint-Paul, 2000).

Moreover, the fluctuations of insider labour force around the steady-state solution implied by the parameter values in table 1 display a clear asymmetric pattern. This result is at odds with the findings of Hizinga and Schiantarelli (1992) who argue that a partial equilibrium insider-outsider model in which the reservation wage is exogenous should be unable to generate asymmetries in the way in which employment adjusts in expansions and recessions.

Actually, the argument put forward by Hizinga and Schiantarelli (1992) holds whenever the firing rate is taken as given. However, as far as the interaction between the firing rate and the other variables usually enclosed in a dynamic insider-outsider model is recognized, the realization of asymmetric employment adjustments becomes a real possibility. An explanation of this pattern can be provided by analyzing in details the effects driven by productivity shocks on w_I, Φ and L_I. As as I argued above, a productivity improvement leads to an increase of the insider wage and a reduction of the firing rate. The latter reduction is the channel that allows the insider workforce expansion. By contrast, a negative productivity shock leads to an increase of the firing rate. In this case, insiders will find in their best interest to adjust their wage downwards in order to offset the consequent contraction of the insider workforce.

More in general, regarding the different movements of the insider labour force after positive and negative productivity shocks, it is possible to conclude that in the model economy developed in section 2, insider workers tend to adjust their wage setting policy in the attempt to influence the hiring and firing decisions of the firm and retain their positions. In this way, upwards employment adjustments occurring during expansions result in being definitely more pronounced than downwards adjustments during recessions (cf. Begg at al., 1989; Galì, 2016).

4 Concluding remarks

In this paper, I develop a dynamic insider-outsider model grounded on optimal control techniques. Specifically, considering a situation in which time continuous and it extends over an infinite horizon, I set forth an inter-temporal framework in which a union of insiders is called in to choose the trajectory of the wage of its members by taking into account the optimal hiring policy of firms that, in turn, are assumed to decide the firing rate of insiders as well as the number of outsiders to hire.

[17]The relative rigidity of wages is a well-know feature of business cycles (e.g. Shimer, 2005; Ravn and Simonelli, 2007).

Preserving the informational requirements of textbook insider-outsider models, i.e., under the assumption that insiders are able to observe their own productivity, the one of outsiders and the labour turnover cost paid by the firms (cf. Lindbeck and Snower, 1989), the implementation of the maximum principle reveals that within this framework the initial stock of incumbents may well pin down an equilibrium of the model economy in which the trajectories of the insider labour force, the insider wage and the firing rate are jointly determined in a well defined manner.

Moreover, analyzing the FOCs of the model and the dynamics of the insider wage, I show that the proposition according to which incumbents have an incentive to keep their numbers small holds both on static dynamic terms. Furthermore, relying on the outcome of a set of numerical simulations, I show that within this model economy insiders tend to adjust their wage in order to retain their positions and such a behaviour leads to asymmetric employment adjustments and wage stickiness even in a partial equilibrium perspective (cf. Begg at al., 1989; Hizinga and Schiantarelli, 1992; Galì, 2016).

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