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Abstract

After the collapse of the Bretton Woods system, the evidence on the purchasing power parity (PPP) in the long run is still a matter of debate. The difficulties of the problem are the possible nonstationarity of relative price indices and nominal exchange rates. The traditional ways to deal with nonstationarity such as unit root model and cointegration have some problems. In this paper, to deal with nonstationarity, we apply the Hodrick－Prescott(HP) trend-cycle filter in real business cycle literature (Hodrick and Prescott，1981) which can give a nonlinear smooth-trend, and we find that after the 1970s float, the monthly HP trends of US dollar/UK sterling and Deutsche marks/US dollar have certain relevance with their corresponding HP trends of relative consumer price indices. This result indicates that there is no strong evidence to directly deny that the PPP is valid in the long run. In this sense, it is not reliable to directly deny the belief of monetary neutrality!

Keywords: HP filter, purchasing power parity, monetary neutrality

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1. Introduction

Purchasing power parity (PPP) is one of most important theoretical concepts in international economics. PPP also remains a valid benchmark and widely used criterion for judging the extent of successful international arbitrage, and a durable yardstick in theoretical and empirical analysis of the equilibrium real exchange rate (Isard 1995).

However, whether PPP is valid in the short-run or over the long-run is a problem. Indeed, much of economists' faith in PPP derives from a belief that over most of the past century, price level movements have been dominated by monetary factors. If price indices movements are dominated by monetary shocks, and if money is neutral in the long run then even if PPP is not valid in the short-run it will valid over the long run. For example, early monetary models of the exchange rate assume continuous purchasing power parity [see survey in Taylor(1995)]. Although sticky-price exchange rate models of the kind originally developed by Dornbusch(1976) allow the exchange rate to deviate from PPP in the short-run, it is retained as long-run equilibrium condition.

After the collapse of the Bretton Woods system, the excessive volatility of nominal exchange rates and the relatively smooth macroeconomic variables in the short run rejects PPP as a hypothesis about the short run relationship between nominal exchange rates and relative price indices. The evidence on long-run PPP is still a matter of debate. [see survey in Froot and Rogoff(1995)，Rogoff(1996)].

The difficulties of the problem are the possible nonstationarity of relative prices indices and nominal exchange rates. In addition, how to distinguish long-run and short-run equilibrium is still a open question in econometric practice. The traditional ways to deal with nonstationarity such as unit root model and cointegration have some problems. In this paper, to deal with nonstationarity, we apply the Hodrick－Prescott(HP) trend-cycle filter in real business cycle literature (Hodrick and Prescott，1981) which can give a nonlinear smooth-trend, and we find that after the 1970s float, the monthly HP trends of US dollar/UK sterling and Deutsche marks/US dollar have certain relevance with their corresponding HP trends of relative consumer price indices. This result indicates that there is no strong evidence to directly deny that the PPP is valid in the long run. In this sense, it is not reliable to directly deny the belief of monetary neutrality!

The remainder of the paper is set out as follows. In section 2 we briefly introduce the PPP hypothesis and criticize the recent testing methods. In dealing with the possible nonstationarity of nominal exchange rates and relative price indices, econometricians recently have used the unit root test and cointegration based on unit root model. However, the unit root model has some problems such as linear simplicity, marginal stable. In addition, a unit root model can statistically be better described by a nonlinear trend (Bierens，1995) . In section 3 we introduce some trend-cycle filters which concern the choice of observation references in economics theory. In section 4 we give our results and conclusion.

2、PPP Hypothesis and Critiques about the Tests Methodology

2.1 Purchasing Power Parity Hypothesis

PPP theory has two main variants. The absolute PPP hypothesis states that the exchange rate between the currencies of two countries should equal the ratio of the price levels of the two countries. Specifically,

$$S = P/P^* \qquad (1)$$

Where S is the nominal exchange rate measured in units of currency A per unit currency B, P is the price level in country A, and P* is price level in country B. The relative PPP hypothesis states that the exchange rate should bear a constant proportionate relationship to the ratio of national price levels, in particular,

$$S = kP/P^* \qquad (2)$$

Where k is a constant parameter. The logarithmic transformations of （1）and（2）have the form

$$s = a + p - p^* \qquad (3)$$

where s、p、p* are the logarithms of S、P、P* and a=0 under absolute PPP.

To the extent that information on national price levels is readily available in the form of price indices but not as absolute price levels, absolute PPP may not be a useful operational hypothesis. Most of the empirical literature, in any case, has relied on price indices in examining the validity of PPP, and has thus focused implicitly on the relative PPP hypothesis.

The PPP hypothesis is frequently restated in terms of the real exchange rate (Q), defined as

$$Q = SP^*/P \qquad (4)$$

Under this terminology, （1） and （2）each imply that the real exchange rate is time-invariant.

2.2 Testing Methods and Critiques

Recently, Froot and Rogoff（1995）give a full review of the methodological issues surrounding the investigation of PPP. In this section, we will give a brief introduction about them. At the same time, we will give some critiques about the methodological issues from the perspective of nonlinear evolution.

The early formal empirical tests about relative PPP was to run regressions of the form (for example, Frenkel 1981),

$$s_t = \alpha + \beta(p_t - p_t^*) + \varepsilon_t \qquad (5)$$

Where the null hypothesis is $\beta=1$. However, the econometrics of the early tests has some problems. Firstly, it is well known that stationarity of residuals in equation (5) is required for standard hypothesis testing, but relative price indices and exchange rates are possible nonstationary and this may cause the error term of equation (5) nonstationary. Secondly, as a proposition in positive economics, the PPP hypothesis does not make any general assertion about the direction of causation between exchange rates and national price levels. It is quite consistent with a process of two-way causation, with exchange rates adjusting to changes in the ratios of national price levels while inflation rates are simultaneously responsive to changes in exchange rates. Neither exchange rates nor national price levels are exogenous variables. Therefore, there are no reasons to assume exchange rate at the left of equation (5), while relative price at the right ex ante. At last, there are still some forces that may cause $\beta=1$ not to hold. For example, according to Balassa－Samuelson hypothesis (1964) and other theories, the existence of nontraded goods will make the transition of price shock from one market to another impossible, which may make $\beta=1$ not hold while relation (5) hold.

Both the stage-two and stage-three tests we consider next are explicitly designed to deal with the nonstationarity. In the stage-two tests, the null hypothesis becomes that the real exchange rate follows a random walk, with the alternative hypothesis being that PPP holds in the long run. These tests stand those from stage-one tests on their head: they impose – rather than estimate –the hypothesis that $\beta=1$, and test – rather than impose – the hypothesis that the (log of the) real exchange rate

$$q_t = s_t - p_t + p_t^*. \qquad (6)$$

is stationary. The focus of this method is whether the real exchange rate is stationary or a random walk with a constant drift. The econometric model of the random walk with a constant drift has a unit root in time series. The unit root tests often amount to a test for mean reversion in the equation

$$\Delta q_t=\theta+\lambda q_{t-1}+\nu_t \qquad (7)$$

where ν_t is $N(0,\sigma^2)$， and λ expected to between zero and minus one, is called the convergence speed. The selected assumption $H_1:\lambda<0$ is the evidence of stationary. Failure to reject the null hypothesis $H_0:\lambda=0$ using the standard unit root tests is viewed as a failure of the PPP hypothesis because q_t then exhibits no tendency toward mean.

In the stage-three tests, the cointegration methods are applied（Engle and Granger,1987). The techniques are designed to test for long-run equilibrium relationships, for which the adjustment mechanism remains unspecified. Cointegration tests are thus liberated from stage-one concerns about endogenety and left-out variables. In other words, stage-two tests ask whether the real exchange rate $q_t = s_t - p_t + p_t^*$ is stationary. Stage-three tests ask only whether

$$s_t - \beta_1 p_t + \beta_2 p_t^*. \qquad (8)$$

is stationary for any constant β_1 and β_2. Any incremental power from stage-three tests over stage-two tests must therefore come from relaxing the symmetry and proportionality restrictions that $\beta_1=\beta_2=1$. However, the stationary tests of $s_t-\beta_1 p_t+\beta_2 p_t^*$ still apply the unit root tests in the stage-two.

The so-called unit-root revolution in econometrics in the 1980s has made the unit-root model very popular in economics because it is the simplest linear model of permanent shocks (Nelson and Plosser 1982). However, We should point out that the unit-root model is only marginally stable in the parameter space, since a slight variation in structural parameters will lead to either damped or explosive behavior. We call this property "pattern instability" which is a common feature for linear models of business cycles.（Chen1996a,1996b,1999）.

In addition, statistic inference based on i.i.d. process may not distinguish deterministic process from stochastic process（Blatt, 1978）. It is true that spectral analysis may produce spurious cycle generated by increasing trend. For example, the unit-root process is difficult to be ruled out by statistic tests （Nelson and Kang，1981）. Statistically, a unit root model can be better described by a nonlinear trend (Bierens，1995). The simplest example is a pieced linear trend shift model. For example, the shift of GNP growth trend caused by the oil shock could lead to the spurious unit root statistically.（Perron，1989）. This is why it is very difficulty to reject the random walk hypothesis in the stage-two and stage-three tests of the PPP hypothesis. Therefore, properly dealing with the deterministic trend is very important to the study of unit root process debate（Campbell and Perron，1991）.

3. Observational Frames of Reference

The most difficult problem in testing the long run PPP hypothesis is the possibility of nonstationarity of relative price indices and nominal exchange rate. In figure 1.1 and 1.2 we can see that during the float rate system, the nominal exchange rates between US dollar and Deutsche mark, UK sterling and US dollar are excessively volatile and nominal exchanges rates as well as relative indices seemed to be nonstationary. In fact, this is the most striking feature of economic movements in most economic indicators about the industrial society. A major challenge in economic time-series analysis is how to deal with time evolution. Accordingly, the core problem is not noise-smoothing but trend-defining in economic observation and decision making, because the observed patterns of business cycles are more sensitive to trend perspectives than to smoothing techniques. A short-time deviation may be important for speculative arbitrageurs, while the shape of the long-term trend can be critical to strategic investors. Certainly, investors in a real economy have diversified strategies and time horizons. The interactive nature of social behavior often forms some consensus on business cycles. This fact suggests that a relatively preferred reference exist in economic studies.（chen1996a）.

Measurement and theory cannot be separated form each other. An oversimplified theory may greatly distort the measurement. The linear detrending approach dominates econometric analysis because of its mathematical simplicity. There are two extreme approaches in econometric analysis: the trend-stationary (TS) approach of log-linear detrending (LLD) and the difference-stationary (DS) approach of first differencing (FD) (Nelson and Plosser 1982).

$$X_{fd}(t)=\log S(t)-\log S(t-1)=\log\left(S(t) \big/ S(t-1) \right) \quad (9)$$

$$X_{lld}(t)=\log S(t)-(a+bt) \quad (10)$$

In econometrics, a linear stochastic filter of first differencing or pre-whitening is widely applied to obtain an equilibrium picture of economic fluctuations（see FD series in Fig.2.1－3）. However, the differencing is not a whitening device but a “violeting” one, because it dampens low-frequency components but amplifies high-frequency components. Differencing generates an erratic time series when the time unit is not small compared with the length of serial correlations. The discontinuity caused by differencing can be described by a step function whose Fourier transform is a delta function (Papoulis， 1977). Therefore, the resulting time series are erratic and short-correlated.

In neo-classical growth theory, the equilibrium path is characterized by an exponential growth. Correspondingly, the logarithm of macroeconomic indicators should follow a linear trend. The resulting cycles are long-correlated （see LLD series in Fig. 2.1－3）. The problem is that the measurement of average growth rate and cycle variance depends on the choice of time boundaries.

An intermediate trend between FD and LLD is a nonlinear smooth-trend obtained by the HP(Hodrick－Prescott) filter (see HP series in Fig.2.1－3）in the real business cycle literature (Hodrick and Prescott, 1981). The HP filter is a linear transformation of the original time series {S(i)} into a smooth time series {G(i)} by minimizing the following objective function

$$\Sigma [S(i) - G(i)]^2 + \lambda \ \Sigma \{ [G(i+1) - G(i)] - [G(i) - G(i-1)] \}^2 \quad （11）$$

deviations from {G(i)} are considered as the cyclic component：

$$C(i)=S(i)-G(i) \quad （12）$$

Empirical time series can be decomposed into smooth growth series {G(i)} and cyclic series {C(i)}. The characteristic period of HP short cycles depends on the penalty parameter of λ， λ is chosen in such a way that the variance of the growth component is much less than that of the cyclic term (Hodrick and Prescott，1981). In practice, the recommended value of λ is 400 for annual data, 1,600 for quarterly data, and 14,400 for monthly data.

The penalty term in equation （11） is the second difference in the growth series. When λ goes to infinity, the growth trend is a linear function. For logarithmic data, log-linear detrending corresponds to the limiting case in HP decomposition. HP growth trends are less rigid than the log-linear function, and HP cycles are less erratic than differencing. Certainly, HP growth trends provide little information about growth cycles and long waves.

Then the question is what kind of trend is proper for catching the pertinent features of the underlying mechanism? We can only solve the issue by comparing empirical information revealed from competing approaches. Chen（1996a，1996b,1999）tests the three detrending methods of FD, LLD and HP by using Standard & Poor 500 stock price composite monthly index FSPCOM (the source of the data is the Citibase and the data covers a period from 1947 to 1992). Below are some of his results.

In Fig.2.1－3 we can see that the pattern and magnitude of correlation and variance depend on the choice of reference trends (Table 1). The LLD indicates the largest time-window of the entire observational period. The FD implies the shortest time-window of one time unit in business observation when macroeconomics trends are completely ignored. The HP implies a medium time-window in the range of business cycles of several years. The nature of business fluctuations from competing detrendings are quite different. For a FD observer, the variance is the smallest among three detrended series, and correlations are the shortest. For a LLD observer, the variance is the largest and the correlation period is close to 29 years. The HP results are between the two extremes. This finding reveals the critical role of the time scale in choosing observation windows and reference trends.

The FD detrending is the bad observation reference in the business cycle study. Economically speaking, the FD detrending in econometrics implies that the level information in price indicators can be ignored in economics behavior. This assertion may conflict with many economic practices, since traders constantly watch economic trends, and no one will make an investment decision based only on the current rate of price changes. However, most patterns of HP cycles show flow and stock variables are all important in the economic dynamics.

The essence of trend-cycle decomposition is finding an appropriate time window, or equivalently, a proper frequency window, for observing time-dependent movements. From the view of signal processing, log-linear detrending is a low-pass filter or wave detector, while first differencing is a high-pass filter or noise amplifier. Obviously, the FD filter is not helpful for detecting low-frequency cycles. The main drawback of LLD detrending is its over-dependence on historical boundaries. The HP filter has two advantages. First, it is a localized approach in detrending, without the problem of boundary dependence. Second, its frequency response is in the range of business cycles（King and Rebelo，1993）. Some economists argue that the HP filter may transform a unit root process into false cycles. A similar argument is also valid for the unit root school because the FD filter obscures complex cycles by amplifying random noise. No numerical experiment can solve a philosophical issue. In the history of science, the choice of a proper reference is only solved as an empirical issue, i.e., whether or not we can discover some patterns and regularities that are relevant to economic reality（Chen, 1996b）. From the view of complex systems, the linear approach is not capable of describing complex patterns of business cycle (Day and Chen，1993), we need a better alternative to detrending. Statistically, a unit root model can be better described by a nonlinear trend（Bierens，1995）.

In this paper, we will demonstrate that after the 1970s float, the monthly HP trends of US dollar/UK sterling and Deutsche marks/US dollar have certain relevance with their corresponding HP trends of relative consumer price indices. The source of this data is the Citibase. The data covers a period from Jan. 1973 to Dec. 1995 for US dollar/ UK sterling, and from Jan. 1973 to Aug. 1995 for Deutsche mark/ US dollar.

4、Results and Conclusion

The key to apply the HP filter is how to choose the penalty parameter of λ. We firstly choose the recommended λ value 400, 1600, 14400 (Hodrick and Prescott, 1981) to observe the relation between the HP trends of nominal exchange rates and their corresponding HP trends of relative consumer price indices. In Fig. 3.1－3 and Fig. 4.1－3， although we can not find certain relevance between the HP trends of nominal exchange rates and their corresponding HP trends of relative consumer price indices, these HP trends are really sensitive to the λ value. Trying again and again, we find when $\lambda = 24883200$， there is certain relevance between the monthly HP trend of Deutsche mark/US dollar and the corresponding HP trend of relative consumer price indices(see Fig. 3.4)； and so is the monthly HP trend of US dollar/UK sterling and the corresponding HP trend of relative consumer price indices when $\lambda = 2073600$ (see Fig. 4.4). This result indicates that there is no strong evidence to directly deny that the PPP is valid in the long run. In this sense, it is not reliable to directly deny the belief of monetary neutrality!

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Table 1. Detrending Statistics for logarithmic FSPCOM (Standard and Poor 500 index) monthly

Detrending	Mean	Std	variance	T_0 (month)	P_{dc} (year)
FD	0.012	0.1123	0.0126	1.94	0.7
HP	0.008	0.2686	0.0722	8.93	3.0
LLD	0.427	0.3265	0.1066	85.6	28.5

Where T_0 is the decorrelation length measured by the time lag of the first zero in autocorrelations, and P_{dc} is the decorrelatioon period for implicit cycles: $P_{dc} = 4 * T_0$。

Fig 1.1Deutsche mark/ US dollar and relaive consumption price indices, from Jan. 1973 to Aug. 1995

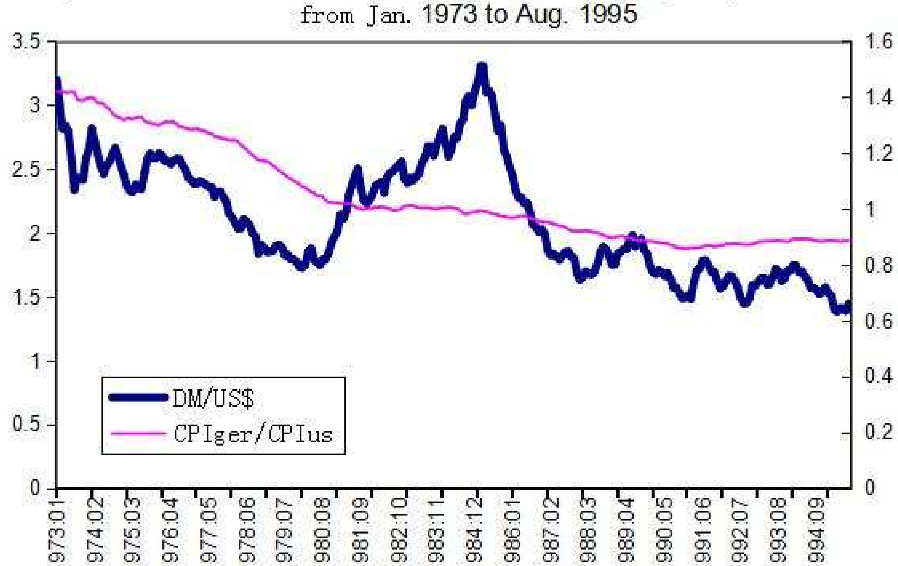

Fig 1.2 US dollar/UK sterling and relaive consumption price indices, from Jan,1973 to Dec. 1995

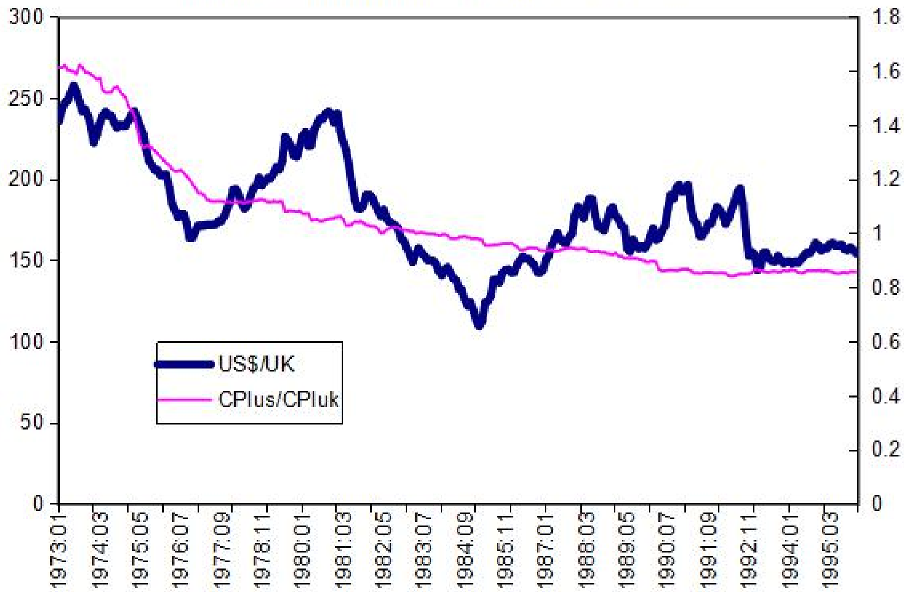

Fig2.1 The HP trend and LLD(log-linear) trend for monthly FSPCOMln

(1947-92,N=552)

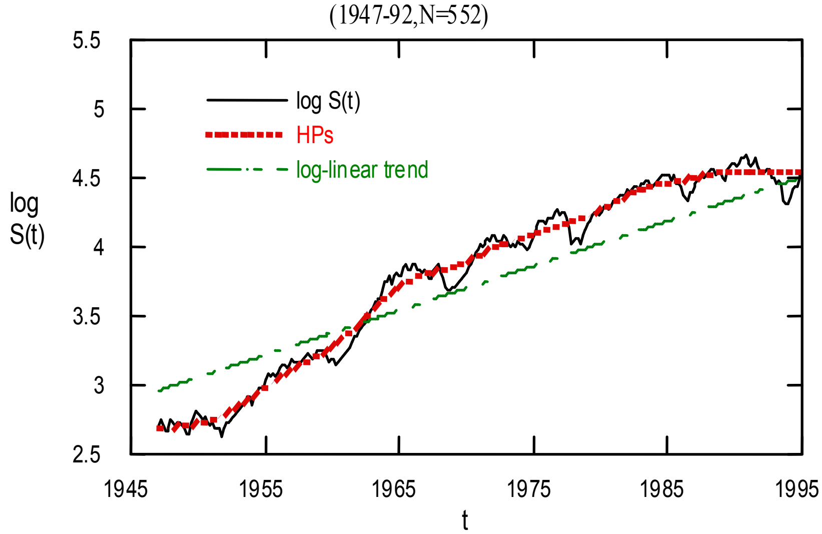

Fig.2.2 Cycles of monthly FSPCOMln index from competing detrending references (1947—92,N=552)

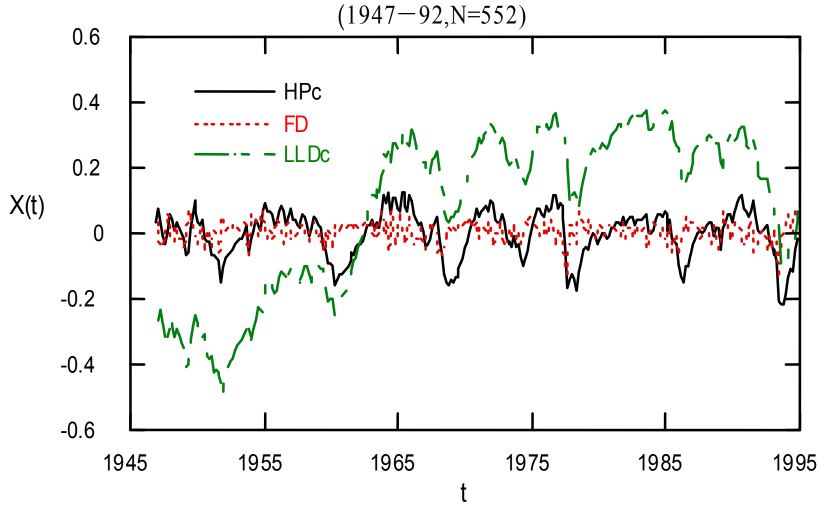

Fig.2.3 Autocorrelations of monthly FSPCOMln cycles (1947—92,N=552)

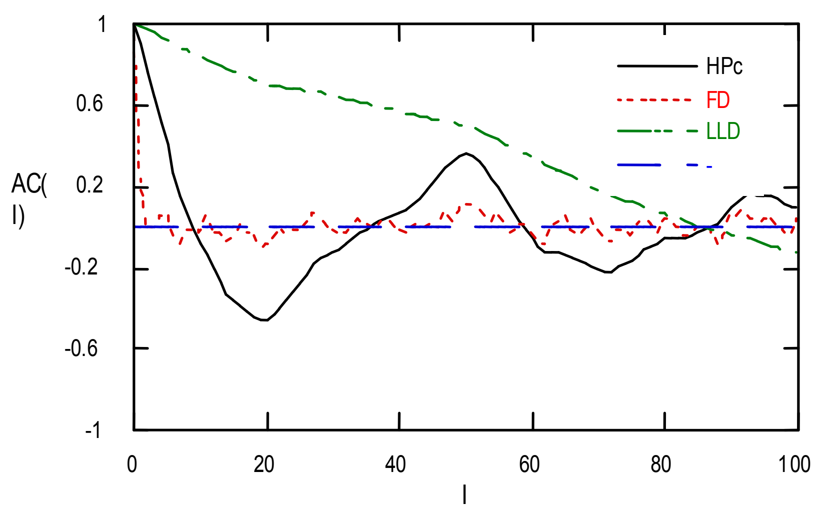

Fig 3.1 HP trends of Deutsche Mark/US dollar and relative consumption price indices, λ=400

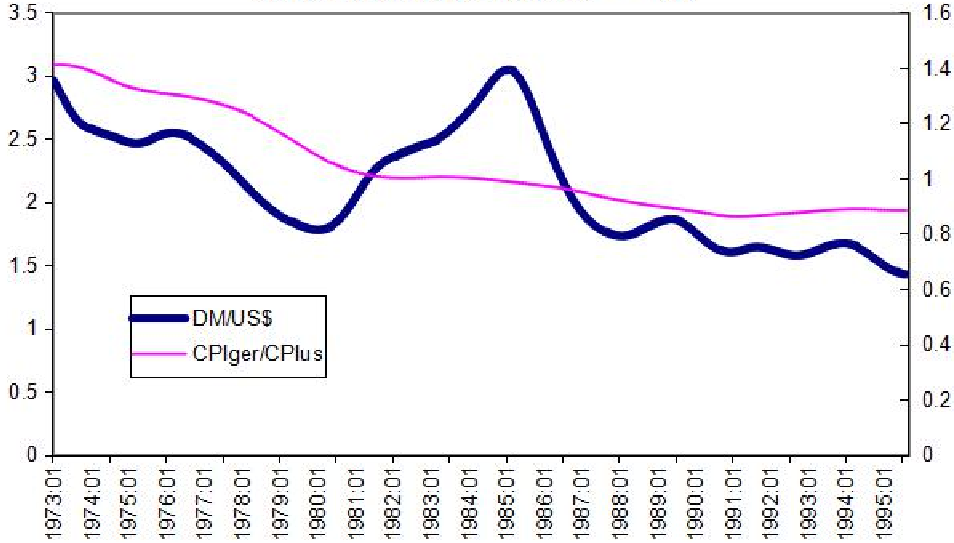

Fig 3.2 HP trends of Deutsche Mark/US dollar and relative consumption price indices, λ=1600

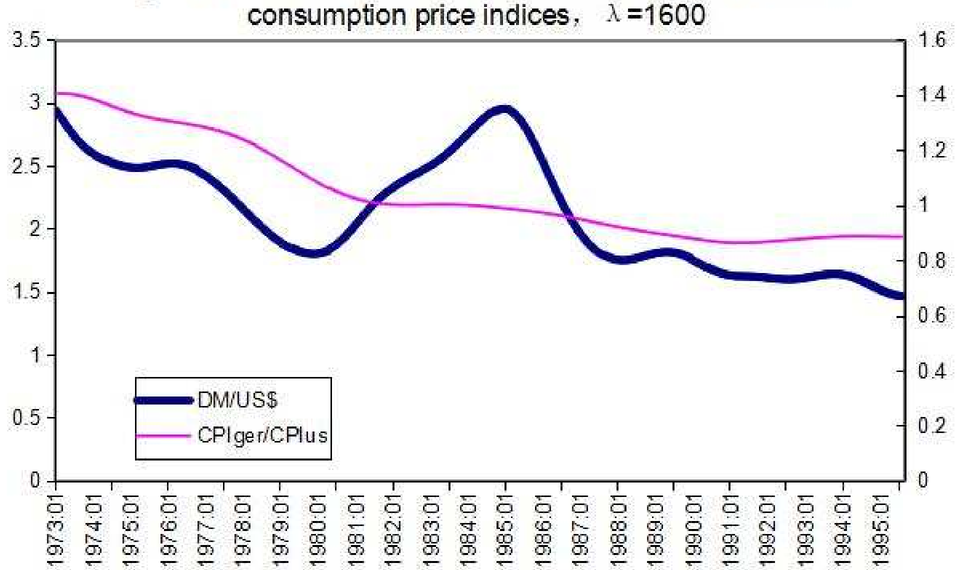

Fig 3.3 HP trends of Deutsche Mark/US dollar and relative consumption price indices, λ=14400

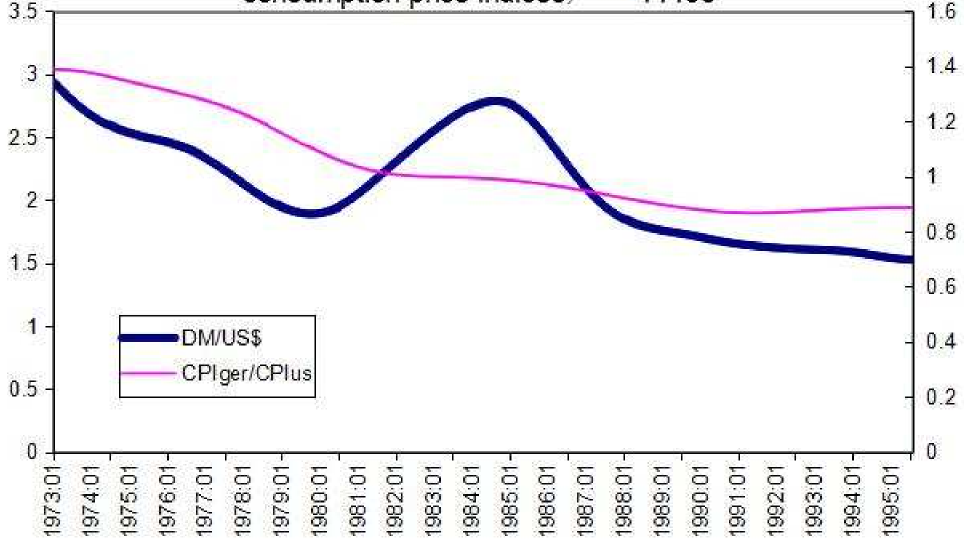

Fig 3.4 HP trends of Deutsche Mark/US dollar and relative consumption price indices，λ=24883200

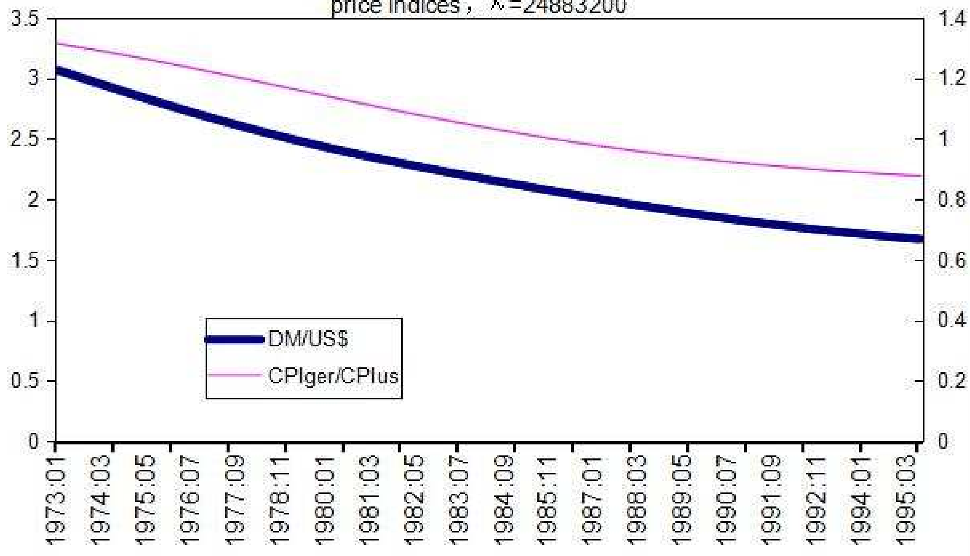

Fig 4.1 HP trends of US dollar/UK sterling and relative consumption price indices, λ =400

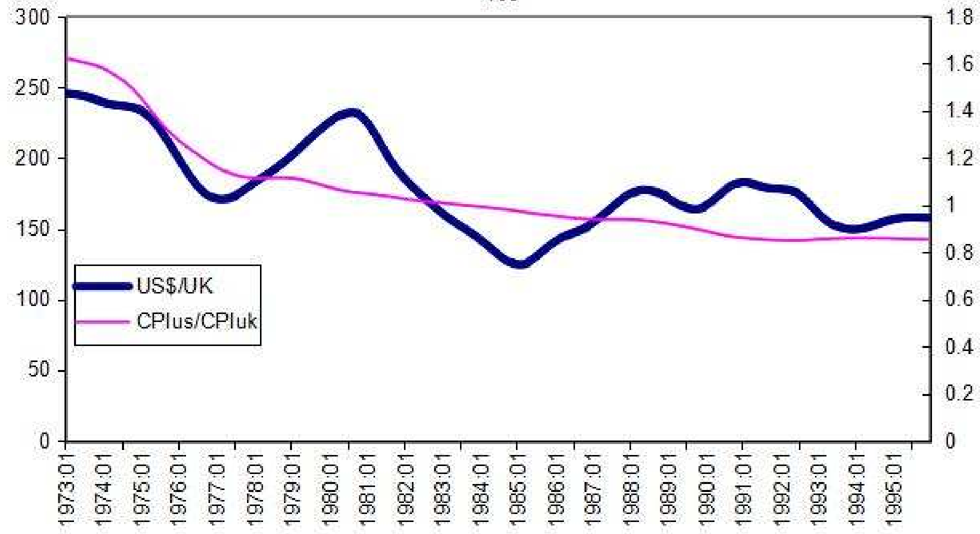

Fig 4.2 HP trends of US dollar/UK sterling and relative consumption price indices, λ =1600

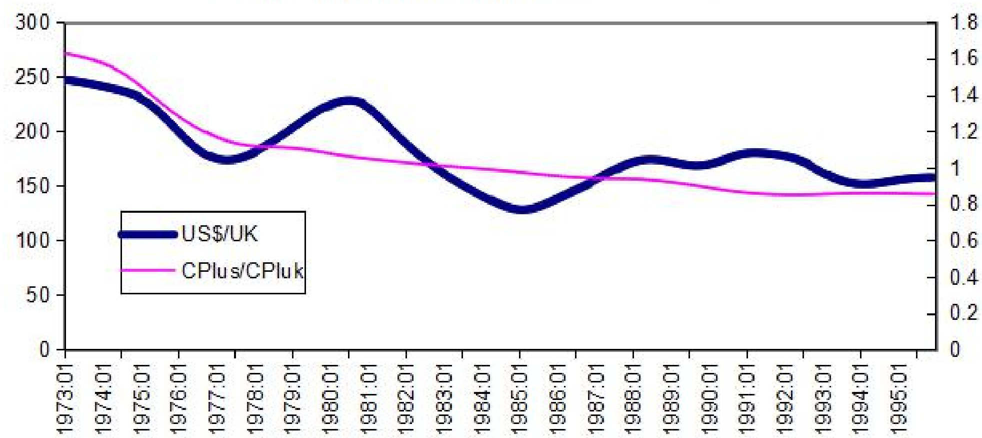

Fig 4.3 HP trends of US dollar/UK sterling and relaive consumption price indices， λ =14400

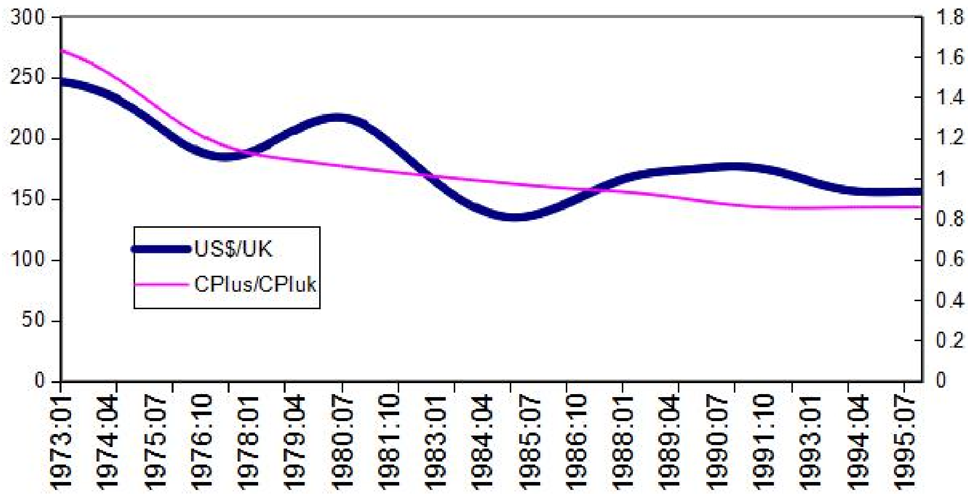

Fig 4.4 HP trends of US dollar/UK sterling and relative consumption price indices， λ=2073600