Risk and Unraveling in Labor Markets

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Abstract

A two period labor market is considered in which workers' quality is revealed in the second period. A signal – revealed to either workers, firms or both at the beginning of the first period – is correlated with the final quality. Under all assumptions about the distribution of information in the first period there exists an equilibrium in which firms only make offers in the second period and workers accept no offer in the first period. Nonetheless, early contracting is also an equilibrium if certain conditions on preferences of firms and workers are met. Workers have to be risk averse or firms risk loving with respect to expectations appropriate to the relevant information structure. Thus the conditions for unraveling depend on the information available to the two sides of the market.

KEYWORDS: Unraveling, Risk Aversion, Asymmetric Information
JEL CLASSIFICATION: C78, D82, D83

1 Introduction

Most markets are open more or less continuously, and unless a market is particularly thin, agents can participate in transactions whenever they wish. The timing of transactions is thus not an issue in itself. Some markets, however, are characterized by their periodic nature, that is transactions are to take place within specified periods of time which are separated by possibly lengthy periods of market inactivity. This constraint may make the timing of transactions a strategic variable, in particular when timing decisions affect the quality or quantity of goods or services available. In some cases, there is a strong tendency for transactions to take place

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ever earlier, even predating the official start of the trading period.

Roth and Xing (1994) provide a detailed survey of a large number of markets where unraveling, that is the move of transactions to increasingly earlier dates, has occurred. Many of those markets are for entry level professional positions, such as medical doctors, lawyers and junior university appointments. Admission into top level colleges also seems to have been moving towards earlier deadlines as admission officials attempt to secure the most promising students. Moreover, they report a rising number of basketball players that are recruited straight from high-school without the once customary delay of college training and experience. While these examples are concentrated in the human resource arena, Roth and Xing (1994) also show that early contracting is a problem in the planning of post-season football games. As an indication that unraveling is not a new concern, they quote medieval legislation outlawing the buying and selling before the official start of periodic goods markets.

Periodic markets – and participants in markets – might suffer for unraveling. This has to do with the purpose of the market institution itself. By providing a well-specified time and location – not necessarily in terms of geographical space – in which buyers and sellers can interact, a market coordinates supply and demand, reduces search and transaction costs and increases the information available to the participants. Since unraveling moves a significant proportion of the transactions outside of the market, it impedes this coordinating function and, in extreme cases, may lead to its dissolution. Another problem arises when unraveling causes trades being executed before all information becomes known. If information is revealed over time, *ex-post* efficiency requires that agents wait as long as possible before they trade. Early trades therefore cannot be *ex-post* efficient. In general, observers and participants in markets agree on this, and attempts are often made to reverse the trend and implement binding transactions dates. In some cases these are adhered to, in others, however, the incentive to contract early seems to be too large. Indeed, risk aversion and uncertainty, for example on the potential output, provide incentives to agents on early contracting. Unraveling can be seen as an insurance phenomenon and a risk sharing attitude. The desirability of unraveling is ambiguous and the role of information seems to be a crucial component. At the same time, Roth and Xing (1994) observe, unraveling is not a universal phenomenon. Indeed, there are many markets where the contract procedures and dates are stable and market participants seem to have no interest in moving early. It is thus reasonable to ask how market institutions or the (behavioral) characteristics of the participants in a market affect the timing decision.

This paper attempts to cast some light on the role of information and risk attitude in the unraveling process. The importance of risk aversion and insurance in unraveling has been pointed out before in theoretical economics.[1]

[1] Other theoretical explanations of this phenomenon are available in the literature. Damiano et al. (2005)

In most of them the labor market is investigated, where firms and workers match pairwise in order to produce output. Matching is possible in two periods and uncertainty, introduced in several different ways, is resolved just prior to the second trading period. Li and Rosen (1998), for example, assume that workers are unproductive with a certain probability while firms are always productive. This individual uncertainty introduces aggregate risk about the relative supply of productive labor in the second period and, in consequence, about the distribution of output between workers and firms. In the unique equilibrium, workers that are productive with high probability contract early while the others prefer to wait for the second-period spot market.[2]

Li and Suen (2004) augment Li and Rosen (1998)'s model to allow for unproductive firms. Since aggregate uncertainty is necessary to induce early contracting, they introduce a random shock to the number of workers in the second period. Multiple equilibria with early contracting are possible due to the non-monotone relationship between the number of early contracts and the probability of being on the long side of the spot-market. This non-monotonicity is caused by the uncertainty over firms' productivity and workers that are more likely to be productive being matched earlier. The higher the uncertainty regarding the number of productive workers in the second period and the more risk-averse workers and firms are, the larger becomes the number of worker-firm pairs that contract early. Restricting the ability of agents to set the distribution of output negotiated in the first period is found to reduce unraveling.

Abandoning the assumption of binary productivity, Li and Suen (2000) examine a model where production is a function of both workers' and firms' quality. Two sided uncertainty is introduced through a continuous distribution of productivity. In equilibrium, matching in the second period is positive assortative in the sense that higher productivity workers are matched with higher productivity firms and output is shared.[3] Since quality is not known with certainty in the first period, early contracting cannot be assortative and reduces the variation of production agents expect albeit at the cost of lower expected output. As in the case of Li and Suen (2004) and Li and Rosen (1998), agents that have a higher expected productivity have a greater incentive to contract early.

Suen (2000) shows that unraveling does not have to start at the top of the type distribution. Productivity is again distributed continuously and final production depends on the quality of both the firm and worker in the final matching. While firm productivity is known by all agents, in the first period the quality of workers is unknown to firms and workers alike. With this set-up and endogenous division of final output, Suen (2000) finds that only mediocre

analyze the effect of costly search, Ostrovsky and Schwarz (2010) determine the link between disclosure information and early contracting, Halaburda (2010) shows that similarity of preferences might lead to early contracting and Fainmesser (2011) investigates how social networks affect the unraveling process.

[2]Output is assumed to be claimed by the side of the market that is in short supply in the spot market. By contracting early, workers and firms can agree to a division of output, removing the risk associated with being on the long side of the market.

[3]See, for example, Becker (1981).

firms have an incentive to contract early. Highly productive firms prefer to wait until workers' productivity is known in order to attain a best possible matching. Low-productivity firms cannot afford the wage demands of workers of *ex-ante* average workers.

Since most of the unraveling seems to occur in labor markets, this focus on matching seems appropriate. Contrary to Suen (2000) and Li and Suen (2000, 2004), in our model, firms and workers do not match in order to produce output. In all these papers early contracting, even given risk aversion, is always related to the distribution of productivity. We investigate a general framework to analyze the role of asymmetric information and risk attitude on early contracting. In our work preferences are common, but there is uncertainty over the quality of agents which is only resolved in the second period. Unlike previous work, preferences are over ranks rather than the quality of agents.[4] This complicates the analysis significantly. In a recent paper, Halaburda (2010) shows that similarity of preferences might drive the unraveling process. Yet, she considers a different setting than ours, using a mechanism design approach.

In our paper every agent (firms and workers) has the ability to compare the quality of workers if they know the quality of the workers. We consider three different assumptions about the information structure of the game and their effect on the matching process examined. Either both sides are informed by a cumulative distribution function on a signal about the quality of the workers or only one side gets this information. An informed agent is then able to compute the ranking probabilities for each worker. In the second period, the quality of workers becomes common knowledge. As a consequence, the set of mixed strategies profiles for firms and workers are related to the information structure. That defines the set of probability profiles made by firms and workers, depending on the firms' quality and their expectations on the resulting workers' expected quality in the second period. Under all three paradigms there exists an equilibrium in which firms only make offers in the second period and workers accept no offer in the first period. However, these equilibria rely on weakly dominated strategies and so are not trembling hand perfect equilibria. As a consequence, unraveling is found to be a possibility in all three information structure. The generality of the model and the very nature of the matching process will preclude very detailed predictions but point to the importance of preferences and information. We determine that early contracting is an equilibrium if workers are risk averse or firms risk loving with respect to expectations appropriate to the relevant information structure. Thus the conditions of unraveling depend on the information available to the two sides of the market given identical preferences.

In Section 2, we introduce notations, define the assumptions on preferences, the matching market and ranking functions. In Section 3, we define the different possible set of available

[4]This assumption is relevant in many matching situations. For example, graduate schools select students on ranks made from their previous degrees. An another example is the labor market for lawyers in Germany. Companies decide to interview new lawyers through their grades in a National exam (named *Staatsexamen*).

information and then investigate the matching market. We determine an equilibrium where there is no match at the first period. Finally, in Section 4, we find out conditions such as early contracting is also an equilibrium for all information structures.

2 Preliminaries

Firms and workers meet in a two period job market. Firms can hire at most one worker and workers can accept no more than one position. Job offers can be made by firms in both periods. If a worker accepts an offer in the first period, the offer is binding and both the firm and the worker exit the job market prior to the second period. Offers do not carry over from the first to the second period, that is, a worker has to accept or decline an offer in the period it is made.[5] Workers cannot propose job offers.

The ranking of firms is common knowledge; the ranking of workers, in contrast, becomes common knowledge between the first and second period in which the market operates. This assumption reflects several characteristics of many actual job markets, in particular for entry level positions. In most cases, the amount of public information about potential employers far exceeds the information available about job seekers.[6] The revelation of information about employees is intended to model the process of unraveling: contracting early implies contracting with less information. Three assumptions are made about the information available in the first period. In the first scenario, both workers and firms receive a signal about workers' rankings. In two other cases the information structure is asymmetric and either workers or firms receive the signal. Both assumptions can be justified with respect to actual market situations. On one hand it can be claimed that agents possess more information about themselves than outside observers do. On the other hand, applicants may lack knowledge about their competitors' quality and therefore their own ranking, while firms have experience in hiring and may be able to rank candidates with some accuracy.

Firms and workers have identical preferences regarding their respective potential matches. While this is clearly an oversimplification, it has the advantage of allowing the analysis to focus on the effects of early contracting. Since common preferences imply the existence of a unique stable matching in the second period market, any stable matching mechanism will result in the same final matching. The precise nature of the second period market will therefore not affect the results obtained. While this assumption might not hold very well in practice, there does seem to be at least *some* agreement over which employers or workers are more desirable in most actual labor markets.

In order to set up the model properly, some notation is needed. Let $I = \{1, \ldots, I\}$ be the set of firms and $N = \{1, \ldots, N\}$ the set of workers attempting to match in the job market.

[5] In the terminology of Niederle and Roth (2009), offers are exploding.

[6] The academic job market, where the ranking of universities is widely published, is a particularly extreme example of this phenomenon.

Denote by $\boldsymbol{s^2}$ the quality vector of workers and S^2 the set of all possible such vectors. Let $\boldsymbol{s^1} \in S^1$ be a vector of signal for $\boldsymbol{s^2}$. This vector of signals could be known by agents. Yet, workers or firms could only known the cumulative distribution function of $\boldsymbol{s^1}$, $G : S^1 \to [0,1]$, and its associated density function g. The conditional distribution and density functions for worker quality given $\boldsymbol{s^1} \in S^1$ are mapping $F(.|\boldsymbol{s^1})$ and $f(.|\boldsymbol{s^1})$ from S^2 to $[0,1]$. The n$^{\text{th}}$ element of $\boldsymbol{s^2}$ is s^2_n. Moreover, agents could compare the quality of workers with a ranking function if they knew the quality vector. Let us define the ranking functions.

Definition 1. *Let r a common knowledge mapping such that for all $i, j \in N$ $r(s^2_i) \leq r(s^2_j)$ if and only if $s^2_i \leq s^2_j$. Additionally, we define a common knowledge ranking function $\boldsymbol{r}(\boldsymbol{s^2})$ which set all permutations of $\{r(s^2_1), ..., r(s^2_N)\}$ such that*

$$\begin{aligned} \boldsymbol{r}(\boldsymbol{s^2}) &= (r(s^2_i), ..., r(s^2_k)) \textit{ for } 1 \leq i, k \leq N, i \neq k \\ &= (r_N, ..., r_1) \textit{ with } r_N \geq .... \geq r_1. \end{aligned}$$

Given a vector of signals $\boldsymbol{s^1}$ and the conditional distribution of $\boldsymbol{s^1}$, the probabilities of a worker n to be ranking as the k$^{\text{th}}$ better worker can be found.

Definition 2. *The individual conditional cumulative distribution function of the worker n to be rank as the k^{th} for every $k \in N$ is defined as*

$$F_n(r_k|\boldsymbol{s^1}) = \int_S f(\boldsymbol{r}(\boldsymbol{s})|\boldsymbol{s^1})d\boldsymbol{s}$$

where $S = \left\{\boldsymbol{s^2} \in S^2 | r(s^2_n) \leq r_k\right\}$. Its associated density, the individual conditional distribution function of the worker n to be rank as the k^{th}, is defined as

$$f_n(r_k|\boldsymbol{s^1}) = \int_S f(\boldsymbol{r}(\boldsymbol{s})|\boldsymbol{s^1})d\boldsymbol{s}$$

where $S = \left\{\boldsymbol{s^2} \in S^2 | r(s^2_n) = r_k\right\}$.

The ranking of workers depends on their quality parameter $\boldsymbol{s^2}$. Without loss of generality, and for the purpose of consistency between $\boldsymbol{s^2}$ and $\boldsymbol{r}(.)$, it is postulated that workers with a higher quality parameter are ranked higher. From the conditional distribution of $\boldsymbol{s^2}$ it is possible to calculate the probability that a particular worker attains a specific rank once her quality becomes known.[7] Due to the nature of the ranking function, the ranking of an individual worker depends on the quality of *all* workers. The definition of the individual conditional distribution function takes this into account.

In contrast, the ranking of firms is common knowledge. The quality ranking of firms is a function $q = \left\{q_1, q_2, \ldots, q_{|I|}\right\}$ from the set of firms I to the set of all permutations on $\{1, 2, \ldots, |I|\}$, such that q_i is the ranking of firm i.

[7] The ranking function translates a continuous density of qualities into a discrete distribution of rankings. It is monotone but not continuous.

Assumption 1. *The ranking of firms* $\boldsymbol{q} = \{q_1, q_2, \ldots, q_{|I|}\}$ *is such as* $q_{|I|} \geq ... \geq q_2 \geq q_1$.

A matching of firms and workers assigns a single worker to every firm. Unmatched agents are assumed to be matched to themselves.

Definition 3. *A* matching μ *for firms and workers is a mapping from* $I \cup N$ *onto itself of order two such that,*

1. $|\mu(i)| = |\mu(n)| = 1$
2. $\mu(i) \in N$ *or* $\mu(i) = i$ *for all* $i \in I$
3. $\mu(n) \in I$ *or* $\mu(n) = n$ *for all* $n \in N$

This definition is the usual one for matching models (see for example Roth and Sotomayor (1990)). A firm (worker) could be match with only one worker (firm). The second (respectively the third) item means that all firms (workers) are matched, either with a worker or with herself.

Preferences are represented by utility functions u and v for workers and firms, respectively. Neither firms nor workers receive utility from the quality of their match; instead they are only interested in obtaining a partner of the highest possible rank. This assumption deviates from the literature where productivity tends to be the crucial characteristic. It is somewhat motivated by job markets, where positions often cannot be held over between years and competition between firms may make them more sensitive to relative quality or ranking of job candidates than overall quality.[8] Since firms and workers are homogeneous except for their quality and ranking, it is convenient to refer to a particular firm i or worker n by its quality or ranking, such that r_n refers to both a ranking and the worker for which $r(s_n^2) = r_n$.[9]

Assumption 2. *Firms and workers have identical utility functions,* $u_i(.) \equiv u(.)$ *and* $v_n(.) \equiv v(.)$*, which are common knowledge.*

Assumption 3. *Both workers and firms prefer to be matched to higher ranked agents,* $v(q_i) \geq v(q_j)$ *whenever* $q_j \leq q_i$ *and* $u(r_m) \geq u(r_n)$ *whenever* $r_n \leq r_m$. *Being unmatched is the least preferred outcome,* $v(n) < min_i\{v(q_i)\}$ *and* $u(i) < min_n\{u(r_n)\}$.

3 The matching market

With these preliminaries the matching market can be described. The market consists of two periods, $t \in \{1, 2\}$. At the beginning of $t = 1$ all agents are unmatched. A signal $\boldsymbol{s^1}$ is drawn from the distribution $G(S^1)$ and is revealed either to all firms, all workers or both firms and

[8] This is not to suggest that the overall quality of candidates does not matter, but to highlight the importance of ranking in some markets.

[9] All results could be establish if we would refer to a worker n by its ranking r_k such that for which $r(s_n^2) = r_k$. The only difference is one do notation.

workers depending on the information scenarios described below. All strategic action takes place in the first period. Firms can make an offer to a single worker or not make any offer at all; workers who receive at least one offer can accept *one* or decline all of them. Analogous to the definition of a matching, firms who do not make an offer to a worker are treated as making an offer to themselves. Thus the set of available actions for a firm i consists of the union of the set of all workers, N, and itself, $\mathcal{A}_i^I = N \cup \{i\}$. Similarly, the set of actions of a worker n is the union of the set of firms, I and itself, $\mathcal{A}_n^N = I \cup \{n\}$. The set of mixed strategy profiles for firms and workers depend on the information structure. If an offer is accepted, the matched pair is removed from the market and a preliminary matching μ_1 can be defined in which $\mu_1(i) = n$ for all firms $i \in I$ whose offer to a worker $n \in N$ has been accepted and $\mu_1(n) = i$ for all workers $n \in N$ who have accepted an offer from a firm $i \in I$. All other agents are matched to themselves: $\mu_1(j) = j$, $\forall j \in I : \mu_1(j) \notin N$ and $\mu_1(m) = m$, $\forall m \in N : \mu_1(m) \notin I$.

The set of unmatched firms after period 1, I' is given by

$$I' = \{i \in I | \mu_1(i) = i\}$$

The set of unmatched workers after period 1, N' is given by

$$N' = \{n \in N | \mu_1(n) = n\}$$

In the second period, the quality of all workers $\boldsymbol{s^2}$ and hence their ranking $\boldsymbol{r}(\boldsymbol{s^2})$ becomes common knowledge. It is then possible to rank the firms and workers still in the market relative to each other. In particular, let r'_n be the n$^{\text{th}}$ element of $\boldsymbol{r}(\boldsymbol{s'})$ where $\boldsymbol{s'} = \boldsymbol{s^2} \times \mathbf{J}$ and $\mathbf{J}$ is a $|N| \times |N'|$ matrix that selects the signals for all workers in N' from $\boldsymbol{s^2}$. Similarly, $\boldsymbol{q'}$ can be defined as the relative ranking of the firms in I'. An external mechanism matches the remaining firms and workers by rank. Let us assume that $|N| \geq |I|$.[10] In period 2,

1. for all $n \in N'$ and $\exists\ i \in I'$ such that $\mu_2(r'_n) = q'_i$ or $\mu_2(r'_n) = r'_n$
2. for all $i \in I'$ and $\exists\ n \in N'$ such that $\mu_2(q'_i) = r'_n$ or $\mu_2(q'_i) = q'_i$
3. $\mu_2(k) = \mu_1(k)\ \forall k \notin I' \cup N'$.

Remark 1. *Assortative matching, that is matching agents by their rank, produces a stable match when preferences on both sides of the market are common. Since this matching is unique and since both rankings and the utility functions are common knowledge at $t = 2$, the external mechanism is outcome equivalent to all stable matching mechanisms. The way in which we consider the matching in period 2 is thus less restrictive than initially apparent. It should be pointed out, however, that the final assignment of workers, $\mu = \mu_1 \times \mu_2$, to firms need not be stable. Although, by our setting (items 1, 2 and 3 before) there exists no worker-firm*

[10]Which implies that $|N'| \geq |I'|$.

pair (i, n) where $i \in I'$ and $n \in N'$ that can block the matching, blocking by a pair that consists of at least one member $k \notin I' \cup N'$ is not ruled out.[11]

The optimal strategy profile of both firms and workers in the first period depends on their respective expectations about the match they can achieve in the second period. These expectations, in turn, are contingent on the information available to the agents. Two information structures are particularly appealing for their simplicity and practical importance. In a symmetric structure, both firms and workers receive the quality signal vector $\boldsymbol{s^1}$. In two asymmetric set-ups, either the workers or the firms obtain information about the signal vector.

Information Structure 1. *The information structure is called symmetric if and only if the vector of signals $\boldsymbol{s^1}$ is common knowledge.*

While it is imaginable that in an actual job market firms have informations about all applicants, the assumption that workers have information about each others' attributes is somewhat strong. Nonetheless, complete symmetry is convenient baseline case against which to compare information structures.

In $t = 1$, a firm i used a mixed strategy vector σ_i, which can be defined as a probability distribution over the set of feasible actions $a_i \in \mathcal{A}_i^I$ as a function of the revealed signal $\boldsymbol{s^1}$, $\sigma_i(a_i|\boldsymbol{s^1})$. Similarly, the mixed strategy vector σ_n for a worker n describes a probability distribution over the set of feasible actions $a_n \in \mathcal{A}_n^N$, contingent on the realized signal $\boldsymbol{s^1}, \sigma_n(a_n|\boldsymbol{s^1})$. In other words, firms make an offer to a particular worker with a certain probability which depends on their own quality and the expected quality of the particular workers derived from the signal $\boldsymbol{s^1}$. Workers accept one of the offers they potentially receive based on the realized signal and the resulting expected quality in the second period.

Information Structure 2. *The information structure is called asymmetric if and only if one side of the market informed about the vector of signals $\boldsymbol{s^1}$. The other side knows only the distribution $G(.)$ of the vector of signals.*

Here, two cases must be distinguished.

- In contrast to the symmetric information case, if a firm makes an early offer to a worker, she has to form beliefs about the realization of $\boldsymbol{s^1}$ before deciding whether to accept or not. Workers' strategies in period 1 can thus no longer be conditional on the observed signal and are given by σ_n for a worker n.

- Instead of firms, workers could obtain information about $\boldsymbol{s^1}$, while firms only know the distribution function $G(\boldsymbol{s^1})$. Thus workers cannot make their strategies contingent on the realized signal.

[11] Adapting the results from Galichon (2012) to our model, as matching in period 1 does not lead to an *ex-post* efficient allocation it cannot lead to an *ex-ante* efficient allocation.

With all that, the equilibrium behavior can be described.

Proposition 1. *There exists a Nash equilibrium in which firms make and workers accept no offer in period 1, such as for all* $i \in I$*,* $n \in N$ *and* $\boldsymbol{s^1} \in S^1$

- *Under symmetric information (Information Structure 1)*

$$\sigma_i(i|\boldsymbol{s^1}) = 1$$
$$\sigma_n(n|\boldsymbol{s^1}) = 1$$

- *If only firms know the vector signal* $\boldsymbol{s^1}$ *(Information Structure 2)*

$$\sigma_i(i|\boldsymbol{s^1}) = 1$$
$$\sigma_n(n) = 1$$

- *If only workers know the vector signal* $\boldsymbol{s^1}$ *(Information Structure 2)*

$$\sigma_i(i) = 1$$
$$\sigma_n(n|\boldsymbol{s^1}) = 1$$

These equilibria are not trembling-hand perfect equilibria.

Proof. The proof is provided only for the Information Structure 1. Indeed the proof for Information Structure 2 is similar with minor modifications.

All agents are matched in the second period, such that $q_i = r_n$. Since any first-period offer by firm will be declined, a firm cannot raise its expected utility by contracting early. A similar argument holds for workers.

Suppose each firm i makes an offer to a particular n worker with probability ϵ_i^n and makes no offer at all with probability $1 - \sum_N \epsilon_i^n$. Then, the optimal strategy for a worker is to play a completely mixed strategy σ_n^ϵ in which an offer from a firm i such that $u(q_i) \geq \int u(r_n) \mathbb{1}_{r_n > |I|} dF_n(r_n|\boldsymbol{s^1})$ is accepted with the maximum probability allowed. As ϵ_i^n decreases, this strategy does not change, ruling out a limit in which workers decline all offers in the first period. ∎

Under symmetric information, the existence of an equilibrium in which both workers and firms wait until the second period to be matched, seems to indicate that unraveling may not be a problem in the type of job market described above. However, as the equilibrium is not trembling-hand perfect, it is not clear that workers – if faced with an offer in period 1 – should automatically refuse being matched before the second period. Intuitively, it makes no sense for workers to decline a first-period offer from a firm which will generate a higher utility than can be expected from being matched in the second period. This, in turn, implies that there may exist a firm that prefers making an offer in period 1. In other words, the Nash equilibrium in which firms and workers never match in the first period relies on weakly dominated strategies and is therefore not trembling-hand perfect. Without restrictions on preferences, it is nevertheless not possible to predict if there is in fact a worker-firm pair that will contract in the first period or not. The conditions necessary to find a worker and a firm that can contract early can be related to their respective risk attitude. Risk aversion by workers has been pointed out as a factor in unraveling labor markets by other authors, but the risk attitude of firms can play an equally important part. Furthermore, since rankings are discrete, and the number of firms and workers is limited, risk aversion has to be strong enough for workers to accept an offer from a firm that is ranked lower than the worker's expected ranking. If workers are not risk averse, early contracting can still take place as long as firms possess risk-loving utility functions.

Matching with full information in the second period is an equilibrium under all three information assumptions. Under Information Structure 2, the shortcomings of the equilibrium are the same as in the symmetric information case. Since workers have no information about signal $\boldsymbol{s^1}$ in the first period, the condition for successful early contracting are different, however. If a worker receives an offer in the first period, she updates her beliefs about the signal $\boldsymbol{s^1}$ taking into account the offer she has received. Specifically, all realizations of $\boldsymbol{s^1}$ which would make the early offer non-profitable for the firm will be assigned zero probability. Then, the worker can calculate the expected utility from waiting and compare this to the proposed match. This can be expressed more technically.

Proposition 2. *A pair of firm and worker, (n, i), can deviate profitably from the equilibrium in Proposition 1, if and only if their respective utility functions fulfill the following conditions,*

(i) Under symmetric information (Information Structure 1)

$$u(q_i) \geq \int u(r_n) \mathbb{1}_{r_n > |I|} dF_n(r_n|\boldsymbol{s^1}) \text{ and } \int v(r_n) dF_n(r_n|\boldsymbol{s^1}) > v(q_i)$$

(ii) If only firms know the vector signal $\boldsymbol{s^1}$ (Information Structure 2)

$$\int v(r) dF_n(r_n|\boldsymbol{s^1}) > v(q_i) \text{ and } u(q_i) \geq \int_{S^\star} \int u(r_n) \mathbb{1}_{r_n > |I|} dF_n(r_n|\boldsymbol{s^1}) dG^\star(\boldsymbol{s^1})$$

where

$$S^\star = \left\{ \boldsymbol{s^1} \in S^1 | \int v(r) dF_n(r_n|\boldsymbol{s^1}) > v(q_i) \right\} \text{ and } \frac{\partial}{\partial \boldsymbol{s^1}} G^\star(\boldsymbol{s^1}) = \frac{g(\boldsymbol{s^1})}{1 - \int_{S^1 \setminus S^\star} g(\boldsymbol{s}) d\boldsymbol{s}}$$

(iii) If only workers know the vector signal $\boldsymbol{s^1}$ (Information Structure 2)

$$\int_{S_\star} \int v(r_n) dF_n(r_n|\boldsymbol{s^1}) dG_\star(\boldsymbol{s^1}) > v(q_i) \text{ and } u(q_i) \geq \int u(r_n) \mathbb{1}_{r_n > |I|} dF_n(r_n|\boldsymbol{s^1})$$

where

$$S_\star = \left\{ s^1 \in S^1 | u(q_i) \geq \int u(r_n) dF_n(r_n|\boldsymbol{s^1}) \right\} \text{ and } \frac{\partial}{\partial s^1} G_\star(\boldsymbol{s^1}) = \frac{g(\boldsymbol{s^1})}{1 - \int_{S^1 \setminus S_\star} g(\boldsymbol{s}) d\boldsymbol{s}}$$

Proof. The following arguments applied for Information Structure 1 and 2 as well. Let us consider the symmetric information. Suppose firm i makes an offer to worker n with expected quality $\mathbb{E}[r_n|\boldsymbol{s^1}]$. As $\int v(r) dF_n(r_n|\boldsymbol{s^1}) > v(q_i)$, the firm prefers being matched to n, by condition 1, worker n will accept the offer. If $u(q_i) \geq \int u(r_n) \mathbb{1}_{r_n > |I|} dF_n(r_n|\boldsymbol{s^1})$ is not fulfilled, a deviating firm cannot find a worker who it would prefer to being matched in the second period and who would accept its proposal. ■

If the the available information is symmetric (case (i)), the result is interesting for two reasons. First, it shows that risk aversion by workers is not necessary for unraveling, as long as firms are sufficiently risk-loving. Moreover, if the conditional density of rankings for workers $f_n(r_n|\boldsymbol{s^1})$ is sufficiently non-degenerate, unraveling is more likely to involve firms and workers in the middle of the ranking distribution. In this case, the expected ranking of workers are found mainly in the middle of the ranking distribution, making early matching unattractive for firms at the top of the distribution unless they exhibit considerable risk-seeking behavior. By the same argument, workers are unwilling to match with low ranked firms. An implication of this observation is the role of the conditional density function $f_n(r_n|\boldsymbol{s^1})$. The more information the signal $\boldsymbol{s^1}$ provides, that is, the more precise the prediction of a workers' final rankings is, the wider will be the distribution of expected rankings and, in consequence, the more firms can make profitable offers in the first period that will be accepted. This suggests that labor markets in which little information is available to potential early contractors are less likely to exhibit unraveling.

Under asymmetric information and if only the firms observe the signal $\boldsymbol{s^1}$, unless the offer originates with the lowest ranked firm, Proposition 2 implies that a worker having received an offer revises her expected ranking upwards. Moreover, the higher the rank of the proposing firm, the higher the expected ranking conditional on the offer. This has two important consequences. A worker who would have accepted an out of equilibrium offer from a firm under the symmetric information assumption, might no longer accept as she forms unrealistic but rational beliefs about her expected ranking. In that sense, withholding information from workers may reduce the likelihood of early contracting and the unraveling of the market. At the same time, however, an offer by a lower ranked firm might convince a worker that his expected ranking is lower than it actually is, leading her to accept an offer she would not have considered under symmetric information. The effect of asymmetric information on the incidence of early contracting is thus not obvious. Since firms possess all relevant information in the first period, the early contracting equilibrium also exists when workers do not know the signal vector $\boldsymbol{s^1}$. Furthermore, the same coalition or a high-ranked firm and a lower ranked worker can deviate. The reason for this is that in equilibrium the firms' offers reveal enough information about the realization of $\boldsymbol{s^1}$ to the workers for them to face incentives similar to the symmetric case.

Let us now consider Information Structure 2 where the workers get the signal. Not surprisingly these conditions are in a sense the opposite from the case where firms possess more information than workers. In order for a firm to make an offer to a worker, it has to evaluate the information it obtains from the decision to accept – or not – by the worker. An acceptance is then almost bad news since it implies that the ranking the worker expects is low enough for her to accept, and the firm updates its belief about the probabilities a particular signal $\boldsymbol{s^1}$ was received by the workers.

Despite the similarities between all three information structures, there does seem to be one important difference. When the signal vector $\boldsymbol{s^1}$ and the preferences of workers are common knowledge, firms can identify workers in the first period who would accept an out of equilibrium offer. Similarly, when only firms have information about $\boldsymbol{s^1}$, they can find workers for who the conditions necessary for early contracting are met. Their proposal then signals to the worker that accepting might be in their own best interest. In contrast, if firms have no information, they might not be able to identify a potential early-matching partner, even though they know that she exists. The crucial difference between the two asymmetric cases is thus the information available to the proposing party. Indeed, if workers rather than firms took the initiative in early contracting the result would be reversed.

4 Unraveling Equilibrium Strategies

With unraveling possibly being a problem, all agents matching in the first period may be an equilibrium in the labor market with symmetric information. The following proposition describes the equilibrium strategies under the assumption $|I| = |N|$.

Proposition 3. *Assume firms and workers are both observing the vector of signal $\boldsymbol{s^1}$. There exists a Nash equilibrium in which firms make offers in period 1 and all workers accept, that is, for all $i \in I$, $n \in N$ and $\boldsymbol{s^1} \in S^1$,*

$$\sigma_i(m|\boldsymbol{s^1}) = 1$$

where $m \in N$ such that $q_i = r(\mathbb{E}[r_m|\boldsymbol{s^1}])$,

$$\sigma_n(j|\boldsymbol{s^1}) = 1$$

where $j = \arg\min_{j \in I} q_j$ subject to $\sigma_j(n|\boldsymbol{s^1}) = 1$.

Proof. Three types of deviations are potentially profitable. First, a firm i can make an offer to a worker m such that $q_i > r(\mathbb{E}[r_m|\boldsymbol{s^1}])$. Since this worker will also receive an offer from a firm j with $q_j > q_i$, firm i's offer will not be accepted. If all other agents follow their equilibrium strategy, the set of unmatched agents after period 1 resulting from this deviation, $U = N' \cup I'$ consists of firm q_i and worker n such that $q_i = r(\mathbb{E}[r_n|\boldsymbol{s^1}])$, $U = \{n, i\}$. Second, assume worker n employs any strategy σ' such that $\sigma'(j|s^1) = 0$ for all j with $q_j \geq r(\mathbb{E}[r_n|\boldsymbol{s^1}])$, the set of unmatched agent after period 1 will also be $U = \{n, i | q_i = r(\mathbb{E}[r_n|\boldsymbol{s^1}]))\}$. Similarly, if either firm i does not make any offer, or worker n declines all offers, $U = \{n, i | q_i = r(\mathbb{E}[r_n|\boldsymbol{s^1}]))\}$.

In all of those cases, the mechanism in period 2 will match firm i and worker n, and the final matching is not affected at all by the deviations.

■

The intuition is simple. As long as only a single agent deviates and all others follow their equilibrium strategy, the worker-firm pair that remains unmatched and will be matched to each other in the second period is the same that would have been matched in equilibrium anyway. Thus they cannot gain by holding out for a contract in the second period. This argument, however, holds only if the set of unmatched agents consists only of single worker and firm. Since the expected ranking of a worker is unlikely to be equal to her realized ranking in period 2, if two or more workers remain unmatched after the first period, their ranking relative to each other may change between the periods. In consequence, their final partner may be different from their equilibrium match. This leads a possible deviation by a coalition of a firm and a worker who are not matched in the equilibrium of Proposition 3.

Proposition 4. *The equilibrium in Proposition 3 is not coalition proof with respect to a coalition consisting of a firm and worker,* $\{i, n\}$ *such that* $q_i < r(\mathbb{E}[r_n|\boldsymbol{s^1}])$.

Proof. Consider strategies $\sigma_i(i|\boldsymbol{s^1}) = 1$ and $\sigma_n(n|\boldsymbol{s^1}) = 1$ for the deviating firm and worker respectively. Then, the set of unmatched agents consists of firms i and j with $q_i < q_j$ and workers n and m with $r(\mathbb{E}[r_m|\boldsymbol{s^1}]) < r(\mathbb{E}[r_n|\boldsymbol{s^1}])$. In equilibrium, firms i and j would have been matched to worker m and n, respectively, $\mu(i) = m, \mu(j) = n$. The mechanism in the second period matches the two top ranked agents with each other, so that $\mu(i) = \arg\min_{k \in N'} r_k \leq r_m$. Hence firm i cannot lose, but possibly obtain a better match by waiting. Similarly, if $r_n > r_m$, worker n will be matched to the same firm as in equilibrium; if $r_n < r_m$, however, n will be matched to the higher-ranked firm. Hence worker n weakly prefers to wait for the second period. ■

Again, the intuition is straightforward. If a high-ranked firm waits and a lower ranked firm remains unmatched, their equilibrium matches are also available in the second period. Since the higher-ranked firm is able to obtain the worker with the higher realized rank it can only gain by waiting. In the worst case it is matched with the worker it would have been matched in equilibrium; in the best case the other firm's equilibrium match turns out to be better and the higher-ranked firm can achieve a better match. The worker with the lower expected rank, similarly, cannot do worse than her equilibrium match, but can be matched to the preferred firm if her realized ranking is higher than the other worker's. By the symmetric argument, the lower-ranked firm and the higher ranked worker can only be made worse off.

These results suggest that there might be equilibria in which some firms match early while others wait until the second period. As long as the set of agents waiting until period 2 consists of only two firms and workers, Proposition 4 has shown this to be the case, indeed. It is not possible to generalize this finding to cases where more than two agents on each side of the market contract late. In fact, if worker preferences exhibit universal risk aversion, that is, for any i and all $\boldsymbol{s^1} \in S^1$, there exists a firm n such that the conditions in Proposition 2 hold, there exists no trembling-hand perfect equilibrium with late matching by more than two worker-firm pairs. Moreover, the only equilibrium with some firms matching in the second period that survives trembling hand perfection for all preferences and quality distributions is early contracting by all firms and workers but the top ranked firm and the lowest ranked worker. For any other worker there exist a combination of preferences and type-distributions such that she would accept a proposal by a slightly lower ranked firm in the first period.

To some extent the instability of the second-period matching equilibrium is due to the fact that workers have as much information about their expected ranking as firms do. This allows them to evaluate an offer in the first period without having to make an inference about their expected ranking.

Let us now consider asymmetric information between firms and workers. If firms possess all relevant information in the first period, the early contracting equilibrium of Proposition 3 also exists when workers do not know the signal vector $\boldsymbol{s^1}$. Furthermore, the same coalition or a high-ranked firm and a lower ranked worker can deviate. The reason for this is that in equilibrium the firms' offers reveal enough information about the realization of $\boldsymbol{s^1}$ to the workers for them to face incentives similar to the symmetric case. This is no longer true when firms have less information than workers.

Proposition 5 shows there is also an equilibrium with early matching under Information Structure 2 when only workers know the vector of signals $\boldsymbol{s^1}$. Unless the conditional densities $f_n(r_n|\boldsymbol{s^1})$ are identical for all workers, firms can arrive at an expected ranking in period 1 even though they lack information about $\boldsymbol{s^1}$.

Proposition 5. *Assume workers are perfectly informed about the vector of signal* $\boldsymbol{s^1}$. *If* $f_n(r_n|\boldsymbol{s^1}) \neq f_m(r_n|\boldsymbol{s^1})\ \forall n, m \in N$, *there exists a Nash equilibrium in which firms make offers in period 1 and all workers accept. The equilibrium strategies are as follows. For all* $i \in I$, $n \in N$ *and* $\boldsymbol{s^1} \in S^1$,

$$\sigma_i(m) = 1$$

where $m \in N$ *such that* $q_i = r(\mathbb{E}[r_m])$,

$$\sigma_n(j|\boldsymbol{s^1}) = 1$$

where $j = \arg\min_{j \in I} q_j$ *subject to* $\sigma_j(n|\boldsymbol{s^1}) = 1$.

Proof. The proof follows the proof of Proposition 3. ■

If firms cannot distinguish the expected quality of worker, that is if $f_n(r_n|\boldsymbol{s^1}) = f(r_n|\boldsymbol{s^1})$, this equilibrium no longer exists. There *does* exist an equilibrium with complete contracting in the first period, however. Firms make random offers and workers accept the highest offer they receive. Yet such an outcome is not attractive since it does not use all information available, even if it is only to one of the parties. Given that workers receive (and accept) an offer from a firm below their expected ranking with non-zero probability, they have an incentive to communicate information about $\boldsymbol{s^1}$ to the firms.[12] Moreover, random contracting implies that firms and workers with expected rankings above the median would prefer matching to be restricted to the second period *ex-ante*. This raises the question about possible deviations from an equilibrium with contracting in period 1.

Proposition 6. *The equilibrium in Proposition 5 is not coalition proof with respect to a coalition consisting of a firm and worker,* $\{i, n\}$ *such that* $q_i < r(\mathbb{E}[r_n|\boldsymbol{s^1}])$.

Proof. Since the deviating coalition and its strategy is independent of the information structure, its existence is not affected by firms' inability to observe $\boldsymbol{s^1}$ and the proof follows Proposition 4. ■

As with the other information structures, the existence of equilibria with partial early contracting depends on the utility functions of workers and firms and without further parameterizing the model little can be said.

5 Conclusion

Unraveling, that is contracting before the official opening of a market, has been a major problem in many real world labor markets. Although this phenomenon may have different reasons, it is generally considered undesirable by the participants in the affected markets (Roth and Xing, 1994). Several authors (see Suen (2000) and Li and Suen (2000, 2004)), have provided theoretical justifications for early contracting, focusing mainly on early contracting as a mechanism to insure against uncertainties in the distribution of surplus from the matching of workers to firms. Risk aversion is thus a crucial aspect of unraveling in labor markets. The present analysis differs in that firms and workers do not match in order to produce output and

[12]Communication between workers and firms, other than related to contracts, are ruled out in this model.

that information can be asymmetric. We determine that asymmetric information matters in early contracting process and understand how it is related to risk aversion.

This paper provides a general framework to examine the effect of risk aversion on early contracting under three different assumptions about the distribution of information between firms and workers. A two period labor market is considered in which workers' quality is revealed in the second period. A signal – revealed to either workers, firms or both at the beginning of the first period – is correlated with the final quality. Preferences over firms are common knowledge and identical for all workers. Although very formalized, this setup reflects the characteristics of many labor markets, where there is general agreement over the desirability of a match and where information about workers is revealed over time through exams, perhaps, or the production of job relevant material such as dissertations or portfolios.

Under all assumptions about the distribution of information in the first period there exists an equilibrium in which firms only make offers in the second and workers accept *no* offer in the first period. As a result all matching takes place under full information. Though desirable from the perspective of efficiency, this equilibrium is not realistic. Since the rejection of all offers in the first period as well as the refusal to make any offers are weakly dominated strategies the resulting equilibrium is not trembling hand perfect. Alternative equilibrium strategies are suggested which include offers in the first period. Nonetheless, early contracting is only an equilibrium outcome if certain conditions on preferences of firms and workers are met. Loosely speaking, workers have to be risk averse or firms risk loving with respect to expectations appropriate to the relevant information structure. That is, the conditions for unraveling depend on the information available to the two sides of the market.

In a second equilibrium firms and workers match in the first period. Since the final rankings of workers are not known at the time the offers are made and accepted, firms use the information implicit in the signal to make an offer. Then the worker with the nth highest expected ranking will be matched to the nth highest ranked firm. As long as firms are able to obtain an expected ranking of workers this equilibrium survives even if they do not possess information about the signal. As the matching is based on expected rankings it is generally not efficient and not stable *ex-post*. Moreover, the equilibrium is not coalitionnally stable, as a high ranked firm and a low ranked worker can profitably deviate by not matching in the first period. The existence of larger deviating coalitions depends on preferences and the densities of the stochastic process, as does the existence of equilibria with partial matching in both periods.

The generality of the analysis, while making precise conclusions elusive, allows the framework to be adapted for further research. First, parameterizing the density functions of the signal and the final quality may allow clearer predictions in the context of the two period matching model. More interestingly perhaps, the effect of the protocol for making and reacting to offers under different information assumptions can be examined. If firms have no information about worker quality, for example, allowing workers to apply rather than wait for offers might lead to a different set of equilibria. Finally, and probably the most important future research could be about welfare maximization. In our current analysis we do not know which information structure is welfare maximizing. That implies to determine which information structure is *ex-ante* better to preclude unraveling and then leads to *ex-post* efficiency. A such analysis would be an interesting companion paper of Halaburda (2010) who determines that some mechanisms are Pareto optimal in markets where unraveling can occur. As a consequence, this investigation would have important policy implications on market design, as well as on the interaction rules in the labor market.

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