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Fedotenkov, Igor and van Groezen, Bas and Meijdam, Lex

Network for Studies on Pensions, Aging and Retirement (Netspar), Tilburg University, Bank of Lithuania

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Igor Fedotenkov[*], Bas van Groezen[†], Lex Meijdam[‡§]

**Abstract**

The central question of this paper is how international trade and specialization are affected by different designs of pension schemes and asymmetric demographic changes. In a model with two goods, two countries and two production factors, we find that countries with a relatively large unfunded pension scheme will specialize in the production of labour intensive goods. If these countries are hit by a negative demographic shock, this specialization will intensify in the long run. Eventually, these countries may even completely specialize in the production of those goods. The effects spill over to other countries, which will move away from complete specialization in capital intensive goods as the relative size of their labour intensive goods sector will also increase.

**JEL Classification:** E27, F11, F16, H55, J11.

**Keywords:** demographics, international spillovers, pensions, trade, specialization.

[*] *Corresponding author.* Economics Department, Bank of Lithuania, Totoriu g. 4, LT-01121 Vilnius, Lithuania. i.fedotenkov@gmail.com, ifedotenkov@lb.lt, +370(5)2680126

[†] Department of Economics and Network for Studies on Pensions, Aging and Retirement (Netspar), Tilburg University, PO Box 90153, 5000 LE Tilburg, Netherlands. Email: b.j.a.m.vangroezen@uvt.nl. Phone: ++31(0)13 4664034

[‡] Department of Economics and Network for Studies on Pensions, Aging and Retirement (Netspar), Tilburg University, PO Box 90153, 5000 LE Tilburg, Netherlands

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# 1 Introduction

In the next decades, the demographic composition in large parts of the world will change, due to population ageing, migration etc. This will lead to changes in the size of the (potential) labour force, for which the number of people aged between 20 and 64 is a good indication. As Figure 1 shows, the differences between countries and regions are pronounced. While the number is steadily rising in the United States (19% between 2015 and 2100), it decreases in Japan (by 46%) and Europe (29%). Within Europe, especially the eastern and southern part will experience a substantial drop.[1] The development for China is shown in Figure 2, indicating that also there the size of the labour force is expected to decrease substantially (by about 48% in 2100).

Table 1 is derived from these figures and shows the relative sizes. For instance, in 2015, the number of people aged 20-64 is 2.7 times higher in the United States compared to Japan. By 2100, this ratio is expected to have risen to almost 6. In other words, the United States' "young" population will have grown by a factor 2.2 compared to that of Japan.

Table 1: Relative number of people aged 20-64

| | 2015 | 2030 | 2045 | 2060 | 2075 | 2100 |
|---|---|---|---|---|---|---|
| USA/Japan | 2.7 | 3.1 | 3.9 | 4.6 | 5.2 | 5.9 |
| China/USA | 4.8 | 4.5 | 3.7 | 2.9 | 2.6 | 2.1 |
| Europe/USA | 2.4 | 2.1 | 1.8 | 1.6 | 1.6 | 1.4 |
| Eastern Europe/Western Europe | 1.7 | 1.5 | 1.5 | 1.3 | 1.3 | 1.2 |
| Northern and Western Europe/ Southern Europe | 1.9 | 2.0 | 2.3 | 2.4 | 2.6 | 2.8 |

Source: *United Nations World Population Prospects: the 2015 Revision* (medium variant)

The numbers clearly show that substantive changes are expected in the relative size of the (potential) labour force in different parts of the world. Undoubtedly, this will put pension systems under pressure, but will also affect

[1] Northern Europe includes Denmark, Estonia, Faeroe Islands, Finland, Iceland, Ireland, Latvia, Lithuania, Norway, Sweden and the United Kingdom. Eastern Europe consists of Belarus, Bulgaria, Czech Republic, Hungary, Poland, Moldova, Romania, Russia, Slovakia, Ukraine. Southern Europe includes Albania, Andorra, Bosnia and Herzegovina, Croatia, Greece, Italy, Malta, Montenegro, Portugal, San Marino, Serbia, Slovenia, Spain and TFYR Macedonia. Western Europe consists of Austria, Belgium, France, Germany, Liechtenstein, Luxembourg, Monaco, Netherlands and Switzerland.

Figure 1: Population aged 20-64 (thousands)

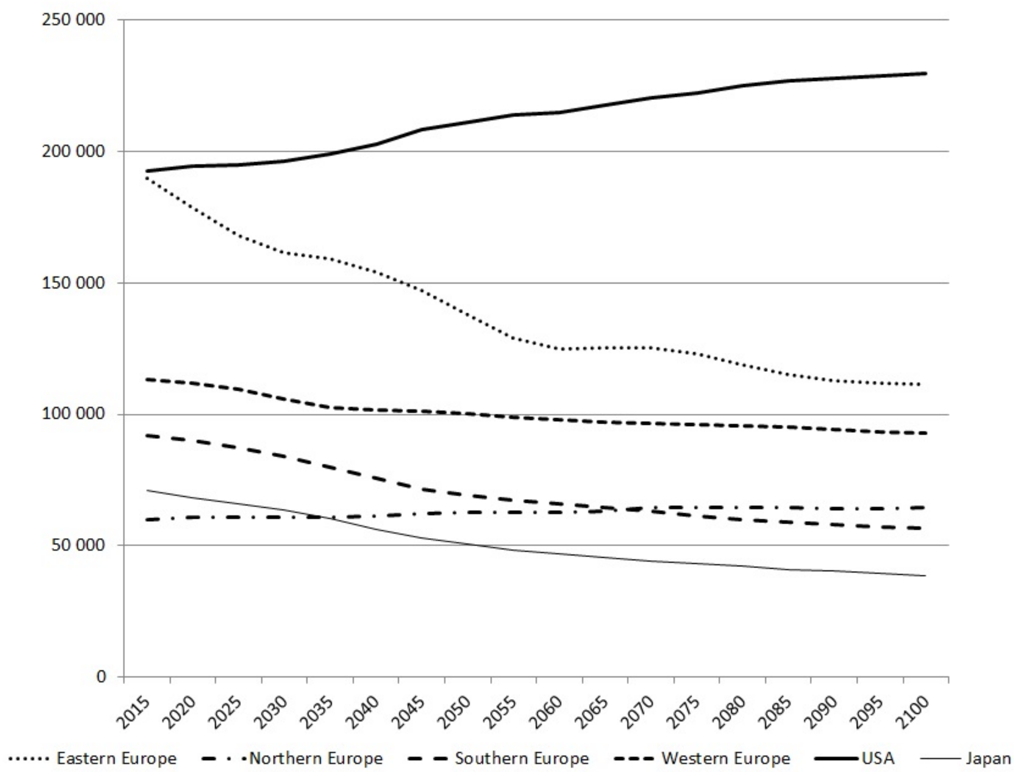
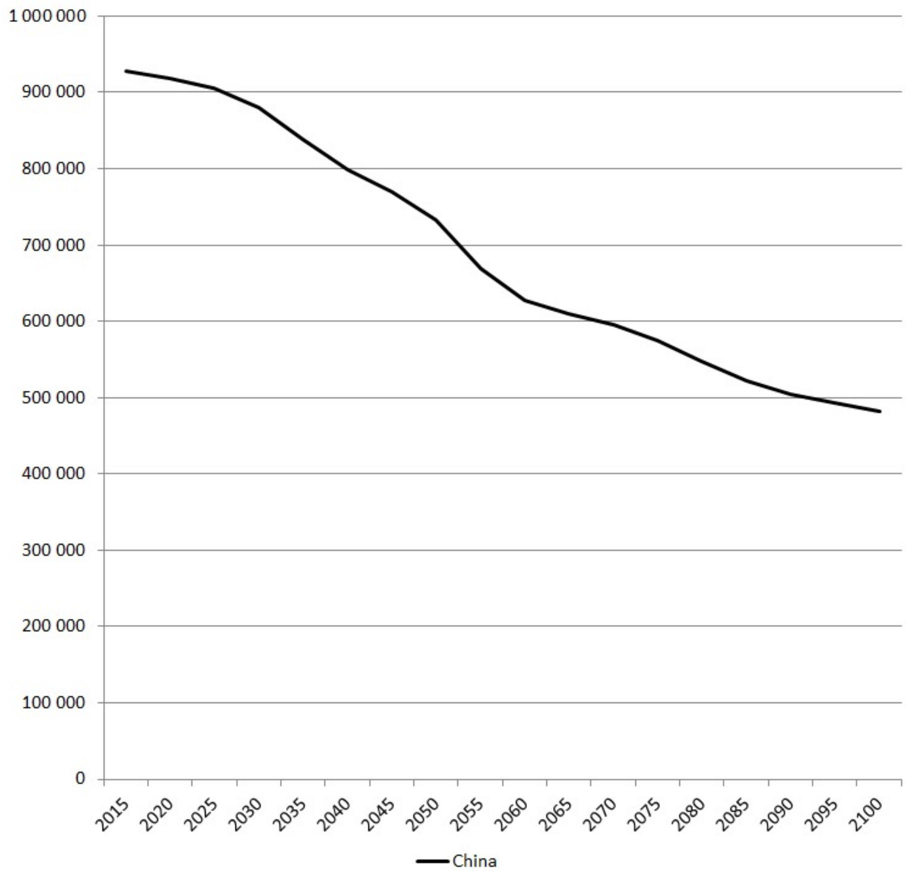


Figure 2: Population aged 20-64 in China (thousands)



the endowments of capital and labour, and therefore have important macroeconomic consequences that may spill over to other countries and regions through international trade, specialization and real exchange rate changes.

According to the classical Heckscher-Ohlin-Samuelson (HOS) model (Samuelson (1948)), a country will specialize in the production of the good that uses its relatively abundant production factor. This would imply, as stated in the Rybczynski theorem (Rybczynsky (1955)) and demonstrated by e.g. Findlay (1970), that a country which experiences a relatively shrinking labour force will specialize in the production of capital intensive goods, and will trade with countries that are endowed with a relatively small capital stock. In these models, the amounts of labour and capital in each country are exogenously given and internationally immobile. This sounds reasonable for analysing the short-run effects of demographic shocks, but this assumption is not appropriate when studying the long-run effects. In the course of time, the amount of capital in an economy will adjust to its new population level. So in the long run specialization also depends on factors that determine people's desire and ability to save and accumulate capital, such as individual thriftiness and institutional arrangements like pension schemes. The research question of this paper therefore is how specialization and international trade are affected by asymmetric demographic changes when countries have different designs of their pension schemes.

Our paper adds to the previous literature in two ways. First, it focuses on the pension system as another important element for international differences in savings, and thereby for specialization and trade. Second, it studies the short-run and long-run effects of demographic developments when differences in pension systems are taken into account. Factor rewards and the real exchange rate are endogenously determined.

To analyse this, we develop a 2 overlapping-generations model with 2 countries and 2 goods/sectors: a consumption good and an investment good. The production of consumption goods is more labour intensive than the production of investment goods. Savings can only be invested in investment goods, which add to the capital stock one period later. The countries, which can trade these goods with each other, differ in two respects: they have a different size of their unfunded pay-as-you-go (PAYG) pension scheme and a different demographic development. In particular, one country experiences a temporary drop in the rate of population growth.

We find that in the long run, the relative size of the two production sectors – and therefore the pattern of trade – depends on the generosity of the PAYG schemes: the larger such a scheme is compared to that of another country, the less capital it will accumulate and the more it will specialize in the labour intensive consumption goods, which it will export. In the steady state, the distribution of labour between the two sectors not only depends on the size of the PAYG scheme, but also on the relative size of both countries in terms of population. As a consequence, temporary demographic shocks that alter the relative size of both countries may have permanent effects, also for the country that is not experiencing the shock. However, this is only the case if the size of the PAYG scheme in both countries is different. If the country with the larger PAYG pension scheme is confronted with a relative decrease in its population size, this may lead to a larger share of the labour force working in the labour intensive goods sector. The intuition for this result is the following: the country with the relatively large pension scheme has smaller savings per capita and, as a result, specializes in labour-intensive goods. A shrinkage of its population makes labour-intensive goods relatively scarce in the two-country world. As a result, after the negative demographic shock hits, this country increases its specialization in labour-intensive goods. This effect also spillovers to the neighbouring country.

The rest of the paper is organised as follows: The next section discusses some related literature. Section 3 describes the model, and Section 4 presents the steady state and elaborates on the effect of the size of the PAYG schemes in the countries on the sector structure of the economies. In Section 5 we analyse the effects of a temporary decrease in fertility in the Home country. First, analytical long-run results are derived, then the short-run dynamics is illustrated using numerical simulation experiments. Section 6 concludes and discusses possible extensions of the model.

## 2 Related literature

So far, the dynamic trade literature can basically be divided in studies that focus on different saving rates and specialization, studies that focus on different demographic developments and specialization, and studies that combine both, but do not consider different kinds of commodities (so specialization is neglected).

Previous studies on multiple goods and trade have already emphasised international differences in saving rates as an important factor for trade flows and patterns of specialization, dating back to the seminal article by Oniki and Uzawa

(1965) in which countries have fixed, but different saving rates. Findlay (1970) also applies exogenous saving rates and finds that a higher saving rate is associated with a higher relative output of the capital intensive good. In Stiglitz (1970) and Deardorff and Hansson (1978), saving rates were endogenised by assuming infinitely-lived consumers whose rates of time preference differ between countries, leading to high-saving countries exporting capital intensive goods. Later extensions of such Ramsey-type trade models focus on relative factor intensities (Engel and Kletzer (1989)), taxation of capital goods (Baxter (1992)), and terms-of-trade effects that influence incentives to save and convergence (Bajona and Kehoe (2010)). In another strand of literature, the overlapping-generations framework is applied, which allows for analysing intergenerational issues, such as in Fried (1980) who finds that transfers between generations are required for a production innovation to be welfare improving. Internationally different saving rates play an important role when analysing the short-run and long-run welfare effects of free trade and capital flows (Buiter (1981), Serra (1991), Fisher (1992), Kemp and Wong (1995), Gokcekus and Tower (1998)) and also in the paper by Galor and Polemarchakis (1987) on transfers between countries, which shows that such transfers will decrease welfare in the recipient country if the marginal propensity to save in this country is lower than in the donor country. A similar conclusion was drawn by Mountford (1998), where trade may move the world toward a low-income steady state. Frenkel and Razin (1996) provide an overview in open-economy macroeconomics, including a two-period, two-good model to study saving behaviour and current account dynamics after shocks in the terms of trade.

Some recent papers focus on demographic factors and trade. Kenc and Sayan (2001) apply an OLG computable general-equilibrium model to analyse the effects of population ageing in the EU on Turkey's key macroeconomic variables with international trade as a transmission mechanism. They find that demographic developments in Europe magnify the economic effects of ageing in Turkey considerably. Sayan (2005) analyses the effects of demographic changes on the pattern of losses and gains from trade in a '2x2x2' HOS model where countries are identical except for the rate of population growth. As a lower population growth rate in autarky would imply a higher capital-labour ratio (due to less capital dilution), this leads to trade with the ageing country that is specializing in capital intensive goods. However, this trade does not necessarily lead to welfare gains for both countries. Naito and Zhao (2009) refine this result and also conclude that an ageing country will export the capital intensive good. The old generation in the ageing country gains from trade, but subsequent generations lose during the transition phase. A compensation scheme consisting of country-specific lump-sum taxes (transfers) and saving subsidies (taxes) can make free trade Pareto superior to autarky. This paper is extended by Yakita (2012) with endogenous fertility and human capital, which finds that ageing, modeled as an increase in life expectancy, does not necessarily make the country a net exporter of capital intensive goods.

The combination of international differences in savings/pension schemes and population growth rates have been analysed with one-good models. For instance, Börsch-Supan et al. (2006) study the consequences of asymmetric population ageing in five regions of the world using a multi-generations simulation model. In most scenarios in this model, ageing countries export capital in the short run, and start importing capital later. Adema et al. (2008) study the spillover effects of pension schemes under symmetric population ageing in a two-country two-OLG model with perfect capital mobility, and show that a country with a PAYG pension scheme gains from the country with a funded pension scheme, because it generates larger savings. Ito and Tabata (2010) also study international spillover effects through cross-border capital flows, but model ageing as an increase in longevity.

The main difference between these studies and our paper is that we allow for two different goods, which enables us to study the effects of a demographic shock on the relative size of sectors in the countries, specialization, the characteristics of international trade flows and the real-exchange rate. To the best of our knowledge, this not been analysed before in the dynamic trade literature, with the exception of Matsuyama (1988) who analyses the long-run effects of a higher population growth rate. If the country's citizens are relatively patient compared to the rest of the world, this country will export capital intensive goods, but higher population growth will lead to exports of labour intensive goods in the long run. This, however, concerns a small open economy with fixed wages and interest rate.

# 3 The model

The model describes a discrete time two-country two-sector economic union where both countries have the same sectors. So, goods produced in different countries but the same sectors are identical. We assume that one sector (indicated with index $C$) produces goods used for consumption and the other sector (index $I$) produces goods that can only be used for investment. Both countries are identical with respect to the production structure and household preferences, but they may differ with respect to the size of the PAYG pension scheme. To distinguish between the countries one country will be referred to as "Home country" (H), the other being the "Foreign country" (F). We use the same notation for the variables in the Home country and the Foreign country, but variables referring to the latter are indicated by a tilde ($\sim$).

There are two overlapping generations in the model: young and old, and there is no population growth except for one period of temporal change in the fertility rate in one of the countries. Each person lives for two periods. The total number of young in the Home country is indicated by $\Lambda$. These young agents inelastically supply one unit of labour. We denote the number of young in the Home country working in the investment good sector in period $t$ by $L_I(t)$, so labour market clearing implies $L_C(t) = \Lambda(t) - L_I(t)$ persons working in the consumption good sector. Production factors are not internationally mobile. That is, agents cannot migrate and they can only invest in their Home country. However, both investment goods and consumption goods can be traded on the international market. In fact, the presence of international trade equalises interest rates in the countries, and a possibility of capital mobility becomes redundant. This property is discussed in detail by Mundell (1957) and Razin and Sadka (1992).

We suppose that in the initial period the countries are identical apart from the pension schemes. However, as it will be explained later, differences in pension schemes, do cause other differences between the countries, such as savings rates and their specializations.

## 3.1 Firms

The firms in both countries are identical; therefore, we only present the equations for the Home country. In both sectors production at time $t$ is described by a standard Cobb-Douglas function with constant returns to scale:

$$Y_C(t) = K_C(t)^{\alpha} L_C(t)^{1-\alpha}, \tag{1}$$

$$Y_I(t) = K_I(t)^{\beta} L_I(t)^{1-\beta}, \quad 0 < \alpha < \beta < 1, \tag{2}$$

where $K_C(t)$ and $K_I(t)$ denote the capital stock in the consumption good sector and the investment good sector, respectively. We assume that production in the investment good sector is more capital intensive than in the consumption good sector (i.e., $\beta > \alpha$).

Capital fully depreciates in one period. In both sectors, the returns to the production factors are equal to their marginal products. Both interest rates and wages are expressed in terms of consumption goods, and the interest rates are defined as returns to investment measured in consumption goods:

$$w_C(t) = (1-\alpha)k_C(t)^{\alpha} \tag{3}$$

$$w_I(t) = (1-\beta)p(t)k_I(t)^{\beta} \tag{4}$$

$$1 + r_C(t) = \frac{\alpha}{p(t-1)}k_C(t)^{\alpha-1} \tag{5}$$

$$1 + r_I(t) = \frac{\beta p(t)}{p(t-1)}k_I(t)^{\beta-1} \tag{6}$$

where $w_C(t)$ is the wage rate in the consumption good sector, $w_I(t)$ denotes the wage in the investment good sector, $r_C(t)$ and $r_I(t)$ are the interest rates in consumption and investment good sector. $p(t)$ is the price of investment goods in terms of consumption goods at time $t$ and $k_C(t) \equiv K_C(t)/L_C(t)$ and $k_I(t) \equiv K_I(t)/L_I(t)$ denote the capital-labour ratio in sector $C$ and sector $I$, respectively.

## 3.2 Factor mobility between sectors

Agents can freely choose in which sector to work when young. After they have chosen a sector, this cannot be changed any more. As a result, expected lifetime utility at the start of the career should be equal for workers in both sectors.[2] The assumption that labour and capital cannot change their location after they are employed in a particular sector does not affect long run results. Regarding the short run results, under such an assumption, structural changes in the economy will have heterogeneous effects on the agents working in different sectors. However, as will be discussed in the next section, they are pretty similar, implying that qualitative as well as quantitative results do not depend on this

[2]The equations in this section are based on the assumption of rational expectations. Therefore, they may not hold in a period in which a shock takes place after the agents have decided in which sector to work and to invest. In the numerical simulations in Section 4, the actual return to savings in the period of the demographic shock is calculated using the assumption that all the agents have the same portfolio.

assumption much.

A larger budget allows an agent to reach a higher indifference curve; therefore, this implies equality of expected present values of individual incomes. As the pension system is organised on a national level and is identical for workers in both sectors, this boils down to equality of the wages in the sectors:[3]

$$w_C(t) = w_I(t) \equiv w(t). \tag{7}$$

Likewise, agents may freely choose in which sector to invest their savings, implying that interest rates in both sectors are equal as well:

$$r_C(t) = r_I(t) \equiv r(t). \tag{8}$$

Using equations (3)-(8), we can express the capital-labour ratio in the sectors as a function of the relative price of investment goods:

$$k_C(t) = \left(\frac{1-\alpha}{1-\beta}\right)^{\frac{1-\beta}{\beta-\alpha}} \left(\frac{\alpha}{\beta}\right)^{\frac{\beta}{\beta-\alpha}} p(t)^{-\frac{1}{\beta-\alpha}}, \tag{9}$$

$$k_I(t) = \left(\frac{1-\alpha}{1-\beta}\right)^{\frac{1-\alpha}{\beta-\alpha}} \left(\frac{\alpha}{\beta}\right)^{\frac{\alpha}{\beta-\alpha}} p(t)^{-\frac{1}{\beta-\alpha}}. \tag{10}$$

Note that these equations also hold for the Foreign country. That is, the capital-labour ratios in a particular sector are equal in both countries, resulting in equality of factor prices. This is in line with the classical Heckscher-Ohlin-Samuelson model. It also follows that the capital-labour ratios in both sectors are proportional to each other, i.e.:

$$k_C(t) = \kappa k_I(t) \tag{11}$$

where $0 < \kappa \equiv \frac{(1-\beta)\alpha}{(1-\alpha)\beta} < 1$, so $k_I(t) > k_C(t)$.

### 3.3 Households

Because of the free mobility of production factors between sectors, we do not have to distinguish between households working in both sectors (except for the time of a population shock). Utility functions and budget constraints are

[3] Since the same holds for both countries, we again only give the equations for the Home country. It should be noted, however, that the pension schemes are organised on a national level and PAYG tax and benefit levels in the countries may differ.

assumed to be identical in both countries, so again we only present the equations for the Home country.

Agents are assumed to have a logarithmic utility function:

$$U(t) = \log C^y(t) + \frac{1}{1+\rho} \log C^o(t+1), \tag{12}$$

where $C^y$ and $C^o$ denote consumption when young and when old, respectively, and $\rho$ is the rate of time preference. The budget constraints for the first and the second period of life are given by:

$$C^y(t) = w(t)(1-\tau) - s(t), \tag{13}$$

$$C^o(t+1) = \big(1 + r(t+1)\big)s(t) + x(t+1), \tag{14}$$

where $s$ stands for savings, $\tau$ is the tax rate on labour income to finance the PAYG pension scheme, and $x$ is the pension benefit. The pension system is organised on a national level and is identical for workers in both sectors. Because the PAYG tax is fixed, the pension benefit adjusts to e.g. demographic changes according to the following balanced-budget constraint for the government:

$$x(t+1) = \tau \frac{w(t+1)\Lambda(t+1)}{\Lambda(t)}. \tag{15}$$

Optimising the utility function (12) subject to the budget constraints (13) and (14) results in the following expression for savings:

$$s(t) = \frac{1}{2+\rho} \left( w(t)(1-\tau) - \frac{1+\rho}{1+r(t+1)} x(t+1) \right). \tag{16}$$

## 3.4 Equilibrium

The model comprises four markets that are simultaneously in equilibrium each period. Equilibrium in the markets for consumption and investment goods implies:

$$Y_C(t) + \tilde{Y}_C(t) = \Lambda(t)C^y(t) + \Lambda(t-1)C^o(t) + \tilde{\Lambda}(t)\tilde{C}^y(t) + \tilde{\Lambda}(t-1)\tilde{C}^o(t) \tag{17}$$

$$Y_I(t-1) + \tilde{Y}_I(t-1) = K_C(t) + K_I(t) + \tilde{K}_C(t) + \tilde{K}_I(t). \tag{18}$$

We assume that savings are invested within the country of origin. Hence, the total amount of capital in the Home country in period $t$ is equal to total savings in this country at time $t-1$, so capital market equilibrium is given by:

$$K_C(t) + K_I(t) = \Lambda(t-1)s(t-1)/p(t-1) \tag{19}$$

and equivalent for the Foreign country.

Finally, the labour market clears if

$$L_C(t) + L_I(t) = \Lambda(t) \tag{20}$$

Defining $\lambda(t) \equiv \frac{L_I(t)}{\Lambda(t)}$ as the share of the labour force in the Home country working in the investment good sector, this indicates the degree of specialization within the country in the production of investment goods. Using equations (3-8),(15), (16), (19) and (20), equilibrium in the capital market (19) can be written as:

$$\lambda(t) = A(t)z(t) - B \tag{21}$$

where $A(t) \equiv \frac{1-\beta}{\beta-\alpha}\frac{1-\alpha}{2+\rho}\frac{\Lambda(t-1)}{\Lambda(t)}\beta(1-\tau) > 0$, $B \equiv \frac{1-\beta}{\beta-\alpha}[\frac{(1+\rho)(1-\alpha)\tau}{2+\rho} + \alpha] > 0$ and $z(t) \equiv \frac{k_I(t-1)^\beta}{k_I(t)}$. A similar equation holds for the Foreign country.[4]

Substituting expressions (9), (10) and (20) into (18) we get the following condition for equilibrium in the market for investment goods:

$$\begin{aligned} z(t)[\Phi(t-1)\lambda(t-1) + \tilde{\lambda}(t-1)] = \\ \frac{1}{(1-\alpha)\beta}\left\{\alpha(1-\beta)[\Phi(t)+1] + (\beta-\alpha)[\Phi(t)\lambda(t) + \tilde{\lambda}(t)]\right\} \end{aligned} \tag{22}$$

where $\Phi(t) \equiv \Lambda(t)/\tilde{\Lambda}(t)$ is the relative population size of the Home country. Inserting equation (21) and its Foreign counterpart in (22) gives a non-linear first-order difference equation in $z(t)$, which is unstable (as shown in Appendix 1). This difference equation forms the basis of the numerical simulations of the demographic shock presented in Section 4. In the next section the steady state of this system will be discussed.

[4] Note that $z = \tilde{z}$.

# 4 The steady state

In steady state, the equilibrium condition for the capital market (equation (21)) can be rewritten as:[5]

$$\lambda = Ak_I^{\beta-1} - B. \tag{23}$$

It should be noted that $A < (>)\tilde{A}$ and $B > (<)\tilde{B}$ if $\tau > (<)\tilde{\tau}$. Furthermore, the capital-labour ratio in the investment good sector is equal in both countries ($k_I = \tilde{k}_I$), which implies that

$$\lambda(t) = \frac{1-\tau}{1-\tilde{\tau}}\tilde{\lambda}(t) + \tilde{B}\frac{1-\tau}{1-\tilde{\tau}} - B. \tag{24}$$

Equation (23) is represented graphically as the curve indicated "Capital market equilibrium (H)" in Figure 3. Equation (24) is shown as the line called "Capital market equilibrium (F)".

The downward-sloping relation between the capital-labour ratio in the investment good sector ($k_I$) and $\lambda$ can be interpreted as follows. A higher fraction of people working in the investment good sector has two effects on the capital labour ratio of that sector. First, more people working in the investment goods sector directly reduces $k_I$ for a given capital stock. Second, for a given capital-labour ratio, total capital per worker rises if $\lambda$ increases. This immediately follows from equation (19) that can be written as: $\frac{K_I+K_c}{\Lambda} = \lambda k_I + (1-\lambda)k_c = [1+(1-\kappa)\lambda]k_I$ and the fact that $\kappa < 1$ because the investment good sector is relatively capital intensive. More capital per worker will increase wages and thereby savings and capital accumulation, but less than proportionally. As the first effect dominates, the curve is downward sloping. A higher PAYG tax in the home country reduces savings and implies less capital accumulation, so for a given $\lambda$, $k_I$ will be lower. It also immediately follows that the relative share of the investment good sector in the Home country is smaller (larger) than in the Foreign country if the Home country has a larger (smaller) PAYG pension scheme.

In order to determine the value of $\lambda$ in the steady state, we also have to use the condition for equilibrium in the market for investment goods (equation (18)). Substituting the production functions and equation (10) into equation (18), and using the fact that the capital-labour ratio in the investment good

[5] We indicate steady-state values by omitting the time index of a variable.

sector in both countries is equal, we find that:

$$\lambda = \frac{\alpha(1-\beta)}{((1-\alpha)\beta k^{\beta-1} - \beta + \alpha)} \frac{\Phi + 1}{(\Phi + (\tilde{A}/A))} - \frac{(1-\tau)\tilde{B} - (1-\tilde{\tau})B}{\Phi(1-\tau) + (1-\tilde{\tau})}. \tag{25}$$

This curve is presented in Figure 3, indicated as "Investment goods market equilibrium". It has a straightforward interpretation: an increasing capital-labour ratio affects production in both sectors. But, in the capital intensive sector productivity of labour grows more, which attracts labour to this sector. This results in a positive relation between $\lambda$ and $k_I$.

The intersection of both curves in the left-hand panel of Figure 3 determines the steady-state values of $\lambda$ and $k_I$.[6] Given the value of $\lambda$, the value of $\tilde{\lambda}$ can subsequently be determined from the right-hand panel of the figure.

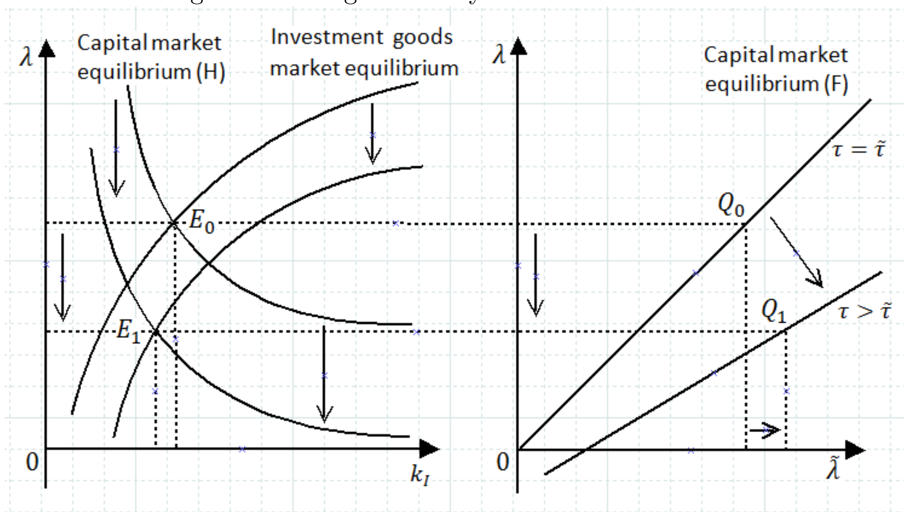
The effect of an extension of the PAYG scheme in one of the countries is shown in Figure 3. In the initial situation ($E_0$ and $Q_0$) the countries have the same size of unfunded pension schemes. Keeping $\tilde{\tau}$ constant and increasing $\tau$, agents in the Home country get a weaker incentive to save, hence, the amount of capital in the country decreases. As a result, the Home country specializes in the production of labour intensive goods ($\lambda$ falls), as indicated by the movement to point $E_1$. The decline in $\lambda$ is very intuitive: higher PAYG contributions reduces savings and the amount of capital in the Home country; therefore, the share of agents working in the capital intensive sector declines. Moreover, the decline in the amount of capital in the economy reduces capital-labour ratio in the capital intensive sector.

The Foreign country is affected as well. First, it has a comparative advantage in the production of capital intensive goods, which creates incentives for the Foreign country to specialize in capital intensive goods. Second, the world demand for capital falls when $\tau$ grows, this negatively affects the share of labour in the capital intensive sector in both countries. The overall effect on $\tilde{\lambda}$ is not clear, as it depends on the exact parameter values: if the slope of the capital-market equilibrium line after the change of $\tau$ is still rather steep, $\tilde{\lambda}$ may even decrease. Anyway, the share of the investment goods sector in the Home country becomes smaller relative to the Foreign country.

It should also be pointed out, that a further increase in $\tau$ shifts the curves in the left panel of Figure 3 even more down, so if the difference between $\tau$ and $\tilde{\tau}$ gets sufficiently large, the intersection of the curves will be below the horizontal

[6]The analytical solution for the steady state is presented in Appendix 1.

Figure 3: Change of steady state when $\tau$ increases




axis, resulting in a corner solution in which the Home country produces labour intensive goods only.

This result coincides with what one would expect in a static HOS-model: the country with a large PAYG scheme has low savings, so capital is relatively scarce and labour relatively abundant. However, it is not completely clear if specialization of countries necessarily leads to international trade. The Home country produces less capital, but it also needs less capital. So the graphs do not provide an answer to the question whether specialization within the countries intensifies international trade.

# 5 An asymmetric decrease in fertility

In this section we study the long-run and short-run effects of a negative demographic shock occurring in one country at period $t = 1$. The timing is as follows. At time $t = 1$ population growth in the Home country is temporarily lower than in the Foreign country, leading to a relatively smaller labour force in the Home country, so $\Phi \equiv \Lambda/\tilde{\Lambda}$ decreases. This can be interpreted as one country ageing relative to the other. This demographic shock is not foreseen in period $t = 0$ at the moment that agents choose a sector to work, but they do take it into account when subsequently making their saving/consumption decisions.[7] In the following periods, the population in both countries grows at the original rate again, so the relative size of the country will not change further and $\Phi$ will remain at the lower level. If instead of the temporal difference in fertilities we assume a permanent one, one country would become infinitely small compared with another, and the model would reduce to the one-country case in the long run.

## 5.1 Long-run effects

The long-run effects of a negative demographic shock in the Home country are summarised in the following proposition:

**Proposition 1.** *Suppose that $\tau + \tilde{\tau} < 1$ and $\alpha > 0.25$. A decrease in the relative size of the Home country's population:*

- *increases the capital-labour ratio in both sectors and decreases the share of labour employed in the investment good sector in both countries if $\tau > \tilde{\tau}$.*
- *decreases the capital-labour ratio in both sectors and increases the share of labour employed in the investment good sector in both countries if $\tau < \tilde{\tau}$.*
- *does not affect the capital-labour ratio and the relative degree of specialization if $\tau = \tilde{\tau}$.*

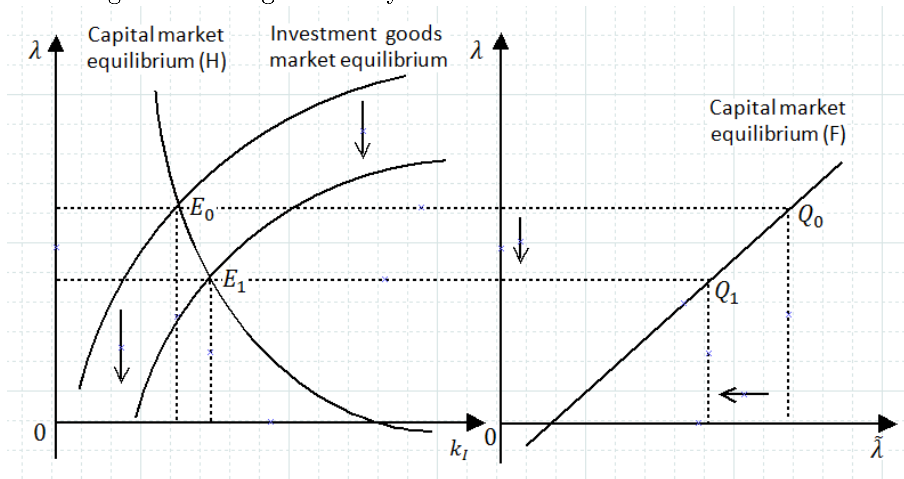
*Proof.* The proof is in Appendix 2. □

In case $\tau > \tilde{\tau}$, agents in the Home country save less than agents in the Foreign country. So decreasing the relative size of the Home country leads to an increase of the average amount of savings per worker in the union as a whole, which drives the capital-labour ratios up. If $\tau < \tilde{\tau}$, agents in the Home country save more than agents in the Foreign country, so if their number decreases, capital-labour ratios in the union will decrease.

This result can also be represented graphically. The decrease in the relative size of the Home country shifts the curve representing equilibrium in the market for investment goods down if the Home country has a relatively large PAYG scheme (see Figure 4), and up if this country has a relatively small PAYG

[7] We made this assumption because the choice in which sector to work often requires a specific education or skills training, which is usually obtained before entering the labour market. However, the saving-consumption decision is usually made during the working period when more information about future developments can be taken into account.

Figure 4: Change of steady state when $\Lambda$ decreases in case $\tau > \tilde{\tau}$




scheme. As the line and curve representing capital market equilibrium are not affected, the result for the capital-labour ratio in the investment good sector and the share of labour employed in this sector immediately follows. According to equations (9) and (10), the capital-labour ratio in the consumption good sector moves in the same direction as $k_I$.

The right-hand panel of Figure 4 shows that $\lambda$ and $\tilde{\lambda}$ move in the same direction. If $\tau > \tilde{\tau}$, the asymmetric demographic shock in the Home country causes the Foreign country to move away from the point of full specialization in investment goods. At the same time, if $\Lambda$ decreases very strongly, a corner solution may result where the Home country is fully specialized in the production of consumption goods and imports all investment goods. This is opposite to the prediction of the HOS model without endogenous capital accumulation that a decrease in the labour force in the Home country will lead to more specialization in the production of capital intensive goods. The difference comes from including pension systems, which determine savings and, hence, the comparative advantage of the country in capital/labour intensive goods production. Indeed, according to the standard HOS theorem the relative abundance and scarcity of production factors determines the specialization pattern of a country; therefore, if there is no endogenous capital accumulation, one may expect that a decline in the labour force in one of the countries will make it labour abundant. However, in the long run, the amount of capital adjusts in the model, leading to a change in abundance and scarcity of production factors, which depends on the size of PAYG contributions.

This result follows from the fact that if $\tau > \tilde{\tau}$, the Home country specialises in labour intensive goods and the Foreign country in capital intensive goods. A decline in $\Lambda$ increases the average world per capita savings, implying relatively more capital and less labour. Without a change in the allocation of labour between the two production sectors, this would make labour intensive goods relatively scarce, and capital intensive goods abundant. At the same time, the demand for both goods increases in per capita terms because income per capita rises due to a higher capital-labour ratio. Consequently, labour will move to the sector producing the labour-intensive good, resulting in further specialization in the Home country and spillover effects to the Foreign country.

In the case that $\tau < \tilde{\tau}$, the opposite result holds: the negative demographic shock in the Home country leads to higher shares of the labour force working in the investment goods sector. The Foreign country moves away from the point of full specialization in labour intensive consumption goods. For the Home country it holds that if the value of $\lambda$ increases very strongly, it may eventually be completely specialized in the production of capital intensive goods.

## 5.2 Short-run effects

In this subsection, we discuss the short-run effects of a temporary decrease in fertility in the Home country. That is, the relative size of the Home country decreases once and for all at time $t = 1$. The short-run effects will be demonstrated with numerical simulations in this section, a more technical discussion can be found in Appendix 1.

### 5.2.1 Calibration

The capital share for a one-sector model is usually estimated to be around 0.3. For example, Howarth (1998) assumes a share of capital income equal to 0.25, Bouzahzah et al. (2002) set it at 0.29, Kydland and Prescott (1982) choose a value of 0.36. In line with this, we chose a value that is a bit above these estimates for the capital intensive sector and a value just below these values for the labour intensive sector, i.e., $\alpha = 0.2$ and $\beta = 0.4$. As the period in the model is approximately equal to 35 years, the discount rate $\rho$ is set at 0.42, which approximately corresponds to the 1% annual discount rate that is widely used in the literature (see for example Jermann (1998) and Börsch-Supan et al.

(2006)). The tax rate is set at 0.25 for the country with the relatively small PAYG scheme and to 0.3 in the other country.[8] Initially, the population size in both countries is equal: $\Lambda = \tilde{\Lambda} = 1$. At time $t = 1$ the population in the Home country decreases to a level that is permanently 5% lower (i.e., from $\Lambda = 1$ to $\Lambda = 0.95$).

The dynamics of this model are summarised by a first-order difference equation in $z(t) \equiv k_I(t-1)^{\beta}/k_I(t)$ with time-dependent coefficients. As shown in the Appendix, this difference equation is unstable.[9]

#### 5.2.2 Simulation results

Figure 5 presents the time path of $\lambda$ both for the case that $\tau < \tilde{\tau}$ and for $\tau > \tilde{\tau}$. First consider the case that $\tau < \tilde{\tau}$. At $t = 1$ $\lambda$ increases in both cases, which corresponds to the result of the static HOS model. Note that with a larger demographic shock this increase may lead to a corner solution where one of the countries completely specializes in one of the goods. Next period, $\lambda$ jumps to its new steady state value. If $\tau < \tilde{\tau}$ this new steady state value is slightly larger and if $\tau > \tilde{\tau}$ it is smaller than the value in the old steady state. In the latter case, the long-run result is opposite to the prediction of the HOS model without an endogenous capital accumulation: the Home country specializes in the production of labour intensive goods, and a negative demographic shock in this country makes it specialize even more in the production of this type of goods.

Figure 6 presents the time path of the capital-labour ratio in the investment good sector. As the amount of investment goods produced in period $t = 0$ cannot be changed any more when it becomes known that the population in the Home country will shrink in period $t = 1$, capital is abundant at $t = 1$. That is, the capital-labour ratios in both sectors and both countries at $t = 1$ are high. Since the low interest rate is already anticipated at $t = 0$, savings and thus the demand for investment goods change at $t = 0$ in both countries. A lower interest rate, and higher wages, affect agents in both countries.[10] In the

[8] In fact, qualitatively the results are not sensitive to the absolute values of PAYG tax rates, but it is important that the two countries have different rates.

[9] Note that $z$ is a non-predetermined variable, as $k_I(t)$ depends on $L_I(t)$, which is non-predetermined.

[10] The presence of a PAYG pension scheme causes savings to decrease when the interest rate decreases and future wages are higher. Given that we have assumed a logarithmic utility function, the income effect and the substitution effect of a change in the interest rate on savings would exactly cancel out if there was no PAYG scheme. This does not hold with a more general utility function as well as in our model due to the presence of a PAYG scheme.

Figure 5: Share of labour in sector I of country H

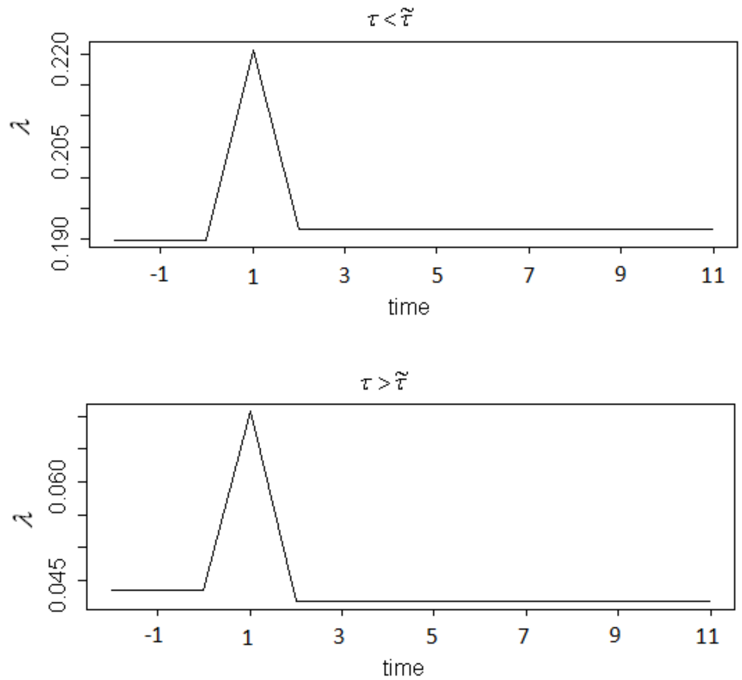
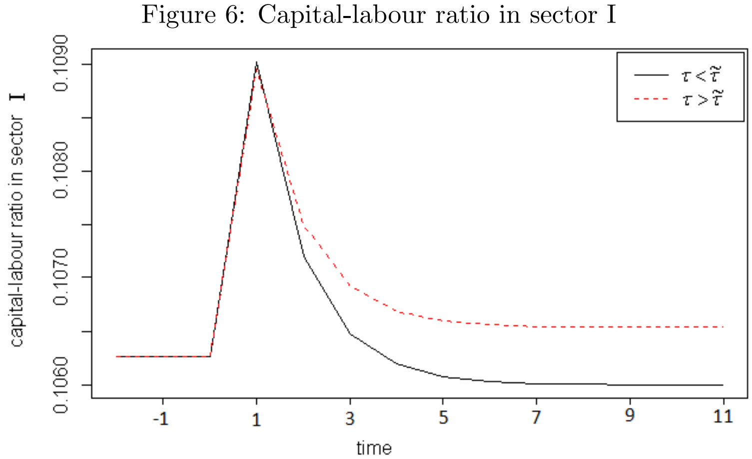



Figure 6: Capital-labour ratio in sector I



Home country, this is counteracted by the decrease in pension benefits due to the negative demographic shock at $t = 1$, which raises savings at $t = 0$ in that country. For the given parameter values, the negative effect via interest rate and wages dominates when $\tau < \tilde{\tau}$, implying that the demand for investment goods falls, leading to a decrease in the relative price of these goods already at $t = 0$ (see Figure 7).

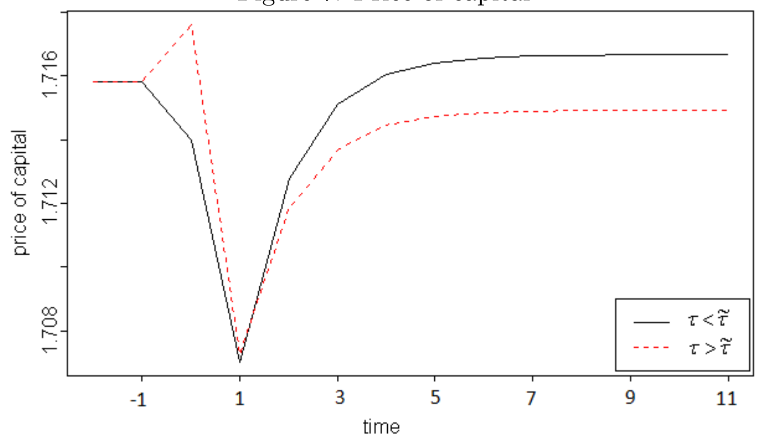
The abundance of capital at $t = 1$ stimulates production in both sectors, but more in the capital intensive investment good sector. Furthermore, workers move from the consumption good sector to the investment good sector. Thus, the production of investment goods will be relatively high at $t = 1$ leading to a fall in the price of investment goods that is even lower than at $t = 0$.

The high supply of investment goods in period $t = 1$ affects capital-labour ratios in the subsequent periods that are lower than at $t = 1$ but still above the new steady-state value. The abundance of capital leads to the same effects as at $t = 1$, but the size of the effects is smaller. This development continues, leading to a gradual decrease in the capital-labour ratios along with a gradual increase in the price of investment goods. As discussed above, in the long run, this results in a capital-labour ratio that is below the initial steady-state value and a price of investment goods that is higher than it initially was.

In case the Home country has a relatively large PAYG scheme, the short-run results are analogous to the ones presented above, the only difference being that in that case, the interest-rate effect on savings at $t = 0$ is dominated by the effect of the decrease in the PAYG benefit due to the demographic shock, so that the relative price of investment goods slightly rises at that time.

Figure 8 shows the effect of the demographic shock on international trade. It presents the difference between the amount of investment goods produced in country H and the actual amount of savings in country H in that period. This difference is equal to the amount of investment goods sold to (bought from) country F, if the value is positive (negative). The demographic shock at $t = 1$ leads to a relatively higher demand for capital goods in country H at $t = 0$ compared to country F, because pension benefits in country H are directly affected by the demographic shock. This leads to a smaller export of investment goods from the Home country when $\tau < \tilde{\tau}$, or a larger import if $\tau > \tilde{\tau}$ at $t = 0$. At $t = 1$ there is relatively more capital in the country affected by the demographic shock, so it starts exporting more capital (or importing less). At later stages the volume of trade decreases in both cases compared to their initial

Figure 7: Price of capital




equilibrium, because of a smaller total number of agents in the world.

Figures 9 and 10 show the effect on lifetime utility of the generations in both sectors and countries when $\tau > \tilde{\tau}$. As the case $\tau < \tilde{\tau}$ is very similar, we omit it. The time on the horizontal axis corresponds to the period of birth of a generation. The increase in the price of investment goods at $t = 0$ leads to higher wages in that sector and hence positively affects the PAYG benefits of the generation born at $t = -1$. Furthermore, the higher price increases the interest rates in the investment good sector at $t = 0$ and thus the return on savings of generation $t - 1$. These effects are the same in both sectors and both countries. However, they are very small. So, the utility of generation $t = -1$ slightly increases. At $t = 1$ the number of agents in the world is smaller and capital is abundant, which drives interest rates down. Smaller interest rates imply a lower return to savings for the generation born at $t = 0$ in both countries. In the Home country, the decrease in the PAYG benefit caused by the rise in the dependency ratio adds an additional negative effect on utility of this generation. Finally, the members of this generation that work in the investment good sector experience a (small) positive effect as the increase in the price of the investment good at $t = 0$ implies a higher wage when young. Note that this is the only generation for which the effects of the shock differ for workers in different sectors. All subsequent generations can take the effects of the shock into account when deciding in which sector to work. Hence, for these generations utility for workers in both sectors must be equal.

Figure 8: Net exports of investment goods by the Home country (total volume)

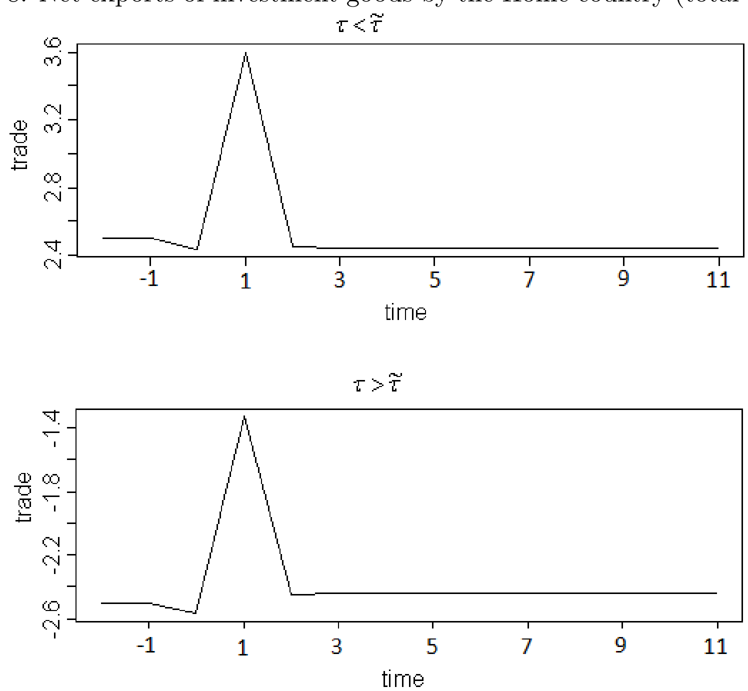



Figure 9: Utility in the Home country when $\tau > \tilde{\tau}$

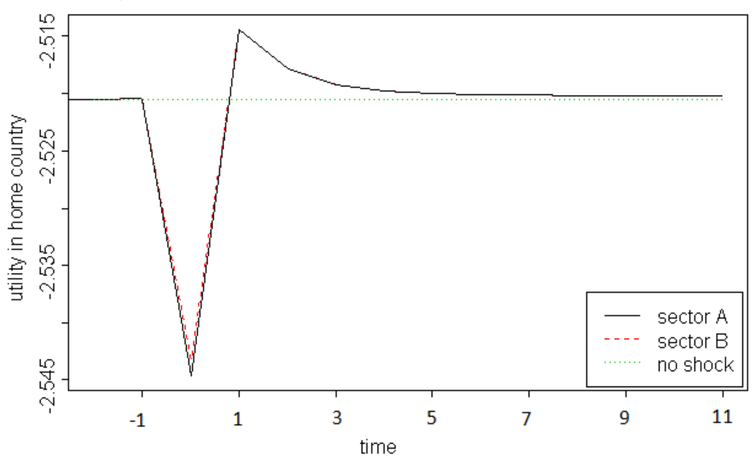
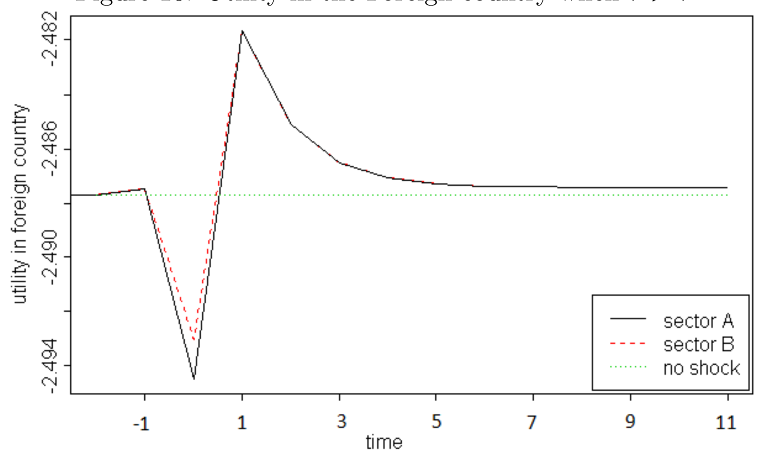


Figure 10: Utility in the Foreign country when $\tau > \tilde{\tau}$




The generation born at $t = 1$ enjoys the large capital stock that results from the investment of their parents, so their utility is affected positively. In subsequent periods, the capital-labour ratio gradually decreases and the same holds for utility. Initially, it is much larger than in the original steady state; in the long run, it decreases but will still be above that level. This time pattern is the same for both countries. However, absolute utility levels differ due to the difference in the size of the PAYG scheme between the countries. Without such difference, there would be no long-run effects of a temporary demographic shock. Furthermore, these differences also imply that the effects of the shock in one country spill over to the other country: as the economy is dynamically efficient, the Foreign country (which has the smallest PAYG scheme in this example) has highest utility. The opposite holds if $\tau < \tilde{\tau}$.

# 6 Conclusions

This paper analysed the effects of demographic shocks in a discrete time dynamic model with two countries, two sectors, two-period overlapping generations and PAYG pensions. We showed that in the long run, the relative size of the sectors in both countries (and therefore the pattern of trade) depends on the relative size of the PAYG schemes in the countries. A smaller PAYG pension scheme leads to higher savings, which gives a country a comparative advantage in the production of capital intensive goods, leading to specialization in these goods. The other country specializes in labour intensive goods. We also showed that, while the short-run effects of demographic shocks are not affected by the presence of PAYG schemes and coincide with the predictions of the classical HOS-model, the long-run effects can be different. Due to the fact that PAYG pensions affect the incentive to save and accumulate capital, the long-run consequences of demographic shocks in the dynamic model with unfunded pensions may be opposite to the predictions of the partial-equilibrium classical HOS model without endogenous capital accumulation. So, a temporal decrease in fertility leads to specialization of the country in capital intensive goods in the short run, but in the long run, specialization of the countries depends on the relative size of the PAYG scheme. Comparable results are likely to hold in the presence of other factors that affect the desire or ability to accumulate capital like, for example, differences in the rate of time preference and government debt.

The current setup could be used as a basis for future research by adding several extensions. In this paper, labour supply was assumed to be exogenous. Allowing for endogenous labour supply could lead to a smaller size of the labour force, making capital the relatively abundant factor, if the PAYG scheme is extended. Furthermore, the PAYG scheme could be designed as a defined-benefit scheme, where the contribution rate is endogenously determined to balance the government budget. This will not affect the long-run results we found in this paper, but it can affect the short-run transitional dynamics if a higher contribution rate, caused by an increase in the old-age dependency ratio, discourages private savings. Finally, the size of the PAYG scheme itself could be considered to be the outcome of a political decision-making process, and hence endogenously determined through a voting process of members of currently living generations.

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## Appendix 1: The dynamic system

To analyse the dynamics of the model, one can express the dynamic equation for the fraction of agents working in the investment good sector. For this reason we take $z(t)$ from equation (21) and $\tilde{\lambda}(t)$ from equation (24), after inserting them in (22) we get:

$$
\begin{aligned}
&\lambda(t)\Big[\lambda(t-1)\Big(\Phi(t-1)+\frac{1-\tilde{\tau}}{1-\tau}\Big)+B\frac{1-\tilde{\tau}}{1-\tau}-\tilde{B}-\frac{A(t)(\beta-\alpha)}{(1-\alpha)\beta}\Big(\Phi(t)+\frac{1-\tilde{\tau}}{1-\tau}\Big)\Big]\\
&+B\lambda(t-1)\big(\Phi(t-1)+\frac{1-\tilde{\tau}}{1-\tau}\big)+B\Big(B\frac{1-\tilde{\tau}}{1-\tau}-\tilde{B}\Big)\\
&=\frac{A(t)}{(1-\alpha)\beta}\Big[\alpha(1-\beta)(\Phi(t)+1)+(\beta-\alpha)\Big(B\frac{1-\tilde{\tau}}{1-\tau}-\tilde{B}\Big)\Big]. \qquad (26)
\end{aligned}
$$

From here, $\lambda(t)$ can be expressed as:

$$\lambda(t)=\frac{a\lambda(t-1)+b}{c\lambda(t-1)+d}, \qquad (27)$$

where

$$
\begin{aligned}
a&=-B\Big(\Phi(t-1)+\frac{1-\tilde{\tau}}{1-\tau}\Big);\\
b&=\frac{A(t)}{(1-\alpha)\beta}\Big[\alpha(1-\beta)(\Phi(t)+1)+(\beta-\alpha)\Big(B\frac{1-\tilde{\tau}}{1-\tau}-\tilde{B}\Big)\Big]-B\Big(B\frac{1-\tilde{\tau}}{1-\tau}-\tilde{B}\Big);\\
c&=\Phi(t-1)+\frac{1-\tilde{\tau}}{1-\tau};\\
d&=B\frac{1-\tilde{\tau}}{1-\tau}-\tilde{B}-\frac{A(t)(\beta-\alpha)}{(1-\alpha)\beta}\Big(\Phi(t)+\frac{1-\tilde{\tau}}{1-\tau}\Big).
\end{aligned}
$$

This function is a hyperbola; $a<0$ and $c>0$. It is easy to note that if $\lambda(t-1)<-d/c$, the denominator of equation (27) is negative. If $\lambda(t-1)>-b/a$, the nominator is negative too. Furthermore, if $\lambda(t-1)=0$, $\lambda(t)=b/d<0$ for most reasonable parameter values.

In order to determine the shape of the hyperbola we shall find out whether $-b/a<-d/c$ or the opposite. Assume that: $-b/a>-d/c$, this equation is equivalent to the following:

$$\frac{A(t)}{B(1-\alpha)\beta}\Big[\alpha(1-\beta)(\Phi(t)+1)+(\beta-\alpha)\Big(B\frac{1-\tilde{\tau}}{1-\tau}-\tilde{B}\Big)\Big] >$$
$$\frac{A(t)(\beta-\alpha)}{(1-\alpha)\beta}\Big(\Phi(t)+\frac{1-\tilde{\tau}}{1-\tau}\Big)$$

After further simplifying this expression, we get:

$$\frac{(1+\rho)(1-\alpha)(\Phi(t)\tau+\tilde{\tau})}{2+\rho} < 0. \tag{28}$$

This is in contradiction to the assumptions about the parameter values; therefore $-b/a < -d/c$.

Figure (11) presents a schematic portrait of the dynamic equation if $b/d < 0$ and $\lambda(t)$ is limited by reasonable values ($\lambda(t) \in [0, 1]$).[11] It is easy to notice that the equilibrium is unstable.

We assumed that initially the system is in its steady-state equilibrium. Figure 11 shows that there is a unique reasonable equilibrium point for $\lambda$. [12] Therefore, equation (21) gives a unique equilibrium value of $z$, resulting in a unique value of $k$ in equilibrium, and equation (10) gives a unique equilibrium value of the price level $p$. Therefore, given the assumptions about the parameter values, all variables of interest are uniquely determined.

*Short-run effects of a demographic shock*

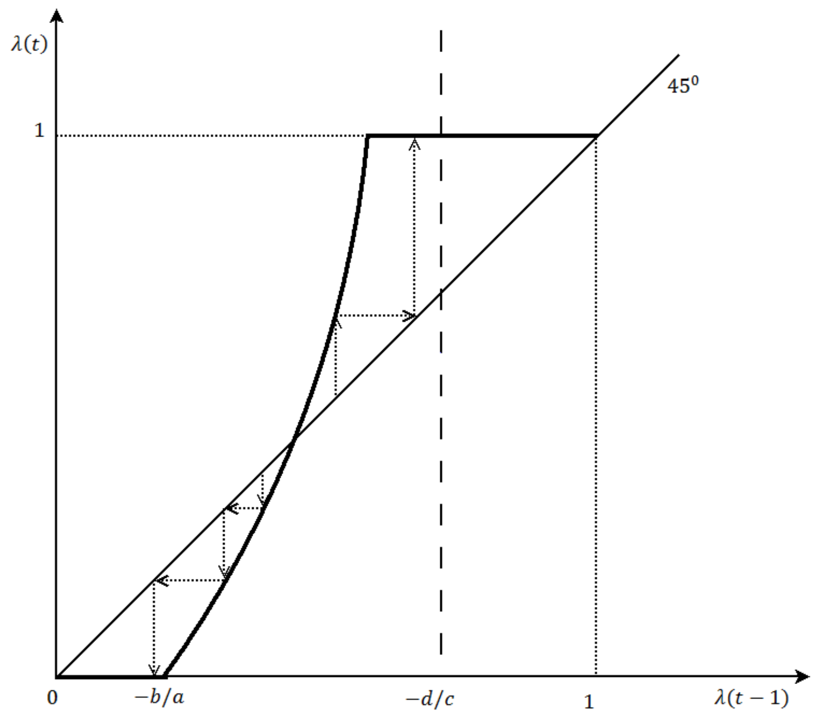
Because difference equation (27) is unstable, we assume that after the demographic change, the system is again in the equilibrium with the incomplete specialization: the value of $\lambda$ jumps from one equilibrium to another.[13] The system should be in its new steady state at time $t = 2$ (when the coefficients of the difference equation do not change any more), i.e. immediately after the shock in period $t = 1$, otherwise it will never stabilise. Given that $\lambda$ attains its new steady-state value at $t = 2$, the value of this variable at the time of the shock ($t = 1$) can easily be calculated using the difference equation. So $\lambda$ goes to the new steady-state in two steps. The same holds for $z$ (see equation (21)).

[11] If $b/d > 0$, an internal solution does not exist, and the Home country completely specializes in consumption goods. In general, the hyperbola can intersect the $45^0$ line when $\lambda(t-1) > 1$; in this case, the country specializes in investment goods.

[12] In general, there is also an equilibrium point with $\lambda < 0$; however, it is meaningless.

[13] In general, the system may not necessarily be in the steady state, and then the system diverges to a point of complete specialization. As the figure indicates, the transition path in this case is also unique.

Figure 11: Dynamics of $\lambda(t)$



Given the initial value $k_I(0)$, the time path for $k_I$ can easily be derived from $z(1)$ and $z(2)$. As $z(t) = k_I^{\beta}(t-1)/k_I(t)$, and $k(t-1)$ is known from the previous period, the value of $k_I(t)$ is unique. Note that the definition of $z(t)$ implies that the capital-labour ratio does not go to the new steady-state in two steps, but jumps to a new value at $t = 1$ and then gradually converges to its new long-run equilibrium. Equation (10) results in a unique price level. All other variables can then easily be derived.

## Appendix 2: Proof of Proposition 1

In order to prove proposition 1, it is more convenient to derive another dynamic equation for $z(t)$ instead of using equation (26). Inserting equation (21) and its Foreign counterpart in (22) gives a non-linear first-order difference equation in $z(t) \equiv \frac{k_I(t-1)^\beta}{k_I(t)}$:

$$z(t)\left\{\Phi(t-1)[A(t-1)z(t-1)-B]+\tilde{A}(t-1)z(t-1)-\tilde{B}\right\} =$$

$$\frac{1}{(1-\alpha)\beta}\left\{\alpha(1-\beta)[\Phi(t)+1]+(\beta-\alpha)\Phi(t)[A(t)z(t)-B]+(\beta-\alpha)[\tilde{A}(t)z(t)-\tilde{B}]\right\}. \quad (29)$$

Note that $z(t)$ is a non-predetermined variable, as it contains $k_I(t)$, which depends on the non-predetermined variable $L_I(t)$.

*The steady state*

We first solve the steady state of this system. In the steady state, equation (29) is equivalent to:

$$\tilde{a}z^2+\tilde{b}z+\tilde{c}=0, \quad (30)$$

where

$$\tilde{a} \equiv \frac{\beta(1-\alpha)\big(\Lambda(1-\tau)+\tilde{\Lambda}(1-\tilde{\tau})\big)}{\beta-\alpha} > 0 \quad (31)$$

$$\tilde{b} \equiv -\frac{(\Lambda+\tilde{\Lambda})(\beta+\alpha+\alpha\rho)+(1+\rho-\alpha\rho-\beta)(\tau\Lambda+\tilde{\tau}\tilde{\Lambda})}{\beta-\alpha} < 0 \quad (32)$$

$$\tilde{c} \equiv \frac{(1+\rho)(\tau\Lambda+\tilde{\tau}\tilde{\Lambda})}{\beta} > 0. \quad (33)$$

The solutions to this quadratic equation are:

$$z_1 = \frac{-\tilde{b}+\sqrt{\tilde{b}^2-4\tilde{a}\tilde{c}}}{2\tilde{a}}. \quad (34)$$

$$z_2 = \frac{-\tilde{b}-\sqrt{\tilde{b}^2-4\tilde{a}\tilde{c}}}{2\tilde{a}}. \quad (35)$$

*Proof.* Consider the long-run effects of $\Lambda$ on the equilibrium point $k_I$. As $k_I$ is a strictly decreasing function of $z_1$ described by equation (34), the sign of the derivative of $k_I$ with respect to $\Lambda$ is the negative of the sign of the derivative of $z_1$ with respect to $\Lambda$ at the point $\Lambda = \tilde{\Lambda}$, which is for simplicity set to 1.

$$\frac{\partial z_1}{\partial \Lambda} = -\frac{\partial \tilde{b}}{\partial \Lambda}\frac{1}{2\tilde{a}} + \frac{1}{2\tilde{a}\sqrt{\tilde{b}^2 - 4\tilde{a}\tilde{c}}}\left(\tilde{b}\frac{\partial \tilde{b}}{\partial \Lambda} - 2\tilde{c}\frac{\partial \tilde{a}}{\partial \Lambda} - 2\tilde{a}\frac{\partial \tilde{c}}{\partial \Lambda}\right) - \frac{z_1}{\tilde{a}}\frac{\partial \tilde{a}}{\partial \Lambda} \tag{36}$$

The derivatives $\partial\tilde{a}/\partial\Lambda$, $\partial\tilde{b}/\partial\Lambda$, $\partial\tilde{c}/\partial\Lambda$ can be easily found from equations (31)-(33). Substituting $\tilde{a}$, $\tilde{b}$, $\tilde{c}$, their partial derivatives and $z$ into equation (36) and making a number of algebraical manipulations the expression simplifies to:

$$\frac{\partial z_1}{\partial \Lambda} = -z_1^2\Omega(\tilde{\tau} - \tau) \tag{37}$$

where

$$\Omega = \frac{1}{2(\beta-\alpha)(\tau+\tilde{\tau})}\Bigg(\beta + \alpha\rho + \alpha + \frac{1}{(\beta-\alpha)\sqrt{\tilde{b}^2 - 4\tilde{a}\tilde{c}}}\Big(2\beta^2 + 2\alpha^2 + 2\alpha^2\rho^2 + 4\alpha\beta + 4\alpha^2\rho + 4\beta\alpha\rho + (\tau+\tilde{\tau})(-\beta - \beta^2 - 2\alpha^2 + 3\alpha - \rho\beta + \rho^2\alpha - \alpha^2\rho^2 - 3\alpha^2\rho + \alpha\beta + 4\alpha\rho)\Big)\Bigg)$$

A sufficient condition for $\Omega$ to be positive is $\tau + \tilde{\tau} < 1$ and $\alpha > 0.25$. □

Despite the proposition is formulated and proved for $\tau + \tilde{\tau} < 1$ and $\alpha > 0.25$, the set of parameters for which the proposition is valid, is much larger. Numerically, with the same two-step L-BFGS-B algorithm as in Appendix 1, we checked that for parameter values $0 < \alpha < \beta < 1$, $\rho > 0$, $0 < \tau + \tilde{\tau} < 2$, $\Omega$ can get nonnegative values only. Hence, from (37), the sign of $\partial k_I/\partial\Lambda$ in equilibrium depends on the sign of $\tilde{\tau} - \tau$: if $\tau < \tilde{\tau}$, the dependence is positive.