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Campbell, Carl

Northern Illinois University

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Efficiency Wage Setting, Labor Demand, and Phillips Curve Microfoundations

Carl M. Campbell III*
Professor
Department of Economics
Northern Illinois University
DeKalb, IL 60115
U.S.A.

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Abstract

This study demonstrates that a model with efficiency wages and mixed rational and adaptive expectations produces both a Phillips curve relationship and the upward-sloping counterpart to the Phillips curve, referred to as the Dynamic Labor Demand (DLD) curve. Empirical estimates with U.S. data support the model’s predictions about the coefficients in both the Phillips curve and the DLD curve, including specific predictions about the magnitudes of several coefficients. The Phillips curve and DLD curve show the transition path of unemployment and wage inflation in response to shocks, and simulations make realistic predictions about the labor market’s behavior in recessions.

Keywords: Phillips curve; Efficiency wages
JEL codes: E24; E31; J64

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* Tel.: +1 815 753 6974; fax: +1 815 752 1019
E-mail address: carlcamp@niu.edu.
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I. Introduction

The Phillips curve was originally developed as a relationship between unemployment and the rate of wage inflation, based on Phillips’s observation of British data from 1861-1957. Subsequent work by Friedman (1968) and Phelps (1968) argued that expected inflation should be included as an independent variable in a Phillips curve, with a predicted coefficient of 1. While researchers have found empirical evidence for the expectations-augmented wage Phillips curve, it has been much more difficult to provide theoretical justification for a negative relationship between wage inflation and unemployment in a model in which firms set wages. In addition, the Phillips curve does not have a counterpart curve in unemployment – inflation space, which means that it shows the combinations of unemployment and wage inflation that are possible but does not predict the actual values of these variables.

This study demonstrates that a downward-sloping short-run wage Phillips curve results from optimizing behavior, if firms pay efficiency wages and their expectations are a mixture of rational and adaptive expectations, as in Levine et al. (2012) and Campbell (2010, 2014). This study also derives the upward-sloping counterpart to the Phillips curve, referred to as the dynamic labor demand (DLD) curve, in wage inflation – unemployment space from the same framework used to derive the Phillips curve. Shifts in the Phillips curve and the DLD curve trace out the paths of wage inflation and unemployment in response to aggregate shocks.

Previous Phillips curve research is discussed in Section II. Section III develops a model in which efficiency depends on workers’ relative wages. The first-order conditions yield equations for labor demand and for the efficiency wage setting condition. Substituting the production function, the unemployment equation, and the equation for aggregate demand into the
first-order conditions yield equations for labor demand and the efficiency wage setting condition in which wages are related to unemployment. If labor demand is substituted into the efficiency wage-setting condition, an equation for a firm’s optimal wage is obtained as a function of the average wage and the unemployment rate. By making the assumptions that a fraction of firms adjusts wages each period and that expectations about future average wages are a mixture of rational and adaptive expectations, a Phillips curve relationship is derived in which wage inflation depends negatively on the unemployment rate and depends positively on expected future wage inflation and on lagged wage inflation, with the sum of coefficients on expected future wage inflation and lagged wage inflation approximately equal to 1 (and exactly equal to 1 if the discount factor equals 1).

In response to shocks to the growth rates of the money supply, real demand, and technology, shifts in the DLD curve and Phillips curve determine the paths over time of wage inflation and unemployment, with unemployment eventually returning to its natural rate. For example, an increase (decrease) in monetary growth initially reduces (raises) unemployment and raises (reduces) inflation by less than money growth changes. Over time, unemployment returns to the natural rate, and wage inflation equals the new growth rate of the money supply.

In Section IV the model is tested with U.S. data. The Phillips curve derived in this study is initially estimated by itself. Then the Phillips curve and the DLD curve are jointly estimated under different assumptions about demand. Demand is first assumed to be determined by the new Keynesian (NK) IS-LM system, although this specification has the drawback that wage inflation depends on expected future GDP growth, which is unobserved. To sidestep this issue, the system is also estimated with just the NK LM curve, which does not depend on any forward-looking variables. In this case, demand shocks are reflected by changes in nominal interest rates.
Finally, the model is estimated under the assumption of constant velocity, a special case of the NK LM specification. With all specifications, the model is estimated both over the 1967:2-2015:4 sample period and also over the period from 1967:2-2007:4, which excludes the Great Recession, since downward nominal wage rigidity may have been binding during this period, possibly skewing the results.

With all three demand specifications, the empirical results support the predictions of the theoretical model. The coefficients in the Phillips curve and the DLD curve always have the expected sign and are generally significant. In addition, the model makes several specific predictions concerning the magnitudes of coefficients in the DLD curve. For example, in the DLD curve the coefficients on current and lagged unemployment should be equal in absolute value but have opposite signs, the coefficient on technology should equal the negative of the coefficient on real demand times the elasticity of output with respect to labor, and the effect of money supply growth (adjusted for changes in trend velocity) on wage inflation should be equal to the ratio over the sample period between the growth rate of wages and the growth rate of per-worker nominal GDP (about 0.85). Also, with the NK LM and the constant velocity specifications, the coefficient on unemployment in the DLD curve should equal the ratio between the percentage point deviations of unemployment and the percentage deviations of hours worked from their steady-state trends. When the model is estimated with pre-2008 data, the specific restrictions implied by the model are almost never rejected at the 5% level. When it is estimated over the entire sample period, the prediction concerning the ratio between the coefficients on technology and real demand is strongly rejected, possibly because of the effect of downward nominal wage rigidity, but the other predictions are supported by the regression results.
The response of the Phillips curve – DLD system to a contractionary shock is simulated in Section V, and the model’s predictions about the time it takes for unemployment to reach its peak value and for it to return to the natural rate are consistent with the behavior of unemployment in recessions that occurred during the sample period. Section VI concludes.4

While the present model assumes efficiency wage setting, Campbell (2016) demonstrates that a similar equation for the wage Phillips curve can be derived from a model that combines efficiency wages, bargaining, and search and matching.

II. Relation to Previous Phillips Curve Models

The model of the Phillips curve that has generated the most research in recent years is the new Keynesian Phillips curve. Roberts (1995) shows that the new Keynesian Phillips curve can be derived from the staggered contract models of Taylor (1979, 1980) and Calvo (1983) and from the quadratic adjustment cost model of Rotemberg (1982). Roberts demonstrates that these sticky price models all yield the prediction that inflation depends on expectations of future inflation and on the output gap.

While the sticky price model is widely used in policy analysis,5 it has been criticized on several grounds. Fuhrer and Moore (1995) find that it predicts much less inflation persistence than is observed in actual data, and Ball (1994) shows that announced, credible disinflations may cause booms in this model. In the new Keynesian Phillips curve, inflation depends on output and expected future inflation, yet Fuhrer (1997) and Rudd and Whelan (2005) find evidence against models in which expectations are purely forward looking.6 In fact, many studies that incorporate the new Keynesian Phillips curve, such as Christiano, Eichenbaum, and Evans (2005), assume that firms index their prices to lagged inflation if they cannot reset them, to enable their models to more accurately describe macroeconomic dynamics. However, studies that examine pricing
behavior, such as Klenow and Kryvtsov (2008) and Nakamura and Steinsson (2008), do not find evidence for this assumption.

Galí and Gertler (1999) develop another variant of the sticky price Phillips curve in which inflation depends on marginal cost, which is measured by labor’s share of national income, and they demonstrate that their model outperforms a model in which inflation depends on the output gap. While they show that price inflation depends on the behavior of wages, their study does not analyze the factors that determine wages.

Models with overlapping wage contracts are developed in Erceg, Henderson, and Levin (2000), Smets and Wouters (2003), Christiano, Eichenbaum, and Evans (2005), and Galí (2011), under the assumption that workers set their own wages. However, this assumption does not characterize wage setting for the vast majority of workers, and it means that unemployment cannot legitimately be considered as involuntary.

The Phillips curve derived in this study differs from the new Keynesian Phillips curve in that it is a relationship between inflation and unemployment, as the Phillips curve was initially specified, rather than between inflation and output. To econometrically estimate the Phillips curve, the former is a more useful specification. The measured output gap depends on the calculated value of potential GDP, which is estimated imprecisely because of uncertainties about the natural rate and about the Okun’s law relationship between unemployment deviations and output deviations. On the other hand, only the first uncertainty is relevant when the Phillips curve is estimated with the difference between actual unemployment and the natural rate on the right-hand side.

Another model of the Phillips curve is Mankiw and Reis’ sticky information model (2002, 2006), in which a firm’s optimal price depends on aggregate prices and output. In each
period, a fraction of firms receives information that enables them to compute optimal prices, while the remaining firms operate with out-of-date information. In Mankiw and Reis (2006), workers are assumed to set their own wages, but to have sticky information concerning the determinants of the optimal wage. The present model is similar to Mankiw and Reis in that economic fluctuations result from imperfect information. However, it differs from Mankiw and Reis by assuming that it is firms, rather than workers, who set wages. In addition, the Phillips curve derived here is a relationship between wage inflation and unemployment, whereas the sticky information Phillips curve relates price inflation to real output.

The present study also differs from both the new Keynesian and the sticky information Phillips curves by deriving the counterpart curve to the Phillips curve and by developing a model in which the economy is characterized by equilibrium unemployment.

III. A Model of the Wage Phillips Curve

The economy is assumed to be populated by a continuum of \textit{ex ante} identical households and firms, in which all output is consumed. The rest of this section considers the assumptions about households’ behavior, firms’ behavior, and the derivations of the labor demand curve, the efficiency wage-setting condition, and the Phillips curve.

\textit{Assumptions about households’ behavior}

This subsection develops the theoretical framework to explain households’ decisions regarding product demand, effort, and money demand. It is assumed that households consist of a large number of individuals, who can diversify unemployment risk. Individuals’ utility is assumed to depend on their consumption, their effort, and their holdings of real money balances. As in Dixit and Stiglitz (1977), total consumption \((c)\) is the composite of the output purchased
from individual firms. Assuming a continuity of firms, indexed from 0 to 1, total consumption can be expressed as

\[ c_{t+1} = \left[ \frac{1}{\beta} \int_0^1 c_{t+1}(f) \frac{\gamma}{\gamma-1} \, df \right]^{\frac{\gamma}{\gamma-1}}. \]

Utility is also assumed to depend on workers’ effort \((e)\), with the marginal utility being negative in equilibrium. While increased effort lowers workers’ current utility, it also reduces the probability of dismissal, increasing expected future utility, and a rational worker balances the costs and benefits of effort in deciding how hard to work. It is also assumed that each individual inelastically supplies one unit of labor. (Allowing utility to depend on leisure, so that labor supply is increasing in the real wage, slightly alters the DLD curve, but has no effect on the Phillips curve.)

A representative member of a household seeks to maximize

\[
E[U] = E_{t+1} \sum_{i=0}^\infty \beta^i \left[ \left( \int_0^1 c_{t+1}(f) \frac{\gamma}{\gamma-1} \, df \right)^{\frac{\gamma}{\gamma-1}} - \Pr[\text{Emp}_{t+1}] \log(e_{t+1}) + \frac{\left( \frac{M_{t+1}}{P_t} \right)^{1-\theta}}{1-\theta} \right],
\]

(1)

where \(\beta\) is the discount factor, \(\Pr[\text{Emp}_{t+1}]\) is the probability that the worker is employed in period \(t+i\), \(M\) is money supply per worker, \(P\) is the aggregate price level, \(g(e_{t+1})\) represents the disutility of effort \((e)\), and \(\theta\) is the coefficient of relative risk aversion, assumed to be the same in both the utility from consumption and from real money holdings.

In each period household resources are pooled and the representative member is subject to the budget constraint,
\[
\int_0^1 P_{r+i}(f) c_{r+i}(f) df = W_{r+i} \Pr[\text{Emp}_{r+i}] + (i^B - B_i) + (i^M M_{i-1} - M_i),
\]

where \( P(f) \) is the price of the \( f \)th firm’s output, \( W \) is the nominal wage, \( B \) represents average bond holdings, \( i^B \) is the interest rate on bonds, and \( i^M \) is the interest rate on money.

Given the assumption that total consumption is the composite of individual goods, Dixit and Stiglitz (1977) demonstrate that the demand curve facing each firm can be expressed as

\[
Q^D_t = Y_t \left( \frac{P_t}{P_t} \right)^{-\gamma},
\]

where \( P \) is the firm’s price, \( \gamma \) is the price elasticity of demand, and \( Y \) is real aggregate demand per firm.

This constrained maximization problem can also be used to derive an expression for workers’ effort, although this derivation is quite complex. Campbell (2006) develops a model of workers’ effort with a similar utility function, with assumptions made about the probability of dismissal as a function of effort and the probability of an unemployed worker being hired. It is demonstrated that workers’ utility maximization results in an efficiency function in which efficiency depends on the ratio of their current wage to the average wage at other firms and on the unemployment rate \( (u_t) \), such that

\[
e = e[W_t / \overline{W}_t, u_t] \quad \text{with } e_w > 0, \quad e_u > 0, \quad e_{ww} < 0, \quad e_{wu} < 0.^{10}
\]

Also derived from the constrained maximization problem are the new Keynesian IS and LM curves. Following Romer (2012), the new Keynesian IS curve can be obtained from the Euler equation,

\[
C_t^{\delta} = \beta(1 + r_t)C_{t+1}^{\delta},
\]
where $r$ is the real interest rate. Under the assumption that all output is consumed, this equation can be expressed as

$$Y_t = \beta \frac{1}{\theta} (1+r_t) \frac{1}{\theta} Y_{t-1}.$$  \hspace{1cm} (5)

For the new Keynesian LM curve, two cases are considered. First, it is assumed that the interest rate on money is 0. In this case, the LM curve is

$$\frac{M_t}{P_t} = Y_t \left(1 + \frac{i_t}{i_t}\right)^{\frac{1}{\theta}},$$  \hspace{1cm} (6)

where $i$ is the nominal interest rate.

In the second case, the interest rate on money and the interest rate on bonds are assumed to move together one-for-one, so that the interest elasticity of money demand is 0. With this assumption, the LM curve is given by the constant velocity specification, $Y_t = \frac{M_t}{P_t}$.

**Assumptions about firms’ behavior**

1. Firms produce output ($Q_t$) with the Cobb-Douglas production function,

$$Q_t = A^\phi L_t^\phi K_t^{1-\phi} e^{W_t/W_t, u_t}$$  \hspace{1cm} (7)

where $A$ represents technology (assumed to be exogenous and labor augmenting), $L$ is labor, and $K$ is capital (assumed to be fixed).

2. Parameters are such that firms pay efficiency wages, yielding excess supply of labor. The unemployment rate can be expressed as

$$u_t = \frac{N - L_t}{N},$$  \hspace{1cm} (8)

where $N$ represents the ratio of workers to firms.
3. In line with the findings of Levine et al. (2012), firms’ expectations of future wage inflation are assumed to be a mixture of rational and adaptive expectations, so that expected wage inflation can be expressed as,

$$\pi_{t+1}^{w,e} = \omega \pi_{t+1}^{w,are} + (1 - \omega) \sum_{i=1}^{T} \lambda_i \pi_{t+1-i}^{w} ,$$

where $\omega$ represents the degree of rational expectations, $(1-\omega)$ represents the degree of adaptive expectation, and $\pi_{t+1}^{w,are}$ represents firms’ unbiased expectations of future wages.

Levine et al. (2012) develop a DSGE model both under the assumption that all firms and households have rational expectations and the assumption that a proportion of households and firms have rational expectations and the rest have adaptive expectations. They find that, “All behavioural models [i.e., models with mixed rational and adaptive expectations] ‘decisively’, in fact very decisively, dominate the purely rational models with very large LL [log likelihood] differences of around 20.” Other studies finding that expectations are partly rational and partly adaptive include Fuhrer (1997), Roberts (1998), and Pfajfar and Santoro (2010).\(^{12}\)

Campbell (2014) provides theoretical support for the assumption of mixed rational and adaptive expectations. A model is developed in which it is costly for agents both to form incorrect expectations of average wages and to acquire information that would yield an unbiased estimate of the average wage. It is demonstrated that cost minimization implies that wage expectations are a mixture of rational and adaptive expectations, with a geometric lag in the adaptive component. The values of $\omega$ and of the weights on each lag are determined by the model’s microeconomic parameters.\(^{13}\)

**Derivations of the DLD, DEWS, and Phillips curves**

By solving (3) for $P_i$ and multiplying by $Q_i$, total revenue is given by
Profits in period $t$ are

$$\Pi_t = \frac{1}{\gamma} \left[ A_t^\alpha L_t^{1-\alpha} e_t \left( \frac{W_t}{\bar{W}_t}, u_t \right) \right]^{\gamma - 1} \bar{P}_t - W_t L_t - r K_0. \tag{10}$$

Differentiating the profit function with respect to wages and employment yields the following first-order conditions for the average firm:

$$\frac{d\Pi_t}{dL_t} = 0 = \frac{\phi(\gamma - 1)}{\gamma} Y_t^\gamma A_t^\alpha L_t^{\gamma - 1} K_0^{\gamma - 1} e_t[A_t^\alpha L_t^{1-\alpha}]^{\gamma - 1} \bar{P}_t - W_t, \tag{11a}$$

and

$$\frac{d\Pi_t}{dW_t} = 0 = \frac{\phi(\gamma - 1)}{\gamma} Y_t^\gamma A_t^\alpha L_t^{\gamma - 1} K_0^{\gamma - 1} e_t[A_t^\alpha L_t^{1-\alpha}]^{\gamma - 1} \frac{1}{W_t} \bar{P}_t - L_t. \tag{11b}$$

Combining (11a) and (11b) and taking steady-state values (i.e., $W = \bar{W}$) results in the equilibrium condition,

$$e_w[I, u] e^{-1}[I, u] = 1. \tag{12}$$

The steady-state condition, $e_w e^{-1} = 1$, determines the economy’s natural rate of unemployment. In addition, this condition will be used to simplify equations expressed in terms of deviations from steady-state values.

Equation (11a) is the labor demand curve, and (11b) is the efficiency wage-setting condition. The Appendix demonstrates that if equations (11a) and (11b) and (5) - (8) are totally differentiated and divided by the original equations and (5) – (8) are substituted into (11a) and (11b), the following equation for wages are obtained:
\[
\hat{W}_t = \frac{(1 + \phi \xi(1 - e^{-1}s_L))s_L^{-1}du_t - \phi \xi \hat{A}_t + \phi \xi \hat{\bar{W}}_t + \hat{\bar{M}}_t + \xi \hat{Y}_{t+1} + s_t \pi^*_t}{1 + \phi \xi},
\] (14a)

and

\[
\hat{W}_t = \frac{1}{1 + \phi \xi - e_{ww} e_w} \left[ (1 + \phi \xi)(1 - e^{-1}s_L)s_L^{-1} + e_{wu} e_w^{-1} \right] du_t - \phi \xi \hat{A}_t + \left[ \phi \xi - e_{ww} e_w^{-1} \right] \hat{\bar{W}}_t + \hat{\bar{M}}_t + \xi \hat{Y}_{t+1} + s_t \pi^*_t \right] \]

(14b)

where a variable with a “\(^\wedge\)“ over it represents percentage deviations from steady-state values (e.g., \(\hat{W}_t = dW_t/W_t\)). The above equations express the relationships between percentage deviations in \(W_t, \bar{W}_t, Y_{t+1}, A_t,\) and \(M_t\) from their equilibrium values and percentage-point deviations in \(u_t\) from its equilibrium value. (Thus, \(du_t = u_t - u^*\), where \(u^*\) is the natural rate.) If small deviations of variables from their steady-state values are considered, the coefficients on these variables can be treated as constants, with these constants determined by the equilibrium values of \(W_t, \bar{W}_t, e, e_w, e_u, e_{ww},\) and \(e_{wu}\). In these equations, \(s_i\) is the absolute value of the elasticity of money demand with respect to nominal interest rates, \(\xi\) is the ratio between \(s_i\) and the absolute value of the elasticity of aggregate spending with respect to real interest rates, and \(s_L\) represents the relationship between percentage deviations in aggregate hours worked and percentage-point deviations in the unemployment rate from their steady-state values. Equations (14a) and (14b) are the labor demand curve and the efficiency wage-setting condition, as expressed as relationships between wages and unemployment.

The Appendix demonstrates that if (14a) is solved for \(\hat{\bar{M}}_t\) and the resulting expression is substituted into (14b), an individual firm’s optimal wage is described by the relationship,

\[
\hat{W}_t = \hat{\bar{W}}_t - \frac{e_{wu} - e_u}{e_{ww}} du_t, 14
\]

(15)
It is assumed that wages are set by multi-period overlapping contracts. Let $\tau$ represent the proportion of firms that can change wages in each period and $\beta$ represent the discount factor. Then the Appendix demonstrates that wage inflation ($\pi_w^t$) can be expressed as,

$$\pi_w^t = \beta \pi_{w,e}^{t+1} - \frac{\tau [1 - \beta (1 - \tau)] e_{wu} - e_u e_{wu}}{1 - \tau} du,$$

(17)

where $\pi_{w,e}^{t+1}$ represents firms’ expectations of next period’s wage inflation at the time they set wages in period $t$. If (9), which is the assumption that firms’ expectations of future wages are a mixture of rational and adaptive expectations, is substituted into (17), current wage inflation can be expressed as,

$$\pi_w^t = \beta \omega \pi_{w,e}^{t+1} + \beta (1 - \omega) \sum_{i=2}^{T} \lambda_i \pi_{w,t+i}^w + \frac{\tau [1 - \beta (1 - \tau)] e_u - e_{wu}}{1 - \beta (1 - \omega) \lambda_i} du.$$

(19)

Since current wage inflation is a function of unemployment and of both expected future and lagged wage inflation, this is an equation for a wage-wage Phillips curve. However, when economists estimate Phillips curves, the right-hand side variable is generally expected price inflation rather than expected wage inflation. While expected price inflation is the independent variable in the vast majority of Phillips curve studies, the right-hand side variable in Phelps’s (1968) seminal paper is expected wage inflation, resulting in a wage-wage Phillips curve. In addition, Perry (1978), using annual data, regresses average hourly earnings ($AHE$) on the inverse of the unemployment rate and on lagged values of either $AHE$, the consumer price index ($CPI$), or the private nonfarm GDP deflator ($GDPD$). When the equations are estimated over the longest sample period (1954-1977), the regression with lagged $AHE$ has the lowest standard error and the Durbin-Watson statistic that is closest to 2. In addition, the sum of coefficients is 1.00 on
lagged $AHE$, 0.64 on the lagged $CPI$, and 0.76 on the lagged $GDPD$. These results suggest that the wage inflation process may be described more accurately by a wage-wage Phillips curve than by a wage-price Phillips curve.

With more recent data, Table 1 presents empirical estimates of the wage-wage and wage-price Phillips curves, with price inflation measured both by the CPI and the GDP deflator. These estimates are purely backward-looking since (19) includes expected future wage inflation, as well as lagged wage inflation, and there is not an obvious reason for why future price inflation should be included in a Phillips curve with wage inflation as the dependent variable. Equations are estimated both over the entire sample period and with data through 2007, because of the possibility that downward nominal wage rigidity may have been binding in the most recent recession. A constant term is included since wage inflation is generally higher than price inflation. Table 1 shows that the $R^2$’s are higher and the sum of coefficients on lagged inflation is closer to 1 with lagged wage inflation than with lagged price inflation, both when price inflation is measured with the $CPI$ and with the $GDPD$. These results suggest that a wage-wage specification is superior to a wage-price specification.

With the model developed in this section, it is still likely that researchers will find evidence for a price-price and wage-price Phillips curve, as well as for a wage-wage Phillips curve. Campbell (2008b) demonstrates that a model with similar assumptions yields asymptotic price-price and wage-price Phillip curves when the economy is subjected to stochastic aggregate demand shocks.\textsuperscript{16}

**IV. Empirical Estimation of the Phillips Curve and the DLD Curve**

This section presents estimates of the Phillips curve and the DLD curve. The growth rate in wages ($\pi^w_t$) is measured by the percentage change in Average Hourly Earnings ($AHE$), a
series available since 1964, and $du_t$ is the deviation of the unemployment rate from the Congressional Budget Office’s estimate of the natural rate. Because expected future wage inflation ($\pi_{t+1}^{w,ae}$) may be correlated with the error term in the Phillips curve, this variable is instrumented. To instrument $\pi_{t+1}^{w,ae}$, the labor demand curve in (14a) is solved for $du_t$ (with $W_t$ set equal to $\bar{W}_t$) and the resulting expression is substituted into (19), which produces a difference equation. In this difference equation, wage inflation depends on the changes in the money supply, technology, future real demand, and expected price inflation. The current and seven lagged values of the first three variables (as well as the change in trend velocity) are used to instrument $\pi_{t+1}^{w,ae}$.17

The Phillips curve is first estimated by itself (i.e., without the DLD curve), and the estimates are reported in Table 2. (The equation instrumenting future wage inflation is not reported.) Twelve lags of wage inflation are included in the regressions. Reported in Table 2 are the value of the first lag, the sum of the 2nd through 4th lag, the sum of the 5th through 8th lag, and the sum of the 9th through 12th lag. Equations are estimated both over the entire 1967:2-2015:4 sample period and with data from 1967:2-2007:4, which excludes the Great Recession and the subsequent recovery. The reason for estimating equations that exclude post-2007 data is that the adjustment to the 2008 recession would likely have entailed notional nominal wage reductions for a significant number of workers. If these reductions did not occur because of downward nominal wage rigidity, the estimated coefficients in the Phillips curve (and in the DLD curve in later regressions) may differ from what they would have been in the absence of wage rigidity.

The two-equation system with the Phillips curve and the equation for expected wage inflation is solved with full information maximum likelihood (FIML) estimation. Lindé (2005)
demonstrates the FIML estimation performs better than GMM estimation in estimating a forward-looking Phillips curve.

The coefficient on the unemployment rate is negative and is significant at the 5% level in both regressions, and is significant at the 1% level with pre-2008 data. As expected, the coefficient on unemployment is lower over the full sample, probably because the high unemployment in the recession starting in 2008 should have eventually resulted in negative wage inflation, which did not occur.

In the regression with pre-2008 data, the coefficient on the unemployment rate is -0.082. This estimate is consistent with Gali’s (2011) estimates of the new Keynesian wage Phillips curve with quarterly data through the end of 2007 (with lagged year-to-year price inflation as a dependent variable), in which the sum of coefficients is −0.096 or −0.099, depending on the measure of wages. In addition, several researchers have estimated conventional wage Phillips curves with quarterly data. The coefficient (or sum of coefficients) on the unemployment rate is −0.085 in Blanchard (1984), −0.148 in Campbell (1991), −0.215 in Fuhrer (1995), between −0.091 and −0.109 in Campbell (1997), −0.105 in Staiger, Stock, and Watson (2001), and −0.172 in Flaschel et al. (2007). Thus, the estimate in the present study with pre-2008 data is consistent with the findings of previous researchers.

Next, the Phillips curve and the DLD curve are estimated as a joint system (along with the equation instrumenting expected future wage inflation, which is not reported) using FIML. Aggregating (14a) across firms, setting \( W_i \) equal to \( \bar{W}_i \), and subtracting the lag of the equation results in a dynamic labor demand (DLD) curve that can be expressed as

\[
\pi_t^{e} = \left(1 + \phi \xi (1 - e^{-e^{-1} s_L})s_L^{-1}(du_t - du_{t-1}) - \phi \xi (\hat{A}_t - \hat{A}_{t-1}) + (\hat{M}_t - \hat{M}_{t-1})
+ \xi (\hat{Y}_{t+1} - \hat{Y}_t) + s_i (\pi_t^{e} - \pi_{t-1}^{e})\right) (20)
\]
In (20) the coefficient on the change in technology is negative, indicating that technology improvements shift the DLD to the right, raising unemployment and lowering wage inflation. These predictions are consistent with Basu, Fernald, and Kimball’s (2006) findings that technological improvements initially result in lower employment and slightly lower nominal wages. While the direct effect of positive technology shocks is to initially reduce employment and nominal wages, these shocks should also increase future demand growth ($\hat{Y}_{t+1} - \hat{Y}_t$) by raising permanent income and the marginal product of capital, which will positively affect future employment and nominal wages.

Technology is measured with the utilization-adjusted series on total factor productivity maintained by the Federal Reserve Bank of San Francisco, based on the method of Basu, Fernald, and Kimball (2006) as updated in Basu, Fernald, Fisher, and Kimball (2013). The real cost of crude oil ($OilPrice$) is also included to capture the effect of an additional type of supply shock.\textsuperscript{19}

In the DLD curve, the measure of the money supply is M2 per worker. To account for long-term changes in money demand, the trend change in velocity, estimated with a Hodrick-Prescott filter with a smoothing parameter of 1600, is also included in the regressions. Expected inflation is measured by the Livingston series of expectations of the GDP deflator, which is available since 1971. Including this variable may be problematic since it is probably highly correlated with past and expected future wage inflation and since it is not available for the entire sample period. In addition, it enters (20) only because it represents the difference between nominal and real interest rates. Because of these potential issues with expected inflation, it is included in only two of the eight regressions in Table 3.
The DLD also depends on expected future real GDP growth, which is unobserved. There are two ways to incorporate future demand growth in the DLD curve. First, future demand is related to current demand and interest rates from the IS relationship in (A6), which implies that
\[ \hat{Y}_{r+1} = \hat{Y} + s_r (\hat{r} - \pi^c_{r+1}). \]
Thus, the current change in GDP and the current change in real interest rates can be used in place of \( \hat{Y}_{r+1} - \hat{Y} \) in both the DLD curve and the equation instrumenting for expected future wage inflation. In these regressions, the real interest rate is measured by the 3-month Treasury bill rate\(^{20}\) minus the percentage change in the GDP deflator.

Second, future GDP growth can be instrumented. The variables used to instrument future GDP growth are the current and eight lagged values of real net wealth per worker, the University of Michigan’s Index of Consumer Expectations, and real government defense spending per worker.

The results are presented in Table 3, with estimates over the 1967:2-2015:4 sample period in the first four columns and estimates over the 1967:2-2007:4 sample period in the last four columns. The equation for the DLD curve includes the current and one lagged value of the unemployment rate. Eq. (20) predicts that the coefficient on the current value is positive, that the coefficient on the lagged value is negative, and that the two are equal in absolute value. Also included in the DLD equation are the changes in technology, the M2 money supply, trend velocity, oil prices, real GDP, the real interest rate (when future GDP is not instrumented), and expected inflation (in two columns). Most of the regressions include the current and three lagged values of these variables, and the sum of these coefficients is reported in Table 3.\(^{21}\) However, one column for each sample period is estimated using only the current value of these variables.

In columns 1-3 and 5-7, future GDP growth is treated as a function of the current change in GDP and the current change in real interest rates. The second and sixth columns also include
expected price inflation, although, as previously discussed, this reduces the sample size and may cause issues with multicollinearity. The third and seventh columns include only the current values of technology, M2, oil prices, the real interest rate, and real GDP. In the fourth and eighth columns, future GDP growth is instrumented with real net wealth per worker, the University of Michigan’s Index of Consumer Expectations, and real government defense spending per worker.

In the Phillips curve, the coefficient on the unemployment rate is always negative and is significant at the 10% level in half of the columns. When the Phillips curve was estimated by itself in Table 2, the coefficients on the unemployment rate were approximately of the same magnitude as in Table 3, but in Table 2 this coefficient was significant at the 5% level over the entire sample period and was significant at the 1% level with the pre-2008 data. Thus, it appears that jointly estimating the Phillips curve and the DLD curve reduces the significance of the coefficient on unemployment in the Phillips curve, although it does not greatly affect the value of this coefficient.

In the DLD curve, all 64 coefficients have the expected sign, and 70% of the coefficients are significant at the 5% level. For all nine variables that are included in the DLD curve, the coefficient on the variable is significant at the 5% level in at least half the columns in which it appears. Thus, the predictions of the theoretical model are strongly supported by the empirical results.

In addition, (20) implies three specific predictions concerning the coefficients in the DLD. First, the model predicts that the coefficient on the current unemployment rate and the coefficient on its lagged value are equal in absolute value. Second, the coefficient on the change in technology is predicted to equal \(-\phi\) times the coefficient on the change in real demand. Third, the coefficient on the growth in the money supply (adjusted for changes in trend velocity) should
equal 1. However, labor’s share of national income fell over both the full sample period and the pre-2008 sample period, so nominal wages grew more slowly than nominal GDP per worker. On average, the ratio between the average rise in \( AHE \) and the average rise in nominal GDP per worker was 0.845 over the pre-2008 sample period and 0.859 over the full sample period. Let \( \nu \) represent this ratio. Then the sum of the coefficients on monetary growth and trend velocity should equal \( \nu \).

The last three rows of Table 3 report Likelihood Ratio tests of these restrictions. The test of the restriction that \( u_{t-1} = -u_t \) can be rejected at the 10% level in only one column (when expected inflation is included), and these coefficients are always within 3% of one another in absolute value. The restriction that \( A = -\phi Y \) can usually be rejected at the 5% level over the entire sample period, but cannot be rejected at even the 10% level with pre-2008 data. A possible reason why it is generally rejected over the full sample period is that the coefficient on \( Y \) is much higher when post-2007 data are included, perhaps because the absence of nominal wage cuts at a time of high unemployment results in inaccurate estimates of this coefficient. The restriction that the sum of coefficients on M2 growth and trend velocity equals \( \nu \) can never be rejected at the 10% level, indicating that monetary growth (adjusted for trend velocity) has the expected effect on nominal wage growth. Thus, in addition to all the coefficients having the predicted signs, the restrictions on the coefficients implied by the theoretical model are generally supported by the empirical estimation.

A problem with the new Keynesian IS-LM specification for the DLD is that it includes the change in future GDP \( (\hat{Y}_{t+1} - \hat{Y}_t) \), which is unobserved. In Table 3, this issue was handled either by including current demand growth and current real interest rates or by instrumenting future demand growth. There is another way to express the DLD curve that avoids this problem.
altogether. The Appendix demonstrates that if demand is represented by LM curve alone, the DLD curve can be expressed as

\[
\pi_t = s^{-1} \left( d\mu_t - d\mu_{t-1} \right) + (\hat{M}_t - \hat{M}_{t-1}) + s_i (\hat{i}_t - \hat{i}_{t-1}),
\]

(21)
a specification that does not rely on any unobserved variables. In this equation, demand shocks are reflected by changes in nominal interest rates. A special case of (21) is when interest rates on money and bonds move together one-for-one, so that \( s_i = 0 \) and velocity is constant. Since the change in nominal GDP equals the change in the money supply if velocity is constant, \( \hat{M}_t - \hat{M}_{t-1} \) can be replaced by the change in per-worker nominal GDP (\( NomGDP \)) in these regressions. Estimates of the Phillips curve and the DLD curve as expressed in (21) are reported in Table 4. Columns 1 and 4 include the current and three lagged values of the money supply (along with the change in trend velocity) and nominal interest rates. The interest rate variable is the percentage-point change in nominal interest rates, rather than the percentage change in nominal interest rates, since a 1% change in interest rates should have approximately the same effect whether interest rates are initially 2% or 10%. In columns 2 and 5, only the current changes in \( M \) and \( i \) are included. Columns 3 and 6 assume a constant velocity specification and are estimated with \( NomGDP \), instead of monetary growth, on the right-hand side.

The variables used in the regression to instrument future wage inflation are the ones that appear in the DLD curve for that specification (excluding the unemployment rate). In columns 1, 2, 4, and 5, future wage inflation is instrumented with the current and eight lagged changes of per-worker M2 and nominal interest rates, along with the change in trend velocity. In columns 3 and 6, it is instrumented with the current and eight lagged changes in per-worker nominal GDP.

As before, the model predicts that the coefficients of \( u_t \) and \( u_{t-1} \) in the DLD curve will have equal but opposite signs. With the DLD equation in (21), the model makes the additional
prediction that the magnitude of both coefficients (in absolute value) equals $s_L^{-1}$. The value of $s_L$ measures the relationship between percentage-point deviations of the unemployment rate from the natural rate and percentage deviations of hours worked from its trend. When the deviation of unemployment from the natural rate is regressed on deviations from trend in aggregate hours worked, the coefficient on hours worked is 0.63, implying that $s_L^{-1} = 1.59$.

In Table 4 the coefficient on the unemployment rate in the Phillips curve is always negative and significant at the 10% level and is usually significant at the 5% level. In the DLD curve, all the coefficients have the predicted signs. Of the 26 coefficients in the DLD curve reported in Table 4, all but one is significant at the 10% level, 23 are significant at the 5% level, and 20 are significant at the 1% level. In terms of the restrictions on the coefficients, the restriction that $u_{t-1} = u_t$ is never rejected, and the more specific restriction that the absolute value of both coefficients equals $s_L^{-1}$ cannot be rejected at the 10% level in three of the six columns and is rejected at the 5% level only in column 2 (when lagged values of M2 and interest rates are not included). This restriction is derived from the model in Section 3, and there is little reason why $s_L^{-1}$ should equal the absolute values of the coefficients on unemployment in the DLD equation if the model is not valid. In addition, the restrictions that $M+V_{trend} = \nu$ or that $NomGDP = \nu$ can never be rejected. Again, the empirical results strongly support the predictions of the theoretical model in Section 3.

In the regressions reported in Tables 2, 3, and 4, the coefficient on future wage inflation lies between 0.231 and 0.401 (excluding the columns in which expected price inflation is included as an independent variable). These estimates of the degree of forward-looking expectations are consistent with the findings of Levine et al. (2012), which estimates the degree
of rational vs. adaptive expectations with quarterly data. With their best specification they estimate that 17-25% of firms and 30-34% of households have rational expectations.

V. Simulations

The DLD-PC framework is used to simulate the economy’s response to a deceleration in the growth rate of nominal demand under the assumption of constant velocity, based on the specification with pre-2008 data in column 6 of Table 4. In simulating this shock, the quarterly discount factor (\( \beta \)) is set at 0.99, the natural rate (\( u^* \)) is assumed to be 5%, and the fraction of firms adjusting wages (\( \tau \)) is assumed to equal 0.25, implying that wages are adjusted, on average, once each year. From the estimated coefficients with pre-2008 data, the slope of the Phillips curve is set at -0.063 and the degree of rational expectations (\( \omega \)) is assumed to equal 0.265 (i.e., the coefficient on \( \pi_{t+1}^{w} \) divided by the sum of coefficients on future and lagged wage inflation). To simplify the difference equation used to simulate the model, a geometric lag structure is assumed for past wage inflation, with the lag parameter (\( \lambda \)) set at 0.29 (i.e., the coefficient on \( \pi_{t-1}^{w} \) divided by the sum of coefficients on lagged wage inflation).

The Phillips curve is shifted by lagged wage inflation and expected future wage inflation. With the assumption of constant velocity, the DLD curve is shifted by changes in the growth rate nominal demand and by lagged unemployment. More generally, when demand is determined from the new Keynesian IS-LM system, the DLD curve is shifted by lagged unemployment and by changes in technology, the money supply, future real demand, and inflationary expectations.

Figure 1 shows how unemployment and wage inflation respond to a deceleration in the growth rate of nominal demand from a 1.25% quarterly rate (i.e., a 5% annual rate) in period 0 and prior to 0% in period 1 and thereafter. The initial equilibrium is marked with “0.” The figure
includes the Phillips curve and the DLD curve from period 0 to period 4, and values of inflation and unemployment are denoted by dots up to the point where a new steady state is reached. In this initial equilibrium, the unemployment rate is slightly below the natural rate consistent with zero inflation because wage inflation is positive and the sum of coefficients on expected future inflation and lagged inflation is slightly less than 1, since $\beta < 1$. (If $\beta = 1$, then $u = 5\%$ in period 0.)

In response to this disinflationary shock, wage inflation gradually declines and unemployment increases. Unemployment reaches its maximum value in period 8, and then it begins to fall, until it is within 0.2% of the natural rate in period 20. In the recessions that occurred in the pre-2008 sample period (i.e., 1967:2-2007:4), the time it took for unemployment to reach its maximum value after the recession began (based on the NBER’s dating) ranged from 3 to 10 quarters, with a median of 6.5 quarters. In addition, the time it took for unemployment to fall to within 0.2% of the natural rate was between 17 and 24 quarters, with a median of 20 quarters. Thus the theoretical model, based on the optimizing behavior of individuals and firms, does a good job of explaining the time between the onset of the typical recession and the point at which unemployment starts to decline, and the time it takes for the economy to return to the natural rate.

VI. Conclusion

This study demonstrates that a wage Phillips curve can be derived from a model with efficiency wages and partly adaptive expectations about future average wages. The model’s friction lies in the labor market, as a result of the assumptions concerning the response of workers’ efficiency to relative wages and unemployment and the way firms form expectations of future average wages.
The maximization problem of firms yields the labor demand curve and the efficiency wage-setting condition. By substituting one of these relationships into the other and assuming staggered wage setting and mixed rational and adaptive expectations, a Phillip curve relationship is obtained in which wage inflation depends on unemployment, lagged wage inflation, and expected future wage inflation. In the new Keynesian Phillips curve, lagged inflation is often included by making the *ad hoc* assumption that wages or prices are indexed to past inflation if they cannot be adjusted in the current period. In the present study, lagged inflation is included in the Phillips curve because inflationary expectations are assumed to be partly adaptive, in line with the empirical evidence in Levine *et al.* (2012).

The Phillips curve and DLD curve are jointly estimated with U.S. data with three different specifications: new Keynesian IS-LM, new Keynesian LM, and constant velocity. Equations are estimated both with data through 2007 and through 2015. In all cases, the coefficients on the variables in the Phillips curve and the DLD curve have the predicted signs and are usually significant at the 5% level.

In addition to making predictions about the signs of the coefficients on the independent variables, the model makes specific predictions about the values of some of the coefficients in the DLD curve. In particular, it is predicted that the coefficients on current and lagged unemployment should be equal in absolute value, that the coefficient on money supply growth (adjusted for changes in trend velocity) should equal the long-run change in the ratio between nominal wages and per-worker nominal GDP, and that the coefficient on technology should equal the elasticity of output with respect to labor ($\phi$) times the negative of the coefficient on real aggregate demand. The first two restrictions can never be rejected at even the 10% level, except in one case in which expected price inflation is included in the DLD curve, and the third cannot
be rejected when the equations are estimated with pre-2008 data. In addition, with the LM or constant velocity specification, the model predicts that the absolute value of the coefficient on current unemployment and lagged unemployment in the DLD curve should equal the ratio between deviations in aggregate hours worked and unemployment from their steady-state values, and reasonable support is found for this prediction.

In conventional specifications, the Phillips curve shows the combinations of inflation and unemployment that are possible, but does not predict the actual values of these variables. The present study uses a consistent framework to derive both the Phillips curve and the dynamic labor demand curve. The intersections of these curves determine the values of wage inflation and unemployment that result from various types of shocks, as the economy transitions from its initial equilibrium to its new equilibrium. Thus, the model not only shows the tradeoff between inflation and unemployment, but also predicts the paths of these variables over time.
Appendix

Derivation of (14a) and (14b)

Totally differentiating (11a) and (11b) and dividing the original equation yields

\[
\hat{W}_t = \frac{1}{\gamma} \hat{Y}_t + \frac{\phi(\gamma - 1)}{\gamma} \hat{A}_t + \frac{\phi \gamma - \phi - \gamma}{\gamma} \hat{L}_t + \frac{\phi(\gamma - 1)}{\gamma} e_w e^{-1} \frac{W_t}{W_t} \hat{W}_t
\]

\[
- \frac{\phi(\gamma - 1)}{\gamma} e_w e^{-1} \frac{W_t}{W_t} \hat{W}_t + \frac{\phi(\gamma - 1)}{\gamma} e_u e^{-1} du_t + \hat{P}_t,
\]

(A1)

and

\[
\frac{\phi + \gamma - \phi \gamma}{\gamma} \hat{L}_t = \frac{1}{\gamma} \hat{Y}_t + \frac{\phi(\gamma - 1)}{\gamma} \hat{A}_t + \frac{\phi \gamma - \phi - \gamma}{\gamma} e^{-1} \left[ e_w \frac{W_t}{W_t} \hat{W}_t - e_w \frac{W_t}{W_t} \hat{W}_t \right]
\]

\[
+ e_u e^{-1} W_t \hat{W}_t - e_{ww} e^{-1} W_t \hat{W}_t + e_w e^{-1} du_t - \hat{W}_t + \hat{P}_t,
\]

(A2)

where a variable with a “^” over it represents percentage deviations from steady-state values (e.g., \(\hat{W}_t = dW_t / W_t\)).

From (8), \(du_t\) can be approximated by

\[
du_t = -\frac{dL_t}{N} \approx -s_L \hat{L}_t,
\]

(A3)

where \(du_t\) represents the percentage-point deviation of unemployment from the natural rate, and \(s_L\) represents the ratio between percentage deviations in hours of work from its steady-state value and percentage-point deviations of unemployment from the natural rate. Accordingly, \(\hat{L}_t \approx -s_L^{-1} du_t\).

Since \(Y_t = Q_t\), totally differentiating (7) and dividing by the original equation results in the following expression for \(\hat{Y}_t\):

\[
\hat{Y}_t = \phi \hat{A}_t + \phi \hat{L}_t + \phi e_w e^{-1} \frac{W_t}{W_t} \hat{W}_t - \phi e_w e^{-1} \frac{W_t}{W_t} \hat{W}_t + \phi e_u e^{-1} du_t.
\]

(A4)
Substituting (A1) and the steady-state relationships, \( W/W = 1 \) and \( e_w e^{-1} = 1 \) (from equation (12)), into (A4) yields

\[
\hat{Y}_t = \phi \hat{\lambda}_t + \phi (1 - e_\omega e^{-1} s_L) \hat{L}_t + \phi \hat{W}_t - \phi \hat{W}_t.
\]  

(A5)

To derive an equation for labor demand, it is necessary to calculate an expression for the price level. Taking percentage deviations from steady-state values of (5) and (6) gives

\[
\hat{Y}_t = \hat{Y}_{t+1} - s_r (\hat{i}_t - \pi^*_r), \quad \text{where } s_r = r / [\theta (1 + r)],
\]  

(A6)

and

\[
\hat{M}_t = \hat{P}_t + \hat{Y}_t - s_i \hat{i}_t, \quad \text{where } s_i = 1 / [\theta (1 + i)].
\]  

(A7)

Two ways to express the price level are considered. In the first, \( \hat{i}_t \) in (A6) is substituted into (A7), yielding

\[
\hat{P}_t = \hat{M}_t - \hat{Y}_t + s_r (\hat{Y}_{t+1} - \hat{Y}_t + s_r \pi^*_r),
\]

which can be expressed as

\[
\hat{P}_t = \hat{M}_t - (1 + \xi) \hat{Y}_t + \xi \hat{Y}_{t+1} + s_i \pi^*_r, \quad \text{where } \xi = \frac{s_i}{s_r} > 0.
\]  

(A8)

In the second, the price level is determined from the LM curve, so that

\[
\hat{P}_t = \hat{M}_t - \hat{Y}_t + s_i \hat{i}_t.
\]  

(A9)

In this case, IS shocks are reflected in changes in the nominal interest rate.

First, equations for labor demand and the efficiency wage setting condition are derived with the first price level specification. To derive an equation for labor demand, the steady-state
conditions \((W/W = 1 \text{ and } e_we^{-1} = 1)\) and (A8) are substituted into (13a). By making these substitutions, the labor demand curve can be expressed as

\[
\hat{W}_t = \frac{1 - \gamma - \xi \gamma}{\gamma} \hat{Y}_t + \frac{\phi(\gamma - 1)}{\gamma} \hat{A}_t + \frac{\phi\gamma - \phi - \gamma}{\gamma} \hat{L}_t - \frac{\phi(\gamma - 1)}{\gamma} \hat{W}_t - \frac{\phi(\gamma - 1)}{\gamma} W_t + \frac{\phi(\gamma - 1)}{\gamma} M_t + \xi \hat{Y}_{t+1} + s_i \pi^e.
\]

(A10)

If (A5) and the relationship, \(\hat{L}_t \approx -s_L^{-1} du_t\), are substituted into (A10), the equation for the wage is

\[
\hat{W}_t = \frac{[1 + \phi \xi (1 - e_we^{-1} s_L)]s_L^{-1} du_t - \phi \xi \hat{A}_t + \phi \xi \hat{W}_t + \hat{M}_t + \xi \hat{Y}_{t+1} + s_i \pi^e}{1 + \phi \xi}.
\]

(A11)

Equation (A11) is the labor demand curve. To derive the efficiency wage-setting condition, (A8) and the steady-state conditions are substituted into (13b), resulting in the expression,

\[
\frac{\phi + \gamma - \phi \gamma}{\gamma} \hat{L}_t = \frac{1 - \gamma - \xi \gamma}{\gamma} \hat{Y}_t + \frac{\phi(\gamma - 1)}{\gamma} \hat{A}_t + \frac{\phi\gamma - \phi - \gamma}{\gamma} \hat{W}_t - \frac{\phi(\gamma - 1)}{\gamma} \hat{W}_t + \frac{\phi\gamma - \phi - \gamma}{\gamma} \hat{W}_t + \frac{\phi\gamma - \phi - \gamma}{\gamma} \hat{W}_t + e^{e^{-1} e} du_t - e_{ww} e_{ww} \hat{W}_t + e_{ww} e_{ww} \hat{W}_t + e_{ww} e_{ww} du_t - \hat{W}_t + \hat{M}_t + \nu \hat{Y}_{t+1} + s_i \pi^e.
\]

(A11)

If (A5) and the relationship, \(\hat{L}_t \approx -s_L^{-1} du_t\), are substituted into (A11), the following equation for the efficiency wage-setting condition is obtained:

\[
-\frac{\phi + \gamma - \phi \gamma}{\gamma} s_L^{-1} du_t = \frac{1 - \gamma - \xi \gamma}{\gamma} \hat{A}_t - \frac{1 - \gamma - \xi \gamma}{\gamma} \phi(1 - e_we^{-1} s_L) s_L^{-1} du_t + \frac{1 - \gamma - \xi \gamma}{\gamma} \phi W_t - \frac{1 - \gamma - \xi \gamma}{\gamma} \phi W_t + \frac{\phi\gamma - \phi - \gamma}{\gamma} \hat{W}_t + \frac{\phi\gamma - \phi - \gamma}{\gamma} \hat{W}_t + e_{ww} e_{ww} \hat{W}_t - e_{ww} e_{ww} \hat{W}_t + \hat{M}_t + \xi \hat{Y}_{t+1} + s_i \pi^e.
\]
\[
\hat{W}_t = \frac{1}{1 + \phi \hat{z} - e_{ww} e^{-1}} \left\{ \left[ (1 + \phi \hat{z})(1 - e_s e^{-1} s_L) + e_{wa} e^{-1} \right] du_t - \phi \hat{A}_t \right. \\
+ \left. \left[ \phi \hat{z} - e_{ww} e^{-1} \right] \hat{W}_t + \hat{M}_t + \xi \hat{Y}_{t+1} + s_t \hat{\pi}_t \right\} 
\]  
(A12)

An equation for labor demand is derived with the second price level specification. To derive an equation for labor demand, the steady-state conditions \( W / \hat{W} = 1 \) and \( e_{ww} e^{-1} = 1 \) and (A9) are substituted into (13a). By making these substitutions, the labor demand curve can be expressed as

\[
\hat{W}_t = \frac{1 - \gamma \hat{Y}_t + \phi (\gamma - 1) \hat{A}_t + \phi \gamma - \phi - \gamma \hat{L}_t + \phi (\gamma - 1) e_{ww} e^{-1}}{\gamma} \hat{W}_t - \omega \gamma e_{ww} e^{-1} \frac{W_t}{\hat{W}_t} \hat{W}_t + \frac{\phi (\gamma - 1)}{\gamma} e_s e^{-1} du_t + \hat{M}_t + s_t \hat{\pi}_t, 
\]  
(A13)

If (A5) and the relationship, \( \hat{L}_t \approx -s_L e^{-1} du_t \), are substituted into (A13), the equation for the wage is

\[
\hat{W}_t = s_L e^{-1} du_t + \hat{M}_t + s_t \hat{\pi}_t, 
\]  
(A14)

**Derivation of (15)**

Solving (A11) for \( \hat{M}_t \) yields

\[
\hat{M}_t = -\left[1 + \phi \hat{z}(1 - e_s e^{-1} s_L) \right] s_L e^{-1} du_t + \phi \hat{z} \hat{A}_t - \phi \hat{z} \hat{W}_t - \xi \hat{Y}_{t+1} - s_t \hat{\pi}_t + [1 + \phi \hat{z}] \hat{W}_t. 
\]  
(A15)

Multiplying both sides of (A12) by the denominator of the right-hand side, substituting (A15) into the resulting expression, and using the equilibrium condition, \( e_{ww} e^{-1} = 1 \), gives
\[ [1 + \phi_x^e - e_{ww}e_{w}^{-1}] \hat{W}_t = [(1 + \phi_x^e)(1 - e_{w}e^{-1}_s)s_L^{-1} - \phi_x^e \hat{A}_t + e_{wu}e_{w}^{-1}] du_i \]
\[ + [\phi_x^e - e_{ww}e_{w}^{-1}] \hat{W}_t + \xi \hat{Y}_{t+1} + s_i \pi_e^e - [1 + \phi_x^e (1 - e_{w}e^{-1}_s)]s_L^{-1} du_i \]
\[ + \phi_x^e \hat{A}_t - \phi_x^e \hat{W}_t - \xi \hat{Y}_{t+1} - s_i \pi_e^e + [1 + \phi_x^e] \hat{W}_t \]

\[
\hat{W}_t = \hat{W}_t^* + \frac{e_u - e_{wu}}{e_{ww}} du_i .
\]  

(A16)

Derivation of (17):

Let \( x \) represent the wage set by firms that are able to adjust their wages. Then,

\[
\hat{W}_t = \pi_t + (1 - \tau) \hat{W}_{t-1} .
\]

Subtracting \( \hat{W}_{t-1} \) from both sides yields

\[
\pi_t^w = \tau (x_t - \hat{W}_{t-1}) ,
\]

(A17)

where \( \pi_t^w \) is the rate of wage inflation. The optimal value of \( x \) is

\[
x_t = \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \beta_j (1 - \tau)^j \hat{W}_{t+j}^* e
\]

\[
= [1 - \beta (1 - \tau)] \sum_{j=0}^{\infty} \beta^j (1 - \tau)^j \hat{W}_{t+j}^* e ,
\]

where \( \beta \) is the discount factor and \( \hat{W}_{t+j}^* e \) is a firm’s expectation of its optimal wage in period \( t+j \).

Then,

\[
x_t = [1 - \beta (1 - \tau)] \hat{W}_t^* + \beta (1 - \tau) x_{t+1}^e .
\]

Substituting (16) for \( \hat{W}_t^* \) and subtracting \( \hat{W}_{t-1} \) from both sides of the above equation yields

\[
x_t - \hat{W}_{t-1} = \beta (1 - \tau) x_{t+1}^e + [1 - \beta (1 - \tau)] \left[ \frac{\hat{W}_t^* + e_u - e_{wu}}{e_{ww}} du_i \right] - \hat{W}_{t-1} \]
\[ x_t - \hat{W}_{t-1} = \beta(1-\tau)(x^e_{t+1} - \hat{W}_t) + \beta(1-\tau)(\hat{W}_t - \hat{W}_{t-1}) \]
\[ + [1 - \beta(1-\tau)](\hat{W}_t - \hat{W}_{t-1}) + [1 - \beta(1-\tau)] \frac{e_u - e_w}{e_{ww}} du_t. \]

From (A17), \( x_t - \hat{W}_{t-1} = \pi^w_t / \tau \), and \( x^e_{t+1} - \hat{W}_t = \pi^w_{t+1} / \tau \). As a result,

\[
\frac{1}{\tau} \pi^w_t = \beta(1-\tau) \frac{\pi^w_{t+1}}{\tau} + \pi^w_t + [1 - \beta(1-\tau)] \frac{e_u - e_w}{e_{ww}} du_t. 
\]

Simplifying the above expression yields the Phillips curve equation,

\[
\pi^w_t = \beta \pi^w_{t+1} - \frac{\tau[1 - \beta(1-\tau)] e_w - e_u}{1-\tau} e_{ww} du_t. \quad \text{(A18)}
\]
References


International Monetary Fund, World Economic Outlook: Recessions and Recoveries, April 2002.


Table 1
Estimates of the Wage-Wage vs. the Wage-Price Phillips Curve

<table>
<thead>
<tr>
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<td>0.418(^a)</td>
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<td>(9.92)</td>
</tr>
<tr>
<td>(\mu_t)</td>
<td>-0.0664(^a)</td>
<td>-0.128(^a)</td>
</tr>
<tr>
<td></td>
<td>(-4.34)</td>
<td>(-8.45)</td>
</tr>
<tr>
<td>(\sum_{j=1}^{12} \pi_{t-j}^w)</td>
<td>0.931(^a)</td>
<td>0.778(^a)</td>
</tr>
<tr>
<td></td>
<td>(576.1)</td>
<td>(365.4)</td>
</tr>
<tr>
<td>(F)-test: (\sum_{j=1}^{12} \pi_{t-j}^w = 1)</td>
<td>3.19(^a)</td>
<td>29.64(^a)</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.798</td>
<td>0.718</td>
</tr>
</tbody>
</table>

\(t\)-values or \(F\)-Statistics are in parentheses

\(^a\) significant at the 1% level  
\(^b\) significant at the 5% level  
\(^c\) significant at the 10% level
Table 2
Estimates of the Phillips Curve

<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>( \mu_t )</td>
<td>-0.0464(^{b}) (-2.35)</td>
<td>-0.0822(^{a}) (-2.84)</td>
</tr>
<tr>
<td>( \pi_{t+1} )</td>
<td>0.361(^{a}) (3.18)</td>
<td>0.351(^{b}) (2.54)</td>
</tr>
<tr>
<td>( \pi_{t-1} )</td>
<td>0.194(^{a}) (3.15)</td>
<td>0.163(^{b}) (2.09)</td>
</tr>
<tr>
<td>( \pi_{t-2} )</td>
<td>0.265(^{a}) (7.44)</td>
<td>0.240(^{b}) (4.95)</td>
</tr>
<tr>
<td>( \pi_{t-3} )</td>
<td>-0.070 (0.47)</td>
<td>-0.054 (0.22)</td>
</tr>
<tr>
<td>( \pi_{t-4} )</td>
<td>0.268(^{a}) (8.98)</td>
<td>0.322(^{a}) (10.66)</td>
</tr>
<tr>
<td>( \sum_{j=1}^{12} \pi_{t-j} )</td>
<td>0.656</td>
<td>0.671</td>
</tr>
<tr>
<td>( \pi_{t+1} + \sum_{j=1}^{12} \pi_{t-j} )</td>
<td>1.017</td>
<td>1.022</td>
</tr>
</tbody>
</table>

\( t \)-statistic or Likelihood Ratio statistic in parentheses

\(^{a}\) significant at the 1% level
\(^{b}\) significant at the 5% level
\(^{c}\) significant at the 10% level
Table 3  
Estimates of the Phillips Curve and Dynamic Labor Demand Curve  
IS-LM Specification

<table>
<thead>
<tr>
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<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
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<tr>
<td>Phillips</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>$u_t$</td>
<td>-0.0470&lt;sup&gt;c&lt;/sup&gt;</td>
<td>-0.0278</td>
<td>-0.0456&lt;sup&gt;b&lt;/sup&gt;</td>
<td>-0.0478&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
<tr>
<td>$\pi_{t}^{w}$</td>
<td>0.297&lt;sup&gt;b&lt;/sup&gt;</td>
<td>0.435&lt;sup&gt;a&lt;/sup&gt;</td>
<td>0.369&lt;sup&gt;a&lt;/sup&gt;</td>
<td>0.337&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
<tr>
<td>$\pi_{t-1}^{w}$</td>
<td>0.221&lt;sup&gt;a&lt;/sup&gt;</td>
<td>0.301&lt;sup&gt;b&lt;/sup&gt;</td>
<td>0.188&lt;sup&gt;a&lt;/sup&gt;</td>
<td>0.203&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
<tr>
<td>$\pi_{t-2}^{w} - \pi_{t-4}^{w}$</td>
<td>0.256&lt;sup&gt;a&lt;/sup&gt;</td>
<td>0.074</td>
<td>0.256&lt;sup&gt;a&lt;/sup&gt;</td>
<td>0.247&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>$\pi_{t-5}^{w} - \pi_{t-8}^{w}$</td>
<td>0.003</td>
<td>0.019</td>
<td>0.003</td>
<td>-0.028</td>
</tr>
<tr>
<td>$\pi_{t-9}^{w} - \pi_{t-12}^{w}$</td>
<td>0.239&lt;sup&gt;a&lt;/sup&gt;</td>
<td>0.183&lt;sup&gt;a&lt;/sup&gt;</td>
<td>0.203&lt;sup&gt;b&lt;/sup&gt;</td>
<td>0.258&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>$\sum_{j=1}^{12} \pi_{t-j}^{w}$</td>
<td>0.719&lt;sup&gt;a&lt;/sup&gt;</td>
<td>0.577&lt;sup&gt;a&lt;/sup&gt;</td>
<td>0.649&lt;sup&gt;a&lt;/sup&gt;</td>
<td>0.681&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>$\pi_{t}^{w} + \sum_{j=1}^{12} \pi_{t-j}^{w}$</td>
<td>1.016&lt;sup&gt;a&lt;/sup&gt;</td>
<td>1.012&lt;sup&gt;a&lt;/sup&gt;</td>
<td>1.018&lt;sup&gt;a&lt;/sup&gt;</td>
<td>1.018&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DLD</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$u_t$</td>
<td>4.83&lt;sup&gt;b&lt;/sup&gt;</td>
<td>6.49</td>
<td>3.07&lt;sup&gt;a&lt;/sup&gt;</td>
<td>6.97</td>
</tr>
<tr>
<td>$u_{t-1}$</td>
<td>-4.79&lt;sup&gt;b&lt;/sup&gt;</td>
<td>-6.38</td>
<td>-3.04&lt;sup&gt;a&lt;/sup&gt;</td>
<td>-6.99</td>
</tr>
<tr>
<td>$M$</td>
<td>0.639&lt;sup&gt;a&lt;/sup&gt;</td>
<td>0.827&lt;sup&gt;a&lt;/sup&gt;</td>
<td>0.250&lt;sup&gt;b&lt;/sup&gt;</td>
<td>0.863&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>$V_{trend}$</td>
<td>0.140&lt;sup&gt;a&lt;/sup&gt;</td>
<td>0.131</td>
<td>0.531&lt;sup&gt;a&lt;/sup&gt;</td>
<td>0.174&lt;sup&gt;c&lt;/sup&gt;</td>
</tr>
<tr>
<td>$Y$</td>
<td>1.914&lt;sup&gt;a&lt;/sup&gt;</td>
<td>2.785&lt;sup&gt;a&lt;/sup&gt;</td>
<td>0.690&lt;sup&gt;a&lt;/sup&gt;</td>
<td>3.831&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>$A$</td>
<td>-0.679&lt;sup&gt;a&lt;/sup&gt;</td>
<td>-1.202&lt;sup&gt;a&lt;/sup&gt;</td>
<td>-0.384&lt;sup&gt;a&lt;/sup&gt;</td>
<td>-1.384&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>$OilPrice$</td>
<td>0.0275&lt;sup&gt;a&lt;/sup&gt;</td>
<td>0.0110</td>
<td>0.0042</td>
<td>0.0391&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>$r$</td>
<td>0.871&lt;sup&gt;a&lt;/sup&gt;</td>
<td>0.912&lt;sup&gt;a&lt;/sup&gt;</td>
<td>0.319&lt;sup&gt;b&lt;/sup&gt;</td>
<td>0.694&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>$\pi^e$</td>
<td></td>
<td>2.260&lt;sup&gt;a&lt;/sup&gt;</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>2.260&lt;sup&gt;a&lt;/sup&gt;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M+ V_{trend}$</td>
<td>0.779&lt;sup&gt;a&lt;/sup&gt;</td>
<td>0.958&lt;sup&gt;a&lt;/sup&gt;</td>
<td>0.781&lt;sup&gt;a&lt;/sup&gt;</td>
<td>1.037&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>LR test: $u_{t-1} = u_t$</td>
<td>0.57</td>
<td>1.97</td>
<td>0.52</td>
<td>0.08</td>
</tr>
<tr>
<td>LR test: $A = -\phi Y$</td>
<td>8.34&lt;sup&gt;a&lt;/sup&gt;</td>
<td>6.09&lt;sup&gt;b&lt;/sup&gt;</td>
<td>1.26</td>
<td>14.02&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>LR test: $M+ V_{trend} = v$</td>
<td>0.27</td>
<td>0.25</td>
<td>0.19</td>
<td>0.73</td>
</tr>
</tbody>
</table>

<sup>a</sup> significant at the 1% level  
<sup>b</sup> significant at the 5% level  
<sup>c</sup> significant at the 10% level
Table 4
Estimates of the Phillips Curve and Dynamic Labor Demand Curve
LM and Constant Velocity Specifications

<table>
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<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>No lags in DLD</td>
<td>Nom GDP</td>
</tr>
<tr>
<td>$u_t$</td>
<td>-0.0371$^c$</td>
<td>-0.0395$^b$</td>
</tr>
<tr>
<td>$\pi_{t+1}$</td>
<td>0.287$^a$</td>
<td>0.253$^a$</td>
</tr>
<tr>
<td>$\pi_{t-1}$</td>
<td>0.191$^a$</td>
<td>0.208$^a$</td>
</tr>
<tr>
<td>$\pi_{t-2} - \pi_{t-4}$</td>
<td>0.348</td>
<td>0.329$^a$</td>
</tr>
<tr>
<td>$\pi_{t-5} - \pi_{t-8}$</td>
<td>-0.006</td>
<td>-0.015</td>
</tr>
<tr>
<td>$\pi_{t-9} - \pi_{t-12}$</td>
<td>0.193</td>
<td>0.237$^b$</td>
</tr>
<tr>
<td>$\sum_{j=1}^{12} \pi_{t-j}$</td>
<td>0.725</td>
<td>0.758$^a$</td>
</tr>
<tr>
<td>$\pi_{t+1} + \sum_{j=1}^{12} \pi_{t-j}$</td>
<td>1.012$^a$</td>
<td>1.011$^a$</td>
</tr>
<tr>
<td>DLD</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$u_t$</td>
<td>1.64$^a$</td>
<td>4.67$^b$</td>
</tr>
<tr>
<td>$u_{t-1}$</td>
<td>-1.62$^a$</td>
<td>-4.59$^b$</td>
</tr>
<tr>
<td>$M$</td>
<td>0.599$^a$</td>
<td>0.258$^c$</td>
</tr>
<tr>
<td>$V^\text{trend}$</td>
<td>0.159$^a$</td>
<td>0.507$^c$</td>
</tr>
<tr>
<td>$i$</td>
<td>0.821$^a$</td>
<td>0.973$^b$</td>
</tr>
<tr>
<td>$\text{NomGDP}$</td>
<td>0.835$^a$</td>
<td></td>
</tr>
<tr>
<td>$M + V^\text{trend}$</td>
<td>0.758$^a$</td>
<td>0.765$^b$</td>
</tr>
<tr>
<td>LR test: $u_{t-1} = u_t$</td>
<td>0.47</td>
<td>1.42</td>
</tr>
<tr>
<td>LR test: $u_t = u_{t-1} = s^{-1}_L$</td>
<td>0.47</td>
<td>23.55$^a$</td>
</tr>
<tr>
<td>LR test: $M + V^\text{trend} = \nu$ or $\text{NomGDP} = \nu$</td>
<td>1.80</td>
<td>0.10</td>
</tr>
</tbody>
</table>

$^a$ significant at the 1% level
$^b$ significant at the 5% level
$^c$ significant at the 10% level
Figure 1

Wage Inflation and Unemployment in Response to a Disinflationary Money Supply Shock (1.25% to 0%) with Staggered Wage Setting
Phillips curve, by repeated substitution, as a relationship between current inflation and the discounted sum of expected future output gaps. They then demonstrate that there is little evidence for forward-looking expectations and that forward-looking behavior is strongly dominated by backward-looking behavior. They first regress current inflation on both this discounted sum and lagged inflation and find that the coefficient on this term has the incorrect sign. They then regress current inflation on the discounted sum of future output gaps and find that the coefficient on this term has the incorrect sign. They then regress current inflation on both this discounted sum and lagged inflation and find that the sum of the coefficients is much higher on lagged inflation than on future inflation, and he cannot reject the hypothesis that expectations are purely backward looking. Rudd and Whelan (2005) rewrite the new Keynesian Phillips curve, by repeated substitution, as a relationship between current inflation and the discounted sum of expected future output gaps. They then demonstrate that there is little evidence for forward-looking expectations and that forward-looking behavior is strongly dominated by backward-looking behavior. They first regress current inflation on the discounted sum of future output gaps and find that the coefficient on this term has the incorrect sign. They then regress current inflation on both this discounted sum and lagged inflation and find that the sum of the coefficients on lagged inflation is highly significant and close to 0.9, suggesting that past inflation matters a great deal, even when controlling for forward-looking expectations.

According to McCallum (1997), the Calvo-Rotemberg model of the Phillips curve has become, ‘the closest thing there is to a standard specification.’

Fuhrer (1997) regresses current inflation on lagged inflation, expected future inflation, and the output gap. He finds that the sum of coefficients is much higher on lagged inflation than on future inflation, and he cannot reject the hypothesis that expectations are purely backward looking. Rudd and Whelan (2005) rewrite the new Keynesian Phillips curve, by repeated substitution, as a relationship between current inflation and the discounted sum of expected future output gaps. They then demonstrate that there is little evidence for forward-looking expectations and that forward-looking behavior is strongly dominated by backward-looking behavior. They first regress current inflation on the discounted sum of future output gaps and find that the coefficient on this term has the incorrect sign. They then regress current inflation on both this discounted sum and lagged inflation and find that the sum of the coefficients on lagged inflation is highly significant and close to 0.9, suggesting that past inflation matters a great deal, even when controlling for forward-looking expectations.


Galí (2011) derives a model of the wage Phillips curve under the assumption that workers set their own wages. Campbell (2010) develops a rudimentary version of the model in this study without staggered wage adjustment. This previous study derives an equation for the wage Phillips curve, but does not derive the dynamic labor demand curve or the efficiency wage-setting condition, nor does it consider staggered wage adjustment or empirically estimate the model. The present study derives the transition paths the economy follows in response to shocks, it allows wages to be adjusted sporadically, and it provides empirical tests of the model.

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Footnotes

1 Samuelson and Solow (1960) show that a Phillips curve relationship can also be derived between price inflation and unemployment.


3 Galí (2011) derives a model of the wage Phillips curve under the assumption that workers set their own wages.

4 Campbell (2010) develops a rudimentary version of the model in this study without staggered wage adjustment. This previous study derives an equation for the wage Phillips curve, but does not derive the dynamic labor demand curve or the efficiency wage-setting condition, nor does it consider staggered wage adjustment or empirically estimate the model. The present study derives the transition paths the economy follows in response to shocks, it allows wages to be adjusted sporadically, and it provides empirical tests of the model.

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7 See Congressional Budget Office (2004) for a description of how the CBO calculates potential GDP, using both estimates of the natural rate and Okun’s law in its procedure.

8 In Campbell (2006), it is assumed that utility is logarithmic, which is a special case of a constant relative risk aversion utility function, with $\theta = 1$.

9 The model of Campbell (2006) is based on the shirking model of Shapiro and Stiglitz (1984) and the gift-exchange model of Akerlof (1982). (While the main results can be derived from the shirking model alone, including gift-exchange results in a more realistic curvature of the effort function.) The variable $W^f$ in Campbell (2006) corresponds to the average wage in the present study.

10 In this model, efficiency depends positively on wages since higher wages raise workers’ effort. Other explanations for a positive effect of wages on efficiency include the labor turnover models of Stiglitz (1974), Schlicht (1978), and Salop (1979) and the adverse selection model of Weiss (1980).

11 Assuming a positive relationship between wages and efficiency does not guarantee that there will be excess supply of labor. Whether a firm operates on its labor supply curve or to the left of its labor supply curve (i.e., pays an efficiency wage) depends on the elasticity of output with respect to the wage, calculated at the market-clearing wage. Parameters are chosen so that firms maximize profits by operating to the left of their labor supply curves.

12 Fuhrer (1997) estimates Phillips curves in which expected inflation depends on a weighted average of lagged inflation and actual future inflation. He can reject the hypothesis that expectations are completely rational, but cannot reject the hypothesis that they are completely adaptive. However, he demonstrates that inflation dynamics are predicted more accurately by a model with mixed rational and adaptive expectations than by a model with completely adaptive expectations. Roberts (1998) shows that survey forecasts of inflation can be explained by a model in which part of the population has rational expectations and the rest has adaptive expectations. Pfajfar and Santoro (2010) examine a cross section of individuals’ inflation forecasts, and they find that some individuals have rational expectations, some form their expectations adaptively, while the expectations of others are based on adaptive learning and sticky information.

13 In Campbell (2014), agents can estimate the average wage by observing lagged average wages at a low fixed cost and by incurring an added variable cost to acquire additional information about the mean of the current wage distribution through obtaining and processing macroeconomic data. Agents acquire the amount of information that minimizes the sum of information acquisition costs and the expected utility loss resulting from imperfect information. A Kalman filtering process is used to predict the average wage, and it is demonstrated that expectations are a mixture of rational and adaptive expectations if previous information is orthogonally updated. The value of $\omega$
depends on the cost of information (which may be affected by the transparency of policymakers) and on the variability of demand. It should be noted that Campbell (2014) models expectations of current average wages, while the present model makes assumptions about expectations of future average wages, but the model of Campbell (2014) could be extended to consider expectations of future average wages.

This derivation makes use of the fact that \( e^{-1} = 1 \) in equilibrium, from (12).

See equation 25 on p. 698 of Phelps (1968).

In this model, equations are derived for the paths of wages, prices, and unemployment in response to nominal demand shocks, and these equations are used as data in a theoretical regression of either price inflation or wage inflation on unemployment and lagged price inflation. In these regressions, the coefficient on lagged price inflation asymptotically approaches 1 as the sample size increases, and it is close to 1 even when the sample size is small.

As discussed later in this section, including expected price inflation may be problematic, so it is not used as an instrument.

In Blanchard (1984), Fuhrer (1995), and Staiger, Stock, and Watson (2001), the variable for quarterly wage inflation is expressed in annualized terms, so the coefficient on the unemployment rate is divided by 4 to make the reported coefficient reflect the effect of unemployment on the actual quarter-to-quarter rate of wage inflation. In addition, Blanchard (1984) estimates the Phillips curve with the log of unemployment, so the estimated coefficient is divided by its equilibrium value, assumed to be 6%, so that the reported coefficient measures the effect of a one percentage-point change in the unemployment rate on wage inflation.

Campbell (2009) shows a more direct rationale for including this variable by treating oil as a factor in the production function and demonstrating that the price of oil affects aggregate supply, independent of pure technology shocks.

The 3-month Treasury Bill rate is used in the regressions since both (6) and (7) suggest a short-term interest rate is the appropriate variable. The 5-year Treasury note rate was also used in some unreported regressions, but this interest rate was found to have less explanatory power.

Because the current and recent lagged values of trend velocity are highly correlated with one another, the sum of the current and three lagged changes in trend velocity is entered as a single variable.

If the unemployment rate in the first quarter of a recession (based on NBER dating) is higher than in the previous quarter, the first quarter is treated as period 1. Otherwise, the subsequent quarter is treated as period 1.

The calculations for the median time for the unemployment rate to return to the natural rate do not include the 1970 recession, since unemployment was below the natural rate for almost the entire recession, and do not include the 1980 recession, since unemployment was not close to the natural rate until after the subsequent recession.

This derivation follows Romer (2012, pp. 329-331).