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Krouglov, Alexei

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Alexei Krouglov

alexkrouglov@gmail.com

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ABSTRACT

Presented here is a simplified mathematical model to reflect a weak recovery after the financial crisis. The model confirms hypothesis that the weak recovery is caused by a decline in investment not compensated by the interest rate decrease. The model explains a transformation of economic trend lines. Graphical representation shows how the transformation of economic trend occurs either with or without fluctuations of short-time variations. The graphical representation agrees with practically observable tendencies.

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1 Introductory Discussion

A thorough understanding of the economic growth is very important for humanity. Especially, significance is emphasized by the weak economic recovery after the Great Recession. Additional gravity is imparted by the desire to alleviate seriousness of the future financial crises.

Let us first try to conceptualize the phenomenon of economic growth. Here is how I understand it following the framework outlined in (Krouglov, 2006; 2009) and others. There is a base supply-demand field formed on three economic quantities: supply, demand, and price. In equilibrium the quantities of supply and demand are equal. When the equality of supply and demand is broken then third quantity, the price, is changed in order to bring the field back into equilibrium by affecting the quantities of both supply and demand.

Here comes the economic growth. Imagine situation where the fractional quantity of supply is permanently removed from the market. (One could consider removed quantity as the portion intended for investment.) *Question:* How would the supply-demand field react to this situation? *Answer:* The equality of supply and demand will be broken, the price will be increasing and the demand will tend to shrink. *Question:* How would the demand in money terms react? (The "demand in money terms" can be viewed as the product of a quantity of demand multiplied by price.) *Answer:* If the price increases faster than the demand shrinks, then the demand in money terms increases. If the price increases slower than the demand shrinks, then the demand in money terms decreases. The increment of demand in money terms I associate with an economic growth while the loss of one is associated with an economic decline.

Thus, general explanation of the economic growth is interpreted as an economic phenomenon caused by a partial use of supply for the investment purposes. Furthermore, few particular applications for the economic growth were examined in (Krouglov, 2014a; 2014b).

The aforesaid framework was expanded in (Krouglov, 2015a; 2015b) by the inclusion of debt sustenance and debt accumulation. Joint employment of the concepts of investment, debt sustenance and accumulation allowed modeling of economic recessions. In particular, mutual application of these concepts was used in an attempt of modeling of the Greek crises (Krouglov, 2015c).

Subsequent interpretation of the foresaid concepts prompted following hypothesis for the changes in an economic growth. Decline in investment causes the decrement in an economic growth (i.e., the economic trend flattens). To counteract the process of economic deterioration monetary authorities decrease the interest rate as a compensatory measure. Decline of the interest rate decreases the cost of debt maintenance and thereafter increases an economic growth (i.e., the economic trend steepens). If the effect of economic trend's steepening prevails over the effect of economic trend's flattening then an increase of the economic growth takes place (i.e., the entire economic trend steepens). On the other hand, if the effect of economic trend's flattening prevails over the effect of economic trend's steepening then a decrease of the economic growth takes place (i.e., the entire economic trend flattens).

The principle aim of the current article was to build a mathematical model in order to either verify or reject the stated above hypothesis.

We have considered an entire economic process situated in the domain of real economy. I presume this domain is enough for understanding the base economic processes pertaining to an economic growth and economic trend. As can be seen in (Krouglov, 2013), inclusion of the monetary market can exaggerate real economic results. Anyhow, the effect of monetary market is not considered in the article. Additional challenge and improvement to the article could be inclusion of the technological advancements, which are also not examined here.

In the next sections I present mathematical models that try to support the expressed above argumentations and reasoning.

Another aim of the article is to compare theoretical results of changes in the economic growth with the economic trends observable in practice after financial crisis. For this purpose, the graphical drawings illustrating changes of the economic trend will be presented.

2 Outline of the Apparatus

I deploy a mathematical model of the market for single-product economy, which can give us an explicit observation of interactions among different economic variables. Economic forces acting on the market reflect inherent market forces of demand and supply complemented with the economic forces caused by an investment, debt accumulation and corresponding debt servicing. The market actions are expressed through the system of ordinary differential equations.

When there are no disturbing economic forces, the market is in equilibrium position, i.e., the supply of and demand for product are equal, the quantities of supply and demand are developing with a constant rate and a price of the product is fixed.

I assume the market had been in an equilibrium until time $t = t_0$, volumes of the product supply $V_S(t)$ and demand $V_D(t)$ on market were equal, and they both were developing with a constant rate r_D^0. The product price $P(t)$ at that time was fixed,

$$V_D(t) = r_D^0 (t - t_0) + V_D^0 \tag{1}$$

$$V_S(t) = V_D(t) \tag{2}$$

$$P(t) = P^0 \tag{3}$$

where $V_D(t_0) = V_D^0$.

When the balance between the volumes of product supply and demand is broken, market is experiencing economic forces, which act to bring the market to a new equilibrium position.

I use a model of the single-product economy where the credit is increasing with a constant rate and the investment is increasing with a constant acceleration (Krouglov, 2015b).

According to the scenario, the credit expansion causes a debt growth where the amount of debt $S_D(t)$ on the market rises since time $t = t_0$ according to following formula,

$$S_D(t) = \begin{cases} 0, & t < t_0 \\ \delta_D (t - t_0), & t \geq t_0 \end{cases} \tag{4}$$

where $S_D(t) = 0$ for $t < t_0$ and $\delta_D > 0$.

Correspondingly, the debt accumulation causes an increase of the debt servicing cost $s_S(t) = \delta_S\, S_D(t)$, $\delta_S > 0$, where the accumulated amount of debt servicing cost $S_S(t)$ rises according to following formula since time $t = t_0$,

$$S_S(t) = \begin{cases} 0, & t < t_0 \\ \dfrac{\delta_S\, \delta_D}{2} (t - t_0)^2, & t \geq t_0 \end{cases} \tag{5}$$

where $S_S(t) = 0$ for $t < t_0$ and $\delta_S > 0$.

Likewise, I assume the amount of investment $S_I(t)$ on the market increases since time $t = t_0$ according to following formula,

$$S_I(t) = \begin{cases} 0, & t < t_0 \\ \delta_I (t - t_0) + \dfrac{\varepsilon_I}{2} (t - t_0)^2, & t \geq t_0 \end{cases} \tag{6}$$

where $S_I(t) = 0$ for $t < t_0$, $\delta_I \geq 0$, and $\varepsilon_I > 0$.

Economic forces trying to bring the market into a new equilibrium position are described by the following ordinary differential equations with regard to the volumes of product supply $V_S(t)$, demand $V_D(t)$, and price $P(t)$ given the accumulated amounts of debt $S_D(t)$, debt servicing cost $S_S(t)$, and investment $S_I(t)$ on the market,

$$\frac{dP(t)}{dt} = -\lambda_P \left(V_S(t) - V_D(t) - S_D(t) + S_S(t) - S_I(t)\right) \tag{7}$$

$$\frac{d^2 V_S(t)}{dt^2} = \lambda_S \frac{dP(t)}{dt} \tag{8}$$

$$\frac{d^2 V_D(t)}{dt^2} = -\lambda_D \frac{d^2 P(t)}{dt^2} \tag{9}$$

In Eqs. (7) – (9) above the values $\lambda_P, \lambda_S, \lambda_D \geq 0$ are constants and they reflect the price inertness, supply inducement, and demand amortization correspondingly.

One may observe that a difference between the volumes of product demand and supply on the market is increasing, in the sense:

$$V_D(t) - V_S(t) + \left(S_I(t) + S_D(t) - S_S(t)\right) \geq V_D(t) - V_S(t)$$

if the sum of accumulated amount of investment $S_I(t)$ and accumulated amount of debt $S_D(t)$ exceeds the accumulated amount of debt servicing cost $S_S(t)$, i.e., $S_I(t) + S_D(t) \geq S_S(t)$.

On the other hand, a difference between the volumes of product demand and supply on the market is decreasing, in the sense:

$$V_D(t) - V_S(t) + \left(S_I(t) + S_D(t) - S_S(t)\right) < V_D(t) - V_S(t)$$

if the sum of accumulated amount of investment $S_I(t)$ and accumulated amount of debt $S_D(t)$ goes below the accumulated amount of debt servicing cost $S_S(t)$, i.e., $S_I(t) + S_D(t) < S_S(t)$.

Thus, if $\delta_S\,\delta_D > \varepsilon_I$ then in time interval $t_0 \le t \le t_0 + \dfrac{2(\delta_I + \delta_D)}{\delta_S\,\delta_D - \varepsilon_I}$ an expansion of the product demand over supply takes place, and in time interval $t_0 + \dfrac{2(\delta_I + \delta_D)}{\delta_S\,\delta_D - \varepsilon_I} < t < +\infty$ a reduction of the product demand over supply occurs. If $\delta_S\,\delta_D \le \varepsilon_I$ then an expansion of the product demand over supply takes place in the entire time interval $t_0 \le t < +\infty$.

Let me use a new variable $D(t) \equiv (V_S(t) - V_D(t) - S_D(t) + S_S(t) - S_I(t))$ representing the volume of product surplus (or shortage) on the market. Therefore, behavior of $D(t)$ is described by the following equation for $t > t_0$,

$$\frac{d^2 D(t)}{dt^2} + \lambda_P\,\lambda_D\,\frac{dD(t)}{dt} + \lambda_P\,\lambda_S\,D(t) + \varepsilon_I - \delta_S\,\delta_D = 0 \tag{10}$$

with the initial conditions, $D(t_0) = 0$, $\dfrac{dD(t_0)}{dt} = -\delta_I - \delta_D$.

If one introduces another variable $D_1(t) \equiv D(t) + \dfrac{\varepsilon_I - \delta_S\,\delta_D}{\lambda_P\,\lambda_S}$, then Eq. (10) becomes,

$$\frac{d^2 D_1(t)}{dt^2} + \lambda_P\,\lambda_D\,\frac{dD_1(t)}{dt} + \lambda_P\,\lambda_S\,D_1(t) = 0 \tag{11}$$

with the initial conditions, $D_1(t_0) = \dfrac{\varepsilon_I - \delta_S\,\delta_D}{\lambda_P\,\lambda_S}$, $\dfrac{dD_1(t_0)}{dt} = -\delta_I - \delta_D$.

Similar to Eq. (10), the product price $P(t)$ is described by the following equation for $t > t_0$,

$$\frac{d^2 P(t)}{dt^2} + \lambda_P\,\lambda_D\,\frac{dP(t)}{dt} + \lambda_P\,\lambda_S\left(P(t) - P^0 - \frac{\delta_I + \delta_D}{\lambda_S} - \frac{\varepsilon_I - \delta_S\,\delta_D}{\lambda_S}(t - t_0)\right) = 0 \tag{12}$$

with the initial conditions, $P(t_0) = P^0$, $\dfrac{dP(t_0)}{dt} = 0$.

Let me use variable $P_1(t) \equiv P(t) - P^0 - \frac{\delta_I + \delta_D}{\lambda_S} - \frac{\varepsilon_I - \delta_S \, \delta_D}{\lambda_S}(t - t_0) + \frac{\lambda_D}{\lambda_S^2}(\varepsilon_I - \delta_S \, \delta_D)$ to simplify analysis of the product price behavior. The behavior of variable $P_1(t)$ is described by following equation for $t > t_0$,

$$\frac{d^2 P_1(t)}{dt^2} + \lambda_P \, \lambda_D \frac{dP_1(t)}{dt} + \lambda_P \, \lambda_S \, P_1(t) = 0 \tag{13}$$

with the initial conditions, $P_1(t_0) = -\frac{\delta_I + \delta_D}{\lambda_S} + \frac{\lambda_D}{\lambda_S^2}(\varepsilon_I - \delta_S \, \delta_D)$, $\frac{dP_1(t_0)}{dt} = -\frac{\varepsilon_I - \delta_S \, \delta_D}{\lambda_S}$.

The behavior of solutions for $D_1(t)$ and $P_1(t)$ described by Eqs. (11) and (13) depends on the roots of the corresponding characteristic equations (Piskunov, 1965; Petrovski, 1966). Also Eqs. (11) and (13) have the same characteristic equations.

When the roots of characteristic equation are complex-valued (i.e., $\frac{\lambda_P^2 \, \lambda_D^2}{4} < \lambda_P \, \lambda_S$) both variables $D_1(t)$ and $P_1(t)$ experience damped oscillations for time $t \geq t_0$. If the roots of characteristic equation are real and different (i.e., $\frac{\lambda_P^2 \, \lambda_D^2}{4} > \lambda_P \, \lambda_S$) both variables $D_1(t)$ and $P_1(t)$ don't oscillate for time $t \geq t_0$. If the roots of characteristic equation are real and equal (i.e., $\frac{\lambda_P^2 \, \lambda_D^2}{4} = \lambda_P \, \lambda_S$) both variables $D_1(t)$ and $P_1(t)$ don't oscillate for time $t \geq t_0$ as well.

It takes place $D_1(t) \to 0$ and $P_1(t) \to 0$ for $t \to +\infty$ if roots of characteristic equations are complex-valued ($\frac{\lambda_P^2 \, \lambda_D^2}{4} < \lambda_P \, \lambda_S$), real and different ($\frac{\lambda_P^2 \, \lambda_D^2}{4} > \lambda_P \, \lambda_S$), or real and equal ($\frac{\lambda_P^2 \, \lambda_D^2}{4} = \lambda_P \, \lambda_S$).

We can observe for the product surplus (shortage) $D(t)$, for the product price $P(t)$, for the product demand $V_D(t)$, for the product supply $V_S(t)$, for the amount of debt $S_D(t)$, for the amount of debt servicing cost $S_S(t)$, and for the amount of investment $S_I(t)$ if $t \to +\infty$,

$$D(t) \to -\frac{\varepsilon_I - \delta_S\,\delta_D}{\lambda_P\,\lambda_S} \tag{14}$$

$$P(t) \to \frac{\varepsilon_I - \delta_S\,\delta_D}{\lambda_S}(t - t_0) + P^0 + \frac{\delta_I + \delta_D}{\lambda_S} - \frac{\lambda_D}{\lambda_S^2}(\varepsilon_I - \delta_S\,\delta_D) \tag{15}$$

$$V_D(t) \to \left(r_D^0 - \frac{\lambda_D}{\lambda_S}(\varepsilon_I - \delta_S\,\delta_D) \right)(t - t_0) + V_D^0 - \frac{\lambda_D}{\lambda_S}(\delta_I + \delta_D) + \frac{\lambda_D^2}{\lambda_S^2}(\varepsilon_I - \delta_S\,\delta_D) \tag{16}$$

$$\begin{aligned} V_S(t) \to & \left(r_D^0 + \delta_I + \delta_D - \frac{\lambda_D}{\lambda_S}(\varepsilon_I - \delta_S\,\delta_D) \right)(t - t_0) + \frac{\varepsilon_I - \delta_S\,\delta_D}{2}(t - t_0)^2 \\ & + V_D^0 - \frac{\lambda_D}{\lambda_S}(\delta_I + \delta_D) - \frac{\varepsilon_I - \delta_S\,\delta_D}{\lambda_P\,\lambda_S} + \frac{\lambda_D^2}{\lambda_S^2}(\varepsilon_I - \delta_S\,\delta_D) \end{aligned} \tag{17}$$

$$S_I(t) = \delta_I\,(t - t_0) + \frac{\varepsilon_I}{2}(t - t_0)^2 \tag{18}$$

$$S_D(t) = \delta_D\,(t - t_0) \tag{19}$$

$$S_S(t) = \frac{\delta_S\,\delta_D}{2}(t - t_0)^2 \tag{20}$$

To analyze an economic growth I use the variable $E_D(t) \equiv P(t) \times r_D(t)$ where $r_D(t) \equiv \frac{dV_D(t)}{dt}$, i.e., a rate of demand in money terms for the product, which reflects the product earning on the market.

The variable $E_D(t)$, a rate of demand in money terms adjusted by the amount of debt $S_D(t)$, amount of debt servicing cost $S_S(t)$, and amount of investment $S_I(t)$, for $t \to +\infty$ converges toward

$E_D(t) \to \left(\frac{\varepsilon_I - \delta_S\,\delta_D}{\lambda_S}(t - t_0) + P^0 + \frac{\delta_I + \delta_D}{\lambda_S} - \frac{\lambda_D}{\lambda_S^2}(\varepsilon_I - \delta_S\,\delta_D) \right)\left(r_D^0 - \frac{\lambda_D}{\lambda_S}(\varepsilon_I - \delta_S\,\delta_D) \right)$, and the variable $\tilde{E}_D(t)$, an original rate of demand in money terms, for $t \to +\infty$ converges toward

$\tilde{E}_D(t) \to P^0 \, r_D^0$.

If the amount of debt $S_D(t)$ is increasing with a constant rate $\delta_D > 0$ and the amount of investment $S_I(t)$ is increasing with a constant-acceleration $\varepsilon_I > 0$ the resulting outcome depends on relationship between the acceleration of debt servicing cost $S_S(t)$, i.e., $\delta_S \, \delta_D > 0$, and the acceleration of investment $S_I(t)$, i.e., $\varepsilon_I > 0$.

If $\delta_S \, \delta_D > \varepsilon_I$, the amount of debt servicing cost $S_S(t)$ is going to exceed amount of investment $S_I(t)$, i.e., $\varepsilon_I - \delta_S \, \delta_D < 0$ and $r_D^0 - \frac{\lambda_D}{\lambda_S}(\varepsilon_I - \delta_S \, \delta_D) > 0$, causes $r_D(t) < 0$, and finally brings unrestricted decrease of the rate of demand in money terms $E_D(t)$ with the passage of time.

We can assess a decrease $e_D(t)$ of the rate of demand in money terms $E_D(t)$ where $e_D(t) \equiv \frac{dE_D(t)}{dt}$, i.e., the decrease of a rate of demand in money terms for product, which reflects the decrease of the product earning on market.

It takes place $e_D(t) \to \frac{\varepsilon_I - \delta_S \, \delta_D}{\lambda_S}\left(r_D^0 - \frac{\lambda_D}{\lambda_S}(\varepsilon_I - \delta_S \, \delta_D)\right) < 0$ when $\delta_S \, \delta_D > \varepsilon_I$ for $t \to +\infty$.

The converged value of variable $e_D(t)$ doesn't have extremal points in the region $\delta_S \, \delta_D > \varepsilon_I$. In fact, the variable $e_D(t)$ has a maximal converged value when $\varepsilon_I = \delta_S \, \delta_D + \frac{\lambda_S}{2\lambda_D} r_D^0$. Respectively, the maximum for converged values $e_D(t)$ is equal to $\frac{1}{4\lambda_D}\left(r_D^0\right)^2$, i.e., $e_D(t)_{MAX} \to \frac{1}{4\lambda_D}\left(r_D^0\right)^2 > 0$ for $t \to +\infty$.

Therefore, the converged value of variable $e_D(t)$ is always negative in the region $\delta_S\,\delta_D > \varepsilon_I$. Maximum for the change of rate of demand in money terms, $e_D(t)_{MAX}$, is attained outside of the region $\delta_S\,\delta_D > \varepsilon_I$ and is equal to $\frac{1}{4\lambda_D}\left(r_D^0\right)^2$, i.e., $e_D(t)_{MAX} \to \frac{1}{4\lambda_D}\left(r_D^0\right)^2 > 0$ for time $t \to +\infty$.

If $\delta_S\,\delta_D < \varepsilon_I < \delta_S\,\delta_D + \frac{\lambda_S}{\lambda_D} r_D^0$, the amount of investment $S_I(t)$ exceeds the amount of debt servicing cost $S_S(t)$, i.e., $\varepsilon_I - \delta_S\,\delta_D > 0$ and $r_D^0 - \frac{\lambda_D}{\lambda_S}\left(\varepsilon_I - \delta_S\,\delta_D\right) > 0$, and causes $r_D(t) > 0$. Therefore, it brings unrestricted increase of the rate of demand in money terms $E_D(t)$ with the passage of time.

We can estimate an increase $e_D(t)$ of the rate of demand in money terms $E_D(t)$, i.e., the increase of a rate of demand in money terms for product, which reflects the increase of the product earning on market.

It takes place $e_D(t) \to \frac{\varepsilon_I - \delta_S\,\delta_D}{\lambda_S}\left(r_D^0 - \frac{\lambda_D}{\lambda_S}\left(\varepsilon_I - \delta_S\,\delta_D\right)\right) > 0$ when $\delta_S\,\delta_D < \varepsilon_I < \delta_S\,\delta_D + \frac{\lambda_S}{\lambda_D} r_D^0$ for $t \to +\infty$.

The variable $e_D(t)$ has a maximal converged value when $\varepsilon_I = \delta_S\,\delta_D + \frac{\lambda_S}{2\lambda_D} r_D^0$. Respectively, it takes place $e_D(t)_{MAX} \to \frac{1}{4\lambda_D}\left(r_D^0\right)^2 > 0$ for $t \to +\infty$.

Therefore, the maximal increase in rate of demand in money terms, $e_D(t)_{MAX}$, which reflects maximal increase of the product earning or maximal economic growth, is equal to $e_D(t)_{MAX} \to \frac{1}{4\lambda_D}\left(r_D^0\right)^2 > 0$ for time $t \to +\infty$.

If $\delta_S\,\delta_D + \frac{\lambda_S}{\lambda_D} r_D^0 < \varepsilon_I < +\infty$, the amount of investment $S_I(t)$ exceeds the amount of debt servicing cost $S_S(t)$, i.e., $\varepsilon_I - \delta_S\,\delta_D > 0$ and $r_D^0 - \frac{\lambda_D}{\lambda_S}\left(\varepsilon_I - \delta_S\,\delta_D\right) < 0$, and causes $r_D(t) < 0$. It ultimately brings unrestricted decrease of the rate of demand in money terms $E_D(t)$ with the passage of time.

We can estimate a decrease $e_D(t)$ of the rate of demand in money terms $E_D(t)$, i.e., the decrease of a rate of demand in money terms for product, which reflects a decrease of the product earning on market.

It takes place $e_D(t) \to \frac{\varepsilon_I - \delta_S\,\delta_D}{\lambda_S}\left(r_D^0 - \frac{\lambda_D}{\lambda_S}\left(\varepsilon_I - \delta_S\,\delta_D\right)\right) < 0$ when $\delta_S\,\delta_D + \frac{\lambda_S}{\lambda_D} r_D^0 < \varepsilon_I < +\infty$ for $t \to +\infty$.

In the region $\delta_S\,\delta_D + \frac{\lambda_S}{\lambda_D} r_D^0 < \varepsilon_I < +\infty$ the converged value of variable $e_D(t)$ doesn't have extremal points. In fact, the variable $e_D(t)$ has a maximal converged value when $\varepsilon_I = \delta_S\,\delta_D + \frac{\lambda_S}{2\lambda_D} r_D^0$. Hence respectively it takes place $e_D(t)_{MAX} \to \frac{1}{4\lambda_D}\left(r_D^0\right)^2 > 0$ for $t \to +\infty$.

Thus, the converged value of variable $e_D(t)$ is always negative in the region $\delta_S\,\delta_D + \frac{\lambda_S}{\lambda_D} r_D^0 < \varepsilon_I < +\infty$.

Maximum for the change of rate of demand in money terms, $e_D(t)_{MAX}$, is attained outside of the region

$\delta_S\,\delta_D + \frac{\lambda_S}{\lambda_D} r_D^0 < \varepsilon_I < +\infty$, and is equal to $e_D(t)_{MAX} \to \frac{1}{4\lambda_D}\left(r_D^0\right)^2 > 0$ for time $t \to +\infty$.

Presented in this section is a single-product economy model that describes economic growth through the interactions between investment and credit expansion as following. Initially, the demand for product and supply of it are equal and the market is undisturbed.

The next scenario considered in the model is following. The demand for product on market is increased by undertaking debt growing with a constant rate. Simultaneously, the supply of product was decreased and partially removed from the market through an investment growing with a constant acceleration. The credit expansion in turn causes an increase of the debt servicing cost. Since the debt servicing cost is proportional to accumulated amount of debt, i.e., an integral over time of the assumed debt, the amount of debt servicing cost is growing with a constant acceleration.

Thus, the sum of accumulated amount of investment and accumulated amount of debt is growing with a constant acceleration and it increases the excess of demand over supply. Likewise, the accumulated amount of debt servicing cost is growing with a constant acceleration and it increases the excess of supply over demand. Here, the outcome depends on relations between quantitative values of two accelerations.

If the acceleration rate of debt servicing cost exceeds the acceleration rate of investment then a temporal axis is divided in two intervals. Originally, there is a limited short-term economic growth in money terms. Later, there is an unrestricted long-term economic decline in money terms.

If the acceleration rate of investment exceeds the acceleration rate of debt servicing cost then an identical process takes place over the entire temporal axis that depends on the difference between two accelerations.

If the acceleration rate of investment slightly exceeds the acceleration rate of debt servicing cost then there is an unrestricted long-term economic growth in money terms. On the other hand, if the acceleration rate of investment considerably exceeds the acceleration rate of debt servicing cost then there is an unrestricted long-term economic decline in money terms.

3 Change of the Trend Line

For simplicity, let me use a new variable $S_\Sigma(t) \equiv S_I(t) + S_D(t) - S_S(t)$ combining the accumulated amounts of debt $S_D(t)$, debt servicing cost $S_S(t)$, and investment $S_I(t)$ on the market and representing the accumulated amount of disruption on the market for $t > t_0$.

The accumulated amount of disruption $S_\Sigma(t)$ on the market since time $t = t_0$ is then equal to,

$$S_\Sigma(t) = \begin{cases} 0, & t < t_0 \\ \delta_0 (t - t_0) + \dfrac{\varepsilon_0}{2}(t - t_0)^2, & t \geq t_0 \end{cases} \tag{21}$$

where $S_\Sigma(t) = 0$ for $t < t_0$, $\delta_0 = \delta_I^0 + \delta_D^0$, and $\varepsilon_0 = \varepsilon_I^0 - \delta_S^0 \, \delta_D^0$.

The volume of product surplus (or shortage) $D(t)$ on market is equal to $D(t) = V_S(t) - V_D(t) - S_\Sigma(t)$.

It takes place for the product surplus (shortage) $D(t)$, for the product price $P(t)$, for the product demand $V_D(t)$, for the product supply $V_S(t)$, and for the accumulated amount of disruption $S_\Sigma(t)$ if $t \to +\infty$,

$$D(t) \to -\frac{\varepsilon_0}{\lambda_P \, \lambda_S} \tag{22}$$

$$P(t) \to \frac{\varepsilon_0}{\lambda_S}(t - t_0) + P^0 + \frac{\delta_0}{\lambda_S} - \frac{\lambda_D}{\lambda_S^2}\varepsilon_0 \tag{23}$$

$$V_D(t) \to \left(r_D^0 - \frac{\lambda_D}{\lambda_S} \varepsilon_0 \right)(t - t_0) + V_D^0 - \frac{\lambda_D}{\lambda_S} \delta_0 + \frac{\lambda_D^2}{\lambda_S^2} \varepsilon_0 \tag{24}$$

$$V_S(t) \to \left(r_D^0 + \delta_0 - \frac{\lambda_D}{\lambda_S} \varepsilon_0 \right)(t - t_0) + \frac{\varepsilon_0}{2}(t - t_0)^2 + V_D^0 - \frac{\lambda_D}{\lambda_S} \delta_0 - \frac{\varepsilon_0}{\lambda_P \lambda_S} + \frac{\lambda_D^2}{\lambda_S^2} \varepsilon_0 \tag{25}$$

$$S_\Sigma(t) = \delta_0 (t - t_0) + \frac{\varepsilon_0}{2}(t - t_0)^2 \tag{26}$$

It also takes place for the rate of product demand in money terms $E_D(t)$ when $t \to +\infty$,

$$E_D(t) \to \left(\frac{\varepsilon_0}{\lambda_S}(t - t_0) + P^0 + \frac{\delta_0}{\lambda_S} - \frac{\lambda_D}{\lambda_S^2} \varepsilon_0 \right)\left(r_D^0 - \frac{\lambda_D}{\lambda_S} \varepsilon_0 \right) \tag{27}$$

We can observe four relevant economic trend lines,

$$\overline{P}^0(t) = \frac{\varepsilon_0}{\lambda_S}(t - t_0) + P^0 + \frac{\delta_0}{\lambda_S} - \frac{\lambda_D}{\lambda_S^2} \varepsilon_0$$

$$\overline{V}_D^0(t) = \left(r_D^0 - \frac{\lambda_D}{\lambda_S} \varepsilon_0 \right)(t - t_0) + V_D^0 - \frac{\lambda_D}{\lambda_S} \delta_0 + \frac{\lambda_D^2}{\lambda_S^2} \varepsilon_0$$

$$\overline{V}_S^0(t) = \left(r_D^0 + \delta_0 - \frac{\lambda_D}{\lambda_S} \varepsilon_0 \right)(t - t_0) + \frac{\varepsilon_0}{2}(t - t_0)^2 + V_D^0 - \frac{\lambda_D}{\lambda_S} \delta_0 - \frac{\varepsilon_0}{\lambda_P \lambda_S} + \frac{\lambda_D^2}{\lambda_S^2} \varepsilon_0$$

$$\overline{E}_D^0(t) = \left(\frac{\varepsilon_0}{\lambda_S}(t - t_0) + P^0 + \frac{\delta_0}{\lambda_S} - \frac{\lambda_D}{\lambda_S^2} \varepsilon_0 \right)\left(r_D^0 - \frac{\lambda_D}{\lambda_S} \varepsilon_0 \right)$$

where $\lim\limits_{t \to +\infty} P(t) = \overline{P}^0(t)$, $\lim\limits_{t \to +\infty} V_D(t) = \overline{V}_D^0(t)$, $\lim\limits_{t \to +\infty} V_S(t) = \overline{V}_S^0(t)$, $\lim\limits_{t \to +\infty} E_D(t) = \overline{E}_D^0(t)$.

Then I introduce more changes on the market since time $t = t_1$.

I assume $t_1 >> t_0$, so the variables $P(t)$, $V_D(t)$, $V_S(t)$, $E_D(t)$ *practically* coincide with the variables $\overline{P}^0(t)$, $\overline{V}_D^0(t)$, $\overline{V}_S^0(t)$, $\overline{E}_D^0(t)$ near time $t = t_1$, i.e., $P(t_1 - 0) \approx \overline{P}^0(t_1 - 0)$, $V_D(t_1 - 0) \approx \overline{V}_D^0(t_1 - 0)$, $V_S(t_1 - 0) \approx \overline{V}_S^0(t_1 - 0)$, $E_D(t_1 - 0) \approx \overline{E}_D^0(t_1 - 0)$.

Let me model now the processes of an economic crisis (a demand shock) and a weak recovery (a decline in investment only partially compensated by the decrease of interest rate).

I assume there is a demand shock at time $t = t_1$ on the market,

$$V_D(t) = \begin{cases} \overline{V}_D(t), & t < t_1 \\ \overline{V}_D(t_1) - \Delta V_D, & t = t_1 \end{cases} \tag{28}$$

where $V_D(t) = \overline{V}_D(t)$ for $t_0 << t < t_1$ and $\Delta V_D > 0$.

I also assume the accumulated amount of disruption $S_\Sigma(t)$ on the market is changed since time $t = t_1$,

$$S_\Sigma(t) = \begin{cases} \delta_0 (t - t_0) + \frac{\varepsilon_0}{2}(t - t_0)^2, & t < t_1 \\ \delta_1 (t - t_1) + \frac{\varepsilon_1}{2}(t - t_1)^2 + S_\Sigma^1, & t \ge t_1 \end{cases} \tag{29}$$

where $S_\Sigma^1 = \delta_0 (t_1 - t_0) + \frac{\varepsilon_0}{2}(t_1 - t_0)^2$, $\delta_1 = \delta_I^1 + \delta_D^1$, and $\varepsilon_1 = \varepsilon_I^1 - \delta_S^1 \, \delta_D^1$.

Here I assume that $\varepsilon_I^1 \le \varepsilon_I^0$ and $\delta_S^1 \le \delta_S^0$ since time $t = t_1$. For simplicity, I also assume that $\delta_1 = \delta_0$ (i.e., $\delta_I^1 = \delta_I^0$ and $\delta_D^1 = \delta_D^0$).

Economic forces trying to bring the market into a new equilibrium position are described by the following ordinary differential equations with regard to the volumes of product supply $V_S(t)$, demand $V_D(t)$, price $P(t)$, and the accumulated amount of disruption $S_\Sigma(t)$ on the market,

$$\frac{dP(t)}{dt} = -\lambda_P \left(V_S(t) - V_D(t) - S_\Sigma(t)\right) \tag{30}$$

$$\frac{d^2V_S(t)}{dt^2} = \lambda_S \frac{dP(t)}{dt} \tag{31}$$

$$\frac{d^2V_D(t)}{dt^2} = -\lambda_D \frac{d^2P(t)}{dt^2} \tag{32}$$

I use again the variable $D(t) = V_S(t) - V_D(t) - S_\Sigma(t)$ representing the volume of product surplus (or shortage) on the market. The behavior of $D(t)$ is described by the following equation for $t > t_0$,

$$\frac{d^2D(t)}{dt^2} + \lambda_P \lambda_D \frac{dD(t)}{dt} + \lambda_P \lambda_S D(t) + \varepsilon_1 = 0 \tag{33}$$

with the following initial conditions, $D(t_1) = \overline{V}_S^0(t_1) - \overline{V}_D^0(t_1) + \Delta V_D - S_\Sigma(t_1) = -\frac{\varepsilon_0}{\lambda_P \lambda_S} + \Delta V_D$,

$\frac{dD(t_1)}{dt} = \frac{d\overline{V}_S^0(t_1)}{dt} - \frac{d\overline{V}_D^0(t_1)}{dt} - \frac{dS_\Sigma(t_1)}{dt} = \varepsilon_0 (t_1 - t_0)$.

I also use the variable $D_1(t) = D(t) + \frac{\varepsilon_1}{\lambda_P \lambda_S}$, then Eq. (33) becomes,

$$\frac{d^2D_1(t)}{dt^2} + \lambda_P \lambda_D \frac{dD_1(t)}{dt} + \lambda_P \lambda_S D_1(t) = 0 \tag{34}$$

with the initial conditions, $D_1(t_1) = -\frac{\varepsilon_0 - \varepsilon_1}{\lambda_P \lambda_S} + \Delta V_D$, $\frac{dD_1(t_1)}{dt} = \varepsilon_0 (t_1 - t_0)$.

Similar to Eq. (33), the product price $P(t)$ is described by the following equation for $t > t_0$,

$$\frac{d^2P(t)}{dt^2} + \lambda_P \lambda_D \frac{dP(t)}{dt} + \lambda_P \lambda_S \left(P(t) - P^0 - \frac{\delta_1}{\lambda_S} - \frac{\varepsilon_1}{\lambda_S}(t - t_1) \right) = 0 \tag{35}$$

with the following initial conditions, $P(t_1) = \overline{P}^0(t_1) = \frac{\varepsilon_0}{\lambda_S}(t_1 - t_0) + P^0 + \frac{\delta_0}{\lambda_S} - \frac{\lambda_D}{\lambda_S^2}\varepsilon_0$,

$\frac{dP(t_1)}{dt} = \frac{d\overline{P}^0(t_1)}{dt} = \frac{\varepsilon_0}{\lambda_S}$.

Let me use the variable $P_1(t) = P(t) - P^0 - \frac{\delta_1}{\lambda_S} - \frac{\varepsilon_1}{\lambda_S}(t - t_1) + \frac{\lambda_D}{\lambda_S^2}\varepsilon_1$ to simplify analysis of the product price behavior. The behavior of variable $P_1(t)$ is described by the following equation for $t > t_0$,

$$\frac{d^2 P_1(t)}{dt^2} + \lambda_P \lambda_D \frac{dP_1(t)}{dt} + \lambda_P \lambda_S P_1(t) = 0 \qquad (36)$$

with the initial conditions, $P_1(t_1) = \frac{\varepsilon_0}{\lambda_S}(t_1 - t_0) - \frac{\lambda_D}{\lambda_S^2}(\varepsilon_0 - \varepsilon_1)$, $\frac{dP_1(t_1)}{dt} = \frac{\varepsilon_0 - \varepsilon_1}{\lambda_S}$.

The behavior of solutions for $D_1(t)$ and $P_1(t)$ described by Eqs. (34) and (36) depends on the roots of the corresponding characteristic equations (Piskunov, 1965; Petrovski, 1966). Note that Eqs. (34) and (36) have the same characteristic equations.

When the roots of characteristic equation are complex-valued (i.e., $\frac{\lambda_P^2 \lambda_D^2}{4} < \lambda_P \lambda_S$) the variables $D_1(t)$ and $P_1(t)$ experience damped oscillations for time $t \geq t_1$. When the roots of characteristic equation are real and different (i.e., $\frac{\lambda_P^2 \lambda_D^2}{4} > \lambda_P \lambda_S$) the variables $D_1(t)$ and $P_1(t)$ don't oscillate for time $t \geq t_1$. When the roots are real and equal (i.e., $\frac{\lambda_P^2 \lambda_D^2}{4} = \lambda_P \lambda_S$) the variables $D_1(t)$ and $P_1(t)$ don't oscillate for time $t \geq t_1$.

It occurs $D_1(t) \to 0$ and $P_1(t) \to 0$ for $t \to +\infty$ if the roots of characteristic equation are complex-valued ($\frac{\lambda_P^2 \lambda_D^2}{4} < \lambda_P \lambda_S$), real and different ($\frac{\lambda_P^2 \lambda_D^2}{4} > \lambda_P \lambda_S$), or real and equal ($\frac{\lambda_P^2 \lambda_D^2}{4} = \lambda_P \lambda_S$).

It takes place for the product surplus (shortage) $D(t)$, for the product price $P(t)$, for the product demand $V_D(t)$, for the product supply $V_S(t)$, and for the accumulated amount of disruption $S_\Sigma(t)$ if $t \to +\infty$,

$$D(t) \to -\frac{\varepsilon_1}{\lambda_P \lambda_S} \tag{37}$$

$$P(t) \to \frac{\varepsilon_1}{\lambda_S}(t - t_1) + \frac{\varepsilon_0}{\lambda_S}(t_1 - t_0) + P^0 + \frac{\delta_0}{\lambda_S} - \frac{\lambda_D}{\lambda_S^2}\varepsilon_0 \tag{38}$$

$$V_D(t) \to \left(r_D^0 - \frac{\lambda_D}{\lambda_S}\varepsilon_1 \right)(t - t_1) + r_D^0 (t_1 - t_0) + V_D^0 - \Delta V_D - \frac{\lambda_D}{\lambda_S}\delta_0 + \frac{\lambda_D^2}{\lambda_S^2}\varepsilon_1 \tag{39}$$

$$\begin{aligned} V_S(t) \to & \left(r_D^0 + \delta_1 - \frac{\lambda_D}{\lambda_S}\varepsilon_1 \right)(t - t_1) + \frac{\varepsilon_1}{2}(t - t_1)^2 + \left(r_D^0 + \delta_0\right)(t_1 - t_0) + \frac{\varepsilon_0}{2}(t_1 - t_0)^2 \\ & + V_D^0 - \Delta V_D - \frac{\lambda_D}{\lambda_S}\delta_0 - \frac{\varepsilon_1}{\lambda_P \lambda_S} + \frac{\lambda_D^2}{\lambda_S^2}\varepsilon_1 \end{aligned} \tag{40}$$

$$S_\Sigma(t) = \delta_1 (t - t_1) + \frac{\varepsilon_1}{2}(t - t_1)^2 + \delta_0 (t_1 - t_0) + \frac{\varepsilon_0}{2}(t_1 - t_0)^2 \tag{41}$$

It also takes place for the rate of product demand in money terms $E_D(t)$ when $t \to +\infty$,

$$E_D(t) \to \left(\frac{\varepsilon_1}{\lambda_S}(t - t_1) + \frac{\varepsilon_0}{\lambda_S}(t_1 - t_0) + P^0 + \frac{\delta_0}{\lambda_S} - \frac{\lambda_D}{\lambda_S^2}\varepsilon_0 \right)\left(r_D^0 - \frac{\lambda_D}{\lambda_S}\varepsilon_1 \right) \tag{42}$$

We can estimate a change $e_D(t) = \frac{dE_D(t)}{dt}$ of the rate of demand in money terms, i.e., increase (decrease) in the rate of demand in money terms for the product, which reflects the increase (decrease) in the product earning on the market.

It takes place, $e_D(t) \to \frac{\varepsilon_1}{\lambda_S}\left(r_D^0 - \frac{\lambda_D}{\lambda_S}\varepsilon_1\right)$ if $t \to +\infty$. The converged value of variable $e_D(t)$ defined as $\gamma(\varepsilon) \equiv \frac{\varepsilon}{\lambda_S}\left(r_D^0 - \frac{\lambda_D}{\lambda_S}\varepsilon\right)$ monotonically increases for $\varepsilon < \frac{\lambda_S}{2\lambda_D} r_D^0$ and monotonically decreases for $\varepsilon > \frac{\lambda_S}{2\lambda_D} r_D^0$. We decided to restrict ourselves to a case of the economic growth that implies $\gamma(\varepsilon) > 0$ and entails $0 < \varepsilon < \frac{\lambda_S}{\lambda_D} r_D^0$. The converged value $\gamma(\varepsilon) > 0$ of variable $e_D(t)$ monotonically increases in interval $0 < \varepsilon \le \frac{\lambda_S}{2\lambda_D} r_D^0$ and monotonically decreases in interval $\frac{\lambda_S}{2\lambda_D} r_D^0 \le \varepsilon < \frac{\lambda_S}{\lambda_D} r_D^0$. The converged value $\gamma(\varepsilon) > 0$ of variable $e_D(t)$ has a maximum when $\varepsilon^* = \frac{\lambda_S}{2\lambda_D} r_D^0$, and the maximal converged value $\gamma(\varepsilon) > 0$ is equal to $\gamma(\varepsilon^*) = \frac{1}{4\lambda_D}\left(r_D^0\right)^2$.

Thus, we have constructed a simplified mathematical model that confirms that changes in economic growth and weak recovery after the financial crisis can be attributed to a decline in investment that is only partially compensated by the decrease of interest rate.[1] More accurately, if $\varepsilon_0 > \varepsilon_1$ where $0 < \varepsilon_1 < \varepsilon_0 \le \frac{\lambda_S}{2\lambda_D} r_D^0$, then it takes place $\gamma(\varepsilon_0) > \gamma(\varepsilon_1)$.

We can observe four another economic trend lines,

$$\overline{P}^1(t) = \frac{\varepsilon_1}{\lambda_S}(t - t_1) + \frac{\varepsilon_0}{\lambda_S}(t_1 - t_0) + P^0 + \frac{\delta_0}{\lambda_S} - \frac{\lambda_D}{\lambda_S^2}\varepsilon_0$$

$$\overline{V}_D^1(t) = \left(r_D^0 - \frac{\lambda_D}{\lambda_S}\varepsilon_1\right)(t - t_1) + r_D^0(t_1 - t_0) + V_D^0 - \Delta V_D - \frac{\lambda_D}{\lambda_S}\delta_0 + \frac{\lambda_D^2}{\lambda_S^2}\varepsilon_1$$

[1] The presented model can simulate a phenomenon of *debt overhang* since its effect is characterized by the gradual repayment of principal through the increase of debt maintenance cost.

$$\overline{V}_S^1(t) = \left(r_D^0 + \delta_1 - \frac{\lambda_D}{\lambda_S} \varepsilon_1 \right)(t - t_1) + \frac{\varepsilon_1}{2}(t - t_1)^2 + \left(r_D^0 + \delta_0\right)(t_1 - t_0) + \frac{\varepsilon_0}{2}(t_1 - t_0)^2$$
$$+ V_D^0 - \Delta V_D - \frac{\lambda_D}{\lambda_S} \delta_0 - \frac{\varepsilon_1}{\lambda_P \lambda_S} + \frac{\lambda_D^2}{\lambda_S^2} \varepsilon_1$$

$$\overline{E}_D^1(t) = \left(\frac{\varepsilon_1}{\lambda_S}(t - t_1) + \frac{\varepsilon_0}{\lambda_S}(t_1 - t_0) + P^0 + \frac{\delta_0}{\lambda_S} - \frac{\lambda_D}{\lambda_S^2} \varepsilon_0 \right)\left(r_D^0 - \frac{\lambda_D}{\lambda_S} \varepsilon_1 \right)$$

where $\lim_{t \to +\infty} P(t) = \overline{P}^1(t)$, $\lim_{t \to +\infty} V_D(t) = \overline{V}_D^1(t)$, $\lim_{t \to +\infty} V_S(t) = \overline{V}_S^1(t)$, $\lim_{t \to +\infty} E_D(t) = \overline{E}_D^1(t)$.

In the next section we want to focus on the changes of trend line representing an economic growth from $\overline{E}_D^0(t)$ to $\overline{E}_D^1(t)$ and for convenience rewrite $\overline{E}_D^0(t)$ and $\overline{E}_D^1(t)$ in the following forms,

$$\overline{E}_D^0(t) = \left(\frac{\varepsilon_0}{\lambda_S}(t - t_1) + P^1 \right)\left(r_D^0 - \frac{\lambda_D}{\lambda_S} \varepsilon_0 \right) \tag{43}$$

$$\overline{E}_D^1(t) = \left(\frac{\varepsilon_1}{\lambda_S}(t - t_1) + P^1 \right)\left(r_D^0 - \frac{\lambda_D}{\lambda_S} \varepsilon_1 \right) \tag{44}$$

where $P^1 = \overline{P}^0(t_1) = \frac{\varepsilon_0}{\lambda_S}(t_1 - t_0) + P^0 + \frac{\delta_0}{\lambda_S} - \frac{\lambda_D}{\lambda_S^2} \varepsilon_0$.[2]

4 Implication and Graphical Representation of Economic Trend Lines

We can note that Eqs. (43) and (44) are equations of a straight line,

$$y = m(x - x_1) + y_1 \tag{45}$$

where m is the slope of the line and (x_1, y_1) is any point on the line.

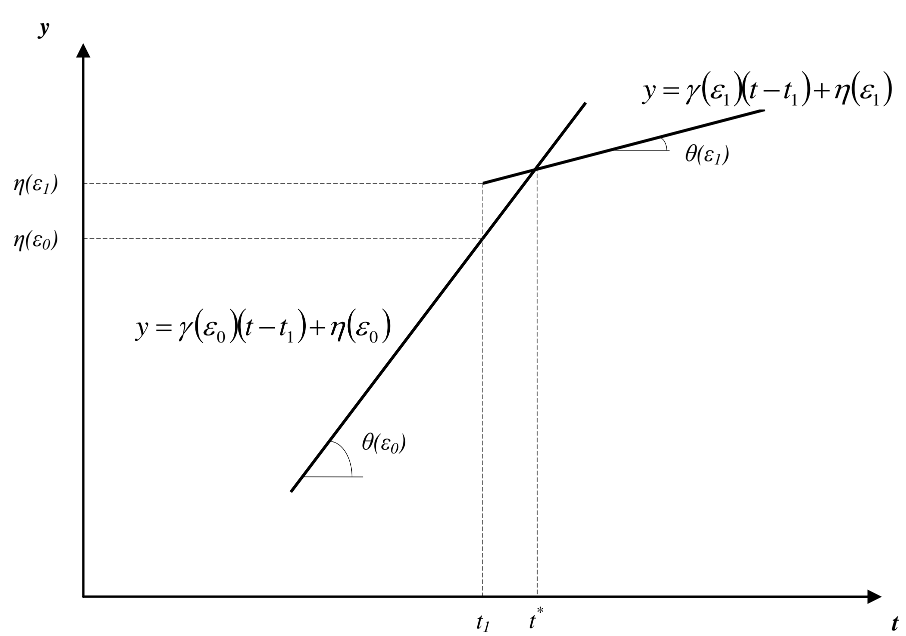
[2] We restrict ourselves with the positive prices, hence $P^1 > 0$.

For Eq. (43) the slope is $\gamma(\varepsilon_0) = \frac{\varepsilon_0}{\lambda_S}\left(r_D^0 - \frac{\lambda_D}{\lambda_S}\varepsilon_0\right)$, and the point on the line is $(t_1, \eta(\varepsilon_0))$ where $\eta(\varepsilon) \equiv \left(r_D^0 - \frac{\lambda_D}{\lambda_S}\varepsilon\right)P^1$. For Eq. (44) the slope is $\gamma(\varepsilon_1)$, and the point on the line is $(t_1, \eta(\varepsilon_1))$.

Since we assumed above that $0 < \varepsilon_1 < \varepsilon_0 \le \frac{\lambda_S}{2\lambda_D} r_D^0$ and $P^1 > 0$, then it takes place $\gamma(\varepsilon_0) > \gamma(\varepsilon_1) > 0$ and $\eta(\varepsilon_1) > \eta(\varepsilon_0) > 0$. It also occurs that $\theta(\varepsilon_0) > \theta(\varepsilon_1) > 0$ where $\theta(\varepsilon)$ is an angle of inclination for the line, $\theta(\varepsilon) \equiv \arctan \gamma(\varepsilon)$.

Fig. 1 below shows graphical representation of the economic trends $\overline{E}_D^0(t)$ and $\overline{E}_D^1(t)$.[3]

Fig. 1 Economic Trends $\overline{E}_D^0(t)$ and $\overline{E}_D^1(t)$



[3] Lines $\overline{E}_D^0(t)$ and $\overline{E}_D^1(t)$ intersect at time $t^* = t_1 + \dfrac{\lambda_D P^1}{r_D^0 - \frac{\lambda_D}{\lambda_S}(\varepsilon_0 + \varepsilon_1)}$.

Economic trend $\overline{E}_D^0(t)$ transforms into economic trend $\overline{E}_D^1(t)$ through the short-time variations. As was noted above, when the roots of characteristic equation are complex-valued (i.e., $\frac{\lambda_P^2\,\lambda_D^2}{4} < \lambda_P\,\lambda_S$) the short-time variations experience damped oscillations for time $t \ge t_1$. When the roots of characteristic equation are either real and different (i.e., $\frac{\lambda_P^2\,\lambda_D^2}{4} > \lambda_P\,\lambda_S$) or real and equal (i.e., $\frac{\lambda_P^2\,\lambda_D^2}{4} = \lambda_P\,\lambda_S$) the short-time variations don't oscillate for time $t \ge t_1$.

Fig. 2 below shows a transformation of the economic trend $\overline{E}_D^0(t)$ into the economic trend $\overline{E}_D^1(t)$ without fluctuations of short-time variations.

Fig. 2 Transformation of Economic Trends without Fluctuations

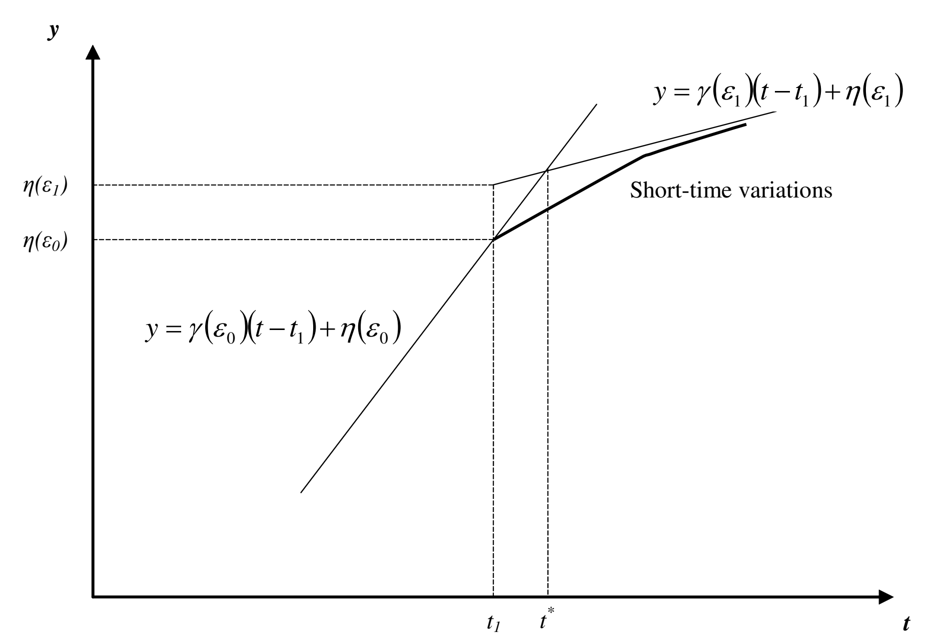
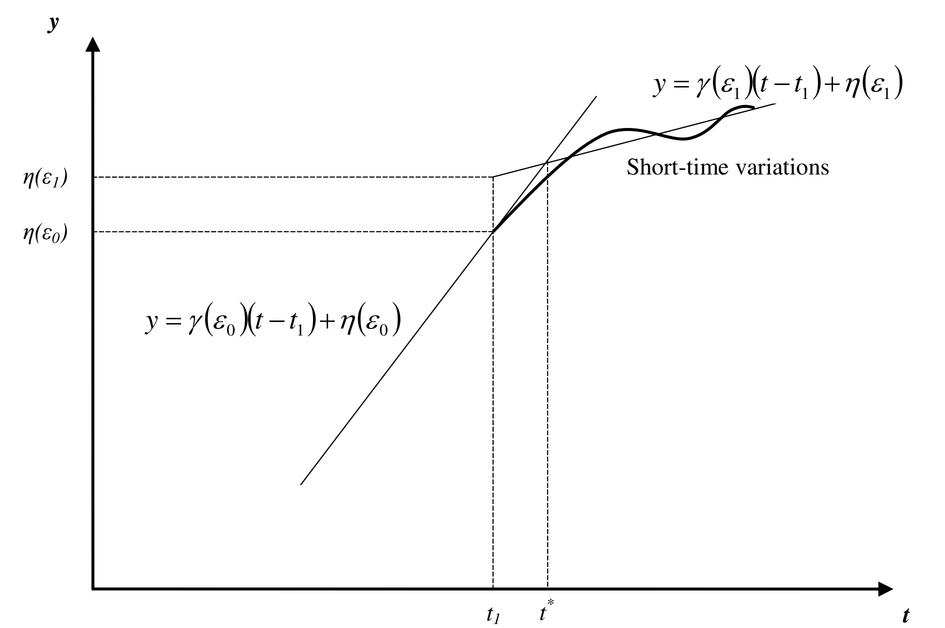


Fig. 3 shows a transformation of the economic trend $\overline{E}_D^0(t)$ into the economic trend $\overline{E}_D^1(t)$ with fluctuations of short-time variations.

Fig. 3 Transformation of Economic Trends with Fluctuations



At first glance the transformation of economic trends presented in article agrees with tendencies showing a weak recovery after the latest financial crisis that are both observable in practice and described in empirical literature.

5 Conclusions

Presented here is a simplified mathematical model that tries to explain the current weak recovery after financial crisis. The author has built the model to either confirm or refute his hypothesis that weak recovery can be attributed to a decline in investment that is not fully compensated by the decrease of interest rate.

The introduced model confirms the hypothesis.

The author also describes an economic process that reflects how the transformation of economic trends occurs. He presents a graphical representation of the transformation and distinguishes cases when the transformation of economic trend occurs either with or without fluctuations of short-time variations. The graphical representation of transformation nearly agrees with the practically observable tendencies that correspond to recovery processes after the latest financial crisis.

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