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1 November 2016

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10th November 2016

Abstract. We formulate economic laws of long-run labor re-allocation across agriculture, manufacturing, and services based on empirical evidence and derive the implications of these laws for the future (transitional and limit) labor allocation dynamics in developed and developing countries. Our approach for deriving these predictions is positivistic in the sense that we try to derive the direct implications of the laws, i.e. we try to minimize the dependence of our predictions on theoretical/ideological arguments. Due to this fact and because the economic laws are qualitative statements, our modeling approach requires the use of geometrical/axiomatic dynamic modeling techniques, set theory and logic.

JEL Codes. C61, C65, O41, O14
Keywords. Labor re-allocation, structural change, sectors, agriculture, manufacturing, services, long run, dynamics, trajectory, geometry, simplex.

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1. INTRODUCTION

In this paper, we study the means and capabilities of a positivistic approach to structural change modeling. Structural change refers here to the long-run labor allocation dynamics in the three-sector framework and, in particular, to the labor re-allocation across agriculture, manufacturing, and services.[1] Our 'positivistic modeling approach' is a three-step procedure.[2] In 'step 1', the empirical evidence is analyzed and the observable empirical regularities (i.e. qualitative statements about the characteristics of structural change that are persistent across countries and time) are elaborated. In 'step 2', these regularities are assumed to be 'laws' (of structural change), which are, among others, valid in future similar to the natural laws in natural sciences.[3] Finally, in 'step 3', (long-run) predictions are made regarding the future dynamics (i.e. future structural change), particularly, transitional and limit dynamics.

The core characteristic of our approach is that in step 3, we try to derive the direct implications of the qualitative laws derived in step 2 and to minimize the dependence of these predictions on other (e.g. theoretical) assumptions.[4] In general, theoretical assumptions are ideological (i.e. not provable by empirical evidence) and, thus, subjective. Often, it is difficult to assess to what extent a prediction is the result of primarily ideological assumptions and to what extent it represents empirical regularities. This is particularly true for the predictions based on complex (micro-founded) dynamic models (cf. the papers listed in Section 4.1.4) that heavily rely on empirically unprovable assumptions and parameter settings. Due to these facts among others, there is disagreement between different schools of economic thought (e.g. Keynesian, neoclassical, and evolutionary school) regarding the causes of economic phenomena, predictions of future dynamics, and policy recommendations. For all these reasons, it seems interesting to separate between the ideologically driven predictions and the direct implications of empirical information or at least to try to minimize the extent of ideological information in economic predictions. Our positivistic modeling approach is an attempt to do so in structural change modeling.

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[1] For an overview of the structural change literature, see, e.g., Schettkat and Yocarini (2006), Krüger (2008), Silva and Teixeira (2008), Stijepic (2011, Chapter IV), and Herrendorf et al. (2014). Recent papers modeling structural change in the three-sector framework are, e.g., Kongsamut et al. (2001), Ngai and Pissarides (2007), Foellmi and Zweimüller (2008), Uy et al. (2013), and Stijepic (2015, 2016).

[2] For a discussion of the usage of the term 'positivism' in the theory of science and the discussion of positivism as a methodological approach, see, e.g., Jackson and Smith (2005).

[3] For usage of 'laws' in natural sciences and economics, see, e.g., Jackson and Smith (2005) and Reutlinger et al. (2015).

[4] Some of our models are solely based on the laws observed and some of our models assume, additionally, that the long-run labor allocation dynamics can be modeled by using continuous or differentiable functions (implying continuous/differentiable functional forms describing the economic relations), which is a typical assumption in long-run economics modeling and, particularly, in the models listed in Section 4.1.4.

Among all the topics in (development) economics, structural change seems to be the most predestined for applying a positivistic (i.e. a law-centric) approach of modeling: as we will see, structural change seems to be one of the most persistent long-run phenomena of economic development having characteristics that are easily identifiable and stable across countries and time. The latter aspect is one of the core characteristics of a (natural) law.

While some sort of positivistic modeling philosophy is the basis of all empirical research in economics, our attempt to primarily work with the observable qualitative laws and to minimize the extent of (additional) theoretical/ideological assumptions is novel to structural change modeling and requires the application of modeling techniques that are not widespread in the long-run economic dynamics literature.[5] In particular, we apply qualitative and, particularly, geometrical (and topological) modeling techniques (e.g. the Poincaré-Bendixson theory) known from dynamic systems analysis, set theory and logic.

The fact that we try to minimize the extent of theoretical assumptions used for prediction does not mean that the laws that we use for prediction in step 3 are theoretically unfounded: in Section 4.1.4, we show that all the laws used in our models are supported by the previous theoretical literature. Nevertheless, our results differ significantly from the results of the standard literature (cf. the papers listed in Section 4.1.4). In particular, our results cover a wider range of possible structural change scenarios than the standard structural change literature does, since our predictions are less restricted by ideological assumptions.[6] For a summary of our models' forecasts, see Section 5.

Since as always in the empirical sciences, there are deviations from the observed regularities (e.g. some countries' dynamics deviate from the regular patterns) and the empirical literature/evidence is sometimes ambiguous regarding the validity of some regularities (due to, e.g., different data sources and measurement problems), it is debatable which of the regularities (cf. step 1) can be regarded as laws (cf. step 2). Thus, we present different models based on different sets of laws/regularities, such that the reader can choose the model that corresponds to his ideology. In particular, our set of models encompasses a conservative model (which is only based on the most/least accepted/disputable laws) and several less conservative models (which rely on a greater number of regularities/laws than the conservative model does).

[5] Stijepic (2015) suggests a structural change model that, in contrast to our contribution, does not try to minimize the extent of theoretical assumptions, but works only with the assumptions that are valid across different schools of thought and are, thus, non-ideological in some sense.

[6] In general, standard structural change models (e.g. the papers listed in Section 4.1.4) predict that in the long run, the labor allocation converges to a steady state allocation dominated by the services sector (cf. Stijepic (2015)).

The rest of the paper is set up as follows. In the next section, we discuss briefly the mathematical prerequisites and, in particular, the geometrical aspects of structural change analysis. In Section 3, we discuss the empirical evidence on structural change (focusing on the World Bank and Maddison (1995) data) and formulate the regularities of structural change. In Section 4, we formulate different models (i.e. sets of laws) on the basis of the regularities postulated in Section 3, study their transitional and limit dynamics, and apply these results for predicting structural change in developed and developing countries. Concluding remarks are provided in Section 5.

2. MATHEMATICAL AND CONCEPTUAL PREREQUISITES

In this section, we define the terms ‘labor allocation’ and ‘structural change’ and discuss their geometrical interpretation via simplexes and trajectories. Furthermore, we discuss the characterization of structural change by referring to the geometrical properties of trajectories and different horizons of analysis (limit vs. transitional dynamics).

The mathematical notation in this paper is as follows: small letters (e.g. x), capital letters (e.g. X), bold capital letters (e.g. **X**), and Greek letters (e.g. α) denote scalars, vectors, sets, and angles, respectively.

2.1 Mathematical Definition of Labor Allocation and Structural Change

As noted in Section 1, we study structural change in the three-sector framework, which is a widespread choice for analyzing structural change empirically and theoretically (cf. Footnote 1). The three-sector framework refers to three sectors: primary or agricultural sector (which we name sector 1), secondary or manufacturing sector (which we name sector 2), and tertiary or services sector (which we name sector 3). Let $l_{1c}(t)$, $l_{2c}(t)$, and $l_{3c}(t)$ denote the employment in sector 1, 2, and 3 at time t in country c, respectively. Furthermore, let $l_c(t):=l_{1c}(t)+l_{2c}(t)+l_{3c}(t)$ denote the aggregate employment at time t in country c. The *employment share of sector i at time t in country c* is defined as follows: $x_{ic}(t):=l_{ic}(t)/l_c(t)$ for i=1,2,3, for all t, and for all c. Since employment cannot be negative and $l_c(t):=l_{1c}(t)+l_{2c}(t)+l_{3c}(t)$ for all t and for all c, the following statements are true

(1) $\forall t\ \forall c\ \forall i\in\{1,2,3\}\ 0 \le x_{ic}(t) \le 1$

(2) $\forall t\ \forall c\ x_{1c}(t)+x_{2c}(t)+x_{3c}(t)=1$

According to these definitions, the vector $X_c(t)$, which is defined as follows

(3) $\forall t\ \forall c\ X_c(t):=(x_{1c}(t),x_{2c}(t),x_{3c}(t))$

represents the *labor allocation* across agriculture, manufacturing, and services at time t in country c.

The term 'structural change' refers to the long-run changes in the labor allocation $X_c(t)$. Thus, per our definition of the term 'labor allocation', *'structural change in country c'* means that at least some of the employment shares $x_{1c}(t)$, $x_{2c}(t)$, and $x_{3c}(t)$ are not constant in the long run in country c. For example, $x_{1c}(t)$ may grow over time, $x_{2c}(t)$ may decline over time, and $x_{3c}(t)$ may be constant over time in country c.

Definition 1 summarizes this discussion, where we do not implement the fact that structural change refers to the long run, since a mathematical formulation of the notion of the long run is not necessary for deriving our results; by omitting such a formulation, the mathematical expressions are significantly abbreviated. Of course, when necessary, we will emphasize that our statements refer to the long-run dynamics.

Definition 1. *The term 'structural change (over the period [a,b]) in country c' refers to the (long-run) dynamics of the labor allocation $X_c(t)$ (over the period [a,b]). In particular, the labor allocation has changed over the period [a,b] in country c, if $\exists t\in(a,b]\ X_c(t)\neq X_c(a)$.*

Simply speaking, Definition 1 states that structural change takes place in country c if $X_c(t)$ is not constant.

2.2 Geometrical Interpretation of Labor Allocation and Structural Change

In this section, we recapitulate some geometrical concepts for analyzing structural change, as discussed by Stijepic (2015, 2016).

Consider the Cartesian coordinate system (x_1,x_2,x_3). We can identify any point in the three-dimensional real space ($\mathbf{R}^3$) with its Cartesian coordinates (x_1,x_2,x_3). Furthermore, let us define the following set of points (in the Cartesian coordinate system)

(4) $\mathbf{S}:=\{X\equiv(x_1,x_2,x_3)\in\mathbf{R}^3: x_1+x_2+x_3=1 \wedge \forall i\in\{1,2,3\}\ 0\leq x_i\leq 1\}$

It is well known that: (a) **S** is a two-dimensional standard simplex (henceforth, 2-simplex); (b) the 2-simplex is a triangle; and (c) the Cartesian coordinates of its vertices are

(5) $(1,0,0)=:V_1 \qquad (0,1,0)=:V_2 \qquad (0,0,1)=:V_3$

For an illustration, see Figures 1 and 2, where we omit the coordinate axes in Figure 2.

Figure 1. *The 2-simplex in the Cartesian coordinate system (x_1,x_2,x_3).*

- insert Figure 1 here -

Figure 2. *The 2-simplex (without coordinate axes).*

- insert Figure 2 here -

This discussion and, in particular, (4) shows that all the points (x_1,x_2,x_3) that satisfy the conditions $x_1+x_2+x_3=1$ and $\forall i\in\{1,2,3\}$ $0 \leq x_i \leq 1$ are located on the 2-simplex **S**, i.e. on the triangle depicted in Figures 1 and 2. These facts and our definitions of labor allocation and structural change (cf. Section 2.1) imply that we can depict the labor allocation $X_c(t)$ and its dynamics (i.e. structural change) on the 2-simplex (cf. (1)-(4)), as explained in the following.

In Section 2.1, we have implicitly assumed that the labor allocation in country c ($X_c(t)$) is a function of time (cf. (3)). Now, we make this assumption explicit by stating that

(6) $X_c(t)$: $\mathbf{D}\times\mathbf{C}\rightarrow\mathbf{S}$

(7) $X_c(t)$: $(t,c)\mapsto(x_1,x_2,x_3)$

(8) $t\in\mathbf{D}\subseteq\mathbf{R} \wedge c\in\mathbf{C}\subset\mathbf{N}$

where **D** is a time interval, i.e. a subset of real numbers (**R**), and **C** is the set of countries, which are indexed by natural numbers (**N**). (6)-(8) state that the function $X_c(t)$ maps time (t) and the country index c to the 2-simplex. In particular, for a given $c\in\mathbf{N}$, the function $X_c(t)$ assigns a point on the 2-simplex **S**, which is located in the coordinate system (x_1,x_2,x_3), to each time point $t\in\mathbf{D}$. Note that due to (1)-(4), we know that the function $X_c(t)$ has values in the set **S** and not elsewhere in $\mathbf{R}^3$.

This discussion and Section 2.1 imply the following geometrical interpretation of the term 'labor allocation'. The labor allocation in the three-sector framework ($X_c(t)$) can be represented by a point on the 2-simplex. This 2-simplex contains all the points that satisfy the definition of the term labor allocation (cf. (1)-(3)). Two different points on the 2-simplex represent two different labor allocations. Thus, if, e.g., $X_c(1) \neq X_c(2)$ (cf. (3)), where $X_c(1),X_c(2)\in\mathbf{S}$, then in country c, the labor allocation at t=2 is not the same as the labor allocation at t=1, i.e. structural change took place over the time period (1,2) in country c (cf. Definition 1).

Overall, per Definition 1, we can derive all the information about structural change by studying the properties of the labor allocation function $X_c(t)$, which is defined in Section 2.1 and by (6)-(8). For the greatest part of our paper, we focus on the geometrical properties of the image of the labor allocation function, which can be analyzed by using the concept of the trajectory ($\mathbf{T}_c$), which we define as follows:

(9) $\forall\, c \in \mathbf{C}\ \mathbf{T}_c(\mathbf{G}) := \{X_c(t) \in \mathbf{S}\colon t \in \mathbf{G}\}$, where $\mathbf{G} \subseteq \mathbf{D}$

In fact, $\mathbf{T}_c(\mathbf{G})$ is the trajectory describing the dynamics of country $c \in \mathbf{C}$ over the time period $\mathbf{G} \subseteq \mathbf{D}$. In other words, $\mathbf{T}_c(\mathbf{G})$ is simply the set of states (or: labor allocations) that the economy experiences (or: realizes) over the time period $\mathbf{G}$. Geometrically speaking, economy c moves along $\mathbf{T}_c(\mathbf{G})$ over the time period $\mathbf{G}$. Note that (9) implies that the labor allocation trajectory $\mathbf{T}_c(\mathbf{G})$ is always located on the 2-simplex $\mathbf{S}$, i.e. $\mathbf{S}$ is the *domain* of $\mathbf{T}_c(\mathbf{G})$.

Figure 3 depicts an example of a trajectory given by (6)-(9), where we assume that $X_c(t)$ is continuous in t. Note that the arrow in Figure 3 indicates the direction of the movement along the trajectory. Let $X_c(a) \equiv (x_{1c}(a), x_{2c}(a), x_{3c}(a))$ denote the initial point and $X_c(b) \equiv (x_{1c}(b), x_{2c}(b), x_{3c}(b))$ be the end-point of the trajectory depicted Figure 3. Obviously, Figure 3 shows that these points differ. Thus, the trajectory in Figure 3 depicts structural change, according to Definition 1. In more detail, by recalling the position of the 2-simplex in the Cartesian coordinate system (x_1, x_2, x_3) (cf. Figure 1), we can see that the trajectory in Figure 3 implies that $x_{1c}(a) > x_{1c}(b)$, $x_{2c}(a) < x_{2c}(b)$, and $x_{3c}(a) < x_{3c}(b)$. That is, x_{1c} decreased and x_{2c} and x_{3c} increased over the time period [a,b] in country c.

Figure 3. *An example of a (continuous) trajectory on the 2-simplex.*

- insert Figure 3 here -

2.3 Geometrical Characterization of Trajectories

Trajectories can be characterized by using the concepts of closeness (to the vertices of the simplex), continuity, monotonicity, self-intersection, and in the case of two or more trajectories (where each trajectory represents the structural change in a different country), (mutual) intersection. In Sections 3 to 5, we use these concepts to characterize the empirically observable trajectories and to formulate economic laws and models based on evidence.

The intuitive/geometrical notion of continuity, self-intersection, and mutual intersection is more or less obvious. For a *continuous* and *non-self-intersecting* trajectory, see, e.g., Figure 3; in contrast, Figures 4 and 5 depict examples of non-continuous and self-intersecting trajectories, respectively. Figure 6 depicts two (mutually) intersecting trajectories, whereas Figure 7 depicts (mutually) non-intersecting trajectories.

Figure 4. *An example of a non-continuous trajectory on the 2-simplex.*

- insert Figure 4 here -

Figure 5. *An example of a self-intersecting (and continuous) trajectory on the 2-simplex.*

- insert Figure 5 here -

Figure 6. *An example of intersecting (and continuous) trajectories on the 2-simplex.*

- insert Figure 6 here -

Figure 7. *An example of non-intersecting (and continuous) trajectories on the 2-simplex.*

- insert Figure 7 here -

In our paper, we apply the following formal definitions of continuity, non-self-intersection, and non-intersection (cf. Stijepic (2016)).

Definition 2. *The trajectory (9) is continuous on* $\mathbf{S}$ *(for a given* $c \in \mathbf{C}$*) if the corresponding function* $X_c(t)$ *(cf. (6)-(8)) is continuous (in t) on the interval* $\mathbf{G}$ *(for the given c).*

Definition 3. *The (continuous) trajectory (9) is non-self-intersecting (for a given* $c \in \mathbf{C}$*) if* $\nexists (t_1,t_2,t_3) \in \mathbf{G}^3 \colon t_1 < t_2 < t_3 \wedge X_c(t_1) = X_c(t_3) \neq X_c(t_2)$.

Definition 4. *Two trajectories (*$\mathbf{T}_a(\mathbf{A})$ *and* $\mathbf{T}_b(\mathbf{B})$*, where* $a,b \in \mathbf{C}$ *and* $\mathbf{A},\mathbf{B} \subseteq \mathbf{D}$*) intersect if* $\mathbf{T}_a(\mathbf{A}) \cap \mathbf{T}_b(\mathbf{B}) \neq \emptyset$. *Otherwise, if* $\mathbf{T}_a(\mathbf{A}) \cap \mathbf{T}_b(\mathbf{B}) = \emptyset$, *the trajectories* $\mathbf{T}_a(\mathbf{A})$ *and* $\mathbf{T}_b(\mathbf{B})$ *do not intersect.*

Note that per Definition 3, a self-intersection requires that the economy leaves the point $X_c(t_1)$ at least for some instant of time (t_2) before it returns to it (at t_3). Thus, per Definition 3, a self-intersection does not occur if the economy reaches some point on **S** (in finite time) and stays there forever.

Later, we will need some notion of closeness to the vertices of the 2-simplex. We use the following definition.

Definition 5. *A point $X_c(t)\equiv(x_{1c}(t),x_{2c}(t),x_{3c}(t))\in\mathbf{S}$ is close to the vertex V_i (cf. (5)) if and only if $x_{ic}(t)>0.5$, where $i\in\{1,2,3\}$, $c\in\mathbf{C}$, and $t\in\mathbf{D}$.*

Note that (4) and Definition 5 imply that a point can be close to only one of the three vertices of the 2-simplex. That is, a point can never be close to two or more vertices at the same time. A geometrical interpretation of Definition 5 is given by the following partitioning of the simplex **S** (cf. Figure 8):

(10) $\mathbf{S^1}:=\{(x_1,x_2,x_3)\in\mathbf{S}: x_1 > 0.5\}$

(11) $\mathbf{S^2}:=\{(x_1,x_2,x_3)\in\mathbf{S}: x_2 > 0.5\}$

(12) $\mathbf{S^3}:=\{(x_1,x_2,x_3)\in\mathbf{S}: x_3 > 0.5\}$

(13) $\mathbf{S^4}:=\mathbf{S}/(\mathbf{S^1}\cup\mathbf{S^2}\cup\mathbf{S^3})$

Definition 5 and (10)-(13) imply the following statements: a point is close to the vertex V_1 if and only if it is located in the partition $\mathbf{S^1}$; a point is close to the vertex V_2 (V_3) if and only if it is located in the partition $\mathbf{S^2}$ ($\mathbf{S^3}$); a point is not close to any of the vertices, if and only if it is located in the partition $\mathbf{S^4}$ (cf. Figure 8).

Figure 8. *Partitioning of the 2-simplex according to Definition 5.*

- insert Figure 8 here -

To economically interpret the notion of closeness given by Definition 5, recall that x_1, x_2, and x_3 stand for the employment shares of agriculture, manufacturing, and services, respectively (cf. Section 2.1 and (6)-(8)). Thus, according to Definition 5, if the labor allocation in country c is represented by a point close to the vertex V_i, sector i employs more than 50% of the country c labor force, i.e. country c is dominated by sector i, where $i\in\{1,2,3\}$. For example, if the labor allocation at time t in country c is represented by a point ($X_c(t)$) close to the vertex V_3, country c is dominated by services at time t, i.e. $x_{3c}(t) > 0.5 > x_{2c}(t)+x_{1c}(t)$ (cf. (1)-(3)).

Definition 6. *Let the function (6)-(8) be differentiable with respect to time for $t\in\mathbf{G}\subseteq\mathbf{D}$ and let $R(t)$ be the tangential vector associated with the point $X_c(t)\equiv(x_{1c}(t),x_{2c}(t),x_{3c}(t))\in\mathbf{T}_c(\mathbf{G})\subset\mathbf{S}$, where $t\in\mathbf{G}\subseteq\mathbf{D}$ and $c\in\mathbf{C}$ (cf. (9)). The vector angle $\alpha(t)$ is the angle between $R(t)$ and the simplex-edge V_1V_2 (cf. (5) and Figure 9), i.e. $\alpha(t):= \angle(R(t),\overline{V_1V_2})$.*

Figure 9. *The vector angle α.*

- insert Figure 9 here -

We can use Definition 6 to geometrically interpret *monotonous* dynamics of sectors, as shown in the following three properties of the 2-simplex.

Property 1 *(cf. Definition 6).* ***a)*** $dx_{1c}(t)/dt>0 \Leftrightarrow 120°<\alpha(t)<300°$. ***b)*** $dx_{1c}(t)/dt<0 \Leftrightarrow 0°<\alpha(t)< 120° \vee 300°<\alpha(t)<360°$. ***c)*** $dx_1(t)/dt=0 \Leftrightarrow \alpha(t)\in\{120°,300°\}$.

Property 1 becomes evident when studying the 2-simplex in the Cartesian coordinate system (cf. Figure 1). For example, according to Property 1, the employment share of the agricultural sector decreases monotonously along the trajectory (9) if each of the tangential vectors associated with the trajectory (9) is characterized by a vector angle between 0° and 120° or between 300° and 360°. Thus, e.g., the employment share of the agricultural sector declines strictly monotonously along the trajectory depicted in Figure 3. The following Properties 2 and 3 are analogous to Property 1. For a detailed discussion of the economic interpretation of the tangential vector angles associated with labor allocation trajectories, see Stijepic (2015).

Property 2 *(cf. Definition 6).* ***a)*** $dx_{2c}(t)/dt>0 \Leftrightarrow 0°<\alpha(t)<60° \vee 240°<\alpha(t)<360°$. ***b)*** $dx_{2c}(t)/dt<0 \Leftrightarrow 60°<\alpha(t)<240°$. ***c)*** $dx_{2c}(t)/dt=0 \Leftrightarrow \alpha(t)\in\{60°,240°\}$.

For example, according to Property 2, the employment share of the manufacturing sector is constant along the trajectory (9) if each of the tangential vectors associated with the trajectory (9) is characterized by a vector angle of 60° or 240°. Thus, the employment share of the manufacturing sector is constant along a trajectory if the trajectory is linear and parallel to the V_3V_1-edge of the 2-simplex (cf. Figure 1).

Property 3 *(cf. Definition 6).* ***a)*** $dx_{3c}(t)/dt>0 \Leftrightarrow 0°<\alpha(t)<180°$. ***b)*** $dx_{3c}(t)/dt<0 \Leftrightarrow 180°<\alpha(t)< 360°$. ***c)*** $dx_{3c}(t)/dt=0 \Leftrightarrow \alpha(t)\in\{0°,180°\}$.

For example, according to Property 3, the employment share of the services sector increases monotonously along the trajectory (9) if each of the tangential vectors associated with the trajectory (9) is characterized by a vector angle between 0° and 180°. Thus, e.g., the employment share of services increases along the trajectory depicted in Figure 3.

2.4 Horizons of Analysis

When characterizing the labor allocation dynamics of a country, we distinguish between limit dynamics (and set of attraction) and transitional dynamics (and range of fluctuation). At least, the difference between the limit dynamics and the transitional dynamics should be known by the most economists dealing with economic dynamics. Nevertheless, we recapitulate these notions briefly, since they are an integral part of our argumentation in Section 4.2.
The term *'limit dynamics'* refers to the dynamics for $t \to \infty$. A standard concept for studying and describing the limit dynamics is the 'omega limit set'.

Definition 7. *Let the function $X_c(t)$ satisfy the conditions (1)-(3), (6)-(8), and $\mathbf{D} \supseteq [0,\infty)$. The point X_c^* is an omega limit point of the trajectory $\mathbf{T}_c([0,\infty))$ (cf. (9)) if there exists a sequence of time points t_k (where k=0,1,2,...) and this sequence satisfies two conditions: (a) t_k converges to infinity (i.e. $t_k \to \infty$ for $k \to \infty$), and (b) the corresponding sequence $X_c(t_k)$ converges to X_c^* (i.e. $X_c(t_k) \to X_c^*$ as $t_k \to \infty$). The omega limit set ($\mathbf{O}(\mathbf{T}_c([0,\infty)))$) of the trajectory $\mathbf{T}_c([0,\infty))$ is the union of all omega limit points of the trajectory $\mathbf{T}_c([0,\infty))$.*

For a discussion and explanation of the omega limit set, see, e.g., Andronov et al. (1987), p.353f, Walter (1998), p.322, and Hale (2009), p.46f. The (type of the) limit dynamics of an economy that moves along the trajectory $\mathbf{T}_c$ is indicated by the omega limit set of the trajectory $\mathbf{T}_c$. Intuitively speaking, in the cases discussed by us, the omega limit set $\mathbf{O}(\mathbf{T}_c([0,\infty)))$ is the set to which the labor allocation in country c ($X_c(t)$) converges along the trajectory $\mathbf{T}_c([0,\infty))$ for $t \to \infty$. The omega limit set may consist of only one point, i.e. a fixed point (X_c^*); in this case, the labor allocation in economy c converges along the trajectory $\mathbf{T}_c$ to the fixed point X_c^* (i.e. the labor allocation converges to a 'steady state' labor allocation) for $t \to \infty$; the proof of existence of such steady states in long-run labor allocation models is interesting, since structural change is transitory if the labor allocation converges to a steady state. Moreover, the omega limit set may be more complex; e.g. it may consist of the image of a (Jordan) curve, such that the labor allocation converges to a limit cycle, i.e. the labor allocation dynamics are cyclical for $t \to \infty$. Overall, the concept of the omega limit set allows us to describe the *type* of the labor allocation dynamics (or: their dynamic pattern) as time goes to infinity.

In general, a model (and, in particular, each of the five models discussed in Section 4.2) generates different trajectories depending on the initial conditions and the model parameters. We define *a model's 'set of attraction'* as the union of the omega limit sets of all the trajectories generated by the model. That is, the set of attraction refers to all the possible 'end-states' predicted by a model (i.e. the states to which the economy may converge for $t\to\infty$ according to the model). As demonstrated in Section 4.2, the size of the set of attraction allows us to estimate the (potential) *strength* of structural change as time goes to infinity.
While the concepts of limit dynamics and set of attraction refer to the dynamics for $t\to\infty$, *transitional dynamics* refers to the dynamics over the period $[0,\infty)$, i.e. this concept does not refer only to the limit ($\lim_{t\to\infty}$), but to the time 'before the limit'. In general, the transitional dynamics can be characterized by the shape of the trajectory, as we will see in Section 4.2.
To express the strength of transitional dynamics (but also the strength of limit dynamics), we use the concept of 'range of fluctuation'.

Definition 8. *Assume that Model y generates different functions $x_{ic}^{j}(t)$ (indicating the employment share of sector i at time t in country c) on the interval $\boldsymbol{G}\subseteq\boldsymbol{D}$, which are indexed by $j\in\boldsymbol{J}$ and satisfy (1) and (2). Let $\boldsymbol{K}^{y}(\boldsymbol{G}) := \bigcup_{j\in\boldsymbol{J}}\bigcup_{t\in\boldsymbol{G}} x_{ic}^{j}(t) \subseteq [0,1]$ denote the set of all values $x_{ic}^{j}(t)$ generated by Model y over the period $\boldsymbol{G}$ (cf. Axiom 1 and, in particular, (1)). The potential range of fluctuation of the employment share of sector i over the period $\boldsymbol{G}$ in Model y is defined as $\boldsymbol{M}_i^{y}(\boldsymbol{G}):=[min(cl(\boldsymbol{K}_y(\boldsymbol{G}))),max(cl(\boldsymbol{K}_y(\boldsymbol{G})))]$, where $cl(\boldsymbol{K}_y(\boldsymbol{G}))$ denotes the closure of the set $\boldsymbol{K}_y(\boldsymbol{G})$. The potential strength of fluctuation of the employment share of sector i in Model y is defined as the length $|\boldsymbol{M}_i^{y}(\boldsymbol{G})|:=max(cl(\boldsymbol{K}_y(\boldsymbol{G}))) - min(cl(\boldsymbol{K}_y(\boldsymbol{G})))$ of the interval $\boldsymbol{M}_i^{y}(\boldsymbol{G})$.*

Although Definition 8 refers to 'fluctuation', $|\mathbf{M}_i^{y}(\mathbf{G})|$ is also defined for non-cyclical and, in particular, monotonous functions $x_{ic}(t)$. If $|\mathbf{M}_i^{y}(\mathbf{G})|$ is small, the strength of fluctuation (or the strength of monotonous dynamics) of sector i's employment share over the period $\mathbf{G}$ cannot be great, where $i\in\{1,2,3\}$. Obviously, $|\mathbf{M}_i^{y}(\mathbf{G})|$ is a relatively crude index: it represents the upper limit of the strength of structural change (with respect to sector i) in Model y over the period $\mathbf{G}$. Nevertheless, it proves useful in Section 4.2, since it allows us to compare different countries and different models based on qualitative empirical information.

Both, the limit dynamics and the transitional dynamics, are important, since a priori, it cannot be decided whether an economy is close to its dynamic equilibrium (and, thus, limit dynamics is prevalent) or not (and, thus, transitory dynamics is prevalent).

3. EMPIRICAL EVIDENCE AND REGULARITIES OF STRUCTURAL CHANGE

In this section, first, we discuss the data and the method of data presentation. Then, we discuss the regularities (or: stylized facts) that can be derived from this data.

3.1 Data Description and Presentation

As explained in Section 2.1, the allocation of labor across agriculture, manufacturing, and services in country c can be represented by the vector $X_c(t):=(x_{1c}(t),x_{2c}(t),x_{3c}(t))$, where $x_{1c}(t)$, $x_{2c}(t)$, and $x_{3c}(t)$ stand for the employment shares of agriculture, manufacturing, and services at time t in country c, respectively.

According to (9), we can construct the labor allocation trajectories of the countries by using empirical data as follows. Assume that we have data on the labor allocation ($X_c(t)$) in county c for the time points t_0, t_1,…t_m. That is, we have the data points $X_c(t_0)$, $X_c(t_1)$,…$X_c(t_m)$ associated with country c. We construct the labor allocation trajectory of country c by depicting the points $X_c(t_0)$, $X_c(t_1)$,…$X_c(t_m)$ on the standard 2-simplex and connecting them by line segments (while preserving their timely order). We indicate the direction of movement (i.e. the timely order of the points) along the trajectory by an arrow at the last observation point. We do this procedure with all the countries for which we have data and depict the trajectories of all the countries belonging to a country group (e.g. OECD countries) on one simplex such that we can identify, among others, (mutual) intersections of the trajectories of the countries belonging to this group.

In Figures 10, 11, and 12, we depict the data on the long-run labor allocation dynamics in the *OECD countries* on the 2-simplex, where the latter refers to the employment shares of agriculture (x_1), manufacturing (x_2), and services (x_3) and the vertices (V_1, V_2, and V_3) are given by (5) (cf. Figure 1 in Section 2.2). For better visibility, Figure 12 depicts the enlarged segment of Figure 11 containing all the trajectories depicted in Figure 11. In Figures 11 and 12, we omit the arrows indicating the direction of movement along the trajectories in the most cases for reasons of clarity. Furthermore, note that while Figure 10 depicts low-frequency data on structural change covering a very long period of time (i.e. 1820–1992), Figures 11 and 12 present high-frequency data on labor allocation dynamics over the period 1980–2015.

Figure 10. *Labor allocation trajectories for the USA, France, Germany, Netherlands, UK, Japan, China, and Russia.*

- insert Figure 10 here -

Notes: *Data source: Maddison (1995). The black dot represents the barycenter of the simplex. Abbreviations: C – China, F – France, G – Germany, J – Japan, N – Netherlands, R – Russia, US – United States, UK – United Kingdom. Data points (years in parentheses): USA (1820, 1870, 1913, 1950, 1992), France (1870, 1913, 1950, 1992), Germany (1870, 1913, 1950, 1992), Netherlands (1870, 1913, 1950, 1992), UK (1820, 1870, 1913, 1950, 1992), Japan (1913, 1950, 1992), China (1950, 1992), Russia (1950, 1992).*

Figure 11. *Labor allocation trajectories of OECD countries over the 1980ies, 1990ies, 2000s, and 2010s.*

- insert Figure 11 here -

Notes: *Data source: The World Bank, World Databank. The black dot represents the barycenter of the simplex. Arrows indicating the direction of movement along the trajectories are omitted in the most cases for reasons of clarity of representation.*

Figure 12. *The labor allocation trajectories depicted in Figure 11, enlarged.*

- insert Figure 12 here -

Notes: *The black dot represents the barycenter of the simplex. The edges of the simplex are not visible in Figure 12. Arrows indicating the direction of movement along the trajectories are omitted in the most cases for reasons of clarity of representation.*

Figures 13 to 15 depict the data on less developed countries. Again, we distinguish between low-frequency data (cf. Figure 13) and higher-frequency data (cf. Figures 14 and 15). Figure 15 depicts the enlarged segment of Figure 14 containing all the trajectories depicted in Figure 14.

Figure 13. *Labor allocation in 1950 and 1980 in emerging countries.*

- insert Figure 13 here -

Notes: *Data source: Maddison (1995). The black dot represents the barycenter of the simplex. Countries depicted: Argentina, Bangladesh, Brazil, Chile, China, Columbia, India, Indonesia, Mexico Pakistan, Peru, Philippines, South Korea, Taiwan, and Thailand.*

Figure 14. *Labor allocation in non-OECD countries over the 1980ies, 1990ies, 2000s, and 2010s.*

- insert Figure 14 here -

Notes: *Data source: The World Bank, World Databank. The black dot represents the barycenter of the simplex. Arrows indicating the direction of movement along the trajectories are omitted in the most cases for reasons of clarity of representation. Countries depicted: see APPENDIX A.*

Figure 15. *The labor allocation trajectories depicted in Figure 14, enlarged.*

- insert Figure 15 here -

Notes: *The black dot represents the barycenter of the simplex. The edges of the simplex are not visible in Figure 15. Arrows indicating the direction of movement along the trajectories are omitted in the most cases for reasons of clarity of representation.*

Finally, Figures 16 and 17 depict the labor allocation dynamics in major (geographical) regions of the world and in country groups formed on the basis of income classification, respectively. Both figures present high-frequency data.

Figure 16. *Yearly data on labor allocation in major world regions in the 1990ies, 2000s, and 2010s.*

- insert Figure 16 here -

Notes: *Data source: The World Bank, World Databank. The black dot represents the barycenter of the simplex. Arrows indicating the direction of movement along the trajectories are omitted in the most cases for reasons of clarity of representation. Data for Sub-Saharan Africa is not available. Data points (years in parentheses): Central Europe and the Baltics (1991–2014), East Asia and Pacific (1991–2011), Europe and Central Asia (1991–2014), European Union (1991–2014), Latin America and Caribbean (1992–2013), Middle East and North Africa (2006–2010), North America (1991–2010).*

Figure 17. *Yearly data on labor allocation in lower middle-income, upper middle-income and high-income countries in the 1990ies, 2000s, and 2010s.*

- insert Figure 17 here -

Notes: *Data source: The World Bank, World Databank. The black dot represents the barycenter of the simplex. Data on low-income countries is not available in the World Databank. Data points (years in parentheses): lower middle-income countries (1994, 2000, 2005, 2010, 2012, 2013), upper middle-income countries (yearly data for the period 1991–2011), high-income countries (yearly data for the period 1991–2013).*

3.2 Regularities of Structural Change

Now, we turn to the discussion of the data and the derivation and discussion of the regularities (or: stylized facts) of structural change. As we will see, while Regularities 1, 2 and 7 seem to be quite robust (i.e. strongly supported by empirical evidence), Regularities 3-6 can be regarded as controversial. Nevertheless, it makes sense to discuss them, since they represent, among others, the results of standard structural change models, as discussed in Section 4.1.4.

3.2.1 Dominance of Agriculture in the Early Phases of Development

The following regularity quantifies the well-known fact that initially, 'all' economies were agricultural economies (cf. Stijepic (2015)).

Regularity 1. *In the early phases of development, the agricultural employment share (x_1) is greater than 0.5.*

We can use the results derived in Section 2.3 to immediately find empirical support of Regularity 1 in Figures 10 to 17: if a labor allocation X is characterized by $x_1 > 0.5$, it is located in the partition $\mathbf{S^1}$ (cf. Figure 8). In other words, all the points that are characterized by $x_1 > 0.5$ in Figures 10 to 17 are 'close' to the vertex V_1.

The initial points of the trajectories depicted in Figure 10 are representative of the 'early development phases' of the present-days highly developed countries. As we can see, the initial points of the trajectories of the USA, Japan, China, and Russia are clearly close to the vertex V_1, and, thus, are characterized by $x_1 > 0.5$.[7] France and Germany recording an agricultural employment share of ca. 0.5 in 1870, respectively, were on the frontier of their early development phase (around 1870). Only the early developers, UK ($x_{1UK}(1820)=0.38$) and Netherlands ($x_{1Netherlands}(1870)=0.37$), are *not* close to V_1 in 1820 and 1870, respectively; i.e., at these time points, they were not agricultural countries (anymore).

The initial trajectory segments of the OECD countries depicted in Figures 11 and 12 are not close to the vertex V_1, since the earliest data points in Figures 11 and 12 refer to the 1980ies when all OECD countries were relatively highly developed and, thus, have already had left the early development phase.

We can see that besides Argentina and Chile, all the emerging countries depicted in Figure 13 were close to the vertex V_1 (cf. Definition 5) in the 1950ies. Furthermore, as we can see in

[7] The agricultural employment shares associated with the initial points of the trajectories of these countries are: $x_{1USA}(1820) = 0.7$, $x_{1Japan}(1870) = 0.7$, $x_{1China}(1950) = 0.77$, and $x_{1Russia}(1913) = 0.7$.

Figures 14 and 15, numerous countries of the world were and are close to V_1 and, thus, agricultural economies in the 1980ies and at the present.

Due to data gaps, Figure 16 excludes highly underdeveloped regions of the world (particularly, Sub-Saharan Africa) and depicts the data starting in the 1990ies. Therefore, besides 'East Asia & Pacific' none of the regions is close to V_1 (in the 1990ies). As we can see in Figure 17, the initial state ($x_{1LMIC}(1994)=0.54$) of the present-days lower middle-income countries (abbr. LMIC) is close to V_1; in other words, in 1994, these countries were agricultural economies.

Note that Regularity 1 follows almost immediately from common (anthropological) knowledge: in the very early phases of development (of the mankind), the 'society' focuses on the production (gathering) of food, i.e. is dominated by agriculture. Thus, by going back in time, it should always possible to find a period over which the agricultural share was relatively large in a country, whether it is in 1820 (as in, e.g., the USA) or earlier (as in UK and Netherlands).

3.2.2 Dominance of Services in the Later Phases of Development

The following regularity is clearly supported by all the data presented in Section 3.1.

Regularity 2. *In the later phases of development, the employment share of services (x_3) becomes greater than 0.5.*

Again, we can use the results of Section 2.3 to immediately find empirical support of Regularity 1 in Figures 10 to 17: if a labor allocation X is characterized by $x_3 > 0.5$, it is located in the partition $\mathbf{S^3}$ (cf. Figure 8). In other words, all the points that are characterized by $x_3 > 0.5$ in Figures 10 to 17 are 'close' to the vertex V_3.

As we can see in Figures 10 to 12, the trajectory segments representing the present-days labor allocation in the highly developed and OECD countries are close to V_3 (cf. Definition 5), i.e. these countries are dominated by services at the present. Furthermore, the trajectory segments representing the present-days labor allocations in the world regions depicted in Figure 16 are close to V_3; the same is true for the trajectories representing the labor allocations in high-income and upper middle-income countries depicted in Figure 17. The labor allocations of emerging countries (except for China and India) converged to V_3 (exactly speaking, the countries' services shares increased) between 1950 and 1980 (cf. Figure 13). In general, the dynamics of the world countries depicted in Figures 14 and 15 reveal a convergence to V_3 (i.e. an increase in the services share).

3.2.3 Monotonously Declining Agricultural Share in the Long Run

Regularity 2 implies that the agricultural share is relatively small (cf. Definition 5) in later development phases: if $x_3 > 0.5$ (cf. Regularity 2), then $x_1 < 0.5$ (cf. (4)). Thus, Regularities 1 and 2 jointly imply that the agricultural share (x_1) declines over the period covering the 'early development phase' (cf. Regularity 1) and (some of) the 'later development phases' (cf. Regularity 2).

However, Regularities 1 and 2 do not provide us with information about the *process* of agricultural decline. In other words, Regularities 1 and 2 are consistent with very different types of trajectories depicting the *transition* from the early to the later development phases. For example, the trajectories depicted in Figures 3, 5, and 18,[8] which represent very different dynamic laws,[9] are consistent with Regularities 1 and 2. As we will see in Section 4, the predictions [10] based on the trajectories/laws depicted in Figures 3, 5, and 18 differ significantly. For these reasons, it seems important to describe the transitional dynamics depicted by the empirically observed trajectories in more detail, as done in Regularity 3 (and 4).

Figure 18. *An example of cyclical dynamics.*

- insert Figure 18 here -

Regularity 3. *The employment share of agriculture declines monotonously in the long run.*

Figure 10 depicting the long-run dynamics of labor allocation supports Regularity 3. As we can see, the angles (α) of all the tangential vectors of all the trajectories in Figure 10 are

[8] In Figure 18, we assume that the point P is the observed initial point, and the point Q is the last point observed. This implies that: the points preceding P represent unobserved labor allocations that were realized before the time point associated with the point P; and the points succeeding the point Q represent the labor allocations that will be observed in future.

[9] The trajectory depicted in Figure 3 is characterized by monotonous dynamics of x_1 and x_3 (cf. Properties 1 and 3). The trajectory depicted in Figure 5 is self-intersecting (cf. Definition 3). The trajectory depicted in Figure 18 is non-self-intersecting and non-monotonous (cf. Definition 3 and Properties 1 to 3). In Section 4, we interpret these characteristics (i.e. monotonicity and self-intersection) as laws and use them for prediction of structural change.

[10] Among others, the omega limit sets (cf. Definition 7) of non-monotonous (and cyclical) and monotonous trajectories can differ significantly: in general, the omega limit set of a bounded, continuous, and monotonous trajectory consists of only one point, while cyclical trajectories can have one-dimensional limit sets.

somewhere in the range between ca. 10° and ca. 120°. Thus, according to Property 1, the agricultural employment shares represented by the trajectories depicted in Figure 10 decline strictly monotonously. Note, however, that Figure 10 depicts low-frequency data connecting time points that are separated by periods of ca. 40 years. Thus, some (shorter-run) non-monotonicities may be not viewable in Figure 10.

Indeed, Figures 11, 12, 14, and 15 depicting high-frequency (i.e. yearly) data reveal that there are many non-monotonicities. In general, we could postulate the hypothesis that these non-monotonicities result from short-run fluctuations, i.e. while the agricultural employment share declines over the long run, it increases sporadically over relatively short periods of time. We leave the empirical testing of this hypothesis for further research, since it is not essential for the key topic of our paper (i.e. the discussion of the positivistic approach) and since it can be replaced by other hypotheses (e.g. Regularity 6). However, at least, we can postulate that the data depicted in Figures 11, 12, 14, and 15 does not display systematical cyclical behavior of the type depicted in Figure 18.

It does not make sense to study the monotonicity properties of the dynamics depicted in Figure 13, since this figure depicts only two points in time (1950 and 1980) for each country.

Figure 16, depicting the dynamics of geographical country groups, supports the view that the agricultural employment share declines monotonously in the long run and that there are only short-run fluctuations where the agricultural share inclines sporadically over relatively short periods of time. Only the dynamics of the agricultural share in 'Middle East & North Africa' appears to be highly non-monotonous; note, however, that the trajectory of this region covers only four years and, thus, represents short-run behavior.

As we can see in Figure 17, besides some short-run non-monotonicities, the agricultural employment share increased monotonously in lower middle-income, upper middle-income, and high-income countries.

Overall, it seems that the employment share of agriculture declines monotonously in the long run. However, the empirical support of this regularity (i.e. Regularity 3) is not as strong as the empirical support of Regularities 1 and 2.

3.2.4 Monotonously Growing Services Sector in the Long Run

Figures 10 to 17 reveal the following regularity of structural change.

Regularity 4. *The employment share of services increases monotonously in the long run.*

We omit a detailed discussion of Regularity 4, since it is very similar to the discussion of Regularity 3 (cf. Section 3.2.3): Figures 11, 12, 14, and 15 reveal some short run non-monotonicities of the services employment share dynamics; in general, the empirical support of Regularity 4 is not as strong as the empirical support of Regularities 1 and 2.

3.2.5 Non-Monotonous Manufacturing Employment Dynamics

Several authors (among others, Ngai and Pissarides (2007), Foellmi and Zweimüller (2008), Uy et al. (2013), Herrendorf et al. (2014), and Stijepic (2015)) have emphasized the non-monotonous dynamics of the manufacturing employment share described by Regularity 5.

Regularity 5. *The employment share of manufacturing increases in the early phases of development ('industrialization phases') and declines in the later phases of development ('de-industrialization phases').*

The data depicted in Figure 10 supports Regularity 5. We can see that: (a) the initial segments of all the trajectories depicted in Figure 10 are characterized by $\alpha < 60°$ (cf. Definition 6); and (b) the final segments of the trajectories of the highly developed countries depicted in Figure 10 are characterized by $\alpha > 60°$. This fact implies per Property 2 that the manufacturing employment share (x_2) increases initially and declines later.

Figures 11, 12, 14, and 15 generate the impression that the trajectory portrait (or: the vector field implied by all the trajectories) depicts a non-monotonous movement: the tangential vectors that are located close to the vertex V_1 point away from the V_3V_1-edge of the simplex (i.e. they are characterized by $\alpha < 60°$), while the tangential vectors that are close to the vertex V_3 point rather towards the V_3V_1-edge of the simplex (i.e. they are characterized by $\alpha > 60°$). However, at the same time, we can see that many trajectories deviate from Regularity 5, not only in Figures 11, 12, 14, and 15, but also in Figures 16 and 17.

Overall, it seems that the empirical support of Regularity 5 is mixed (at the country level).

3.2.6 Non-Self-Intersection of the Labor Allocation Trajectory

We can observe numerous self-intersections (cf. Definition 3) in the data presented in Figures 11, 12, 14, 15, and 16.[11] However, these self-intersections seem to be of rather short-run

[11] For example, the trajectories of the following countries self-intersect in the Figures 11 and 12: Australia, Belgium, Chile, Ireland, Island, Latvia, Luxemburg, New Zealand, Norway, Slovakia, Slovenia, Suisse, Sweden, and Turkey (cf. Stijepic (2016)).

nature, since, among others, they are not observable in the long-run data depicted in Figure 10. See Stijepic (2016) for a detailed discussion, which can be summarized by formulating the following regularity.

Regularity 6 *(Stijepic (2016)).* ***a)*** *Labor allocation trajectories self-intersect. The intersections are of short-run nature, i.e. there are no long-run loops (covering long periods of time).* ***b)*** *The long-run dynamics of labor allocation can be represented by non-self-intersecting trajectories.*

Note that non-self-intersection (cf. Definition 3) is a generalization of the notion of monotonicity (cf. Properties 1 to 3): a monotonous trajectory is always non-self-intersecting (cf. Stijepic (2016), p.27), while a non-self-intersecting trajectory needs not being monotonous. For example, the trajectory depicted in Figure 18 is non-self-intersecting and , obviously, non-monotonous (cf. Properties 1 to 3).

3.2.7 Mutual Intersection of Countries' Trajectories

Mutual intersection of countries' trajectories (cf. Definition 4) is one of the most evident empirical facts: mutual intersections are observable in Figures 10 to 16. Even in the long-run data depicted in Figure 10, we can observe mutual intersections.[12] Thus, we can formulate the following regularity.

Regularity 7 *(Stijepic (2016)). The (long-run) labor allocation trajectories of different countries intersect mutually.*

See Stijepic (2016) for a detailed discussion of mutual intersection.

4. POSITIVISTIC MODELS BASED ON THE EMPIRICAL REGULARITIES

In this section, we formulate models based on Regularities 1 to 7. As noted in Section 3.2, while Regularities 1, 2, and 7 may be regarded as 'robust', Regularities 3 to 6 may be regarded as controversial. In general, different readers may find different regularities controversial. Thus, it makes sense to generate different models based on different

[12] For example, in Figure 10, we can observe the intersections of the trajectories of the following countries: (a) Germany and UK, (b) US and France, (c) Netherlands and France, (d) US and France, (e) Netherlands and US, (f) China and US, (g) Russia and France, (h) Russia and Netherlands, (i) Japan and France, (j) Japan and Netherlands, and (k) Japan and US (cf. Stijepic (2016)).

regularities. Our most conservative model only relies on Regularities 1 and 2, i.e. the regularities that are the least controversial. The other models from our model set combine the remaining regularities in different fashion such that the reader can choose the model that corresponds to his ideology.

To keep our modeling approach as positivistic as possible, we try to minimize the use of (purely) ideological assumptions, which we name here 'axioms'. Moreover, in Section 4.1.4, we show that Regularities/Laws 1-7 are theoretically founded, such that the results of our paper need not only being interpreted as (radically) positivistic results.

4.1 Basic Axioms, Observations and Laws

In this section, we formulate the axioms and laws that we use in Section 4.2 to define structural change models, where each model uses a different set of axioms and laws. Furthermore, (in Section 4.1.4) we provide references on the theoretical foundations of the laws.

4.1.1 Axioms

First, we formulate *axioms* by using the concepts introduced in Section 2.1. In our paper, the axioms represent all the assumptions that are not empirically founded. Although we try to keep our discussion as positivistic as possible, our models, like all other thinking constructs of empirical sciences, cannot be formulated without using a minimum of not empirically founded assumptions.

Axiom 1. *The long-run labor allocation dynamics of country* $c \in \boldsymbol{C}$ *over the period* $t \in \boldsymbol{D}$ *are described by the function* $X_c(t) \equiv (x_{1c}(t), x_{2c}(t), x_{3c}(t))$ *as defined by (1)-(3) and (6)-(8).* $x_{1c}(t)$, $x_{2c}(t)$, *and* $x_{3c}(t)$ *represent the employment share of agriculture, manufacturing, and services at time t in country c, respectively.* $\boldsymbol{D} \equiv (d^l, d^u) \subseteq [0, \infty)$ *is the time interval to which the model applies.* $\boldsymbol{C}$ *is the set of countries to which the model applies.*

In Section 4.2, we use Axiom 1 in all our models, while the following two axioms are only used in some of our models.

Axiom 2. $\forall t \in \boldsymbol{D}$ $\forall c \in \boldsymbol{C}$, $X_c(t)$ *is continuous in t.*

The continuity axiom (i.e. Axiom 2) is a typical (long run) modeling convention in development and growth theory. The models presented by Kongsamut et al. (2001), Ngai and Pissarides (2007), Acemoglu and Guerrieri (2008), Foellmi and Zweimüller (2008), and Boppart (2014) are typical examples of multi-sector models satisfying Axiom 2.

Axiom 3. *$\forall t \in \boldsymbol{D}$ $\forall c \in \boldsymbol{C}$, $X_c(t)$ is differentiable with respect to t.*

Axiom 3 is not necessary for formulating our assumptions, describing the empirical evidence, deriving the predictions in Section 4.2, or any of our other results. That is, we could write an alternative version of our paper that does *not* rely on differentiable functions or the notion of the derivative and generates the same results. However, the use of derivations abbreviates, among others, the formulation of Properties 1 to 3, Definition 6, as well as Laws 3 and 4, significantly. Therefore, we use Axiom 3.

4.1.2 Observations

Now, we suggest mathematical expressions of the verbal statements of Regularities 1-7, where we name these mathematical expressions ‘observations’. Later, we use these ‘observations’ to formulate ‘laws’.

As typical for the discussion of stylized facts in the economic literature, the verbal statements of Regularities 1-7 are relatively imprecise. They do not state explicitly for which countries each regularity is true. In general, (some of) the regularities are valid for some but not for all countries, as discussed in Section 3.2. The following mathematical formulations of the regularities (i.e. Observations 1-7) rely on the term ‘$\exists c \in \mathbf{C}$’ to express the fact that the regularities are possibly not true for all the countries considered but only for some of them. In Section 4.1.3, we discuss this topic in more detail.

By relying on Axiom 1 (and Axioms 2 and 3, in part) and the concepts of elementary calculus and set theory, we can express the verbal statements of Regularities 1-7 as follows.

Observation 1 *(cf. Regularity 1 and Axiom 1). $\exists c \in \boldsymbol{C}$ $\exists a \in \boldsymbol{D}$: $x_{1c}(a)>0.5$.*

Observation 2 *(cf. Regularity 2 and Axiom 1). $\exists c \in \boldsymbol{C}$ $\exists b \in \boldsymbol{D}$: $x_{3c}(b)>0.5 \wedge b>a$.*

Observation 3 *(cf. Regularity 3 and Axioms 1-3). $\exists c \in \boldsymbol{C}$: $\forall t \in [a,b]$ $dx_{1c}/dt \leq 0$.*

Observation 4 *(cf. Regularity 4 and Axioms 1-3). $\exists c \in \boldsymbol{C}$: $\forall t \in [a,b]$ $dx_{3c}/dt \geq 0$*

Observation 5 *(cf. Regularity 5 and Axioms 1-3).* $\exists c \in \boldsymbol{C}\ \exists z \in (a,b)$: $(\forall t \in [a,z)\ dx_{2c}(t)/dt \geq 0) \wedge (\forall t \in (z,b]\ dx_{2c}(t)/dt \leq 0)$.

Observation 6 *(cf. Regularity 6 and Axiom 1).* $\exists c \in \boldsymbol{C}\ \nexists (t_1,t_2,t_3) \in [a,b]^3$: $t_1 < t_2 < t_3 \wedge X_c(t_1) = X_c(t_3) \neq X_c(t_2)$ *(cf. Definition 3).*

Observation 7 *(cf. Regularity 7 and Axiom 1).* $\exists (o,p) \in \boldsymbol{C}^2$: $\boldsymbol{T}_o([a,b]) \cap \boldsymbol{T}_p([a,b]) \neq \emptyset$ *(cf. Definition 4).*

Recall that Axiom 1 refers to the long run; thus, all the statements of Observations 1 to 7 are statements about the long-run dynamics. While Observations 1 and 2 describe the state of the country at the time points a and b, Observations 3 to 7 describe the (transitional) dynamics of the country between these time points. The verbal interpretation of Observations 1 to 7 is given by Regularities 1 to 7, respectively. The notation used in Observations 1-7 and the verbal statements of Regularities 1-7 jointly imply the following interpretation of the time points a, b, and z (where **D** is the time interval to which the analysis applies; cf. Axiom 1): 'a' is an 'early point in development'; 'b' is a 'later point in development'; and 'z' is the turning point in manufacturing sector dynamics (from the industrialization period to the de-industrialization period).

4.1.3 Laws

In general, Observations 1-7 are true statements (if they are regarded as statements about the characteristics of the data sample described in Section 3.1): each of the observations states that there exists a country (in our sample) having certain characteristics at certain time points or over certain periods of time; we have proven in Section 3.2 that Observations 1-7 are true by providing examples of countries that can be characterized by Observations 1-7.

In contrast, 'laws' are statements of greater generality. In general, a law is defined as a regularity that is valid across *time* and *space*.[13] The fact that laws are valid across *time* (cf. **D**) means, among others, that they are valid in future to some extent; thus, we can use them for prediction; we will see below that there are different ways to extend Observations 1-7 across time (i.e. across the time interval (**D**) to which our models refer). '*Space*' refers here to countries, where we can distinguish between general laws (i.e. laws that are valid across all countries) and ceteris paribus laws, which are valid only for some countries (see Stijepic (2016), p.20ff. for a discussion). This distinction is, however, not important in our paper,

[13] For a discussion of laws in economics and natural sciences, see, e.g., Jackson and Smith (2005) and Reutlinger et al. (2015). Stijepic (2016), p.20ff., discusses the application of the term 'law' in structural change modeling.

since our mathematical/logical derivations are the same irrespective of the type of law to which they refer (general vs. ceteris paribus law). Therefore, we assume, henceforth, that the laws are valid for the country set **C**. This set may represent all countries of the world or only a subset of them (e.g., only the countries that are characterized by Regularities 1-7 according to the data discussed in Section 3). The reader may decide on his own whether he considers the laws discussed in our paper as general laws or as ceteris paribus laws and, thus, whether our results/predictions are valid for all countries or only for the subset of countries he thinks they are valid for.

We can summarize this discussion as follows: Laws 1-7 are generalizations of the Observations 1-7 across time (**D**) and space (**C**). We let the reader decide to which countries ('space') the laws apply, and focus now on the discussion of the time period (**D**) to which the laws apply. We start with Law 1.

Law 1 *(cf. Observation 1 and Axiom 1).* $\forall c \in \boldsymbol{C}\ \exists a_c \in \boldsymbol{D}\colon \forall t \in (d^l, a_c]\ x_{1c}(t) > 0.5.$

Law 1 states that each country belonging to the group **C** is an agricultural economy (i.e. is characterized by $x_1 > 0.5$) over the period of time $(d^l,a_c]$. In other words, Law 1 extends Observation 1 backwards in time to the lower limit (d^l) of the time period considered (**D**) (cf. Axiom 1). As discussed in Section 3.2.1, this makes sense, since primitive economies are agricultural economies. This fact may also be relevant for long-run predictions where the backward extension of the trajectory, i.e. $\{X_c(t) \in \mathbf{S}\colon d^l \leq t < a_c\}$, is relevant (cf. Stijepic (2015), p.81).

Note that the period $(d^l,a_c]$ is country-specific (i.e. a_c depends on c, where $c \in \mathbf{C}$) and it represents (a part of) the 'early development phase' (cf. Section 4.2.1 and Regularity 1) of country c, where $c \in \mathbf{C}$. In other words, Law 1 states that each country c has its own (country-specific) 'early development phase' $(d^l,a_c]$. This makes sense, since different countries overcome the early development phase at different points of time, as shown in Section 3.2.1.

Law 1 is formulated by using the expression $\forall c \in \mathbf{C}$, whereas Observation 1 is formulated by using the expression $\exists\, c \in \mathbf{C}$. This reflects our discussion of the fact that laws are generalizations of observations and, in particular, (our) laws are valid for a group of countries (**C**) and not only for one country (c). We keep this view, i.e. we replace '$\exists \mathbf{c}$' by '$\forall \mathbf{c}$', when transforming the Observations 2-7 into Laws 2-7.

Law 2 *(cf. Observation 2 and Axiom 1).* ***a)*** $\forall c \in \mathbf{C}\ \exists b_c \in \mathbf{D}$: $x_{3c}(b_c) > 0.5 \wedge b_c > a_c$. ***b)*** $\forall c \in \mathbf{C}\ \exists b_c \in \mathbf{D}$: $\forall t \in [b_c, d^u)\ x_{3c}(t) > 0.5 \wedge b_c > a_c$.

Law 2 states that each country c belonging to the country group **C** is a services economy (i.e. is characterized by $x_3 > 0.5$) at the time point $b_c > a_c$, where the time point a_c is defined in Law 1. Laws 1 and 2 jointly state that each country from the country group **C** is, first, an agricultural economy (at time a_c) and, later, a services economy (at time b_c). Furthermore, note that Law 2 states that each country c has its own (country-specific) point b_c, where $c \in \mathbf{C}$. This point represents a point in the later phases of development of country c (cf. the explanation of 'b' in Section 4.1.2 and Regularity 2). This is consistent with the empirical evidence discussed in Section 3.2.2, which shows that some countries reach the status of a services economy earlier than others.
The difference between Law 2a and 2b is simple: Law 2b states that country c becomes a services economy at time b_c and continues to be a services economy for the rest of the time period **D**; in contrast, Law 2a does not state what happens after the time point b_c (i.e. economy c may be a services economy or not for $t > b_c$). This fact is of importance in the models of Section 4.2, where Law 2a implies that there is a 'gravity point' 'close' (cf. Definition 5) to the vertex V_3, and Law 2b implies that there is a set of attraction 'close' to the vertex V_3, which is important for predicting the limit dynamics of labor allocation, as we will see.

Law 3 *(cf. Observation 3 and Axioms 1-3).* $\forall c \in \mathbf{C}\ \forall t \in [a_c, d^u)\ dx_{1c}/dt \leq 0$.

By transforming Observation 3 into Law 3, we do not only replace '$\exists$**c**' by '$\forall$**c**', but also assume that each country c has its own (country-specific) period $[a_c, d^u)$ of monotonously decreasing agricultural share ($dx_{1c}/dt \leq 0$), where a_c and d^u are defined in Law 1 and Axiom 1. In other words, Law 3 extends Observation 3 to the end (d^u) of the time period considered (**D**) and to all countries belonging to the group **C**.

Law 4 *(cf. Observation 4 and Axioms 1-3).* $\forall c \in \mathbf{C}\ \forall t \in [a_c, d^u)\ dx_{3c}/dt \geq 0$.

The discussion of Law 4 is analogous to the discussion of Law 3. Law 4 states that each of the countries belonging to the group **C** is characterized by a monotonously growing services share over the (country-specific) period $[a_c, d^u)$.

Law 5 *(cf. Observation 5 and Axioms 1-3).* $\forall c \in \boldsymbol{C}\ \exists z_c \in (a_c, b_c)$: $(\forall t \in [a_c, z_c)\ dx_{2c}(t)/dt \geq 0) \wedge (\forall t \in (z_c, b_c]\ dx_{2c}(t)/dt \leq 0)$.

By transforming Observation 5 into Law 5, we do not only replace '∃**c**' by '∀**c**', but also assume that each country c has its own (country-specific) 'turning point' z_c, where c∈**C**. Law 5 states that this 'turning point' is the time point at which the country c's manufacturing employment share ($x_{2c}(t)$) stops increasing and starts decreasing (cf. the explanation of 'z' in Section 4.1.2). We do not generalize Observation 5 further, i.e. we do not assume that there are many future turning points, since this does not yield additional results in Section 4.2.

Law 6 *(cf. Observation 6 and Axiom 1).* $\forall c \in \boldsymbol{C}\ \nexists (t_{1c}, t_{2c}, t_{3c}) \in \boldsymbol{D}^3$: $t_{1c} < t_{2c} < t_{3c} \wedge X_c(t_{1c}) = X_c(t_{3c}) \neq X_c(t_{2c})$.

Law 6 refers to the time period **D** and states that in this period, each country c∈**C** does not have a (country-specific) point of self-intersection (cf. Definition 3). In other words, Law 6 extends Observation 6 to all the countries belonging to the group **C** and to the whole period **D**.

Law 7 *(cf. Observation 7 and Axiom 1).* $\exists (o,p) \in \boldsymbol{C}^2$: $\boldsymbol{T}_o([b_o, d^u)) \cap \boldsymbol{T}_p([b_p, d^u)) \neq \emptyset$.

Law 7 states that at least two of the trajectories (cf. (9)) belonging to the group **C** intersect mutually, where the (country-specific) periods ($[b_c, d^u)$, c∈**C**) to which the trajectories refer are defined in Law 2 and Axiom 1. In particular, Law 7 states that the country-specific trajectory segments describing the dynamics after the country-specific time points b_c (where c∈**C**) intersect mutually. As we will see in Section 4.2.5, depending on the interpretation of Law 7, interesting predictions of limit dynamics can be made.
Recall that Laws 1-7 are formulated on the basis of Axiom 1 (and Observations 1-7) and, thus, they refer to the *long-run dynamics*.

4.1.4 The Theoretical Foundations of Laws 1-7

As discussed in Sections 3.2.1 and 4.1.3, Law 1 is partly an anthropological fact. The theoretical foundations of Laws 2, 3, 4, and 6 are provided by, e.g., Kongsamut et al. (2001), Ngai and Pissarides (2007), Foellmi and Zweimüller (2008), Stijepic (2011), and Herrendorf et al. (2014); these papers present models that generate structural change trajectories that have the characteristics described in Laws 2, 3, 4, and 6 and can, therefore, be regarded as intuitive/theoretical explanations of these laws. The theoretical foundations of Law 5 are provided by, e.g., Ngai and Pissarides (2007), Foellmi and Zweimüller (2008), Stijepic (2011), Uy et al. (2013), and Herrendorf et al. (2014); these papers focus among others on the explanation of the hump-shaped manufacturing sector dynamics. The theoretical foundations and explanations of Laws 6 and 7 are extensively discussed by Stijepic (2016).

4.2 Long-Run Models of Structural Change

In this section, we formulate models of structural change on the basis of the laws and axioms formulated in Section 4.1. Laws 1-7 and Axioms 1-3 are nothing else than logical statements; we can perform logical operations on them to derive their implications. Each of our models (i.e. Models 1-5) assumes that a subset of Laws 1-7 and Axioms 1-3 is true; each model's predictions (i.e. the statements that refer to future labor allocation dynamics) are the implications of the laws and axioms that are assumed to be valid within the model.

We start with the most conservative model (i.e. Model 1), which is only based on Axiom 1 and Laws 1 and 2. The subsequent models (i.e. Models 2 to 5) add more and more of the more controversial laws (i.e. Laws 3-6) or axioms (i.e. Axioms 2 to 3). For each of the models, we discuss the transitional and limit dynamics (as usual in growth theory) and the set of attraction (cf. Section 2.4), and derive the predictions of structural change for developing and developed countries. Of course, we cannot discuss here all the possible combinations of Axioms 1-3 and Laws 1-7 due to space restrictions. Therefore, we only focus on some examples, which demonstrate the capabilities of the positivistic approach and the implications of the laws.

In all the models of Section 4.2, we assume that the present-days labor allocation is given and we aim to predict the future dynamics. We define the corresponding time points as follows.

Definition 9. *The time point $t=0$ stands for the present and $X_c(0)\equiv(x_{1c}(0),x_{2c}(0),x_{3c}(0))$ stands for the present-day allocation in country c (cf. Axiom 1). The future is represented by $t\in(0,\infty)$ and the future labor allocation dynamics in country c is represented by $X_c(t)$ for $t\in(0,\infty)$.*

4.2.1 Model 1 – the Implications of Laws 1 and 2a or 2b

Model 1 is relatively rudimentary; its predictions follow almost directly from its assumptions (i.e. laws and axioms). Nevertheless, it makes sense to discuss these predictions, since they seem to be the most reliable predictions that we can make. In some sense, this model elucidates what we 'really know' about the future structural change in developing and developed countries. The predictions of Models 2-5 require more mathematics; at the same time, they are more controversial due to the many additional assumptions they require.

We distinguish between two versions of Model 1 (Model 1a and Model 1b), depending on whether Law 2a or Law 2b is assumed to be true.

Assumptions of Model 1a

Assume that country c belongs to the group **C** and satisfies Axiom 1 and Laws 1 and 2a. We are interested in predicting the future dynamics of country c (cf. Definition 9 and Axiom 1).

Predictions of Model 1a

If country c is relatively underdeveloped at the present (cf. Definition 9), i.e. if

(14) $x_{1c}(t) > 0.5$ for $t \leq 0$

is true, the following predictions (of the dynamics for $t > 0$) can be made based on Model 1a.

Law 1 and (14) imply that at the present (cf. Definition 9), country c is in the early development phase $(d^1,a_c]$, i.e. $a_c \leq 0$. Thus, per Law 2a, there exists a future time point (cf. Definition 9) $b_c > 0$ that is characterized by $x_{3c}(b_c) > 0.5$. In other words, (14) and Laws 1 and 2a imply that the country will become a services economy in future. This is all we can say about the *transitional dynamics* (cf. Section 2.4) of Model 1. Any imaginable transitional behavior (e.g. non-continuous, erratic, cyclical, etc.) is possible in Model 1 as long as economy c reaches at least temporarily the state of $x_3 > 0.5$ in finite time.

Similarly, we cannot say anything about the *limit dynamics* (cf. Section 2.4) of economy c based on Model 1a, since Model 1a does not state anything specific about the nature of the function $X_c(t)$. That is, the labor allocation in country c may converge to a fixed point (steady state) or to a limit cycle, or may exhibit any other imaginable limit dynamics (e.g. resulting from some sort of chaotic behavior) on **S**. Obviously, Model 1a's *set of attraction* (cf. Section 2.4) cannot be greater than **S** (cf. Axiom 1) and $\mathbf{O}(\mathbf{T}_c([0,\infty)))\subseteq\mathbf{S}$ (cf. Definition 7 and Axiom 1).

If we replace (14) by

(15) $x_{3c}(0) > 0.5$

reflecting the initial state of a developed economy, we cannot say anything about the future (cf. Definition 9) dynamics of economy c, except that $\mathbf{O}(\mathbf{T}_c([0,\infty)))\subseteq\mathbf{S}$ (cf. Axiom 1 and Definition 7).

Application of Model 1a

Obviously, this discussion can be applied for predicting the future labor allocation dynamics of present-days *developing countries*, which satisfy condition (14). Model 1a implies that these countries will become services economies at some time in future. Afterwards, everything can happen according to Model 1a, i.e. the economies may become agricultural or manufacturing economies again or remain services economies forever. In general, Model 1a may be regarded as an optimistic model, since it states that all economies (belonging to the group **C**) will become services economies at some point in time.

On the basis of Model 1a, we cannot make any predictions of future structural change in present-days *developed countries*, which satisfy (15). Note, however, that in contrast to the standard theoretical literature (cf. Section 4.1.4), Model 1a allows for strong structural change in the future of the present-days developed economies: they may become manufacturing or agricultural economies again or stay services economies forever, i.e. they may reach any point on **S** in future.

Assumptions of Model 1b

Assume that country c belongs to the group **C** and satisfies Axiom 1 and Laws 1 and 2b. That is, in contrast to Model 1a, Model 1b assumes that Law 2b is valid. We are interested in predicting the future dynamics of country c (cf. Definition 9 and Axiom 1).

Predictions of Model 1b

The time period to which the predictions of Models 1a and 1b apply ($[0,\infty)$) can be divided into two subperiods: $[0,b_c)$ and $[b_c,\infty)$ (cf. Law 2 and Axiom 1). While Model 1a's and Model 1b's predictions of the dynamics over the period $[0,b_c)$ do not differ, Model 1b provides interesting predictions of the dynamics over the period $[b_c,\infty)$. Therefore, we focus on this period.

Law 2b and (12) imply that $\forall t\in[b_c,\infty)\ X_c(t)\in\mathbf{S^3}$; Axiom 1 (and, in particular, (1) and (2)) and (12) imply that $X_c(t)\in\mathbf{S^3} \Rightarrow x_{1c}(t)\in[0,0.5] \wedge x_{2c}(t)\in[0,0.5] \wedge x_{3c}(t)\in(0.5,1]$; thus, the following statement is true:

(16) $\forall t \in [b_c,\infty)\ x_{1c}(t) \in [0,0.5] \wedge x_{2c}(t) \in [0,0.5] \wedge x_{3c}(t) \in (0.5,1]$

The assumptions of Model 1b do not impose any further restrictions on the dynamics over the period $[b_c,\infty)$, i.e. economy c may experience any imaginable sort of labor allocation dynamics (on $\mathbf{S^3}$) over the period $[b_c,\infty)$, e.g. transitory, non-continuous, erratic, cyclical, etc. This fact and (16) imply that over the *transitional* phase ($[b_c,\infty)$) and in the *limit* ($\lim_{t\to\infty}$) (cf. Section 2.4), economy c may experience any imaginable sort of labor allocation dynamics on $\mathbf{S^3}$, where the employment shares may change (or fluctuate) strongly over time and, in particular, the potential ranges of fluctuation of the agricultural, manufacturing, and services shares in Model 1b are $\mathbf{M}_1^{1b}([b_c,\infty)) \subseteq [0,0.5]$, $\mathbf{M}_2^{1b}([b_c,\infty)) \subseteq [0,0.5]$, and $\mathbf{M}_3^{1b}([b_c,\infty)) \subseteq (0.5,1]$, respectively (cf. (16) and Definition 8). Thus, each of the employment shares may change or fluctuate by 0.5 over the transitional period $[b_c,\infty)$ and in the limit, i.e. the potential strength of fluctuation of the agricultural, manufacturing, and services shares in Model 1b is given by: $|\mathbf{M}_1^{1b}([b_c,\infty))| \leq 0.5$, $|\mathbf{M}_2^{1b}([b_c,\infty))| \leq 0.5$, and $|\mathbf{M}_3^{1b}([b_c,\infty))| \leq 0.5$. Thus, the structural change predicted by Model 1b can be relatively strong in comparison to the structural change observed in the past (cf. Section 3): for example, the agricultural employment shares in France, Germany, Netherlands, and UK have decreased by less than 0.5 since 1870 (cf. Section 3).

This discussion implies that Model 1b's *set of attraction* (cf. Section 2.4) is a subset of $\mathbf{S^3}$ (cf. (12)). In contrast, Model 1a's set of attraction is a subset of **S**, as shown above. Thus, Model 1b allows us to specify the set of attraction much more precisely than Model 1a does; $\mathbf{S^3}$ covers only 25% of the area of **S** (cf. (4), (12), and Figure 8).

Moreover, this discussion, Definition 7, and the definition of the set of attraction (cf. Section 2.4) imply, obviously, that the *omega limit set* of a trajectory generated by Model 1b is located in the partition $\mathbf{S_3}$, i.e. $\mathbf{O}(\mathbf{T}_c([0,\infty))) \subseteq \mathbf{S^3}$.

Application of Model 1b

This discussion can be applied for predicting the future structural change dynamics of present-days *developing countries*, which satisfy condition (14). Like Model 1a, Model 1b predicts that these countries will become services economies at some time in future (b_c). Moreover, Model 1b predicts that from then on (i.e. for $t > b_c$) the dynamics of these economies will be the same as the future dynamics of the present-days developed countries (see below for a discussion of Model 1b's predictions of the future dynamics of developed economies). In general, Model 1b is even more optimistic model than Model 1a is: it does not only state that all economies (belonging to the group **C**) will become services economies at some point in time but also that they will be able to sustain this development (i.e. stay services economies forever).

Furthermore, Model 1b can be used to predict the future dynamics of the present-days *developed economies*, which are, of course, services economies and satisfy (15). Model 1b predicts that the present-days developed economies will remain services economies forever and may, nevertheless, experience strong structural change in future: each of their sectoral employment shares may fluctuate by 0.5 over time (even in the limit), which is comparable to the magnitude of the structural change over the last 150 years in the present-days highly developed countries. In general, Model 1b states that structural change need not coming to a halt (in highly developed economies), in contrast to the predictions of the standard literature (cf. Section 4.1.4).

4.2.2 Model 2 – the Implications of Law 3 or 4

We distinguish between two versions of Model 2 (Model 2a and Model 2b) depending on whether Law 3 or Law 4 is true.

Assumptions of Model 2a

Assume that country c belongs to the group **C** and satisfies Axioms 1 to 3 and Law 3. We are interested in predicting the future dynamics of country c (cf. Definition 9 and Axiom 1).

Predictions of Model 2a

We begin with the *transitional dynamics* (cf. Section 2.4). Obviously, the employment share of agriculture (x_1) is constant or decreases, according to Law 3. The services and manufacturing employment shares may be increasing, decreasing, constant, or non-monotonous (e.g. cyclical), as long as their sum (x_2+x_3) is constant (if x_1 is constant) or increases over time (if x_1 decreases over time), since, otherwise, (1)-(2) is violated (cf. Axiom 1). The vector angles of the trajectory generated by Model 2a are stated in Property 1b/c. Moreover, since the agricultural employment share decreases monotonously over time, the trajectory does not intersect itself according to Definition 3 (cf. Stijepic (2016), p.27, and Stijepic (2015), p.82f.). Examples of transitional dynamics consistent with Model 2a are depicted in Figures 3 and 20. The trajectories depicted in Figures 4, 5, and 18 are not consistent with Model 2a.

Law 3 implies that $x_{1c}(t)\in[0,x_{1c}(s)]$ for $t\in[s,\infty)$, where $s\in\mathbf{D}\supseteq[0,\infty)$. Thus, Definition 8 implies $\mathbf{M}_1^{2a}([t,\infty))\subseteq[0,x_{1c}(t)]$ for $t\in[0,\infty)$. Therefore, Axiom 1 (and, in particular, (1) and (2)) and Definition 8 imply $\mathbf{M}_2^{2a}([t,\infty))\subseteq[0,1]$ and $\mathbf{M}_3^{2a}([t,\infty))\subseteq[0,1]$ for $t\in[0,\infty)$. Thus, $|\mathbf{M}_1^{2a}([t,\infty))|\leq x_{1c}(t)$, $|\mathbf{M}_2^{2a}([t,\infty))|\leq 1$, and $|\mathbf{M}_3^{2a}([t,\infty))|\leq 1$. Overall, Model 2a allows for stronger future fluctuations of the manufacturing and services share than Model 1b (cf. Section 4.2.1) does (cf. $\mathbf{M}_2^{2a}([t,\infty))|$, $|\mathbf{M}_3^{2a}([t,\infty))|$, $|\mathbf{M}_2^{1b}([b_c,\infty))|$, and $|\mathbf{M}_3^{1b}([b_c,\infty))|$).

The *limit dynamics* (cf. Section 2.4) are relatively easy to predict in Model 2, as shown in the following proposition.

Proposition 1. *Assume that Axioms 1 to 3 and Law 3 are valid. Then,* $\boldsymbol{O}(\boldsymbol{T}_c([0,\infty)))\subseteq\boldsymbol{S}^f:=\{(x_1,x_2,x_3)\in\boldsymbol{S}:x_1=f\}\subset\boldsymbol{S}^{A2a}:=\{(x_1,x_2,x_3)\in\boldsymbol{S}:0\leq x_1\leq x_{1c}(0)\}$, *where* $f\in[0,x_{1c}(0))$ *(cf. (9) and Definition 7). Among others,* $\boldsymbol{O}(\boldsymbol{T}_c([0,\infty)))$ *can consist of only one point, i.e.* $\boldsymbol{O}(\boldsymbol{T}_c([0,\infty)))\equiv X_c^*\in\boldsymbol{S}^{A2a}$.

Proof. The following fact is known from elementary analysis: if $dx_{1c}(t)/dt\leq 0$ for $t\in[0,\infty)$ (cf. Law 3) and $x_{1c}(t)\geq 0$ for $t\in[0,\infty)$ (cf. (1)), then $\lim_{t\to\infty}x_{1c}(t)\equiv f\in[0,x_{1c}(0)]$, i.e. $x_{1c}(t)$ converges to a fixed point (f). (Recall Axioms 2 and 3.) The set of all points on $\mathbf{S}$ that satisfy condition $x_{1c}=f$ is $\mathbf{S}^f$, which is defined in Proposition 1. These facts imply that $X_c(t)\equiv(x_{1c}(t),x_{2c}(t),x_{3c}(t))$ converges to some subset of $\mathbf{S}^f$. In other words, the omega limit set (cf. Definition 7) of the trajectory associated with $X_c(t)$ (cf. (9)) is a subset of $\mathbf{S}^f$. On the one hand, this set (i.e. $\mathbf{O}(\mathbf{T}_c([0,\infty)))$) can consist of only one point (i.e. a fixed point) given the assumptions of Proposition 1; in this case, $\lim_{t\to\infty}X_c(t)\equiv X_c^*\in\mathbf{S}$, i.e. country c converges to a fixed point (X_c^*), as proven by the following example: assume that $\forall t\in[0,\infty)$, $x_{1c}(t)=7/9\exp(-2t)$, $x_{2c}(t)=1/9$, and $x_{3c}(t)=8/9-x_{1c}(t)$; it is easy to prove that these equations satisfy the assumptions of Proposition 1 (i.e. Axioms 1-3 and Law 3) and imply that economy c converges to the fixed point $X_c^*=(0, 1/9, 8/9)$ for $t\to\infty$. On the other hand, given the assumptions of Proposition 1, $\mathbf{O}(\mathbf{T}_c([0,\infty)))$ need not consisting of only one point, as proven by the following example: assume that $\forall t\in[0,\infty)$, $x_{1c}(t)=0.1$, $x_{2c}(t)=1/4\sin(t)+0.4$, $x_{3c}(t)=0.9-x_{2c}(t)$; it is easy to prove that these equations satisfy the assumptions of Proposition 1 (i.e. Axioms 1-3 and Law 3), while the omega limit set (cf. Definition 7) of the corresponding trajectory is $\{(x_1,x_2,x_3)\in\mathbf{R}^3: x_1=0.1 \wedge 0.15\leq x_2\leq 0.65 \wedge x_3=0.9-x_2\}$, i.e. the trajectory converges to a line-segment parallel to the V_2V_3-edge of the 2-simplex. The fact that $\mathbf{S}^{\mathbf{A2a}}\supset\mathbf{S}^{\mathbf{f}}$ for $f\in[0,x_{1c}(0))$ follows immediately from the definitions of the sets $\mathbf{S}^{\mathbf{A2a}}$ and $\mathbf{S}^{\mathbf{f}}$ (cf. the assumptions of Proposition 1).■

In fact, Proposition 1 states that as time goes to infinity, the labor allocation in the economy described by Model 2a converges to (i) a fixed point or (ii) a line-segment (on $\mathbf{S^f}$) parallel to the V_2V_3-edge of the simplex **S**. (The latter fact follows from Proposition 1 and Property 1, where the latter implies that x_1=f=const. only if the economy moves along a line-segment parallel to the V_2V_3-edge of the simplex **S**.) The fixed point or line-segment must be located in $\mathbf{S^{A2a}}$, which is defined in Proposition 1 and depicted in Figure 19. If economy c converges to a fixed point (case i), structural change comes to a halt (in the limit), i.e. the labor allocation converges to a steady state allocation (X_c^*). Figure 3 represents an example of these dynamics where the trajectory-end represents the fixed point. If economy c converges to a line-segment parallel to the V_2V_3-edge (case ii), structural change has a cyclical component (in the limit) where only the employment shares of manufacturing (x_2) and services (x_3) behave cyclically, while x_1 decreases monotonously to its fixed-point value (f). For an example of such a cyclical trajectory, see Figure 20.

Figure 19. *An example for the set S^{A2a}.*

- insert Figure 19 here -

Figure 20. *An example of a trajectory and its omega limit set generated by Model 2a.*

- insert Figure 20 here -

Overall, while Model 2a allows for a much exacter prediction of the limit dynamics than Model 1b does, it does not necessarily allow for an exacter specification of the *set of attraction* (cf. Section 2.4): as shown in Section 4.2.1, Model 1b implies that for t→∞, economy c is located in a subset of $\mathbf{S^3}$; Model 2b (i.e. Proposition 1) predicts that for t→∞, economy c is located in a subset of $\mathbf{S^{A2a}}$; $\mathbf{S^{A2a}}$ may be larger (i.e. may cover a larger area of the simplex **S**) than $\mathbf{S^3}$, depending on the steady state value (f) of the agricultural share.
Analogously, while Model 2a implies that the employment share of agriculture is fixed in the limit (i.e. $\lim_{t\to\infty} x_{1c}(t) \equiv f \in [0, x_{1c}(0)]$), it allows for stronger *limit*-fluctuation of the services and manufacturing shares than Model 1b does (see the discussion of the potential ranges of fluctuation $\mathbf{M}_2^{2a}([t,\infty))|$, $|\mathbf{M}_3^{2a}([t,\infty))|$, $|\mathbf{M}_2^{1b}([b_c,\infty))|$, and $|\mathbf{M}_3^{1b}([b_c,\infty))|$).

Application of Model 2a

The statements of Model 2a are equally applicable to *developed and developing countries*. Model 2a allows for cyclical behavior of the manufacturing and services employment shares even in the limit, i.e. the labor allocation need not converging to a fixed labor allocation but may be characterized by cyclical behavior of the manufacturing and services employment shares in the limit (i.e. 'forever'). Thus, Model 2a (like Models 1a and 1b) allows for the possibility that structural change never comes to a halt, neither in *developed* nor in *developing economies*.

This is relevant for the prediction of structural change in the present-days *highly developed countries* (e.g. the USA), where the present-days structural change is relatively slow, the services employment share is relatively great, and the agricultural and manufacturing shares are relatively small. Model 2 states that these economies may experience strong structural change in future and, in particular, they may re-industrialize.[14] Since Model 2a allows for even stronger (limit) structural change than Model 1b does, the future structural change (i.e. the changes in the manufacturing and services shares) in developed economies may be stronger than the structural change that they experienced over the last 150 years (cf. Section 4.2.1), according to Model 2a.

Furthermore, in contrast to Model 1, Model 2a does not state that all countries (belonging to the group **C**) must become services economies at some point in time, since Law 3 does not state that the agricultural employment share must decline below 0.5. Thus, Model 2a is consistent with the pessimistic view that some *developing economies* may never develop beyond the agricultural stage.

Assumptions of Model 2b

Assume that country c belongs to the group **C** and satisfies Axioms 1 to 3 and Law 4. That is, in contrast to Model 2a, Model 2b assumes that Law 4 and not Law 3 is true. We are interested in predicting the future dynamics of country c (cf. Definition 9 and Axiom 1).

Predictions of Model 2b

It can be shown that the most results of Model 2b are analogous to the results of Model 2a. In particular: (a) the services employment share (x_3) increases monotonously over time and

[14] The situation in highly developed economies described by Model 2a is as follows (cf. Section 3): (a) x_2 and x_1 are relatively small and cannot fall below 0 (cf. (1)); (b) x_1 cannot decrease (cf. Law 3); and (c) x_3 is relatively great and cannot grow beyond 1 (cf. (1)). In other words: the economy is located close to vertex V_3 (since x_3 is very great); thus, it cannot move towards vertex V_3 much (i.e. it cannot increase the services share significantly); moreover, the economy cannot move towards vertex V_1 (i.e. it cannot increase the agricultural share) due to Law 3 (cf. Figure 2). Thus, the only way to achieve a strong labor re-allocation is to move towards vertex V_2 (i.e. to increase the manufacturing share) and away from vertex V_3 (i.e. to decrease the services share). This process may be described as *re-industrialization*.

converges to its steady state value; (b) the agricultural and manufacturing employment shares may exhibit any type of smooth (transitional) dynamics as long as their sum (x_1+x_2) decreases monotonously over time; and (c) cyclical limit dynamics of the manufacturing and agricultural shares are possible, i.e. structural change need not coming to a halt in the limit. We omit a detailed discussion of these aspects, since their proofs and the mathematical techniques used are analogous to the proofs and the techniques used in the discussion of Model 2. Rather, we focus on the key difference between Models 2a and 2b, namely, the strength of structural change over the transitional period and in the limit.

Law 4 implies that $x_{3c}(t)\in[x_{3c}(s),1]$ for $t\in[s,\infty)$, where $s\in\mathbf{D}\supseteq[0,\infty)$. Thus, Definition 8 implies: $\mathbf{M}_3^{2b}([t,\infty))\subseteq[x_{3c}(t),1]$ for $t\in[0,\infty)$. This fact, Definition 8, and Axiom 1 (and, in particular, (1) and (2)) imply $\mathbf{M}_1^{2b}([t,\infty))\subseteq[0,(1-x_{3c}(t))]$ and $\mathbf{M}_2^{2b}([t,\infty))\subseteq[0,(1-x_{3c}(t))]$ for $t\in[0,\infty)$. Thus, according to Definition (8):

(17) $\forall t\in[0,\infty)\ |\mathbf{M}_1^{2b}([t,\infty))|\leq 1-x_{3c}(t) \wedge |\mathbf{M}_2^{2b}([t,\infty))|\leq 1-x_{3c}(t) \wedge |\mathbf{M}_3^{2b}([t,\infty))|\leq 1-x_{3c}(t)$

(17) implies that $|\mathbf{M}_1^{2b}([0,\infty))|$, $|\mathbf{M}_2^{2b}([0,\infty))|$, and $|\mathbf{M}_3^{2b}([0,\infty))|$ are very small if $x_{3c}(0)$ is very great; that is, according to Model 2b, structural change is very weak in future (i.e. for $t\in[0,\infty)$) if the present-days services share ($x_{3c}(0)$) is very great (cf. Definitions 8 and 9). We can apply this result as follows.

Application of Model 2b

Model 2b predicts that in the highly *developed economies*, which are characterized by a very great services share (x_3) at the present, structural change will be relatively weak in future; in particular, Model 2b predicts that these economies will not be able to re-industrialize significantly (cf. Footnote 14).[15] These predictions contradict the predictions of Models 1a, 1b, and 2a, where the latter state that in future, the developed economies may experience changes/fluctuations of the sectoral employment shares that are comparable to or even much stronger than the changes that they experienced over the last 150 years (cf. Section 4.2.1).

Furthermore, like Model 2a, Model 2b does not state that all countries (belonging to the group **C**) must become services economies at some point in time, since Law 4 does not state that the services employment share must grow above 0.5. Thus, Model 2b is consistent with the pessimistic view that some *developing economies* may never become services economies.

[15] Recall that Law 4 states that the services share (x_3) cannot start declining at some time in future; i.e. if it is very great at the present, it remains very great forever.

4.2.3 Model 3 – the Implications of Laws 3 and 4

Assumptions of Model 3

Assume that country c belongs to the group **C** and satisfies Axioms 1-3 and Laws 3 and 4. We are interested in predicting the future dynamics of country c (cf. Definition 9 and Axiom 1).

Predictions of Model 3

The *limit dynamics* (cf. Section 2.4) are relatively easy to predict in Model 3, as shown in the following proposition.

Proposition 2. *Assume that Axioms 1 to 3 and Laws 3 and 4 are valid. Then,* $\boldsymbol{O}(\boldsymbol{T}_c([0,\infty)))\equiv X_c^* \in \boldsymbol{S}^{A3} := \{(x_1,x_2,x_3)\in \boldsymbol{S}: 0\leq x_1 \leq x_{1c}(0) \wedge x_{3c}(0)\leq x_3 \leq 1\}$*, i.e. country c's labor allocation converges to a steady state allocation* (X_c^*)*.*

Proof. The assumptions of Proposition 2 imply that $\lim_{t\to\infty} x_{1c}(t)\equiv f\in[0,x_{1c}(0)]$, i.e. $x_{1c}(t)$ converges to the fixed point f (cf. Proof of Proposition 1). Analogously, it can be shown that $\lim_{t\to\infty} x_{3c}(t)\equiv g\in[x_{3c}(0),1]$, i.e. $x_{3c}(t)$ converges to the fixed point g. Since $\forall t\ x_{2c}(t)=1-x_{1c}(t)-x_{3c}(t)$ (cf. (2)), these two facts imply that $\lim_{t\to\infty} x_{2c}(t)=1-f-g\equiv h\in[0,1-x_{3c}(0)]$, i.e. $x_{2c}(t)$ converges to the fixed point h. These facts imply that $X_c(t)\equiv(x_{1c}(t),x_{2c}(t),x_{3c}(t))$ converges to the fixed point $(f,g,h)\equiv X_c^*\in[0,x_{1c}(0)]\times[0,1-x_{3c}(0)]\times[x_{3c}(0),1]$. The rest of the proof follows from (9) (and (4)) and Definition 7.■

Thus, Model 3 is much more specific about the limit dynamics of labor allocation than Models 1 and 2 are. Model 3 excludes, e.g., cyclical limit dynamics and predicts that structural change is transitory, i.e. comes to a halt (in the limit).

Now, we turn to the *transitional dynamics* (cf. Section 2.4). Laws 3 and 4 imply that if the economy develops (i.e. if the labor allocation is not constant) then the employment share of agriculture decreases over time and/or the services employment share grows over time. Note that Laws 3 and 4 are consistent with the scenario where the economy does not change for all $t > 0$, i.e. no-structural change scenario. This scenario may reflect a development trap or a very mature economy that has converged close to its steady state. Overall, Model 3 is consistent with the following scenarios of transitional dynamics:

(18) $dx_{1c}/dt = dx_{2c}/dt = dx_{3c}/dt = 0.$

(19) $dx_{1c}/dt < 0 \wedge dx_{2c}/dt = 0 \wedge dx_{3c}/dt > 0.$

(20) $dx_{1c}/dt < 0 \wedge dx_{2c}/dt > 0 \wedge dx_{3c}/dt = 0.$

(21) $dx_{1c}/dt = 0 \wedge dx_{2c}/dt > 0 \wedge dx_{3c}/dt > 0$.

(22) $dx_{1c}/dt < 0 \wedge dx_{2c}/dt > 0 \wedge dx_{3c}/dt > 0$. ($\Rightarrow |dx_{1c}/dt| > |dx_{3c}/dt|$)

(23) $dx_{1c}/dt < 0 \wedge dx_{2c}/dt < 0 \wedge dx_{3c}/dt > 0$. ($\Rightarrow |dx_{1c}/dt| < |dx_{3c}/dt|$)

In other words, Model 3 predicts that at any point of time $t \in [0,\infty)$, one (and only one) of the statements (18)-(23) is true. Otherwise, one of the axioms or laws of Model 3 is violated. Of course, the dynamics over the period $[0,\infty)$ can be a mixture of these archetypes. For example, there may exist a $z_c \in (0,\infty)$ such that $\forall t \in [0,z_c)$ statement (22) is true, at $t=z_c$ statement (19) is true, and for $\forall t \in (z_c,\infty)$ statement (23) is true.

The transitional dynamics predicted by Model 3 cover different structural change models: the Kongsamut et al. (2001) model predicts that (19) is true for all t; the Ngai and Pissarides (2007) model generating hump-shaped manufacturing dynamics (cf. Law 5) predicts dynamics that first follow (22) and then (23).

Properties 1-3 can be used to geometrically summarize all the transitional dynamics scenarios ((18)-(23)) of Model 3: Laws 3 and 4 and Properties 1-3 imply that the angles of the tangential vectors of the trajectory generated by Model 3 are between 0° and 120°, i.e. $\forall t \in [0,\infty)\ 0° \leq \alpha(t) \leq 120°$. For example, scenario (19) can be represented by a linear trajectory that is characterized by an angle $\alpha(t)=60°\ \forall t \in [0,\infty)$ and represents a movement away from the vertex V_1. Obviously, the vector angle condition $\forall t \in [0,\infty)\ 0° \leq \alpha(t) \leq 120°$ implies that Model 3 does not predict any self-intersections (cf. Definition 3). Furthermore, due to Axioms 2 and 3, there is no 'erratic' (exactly speaking, discontinuous or non-smooth) behavior in the long run.

The strength of structural change over the transitional period in Model 3 can be studied as follows. It can be shown that $\mathbf{M}_1{}^3([t,\infty)) \subseteq [0,x_{1c}(t)]$, $\mathbf{M}_2{}^3([t,\infty)) \subseteq [0,(1-x_{3c}(t))]$, and $\mathbf{M}_3{}^3([t,\infty)) \subseteq [x_{3c}(t),1]$ for $t \in [0,\infty)$. The proof of this fact is the same as the proof of $\mathbf{M}_1{}^{2a}([t,\infty))$, $\mathbf{M}_2{}^{2b}([t,\infty))$, and $\mathbf{M}_3{}^{2b}([t,\infty))$ (cf. Section 4.2.2). These facts imply: $|\mathbf{M}_1{}^3([t,\infty))| \leq x_{1c}(t)$, $|\mathbf{M}_2{}^3([t,\infty))| \leq 1-x_{3c}(t)$, and $|\mathbf{M}_3{}^3([t,\infty))| \leq 1-x_{3c}(t)$ for $t \in [0,\infty)$. Thus, the results of Model 3 regarding the potential strength of future fluctuation of the agricultural share (the manufacturing and services shares) are the same as the results of Model 2a (Model 2b).

Finally, we turn to the *set of attraction* (cf. Section 2.4) of Model 3. As implied by Proposition 1 (Proposition 2), the set of attraction of Model 2a (Model 3) is a subset of $\mathbf{S}^{\mathbf{A2a}}$ ($\mathbf{S}^{\mathbf{A3}}$). $\mathbf{S}^{\mathbf{A2a}}$ (cf. Proposition 1) is not larger (i.e. does not cover a larger area of the simplex **S**) than $\mathbf{S}^{\mathbf{A3}}$ (cf. Proposition 2), ceteris paribus. Exactly speaking, Propositions 1 and 2 imply that if $x_{1c}(0)$ is assumed to be equal in Models 2a and 3, then $\mathbf{S}^{\mathbf{A3}}$ is not larger (and can be smaller) than $\mathbf{S}^{\mathbf{A2a}}$. That is, Model 3 allows for an exacter specification of the set of attraction. This is not surprising, since the set of restrictions/laws imposed on the dynamics by Model 3 is greater than the set of restrictions/laws imposed on the dynamics by Model 2a.

Application of Model 3

Model 3 implies that the structural change in *developed economies* (belonging to the group **C**) is close to the end, i.e. developed economies will not experience significant (long-run) labor re-allocation in future. The reason for this fact is that in the highly developed countries (e.g. in the USA), the services (agricultural) employment share has already reached a very high (low) level and, therefore, cannot grow (decrease) much anymore, where Law 4 (Law 3) prohibits a decrease (an increase) in the services (agricultural) share. In other words, according to Law 4 (Law 3), the services (agricultural) share must grow (decrease) or be constant; it cannot grow (decrease) significantly, since it is restricted by its upper (lower) limit 1 (0) (cf. (1)); thus, it must be approximately constant. Due to Axiom 1 (and, in particular, (1)), the manufacturing employment share cannot change significantly if the agricultural or services share does not change significantly. Overall, Model 3 does not allow for (significant) structural change in highly developed economies.

Moreover, Model 3 implies that the *developing countries* (belonging to the group **C**) may experience structural change or not. Particularly, scenario (18) represents a stagnating labor allocation (for all future time points). Even if the economy develops, it need not becoming a services economy in future (but may remain an agricultural or become a manufacturing economy); that is, Model 3 is not as optimistic as Model 1 is. If the economy develops, its long-run dynamics are relatively smooth and monotonous: the employment share of services grows and/or the agricultural share shrinks; the manufacturing employment share may exhibit any sort of (smooth and) monotonous or non-monotonous dynamics (e.g. the ‘hump-shaped’ dynamics described in Law/Regularity 5 or transitory cyclical dynamics). Nevertheless, Model 3 predicts that structural change is transitory; thus, according to Model 3, the labor allocation in developing countries converges to a fixed labor allocation (‘steady state’).

Since we have shown that Model 3 can generate the hump-shaped dynamics of the manufacturing sector postulated in Law 5, we do not dedicate a model to Law 5, but go on with Law 6 in Section 4.2.4.

4.2.4 Model 4 – the Implications of Laws 1, 2a, and 6

We present now a model of non-self-intersecting trajectories. As discussed in Section 3.2.6, non-self-intersection is a generalization of the concept of monotonicity (cf. Laws 3 and 4). Thus, in many ways, Model 4 is a generalization of Models 2 and 3.

Assumptions of Model 4

Assume that country c belongs to the group **C** and satisfies Axioms 1 and 2 and Laws 1, 2a, and 6. We are interested in predicting the future dynamics of country c (cf. Definition 9 and Axiom 1).

Predictions of Model 4

If country c is relatively underdeveloped at the present (cf. Definition 9), i.e. if (14) is true, the following predictions (of the dynamics for $t > 0$) can be made based on Model 4.

Law 1 and (14) imply that at the present (cf. Definition 9), country c is in the early development phase $(d^1,a_c]$, i.e. $a_c \leq 0$. Thus, per Law 2 there exists a future time point (cf. Definition 9) $b_c > 0$ that is characterized by $x_{3c}(b_c) > 0.5$. In other words, (14) and Laws 1 and 2a imply that country c will become a services economy in future. Furthermore, the *transitional dynamics* (cf. Section 2.4) over the time period $[0,b_c]$ can be described by a non-self-intersecting trajectory (cf. Definition 3 and Law 6). In general, this does not mean much, since such a trajectory can represent very different types of dynamics. However, interesting statements can be made about the *transitional dynamics* after b_c, i.e. after the country has become a services economy. These dynamics and their application for predicting the future dynamics of developed economies are discussed by Stijepic (2015) in detail.

Regarding the *limit dynamics* (cf. Section 2.4), we can say that neither a fixed point nor a limit cycle is ruled out by the assumptions of Model 4. A limit cycle does not represent a self-intersection and, thus, does not violate Law 6, since in the case of a limit cycle, the trajectory only converges to the image of a Jordan curve and never becomes the image of a Jordan curve. While an omega limit set consisting of a fixed point means that structural change comes to a halt (in the limit), a limit cycle means that labor allocation dynamics are cyclical in the limit, where the employment shares of all sectors (i=1,2,3) may behave cyclically in the limit (cf. Figure 18). To be able to make more specific statements about the limit dynamics based on mathematical theorems (such as the Poincaré-Bendixson theory), we need to make further assumptions, e.g. assumptions regarding the differentiability/smoothness of the dynamic system describing structural change. The model of Section 4.2.5 is an example of how this can be done.

Application of Model 4

Stijepic (2015) applies Model 4 for predicting the future labor allocation dynamics of *developed economies*. For predicting the future labor allocation dynamics of *developing economies* by using Model 4, we must distinguish between two phases of their development: the phase before b_c and the phase after b_c (cf. Law 2a). The Model 4 predictions regarding the labor allocation dynamics in developing economies until they become services economies (i.e. *until* b_c) are the same as the corresponding predictions of Model 1a (cf. Section 4.2.1). The results derived by Stijepic (2015) can be used for predicting the dynamics *after* b_c.

4.2.5 Model 5 – the Implications of Law 7

While Laws 1-6 refer to only one country, Law 7 refers to the trajectories of at least two countries. In general, the labor allocation trajectories differ across countries (cf. Section 3), i.e. there are cross-country differences in labor allocation dynamics. Moreover, there are different ways to model cross-country differences regarding trajectories. An overview is provided by Stijepic (2016). We choose the way described by Axiom 4.

Axiom 4. a) *The labor allocation dynamics of different countries are modeled by one and the same model.* ***b)*** *The model (i.e. the dynamic system) generates different trajectories each corresponding to a different initial state of the system.* ***c)*** *Each trajectory corresponds to a different country, i.e. countries differ by initial states.*

Alternative ways to model cross-country trajectory differences are, e.g., assuming that each country is described by a different model, or assuming that the parameters of the model differ across countries. We do not discuss all these ways and, instead, refer to Stijepic (2016) for a detailed discussion, since we are restricted in space and since the discussion of only one of these ways is sufficient for demonstrating the aims and capabilities of the positivistic approach.

As we will see in this section, the limit dynamics of dynamic systems that are, among others, characterized by mutually *non*-intersecting trajectories are relatively easy predictable by applying the Poincaré-Bendixson theory. However, Law 7 states that the labor allocation trajectories of different countries mutually intersect and, thus, at first sight, seem to be hard predictable, or at least, not predictable by using the Poincaré-Bendixson theory. In this sense, Law 7 is an anti-law: it does not help us to reduce the set of possible prediction scenarios, as the other laws do, but implies that, potentially, structural change is hard to predict. However, as discussed by Stijepic (2016), the fact that mutual intersection of countries' trajectories (i.e. Law 7) is observable does not mean that the long-run dynamics of these countries cannot be modeled by using dynamic systems that are predictable by the Poincaré-Bendixson theory, if the observable mutual intersections of countries' trajectories are interpreted as the result of cross-country parameter differences or (short-run) parameter perturbations. Moreover, there are theories/models (e.g. the Kongsamut et al. (2001) model and the Ngai and Pissarides (2007) model) that generate mutually non-intersecting labor allocation trajectories in their dynamic equilibriums[16], which represent the long-run dynamics (cf. Stijepic (2016)); these models can be regarded as 'theoretical' arguments for modeling (long-run) structural change in a framework of mutually non-intersecting trajectories.

Overall, we have two sides of arguments. On the one hand, we have the empirical evidence (i.e. Law 7) that states that mutual intersection of countries' trajectories is common (side A). On the other hand, we have the mathematical arguments (discussed by Stijepic (2016)) and the models/theories (i.e., e.g., the Kongsamut et al. (2001) model and the Ngai and Pissarides (2007) model) that imply that long-run labor allocation dynamics may be representable by mutually non-intersecting trajectory families (side B). Within our positivistic modeling approach, we cannot decide which side is right. (Not to mention that it seems that this decision requires significant research efforts, which are not representable within only one paper.) Moreover, Models 1-4 are consistent with mutually-intersecting trajectories and, therefore, pay tribute to side A. In contrast, we have not provided a (positivistic) model representing side B, i.e. our model set seems to be ideologically biased towards side A. Last not least, a model of non-intersecting trajectories allows for the application of the Poincaré-Bendixson theory and, thus, demonstrates the 'technical' potential of our positivistic/axiomatic/geometric approach.

For these reasons, we present in this section a model of mutually non-intersecting trajectories and, thus, assume the following ideological interpretation of Law 7, where we seek to predict the dynamics of country c:

Axiom 5. *$c \in \boldsymbol{H} \subset \boldsymbol{C}$. $\forall (o,p) \in \boldsymbol{H}^2$: $\boldsymbol{T}_o([b_o,d^u)) \cap \boldsymbol{T}_p([b_p,d^u)) \neq \emptyset$ (cf. Definition 4 and Law 7).*

[16] Kongsamut et al. (2001) (Ngai and Pissarides (2007)) name the dynamic equilibrium of their model 'generalized balanced growth path' ('aggregate balanced growth path').

Axiom 5 states that country c belongs to the country group **H** and that the group **H** can be modeled by a family of mutually non-intersecting trajectories.

At first sight, Axiom 5 seems not to be ideological, since it is always possible to find a set of countries (**H**) characterized by mutually non-intersecting trajectories and to state that only these countries are analyzed within our framework. However, such a selective choice of the country set to which the model applies is ideological, since it has impacts on the results and is not justified by empirical evidence.

Assumptions of Model 5

We summarize this discussion as follows: we assume that Axioms 1-5 and Laws 6 and 7 are satisfied; we are interested in predicting the future dynamics of country c (cf. Definition 9 and Axiom 1).

Predictions of Model 5

The assumptions of Model 5 imply that country c's dynamics can be represented by a trajectory that belongs to a family of trajectories (**H**) that has the following characteristics:

(I) all trajectories belonging to the family **H** are non-self-intersecting (cf. Definition 3 and Law 6),

(II) all trajectories belonging to the family **H** are mutually non-intersecting (cf. Definition 4 and Axiom 5),

(III) all trajectories belonging to the family **H** are continuous (cf. Definition 2 and Axiom 2),

(IV) all (functions $X_c(t)$ associated with the) trajectories belonging to the family **H** are differentiable with respect to time (cf. Axiom 3),

(V) all trajectories belonging to the family **H** are bounded (since they are located in a bounded subset (**S**) of the plane (cf. Axiom 1 and (4)), i.e. **S** is their domain).

First, we are interested in the *limit dynamics* (cf. Section 2.4) of the trajectories belonging to the trajectory the family **H** (which is the trajectory family generated by Model 5). In this way, we derive the set of all possible limit dynamics of the trajectory of country c (which belongs to the trajectory family of Model 5; cf. Axiom 5). The characteristics (I)-(V) imply that Model 5 satisfies almost all the requirements that are necessary to apply the Poincaré-Bendixson theory for deriving the limit dynamics.[17] Since (a) Model 5 is only an example among different positivistic models, (b) we are critical about its consistency with the stylized facts (cf. characteristic (II) and Law 7), (c) our space is restricted, and (d) the only aim of this section is to demonstrate the methods applicable within our positivistic/geometrical/axiomatic approach, we shorten the mathematical discussion significantly by assuming that the dynamic system generated by Model 5 has all the characteristics that are necessary to apply the Poincaré-Bendixson theory to it. Appendix B discusses briefly (a) the differences between the characteristics of Model 5 and the typical textbook Poincaré-Bendixson theory requirements and (b) the methodological and mathematical arguments that can be used to complete Model 5 such that the Poincaré-Bendixson theory is applicable to it. The full development of such arguments and the exact mathematical proofs seem to be quite lengthy yet interesting research topics (if further research shows that Axiom 5 and, thus, Model 5 are empirically relevant interpretations of Law 7).

If all the requirements of the Poincaré-Bendixson theory are satisfied (e.g. if the dynamic system generating the trajectory $\mathbf{T}_c([0,\infty))$ is representable by a smooth autonomous planar differential equation system, as discussed in Appendix B), one of the following statements is true:

(i) $\mathbf{O}(\mathbf{T}_c([0,\infty)))$ is a fixed point (critical point).

(ii) $\mathbf{O}(\mathbf{T}_c([0,\infty)))$ is (the image of) a Jordan curve.

(iii) $\mathbf{O}(\mathbf{T}_c([0,\infty)))$ is a homoclinic orbit (including its fixed point).

(iv) $\mathbf{O}(\mathbf{T}_c([0,\infty)))$ is a union of at least two fixed points and the trajectories connecting them ('heteroclinic union').

Note that the term 'heteroclinic union' is not common in the literature; we use it here as an abbreviation. Furthermore, note that a 'heteroclinic union' must contain heteroclinic trajectories and can contain homoclinic trajectories. For detailed proofs and extensive discussion of the Poincaré-Bendixson theory, see the references in Footnote 17.

In case (i), the labor allocation in economy c converges along the trajectory $\mathbf{T}_c$ to a fixed point (X_c^*) for $t\rightarrow\infty$, i.e. $\mathbf{O}(\mathbf{T}_c([0,\infty)))=X_c^*$. Thus, structural change is transitory.

Case (ii) is known from the Poincaré-Bendixson *theorem*. See, e.g., Miller and Michel (2007), p.290ff, or Hale (2009), p.51ff. $\mathbf{O}(\mathbf{T}_c([0,\infty)))$ is (the image of) a Jordan curve in case (ii). Thus, $\mathbf{T}_c$ is either (a) a closed trajectory (i.e. the image of a Jordan curve) or (b) a non-closed

[17] For a discussion of these requirements and of the Poincaré-Bendixson theory, in general, see, e.g., Andronov et al. (1987), p.351ff, Guckenheimer and Holmes (1990), p.45, Hale (2009), p.55 (and, in particular, Theorem 1.5), and Teschl (2011), Chapter 7.3 (and, in particular, Theorem 7.16).

trajectory converging to a closed trajectory (i.e. the labor allocation in economy c converges to a limit cycle). (a) is excluded by characteristic (I) (cf. Definition 3 and Law 6). Thus, in case (ii), structural change is (only) cyclical in the limit.

In cases (iii) and (iv), for $t \rightarrow \infty$, the labor allocation in economy c converges along the trajectory $\mathbf{T}_c$ to (all) the points of the homoclinic orbit or to (all) the points of the 'heteroclinic union', per definition of the term omega limit set (cf. Definition 7). Therefore, structural change is cyclical in the limit (cf. Figures 21 and 22). As we can see in the examples depicted in Figures 21 and 22, the cyclicality of structural change means here that the employment shares of all three sectors behave cyclically, in contrast to Model 2a and Figure 20, where only the manufacturing and services sectors are cyclical in the limit but not the agricultural sector. Overall, Model 5 allows for transitory and cyclical limit dynamics.

Figure 21. *Example: $\boldsymbol{O}(\boldsymbol{T}_c([0,\infty)))$ is a homoclinic orbit.*

- insert Figure 20 here -

Figure 22. *Example: $\boldsymbol{O}(\boldsymbol{T}_c([0,\infty)))$ is a 'heteroclinic union'.*

- insert Figure 21 here -

We cannot say much about the *set of attraction* and the *transitional dynamics* (cf. Section 2.4) of the trajectories generated by Model 5: the set of attraction of Model 5 is (some subset of) **S**; the transitional dynamics are relatively 'smooth' due to Axioms 2 and 3; the trajectory is non-self-intersecting, which can restrict the number of transitional scenarios significantly, as discussed in Section 4.2.4. Of course, we could add further laws to the assumption set of Model 5, e.g. Law 2a, such the transitional dynamics become more predictable (cf. Section 4.2.4). However, then, the discussion would be very similar to the discussion of the sets of attraction and the transitional dynamics of the other models discussed in Section 4.2. Therefore, we omit it here.

Application of Model 5

In comparison to most of our other models, Model 5 allows us to specify more precisely the limit dynamic of the economy. Its results are applicable for predicting the future dynamics of *developed and developing economies*. Simply stating, Model 5 predicts that structural change comes to a halt or is cyclical in the limit. Thus, in contrast to the standard structural change literature (cf. Section 4.1.4), Model 5 allows for long-run cyclical behavior of all sectors (or of only two sectors). This aspect is particularly interesting for the prediction of structural change dynamics in present-days developed economies, which have already converged close to the vertex V_3, which is a point in the frontier of the labor allocation domain **S**. In other words, the (highly) developed economies cannot significantly increase their services shares, and the question arises, whether their structural change is now close to the end, i.e. whether they have already converged close to their steady state labor allocation. Model 5 states that structural change need not coming to an end but may continue in cyclical fashion (e.g., the trend of the labor allocation dynamics may be reversed over the next years).

5. CONCLUDING REMARKS

Discussion of the method

Standard/quantitative approaches for prediction of economic dynamics heavily rely on: (a) theoretical information, which is ideological for the greatest part, as in the case of predictions based on theoretical models; (b) complex quantitative empirical patterns/relationships, which are difficult to interpret intuitively, as in the case of, e.g., vector auto-regressions or non-linear regressions; (c) oversimplifying (e.g. linear) estimation equations, which are ideological, yet often loosely related to theoretical arguments, as in the case of linear regression; or (d) in general, quantitative statements that are often restricted in validity to relatively small country groups. In contrast, a great deal of economic knowledge ('economic laws') is rather of qualitative or non-linear nature. In particular, many economic phenomena seem to follow qualitative economic laws that are relatively robust in the sense that they are persistent across time and space. This is particularly true for many topics associated with long-run dynamics and, in particular, long-run labor allocation dynamics. Thus, the idea of our paper is to try to (a) use only such robust (qualitative) information for predicting labor allocation dynamics and (b) reduce the extent of ideological information used, which seems to be a valuable directive (cf. Section 1). Of course, in economics, it is not possible to make predictions without relying on ideological information and to find laws that are true for *all* countries and for *all* time periods. Nevertheless, the reader may agree that there are 'more' ideological statements and 'less' ideological statements as well as more reliable regularities and less reliable regularities. In Section 4.2, we pay tribute to this fact by suggesting not only one model but a set of models, where the models differ by the number of axioms (which represent merely ideological information) and the sets of laws (which represent the empirical information and differ by 'reliability') that they assume to be true.

Mathematics provides us with many tools and concepts (e.g. set theory and predicate logic) that can be used to derive statements/predictions on the basis of qualitative information (on empirical regularities). For using these concepts, we must translate the observed regularities/laws, which are verbal statements that refer to the labor allocation dynamics, into geometrical and topological notions by using the concepts of the trajectory and its domain. Then, we can use logic and set theory to perform logical operations on these transformed statements and, thus, derive implications, which can be interpreted as predictions of future labor allocation dynamics. In this sense, the predictions made in Section 4.2 are logical implications of the regularities/laws observed. Each of our models focuses on one or two regularities and, thus, derives more or less the direct implications of each of the regularities. Thus, the readers of this paper, who have their own opinion on the reliability/validity of the different regularities/laws discussed in Section 3, can use this paper to identify the direct implications of their preferred regularities/laws for future dynamics in developed and developing economies. Of course, these implications are based on ideological information (cf. Axioms 1-5). However, we tried to minimize the use of this type of information (e.g. the predictions of Model 1 do not depend on Axioms 2-5) and formulated the axioms such that they do not differ from the ideological assumptions of standard structural change, growth, and, in general, long-run dynamic models. Thus, our models seem to be less ideological or at least not more ideological than the standard (empirical and theoretical) dynamic models.

Summary of the predictions

In general, we have shown in our paper that simple statements (such as ‘the services employment share increases monotonously over time’) can have interesting implications in the three-sector framework, which can be used for prediction of future structural change if they are regarded as economic laws. In particular, we can specify the type of transitional and limit dynamics (e.g. steady state, limit cycle, or chaotic dynamics), the potential strength of structural change over the transitional period and in the limit, and the location of the economy in its dynamic equilibrium (i.e. the set of attraction).

Moreover, we have shown that apparently very similar statements/laws can have very different implications. For example, as shown in Section 4.2.2, the statement ‘the agricultural share decreases monotonously over time’ (cf. Law 3) implies that highly developed countries may experience very strong structural change in future, while the statement ‘the services share grows monotonously over time’ (cf. Law 4) implies that the highly developed economies will not experience any significant structural change in future. In general, our models generate very different predictions of structural change, as discussed in the following.

Our results regarding the *strength of future structural change in present-days developed economies* cover a wide range of predictions: while Models 2b and 3 predict that developed economies will not experience significant structural change in future, Model 1b predicts that the potential for future labor re-allocation/fluctuation in developed economies is comparable to the cumulative amount of labor re-allocated in these countries over the last 150 years; the remaining models allow for much stronger future structural change in developed economies.

Moreover, the *type of predicted structural change* differs significantly across models. For example, Model 3 predicts that structural change comes to a halt in the limit, i.e. structural change is transitory; Model 2a (Model 5) allows, additionally, for cyclical limit-dynamics of the manufacturing and services sectors (of all sectors); and in Model 1a, irregular dynamics (erratic dynamics or chaos) may arise.

In general, we show that the empirical regularities ('stylized facts') of structural change do not necessarily imply that the structural change in *present-days developed economies* is near to its end: some of our models imply that the developed economies may re-industrialize significantly or may be characterized by limit fluctuations of sectoral employment shares in future.

Our predictions of structural change in *present-days developing economies* range from pessimistic to optimistic predictions stating that the present-days developing economies (a) may never become services economies, (b) may not sustain their development (i.e. become agricultural economies again), or (c) develop as the present-days developed economies.

Topics for further research

Our discussion implies a lot of topics for further research, as discussed in the following.

The five models of labor allocation dynamics presented in Section 4.2 are only examples of models that can be formulated on the basis of Axioms 1-5 and Laws 1-7; they are aimed to demonstrate some major implications of each of the laws and the range of the mathematical methods that applicable when the positivistic approach of modeling structural change is taken. Further research could study other combinations of Axioms 1-5 and Laws 1-7 and their implications. Of course, alternative laws and axioms could be formulated (referring to long-run labor allocation dynamics) and models could be based on them.

The (limit-)fluctuations in the employment shares seem to be an interesting topic. As shown in Section 3, the employment shares fluctuate in the short run. Although our paper does not focus on explaining short-run employment share dynamics, we have shown that (a) some of our models allow for such fluctuations over the transitional period and in the limit, (b) some of our models allow only for transitional fluctuations, and (c) Model 3 does not allow for any fluctuations. Further research could focus on these aspects.

While our paper focuses on labor allocation dynamics, other types of structural change could be studied by using the method and the techniques discussed in our paper. For some examples of the topics that are covered by our method, see Stijepic (2016).

Last not least, our discussion of (a) Model 5, (b) the interpretation of Law 7, and (c) the applicability of the Poincaré-Bendixson theory in Model 5 (cf. Section 4.2.5 and Appendix B) implies many interesting (yet lengthy) empirical and methodological research topics.

These topics are left for further research.

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APPENDIX A. Countries depicted in Figures 14 and 15.

Albania, Algeria, American Samoa, Antigua and Barbuda, Argentina, Armenia, Aruba, Azerbaijan, Bahamas, Bahrain, Bangladesh, Barbados, Belarus, Belize, Benin, Bermuda, Bhutan, Bolivia, Botswana, Brazil, British Virgin Islands, Brunei, Bulgaria, Burkina Faso, Cambodia, Cameroon, Cayman Islands, China, Colombia, Costa Rica, Croatia, Cuba, Cyprus, Dominica, Dominican Republic, Ecuador, Egypt , El Salvador, Estonia, Ethiopia, Fiji, French

Polynesia, Gabon, Gambia, Georgia, Ghana, Grenada, Guam, Guatemala, Guinea, Guyana, Haiti, Honduras, Hong Kong, India, Indonesia, Iran, Iraq, Ireland, Isle of Man, Jamaica, Jordan, Kazakhstan, Kiribati, Kuwait, Kyrgyz Republic, Laos, Latvia, Lesotho, Liberia, Libya, Lithuania, Macao, Macedonia, Madagascar, Malaysia, Maldives, Mali, Malta, Marshall Islands, Mauritius, Moldova, Mongolia, Montenegro, Morocco, Myanmar, Namibia, Nepal, New Caledonia, Nicaragua, Nigeria, Northern Mariana Islands, Oman, Pakistan, Palau, Panama, Paraguay, Peru, Philippines, Puerto Rico, Qatar, Romania, Russian Federation, Rwanda, Samoa, San Marino, Sao Tome and Principe, Saudi Arabia, Senegal, Serbia, Sierra Leone, Singapore, South Africa, Sri Lanka, St. Kitts and Nevis, St. Lucia, St. Vincent and the Grenadines, Suriname, Syrian Arabic Republic, Tajikistan, Tanzania, Thailand, Timor Lest, Tonga, Trinidad and Tobago, Tunisia, Turks and Caicos Islands, Uganda, Ukraine, United Arab Emirates, Uruguay, Uzbekistan, Venezuela, Venezuela, Vietnam, West Bank and Gaza, Yemen, Zambia, Zimbabwe.

APPENDIX B. On the Poincaré-Bendixson Theory and Model 5.

An example of arguments that can be used to show that the Poincaré-Bendixson theory is applicable to Model 5 can be based on the fact that the limit dynamics of unique solutions of autonomous differential equations systems are predictable by using the Poincaré-Bendixson theory and the fact that Model 5 is representable by such systems if certain assumptions regarding the properties of the economic laws it represents are made. In detail, these arguments are as follows.

Obviously, the characteristics (I)-(V) discussed in Section 4.2.5 imply that the family of trajectories generated by Model 5 is representable by a planar system of autonomous differential equations and, in particular, corresponds to a subset of (bounded) trajectories representing the unique solutions of a (sufficiently) smooth [18] autonomous differential equation system in the plane. [19] The Poincaré-Bendixson theory applies to the bounded trajectories generated by the latter.[20] However, the family of trajectories generated by Model 5 consists of a countable number of trajectories (i.e. the trajectories are indexed by the set **H**, which is a subset of natural numbers), while the trajectory family representing the unique

[18] Different conditions can be imposed on autonomous differential equation systems such that they generate unique solutions. See Stijepic (2015), p.84f, for some examples of such conditions. In general, these conditions are related to some sort of smoothness of the differential equation system.

[19] For an explanation of the properties of the families of trajectories generated by the unique solutions of smooth autonomous differential equation systems in the plane, see, e.g., Walter (1998), p.110f, Hale (2009), p.18f. and p.38f, Stijepic (2015), p.84f, and Stijepic (2016), p.22.

[20] For a discussion of the Poincaré-Bendixson theory, see, e.g., the references listed in Footnote 17.

solutions of a planar autonomous differential equation system is a simple covering of (a subset of) the plane (cf. Walter (1998), p.10f. and p.36), i.e. encompasses an uncountable number of trajectories (corresponding to different initial states of the phase space, which is a subset of the two-dimensional real space). The latter fact follows from the continuous dependence of solutions upon initial states/conditions (cf., e.g., Walter (1998), p.108, and Andronov et al. (1987), p.796.). For applying the Poincaré-Bendixson theory to Model 5, we need to provide methodological/theoretical arguments that Model 5 is extendible to a (connected) real subset of **S** containing **H**, or, in other words, that Model 5 generates a family of trajectories that is a simple covering of a subset of **S** containing **H**.

In general, it makes sense to assume that Model 5 generates a family of trajectories that is a (simple) covering of its domain (where the domain of Model 5 is a subset of **S**). The index c does not only represent the countries but also the initial states (on **S**) covered by Model 5 (cf. Axioms 4 and 5). If Model 5 represents economic laws (which are regularities that are persistent across space and, thus, across initial conditions), it should not only be valid for the countable set (**H**) of initial conditions but also for some marginal deviations from this set.[21] This argument could be used to extend Model 5 to a (connected) subset of **S** (containing the set **H**).

We leave the elaboration of this argument for further research for the reasons discussed in Section 4.2.5.

[21] In other words, a model that can explain the dynamics of country c if the initial agricultural share in country c is, e.g., equal to 0.78, but not if it is equal to 0.781, is a knife-edge model. In general, knife-edge models are criticized (see, e.g., Temple (2003)).

FIGURES

Figure 1

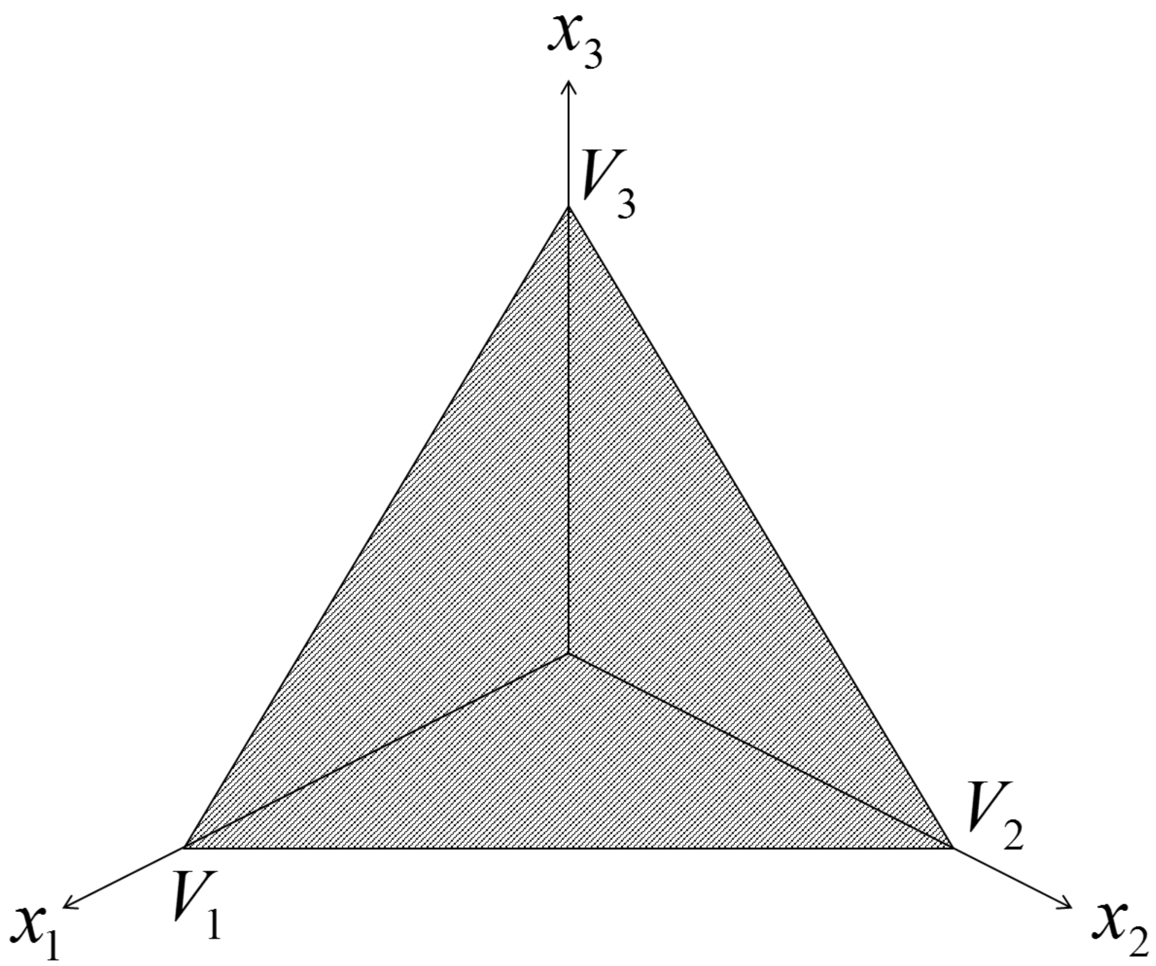


Figure 2

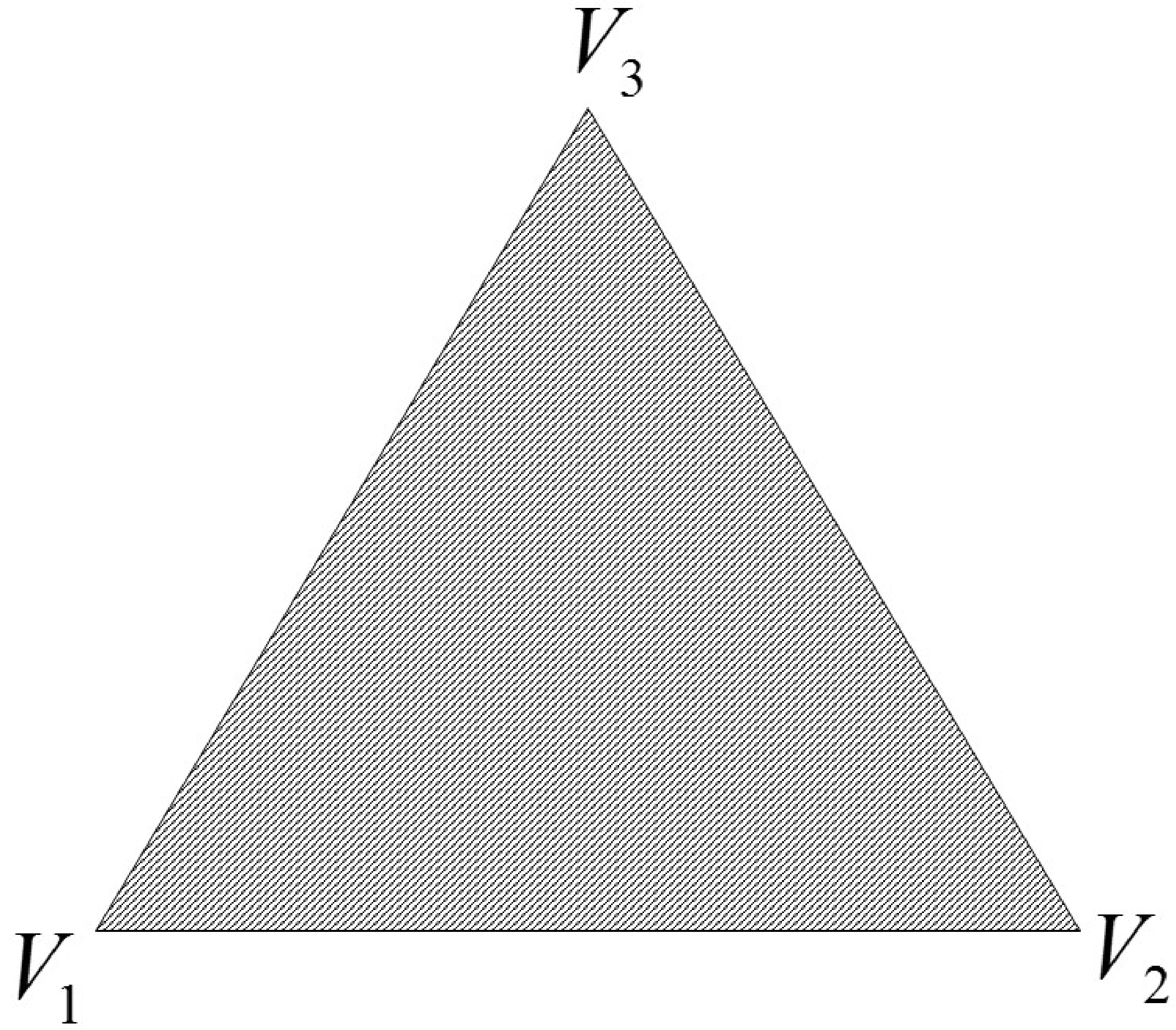


Figure 3

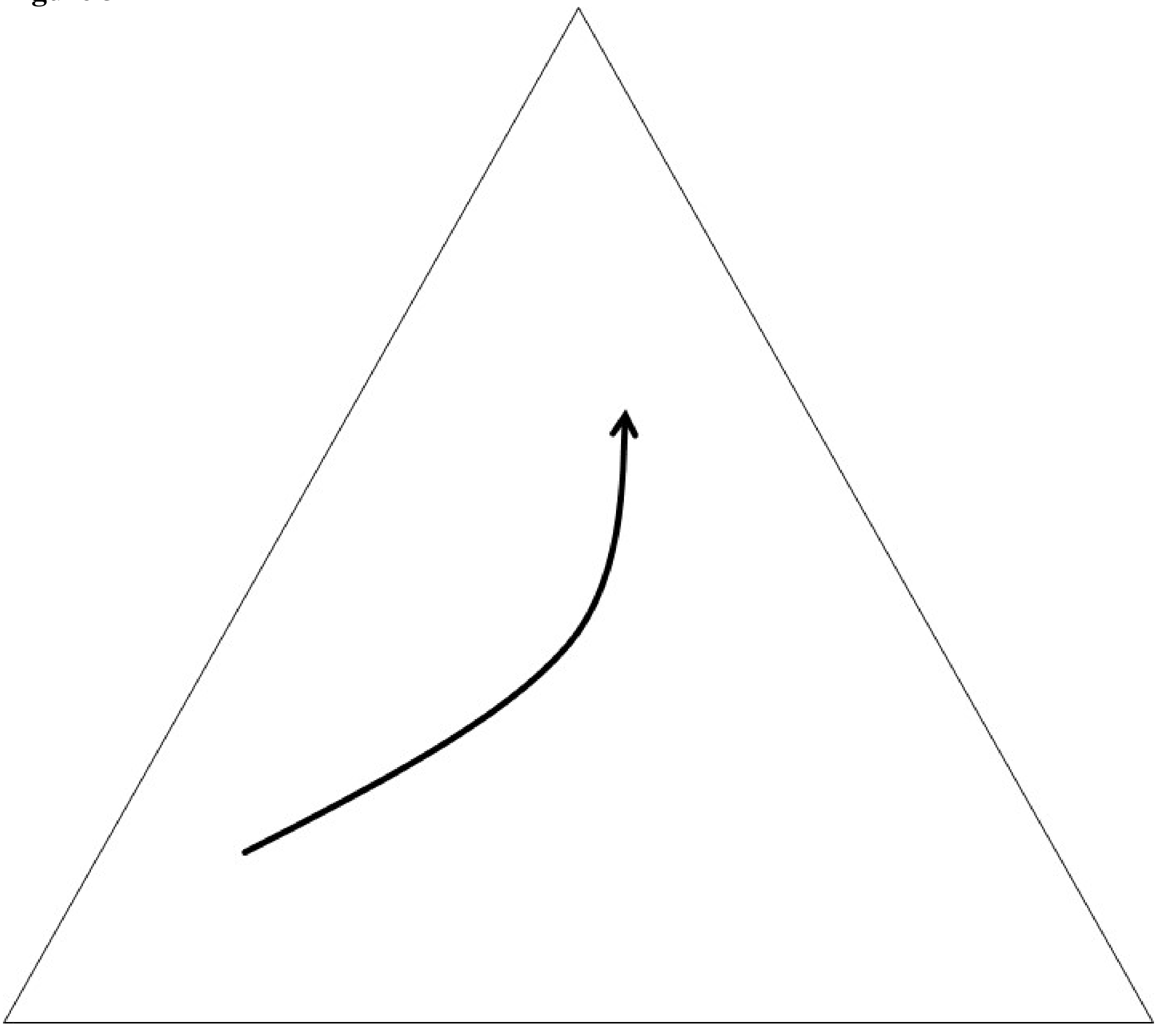


Figure 4

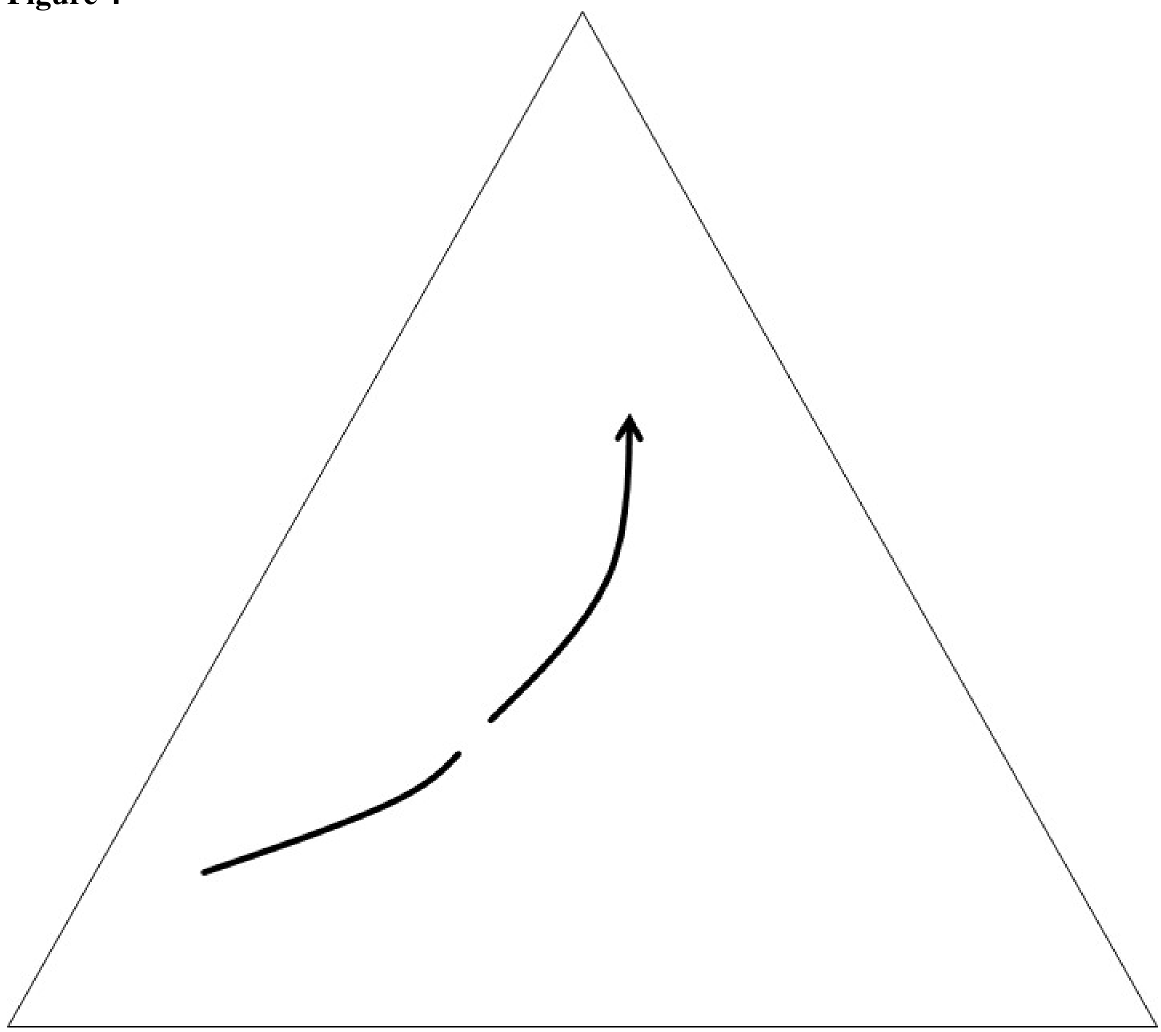
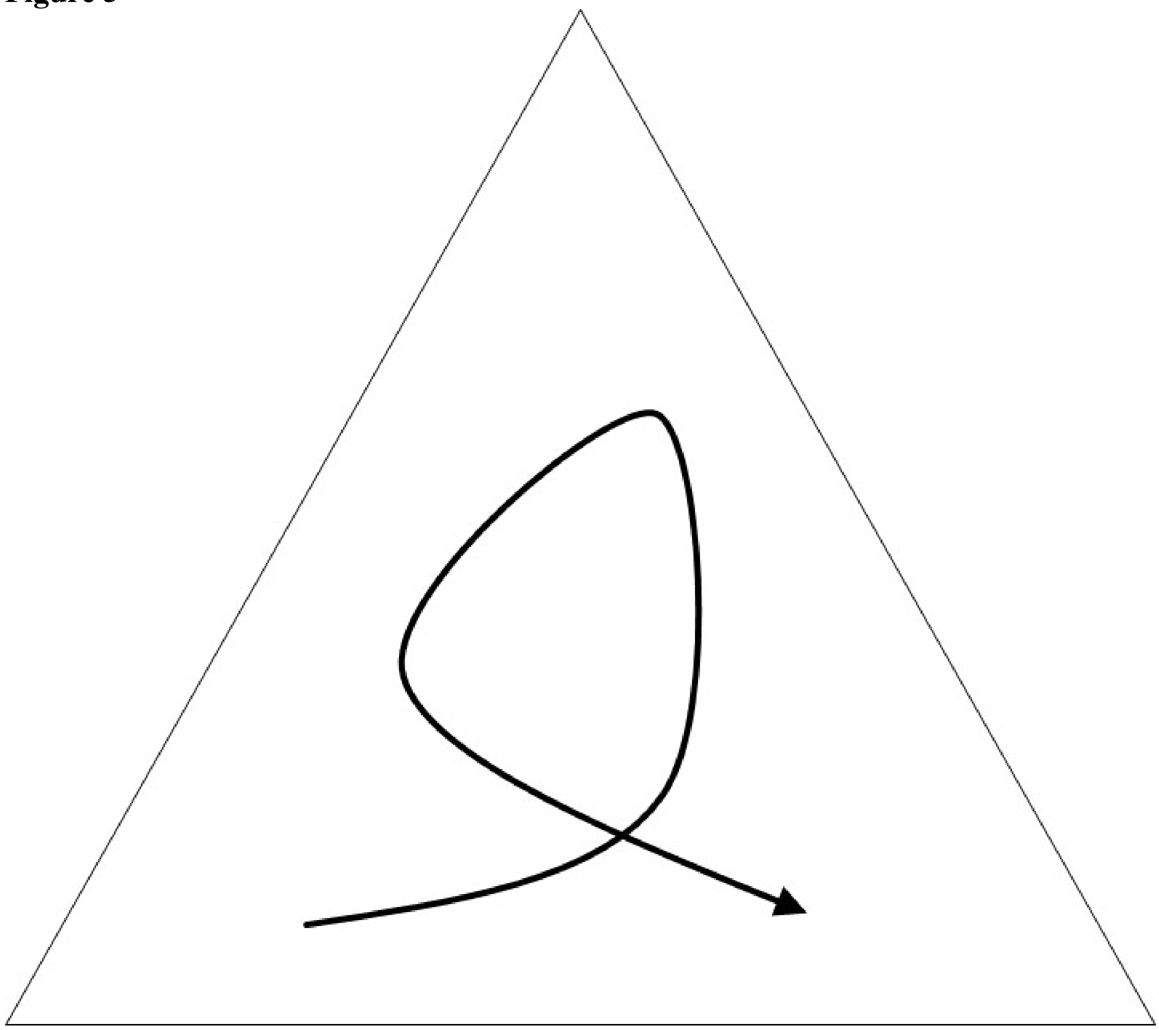


Figure 5



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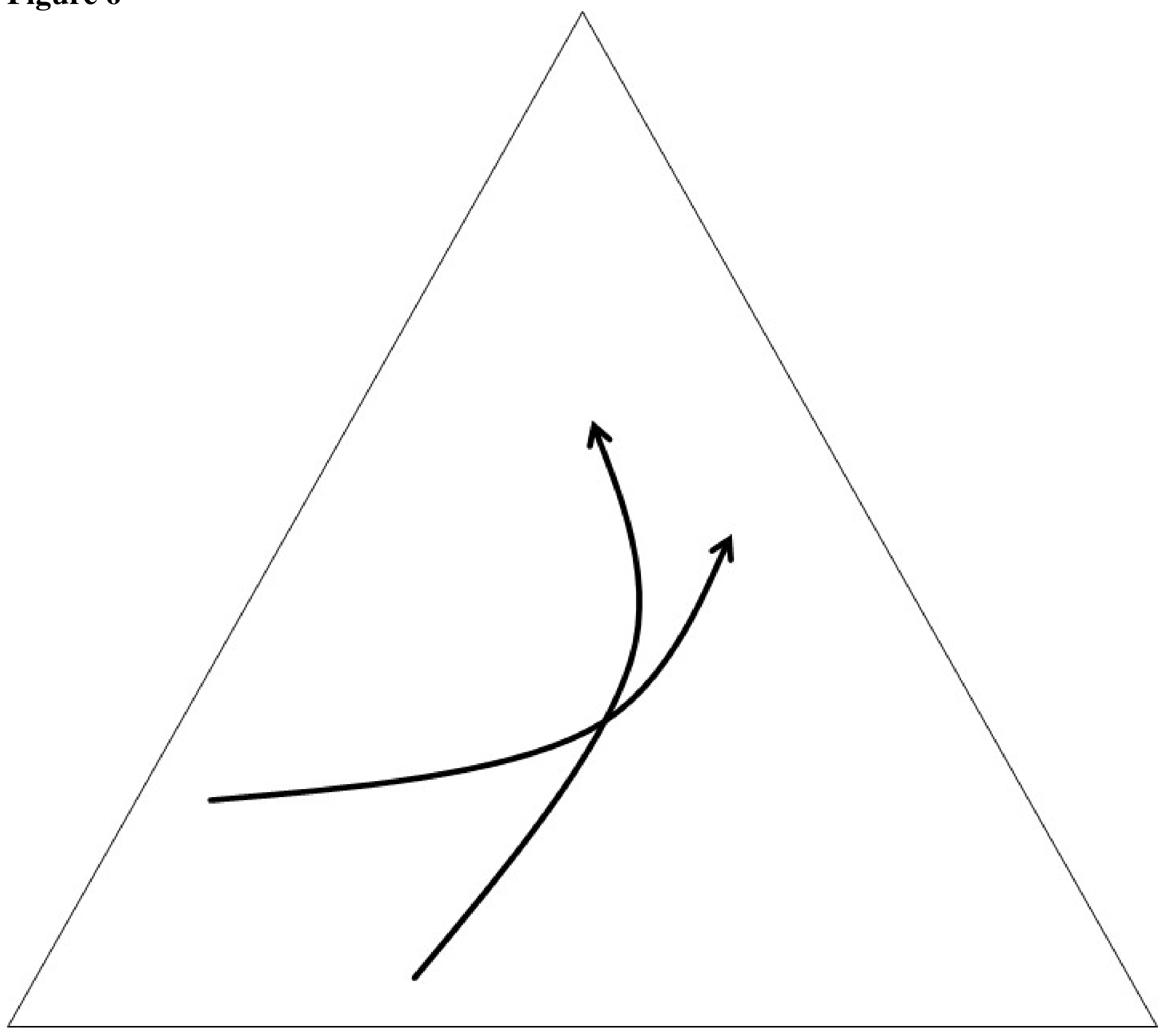


Figure 7

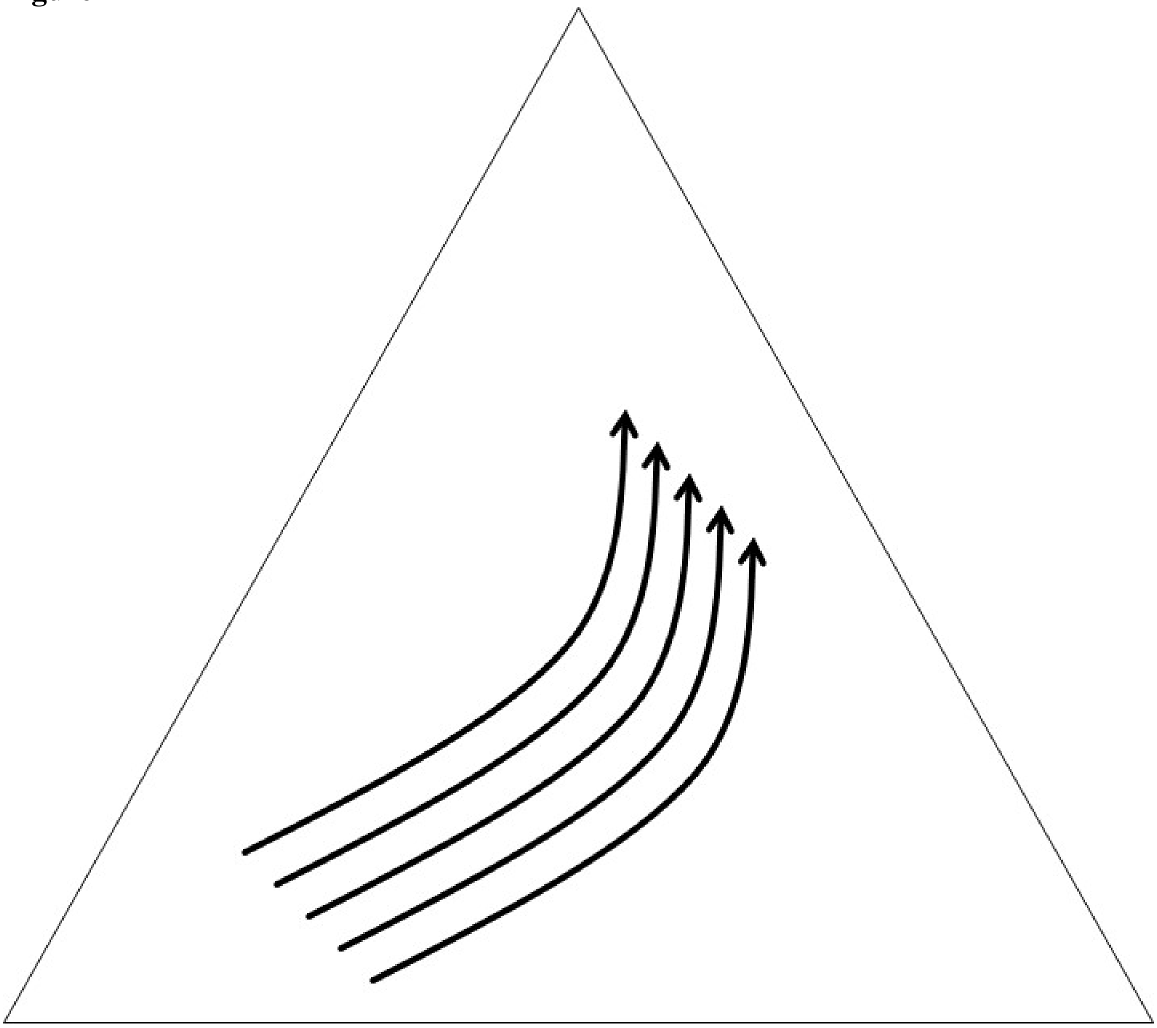


Figure 8

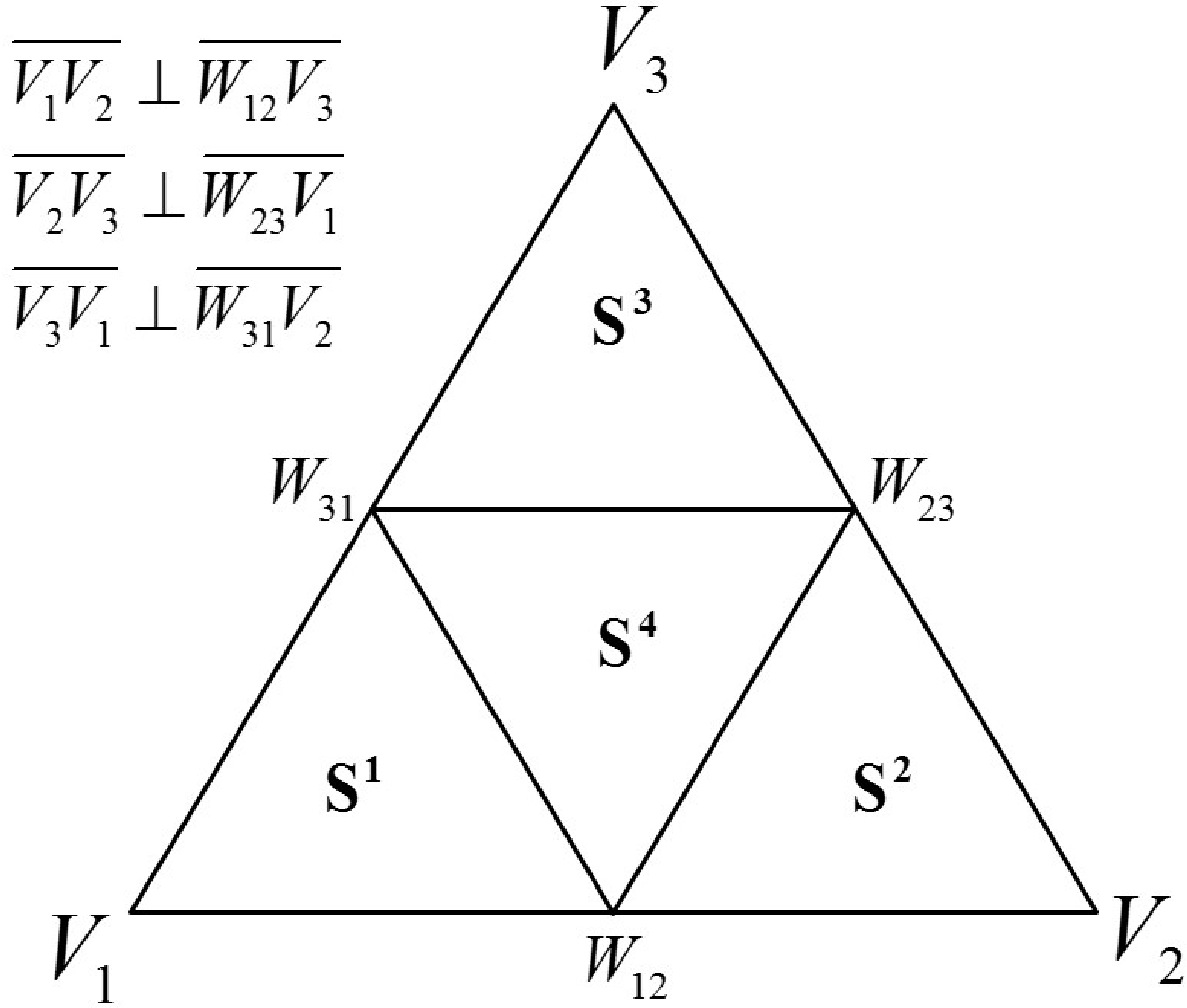
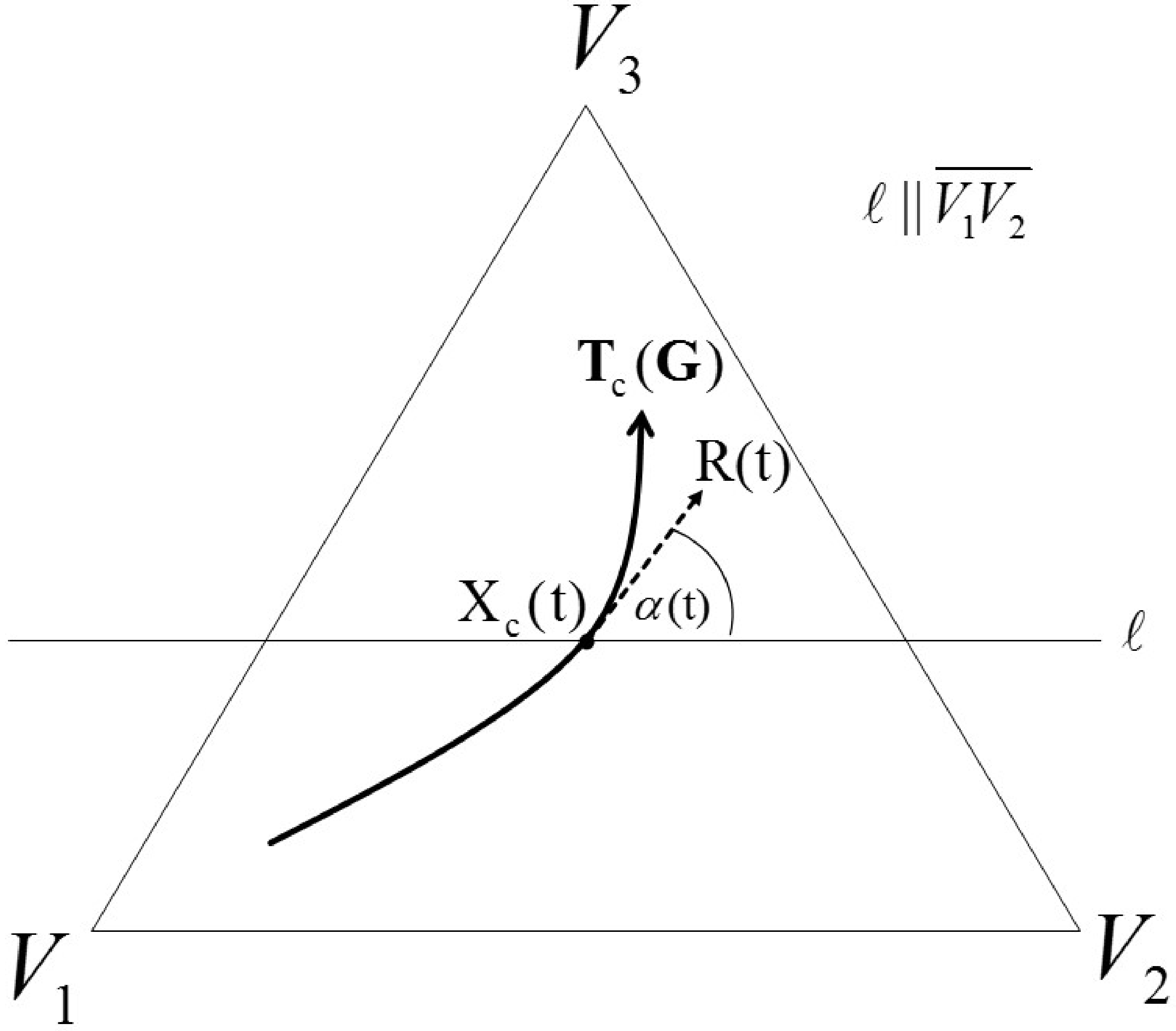

$\overline{V_1V_2} \perp \overline{W_{12}V_3}$
$\overline{V_2V_3} \perp \overline{W_{23}V_1}$
$\overline{V_3V_1} \perp \overline{W_{31}V_2}$
V_3
$\mathbf{S}^3$
W_{31}
W_{23}
$\mathbf{S}^4$
$\mathbf{S}^1$
$\mathbf{S}^2$
V_1
W_{12}
V_2

Figure 9



V_3
$\ell \parallel \overline{V_1 V_2}$
$\mathbf{T}_c(\mathbf{G})$
R(t)
$X_c(t)$
$\alpha(t)$
ℓ
V_1
V_2

Figure 10

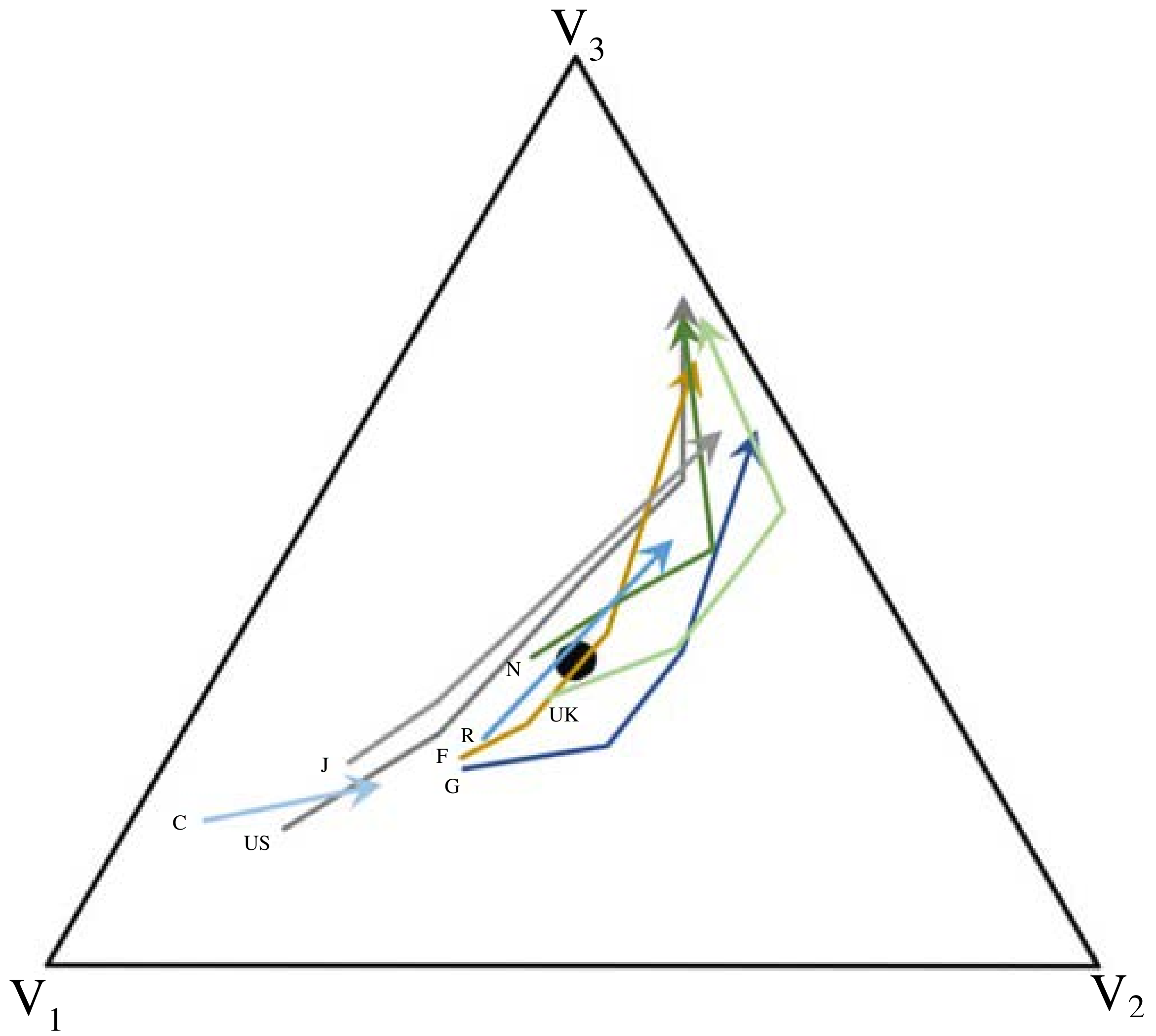


Figure 11

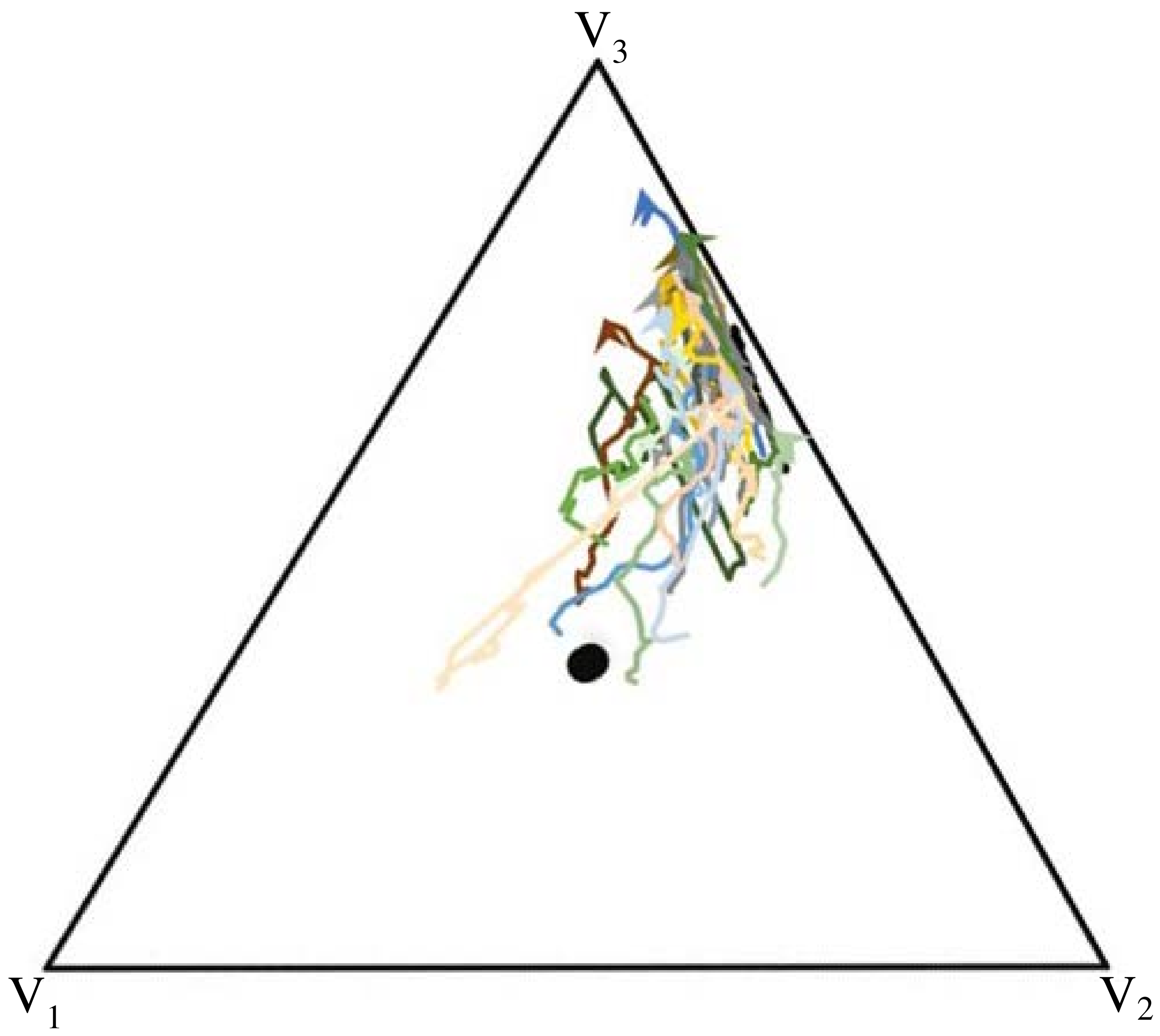


Figure 12

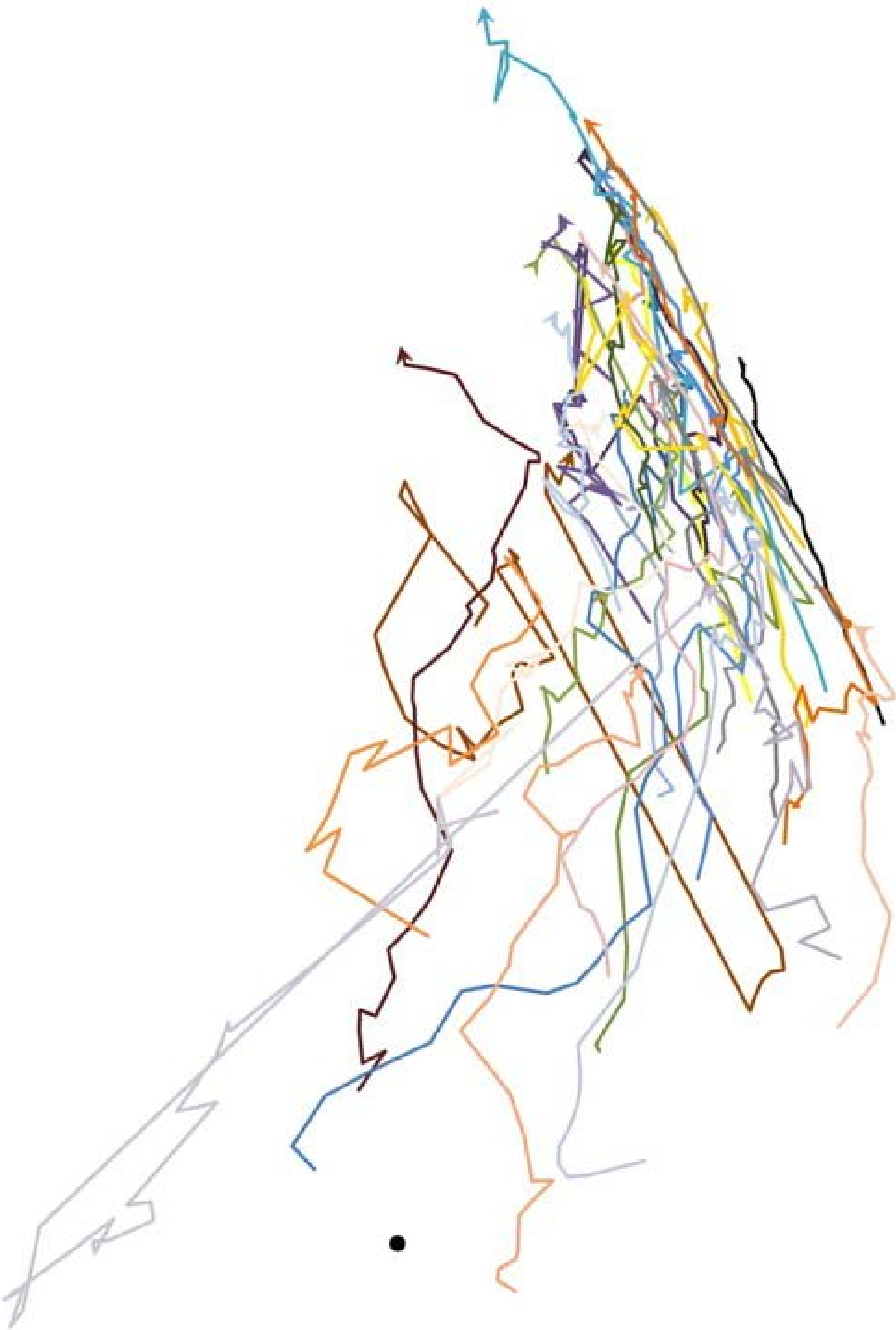


Figure 13

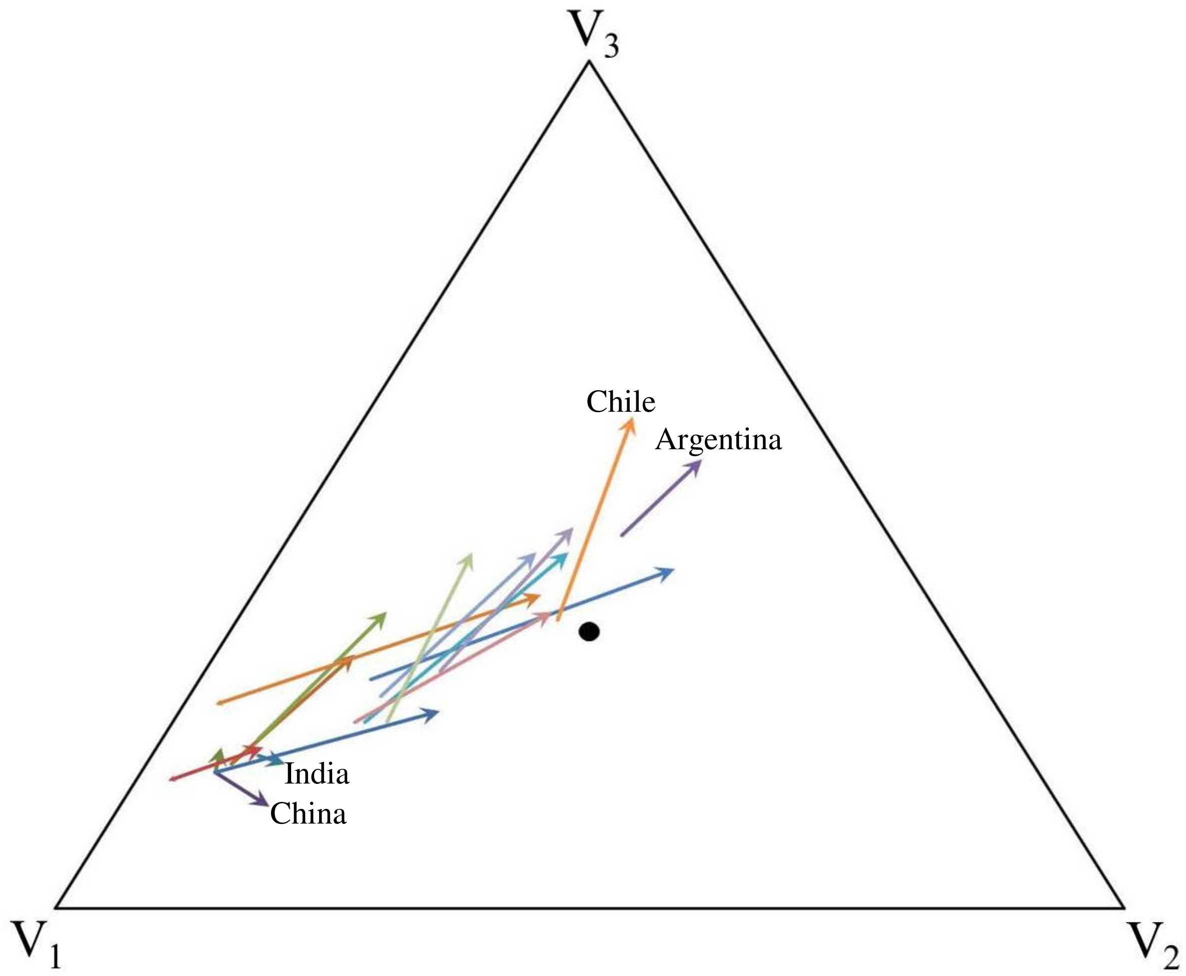


Figure 14

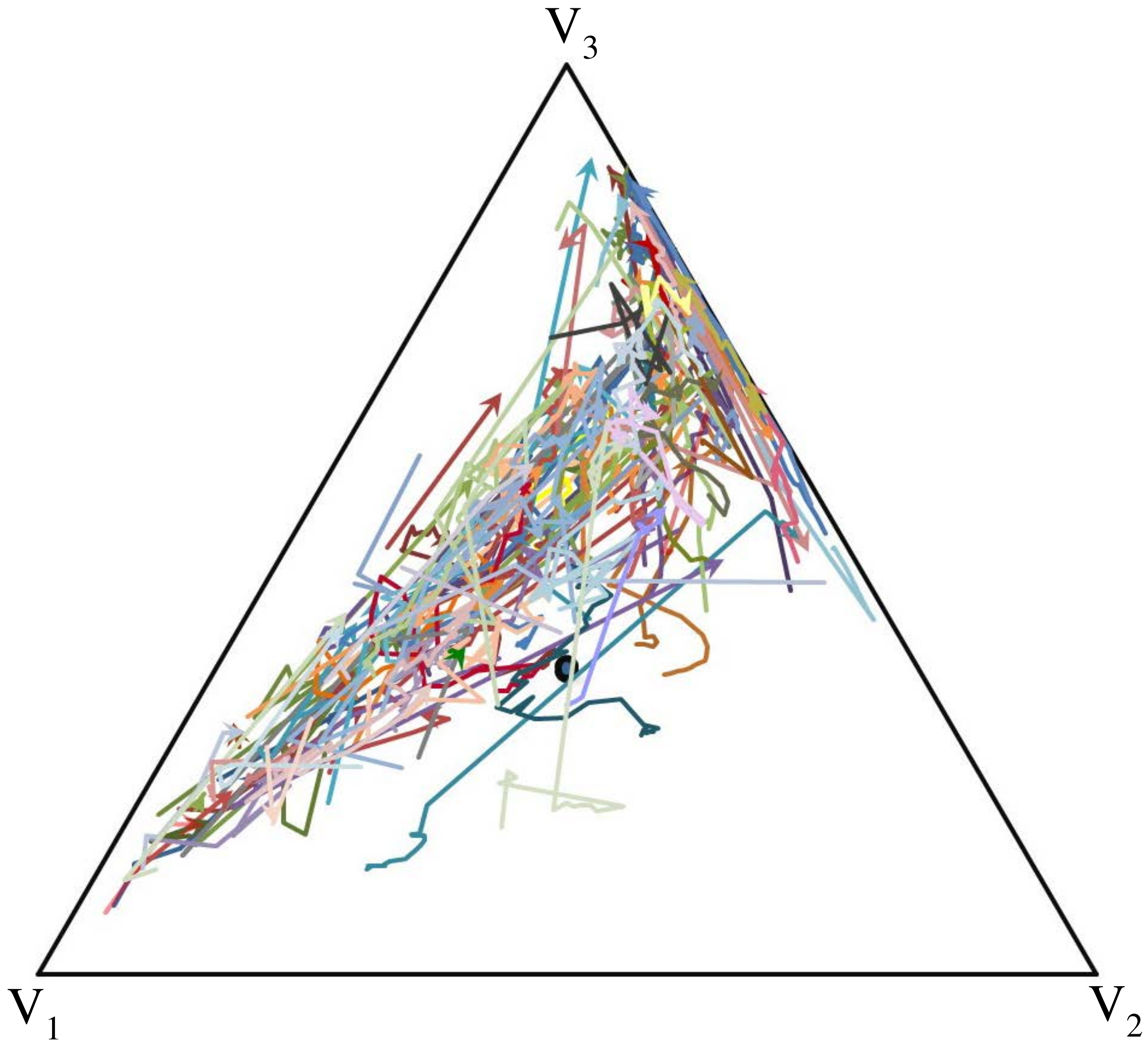


Figure 15

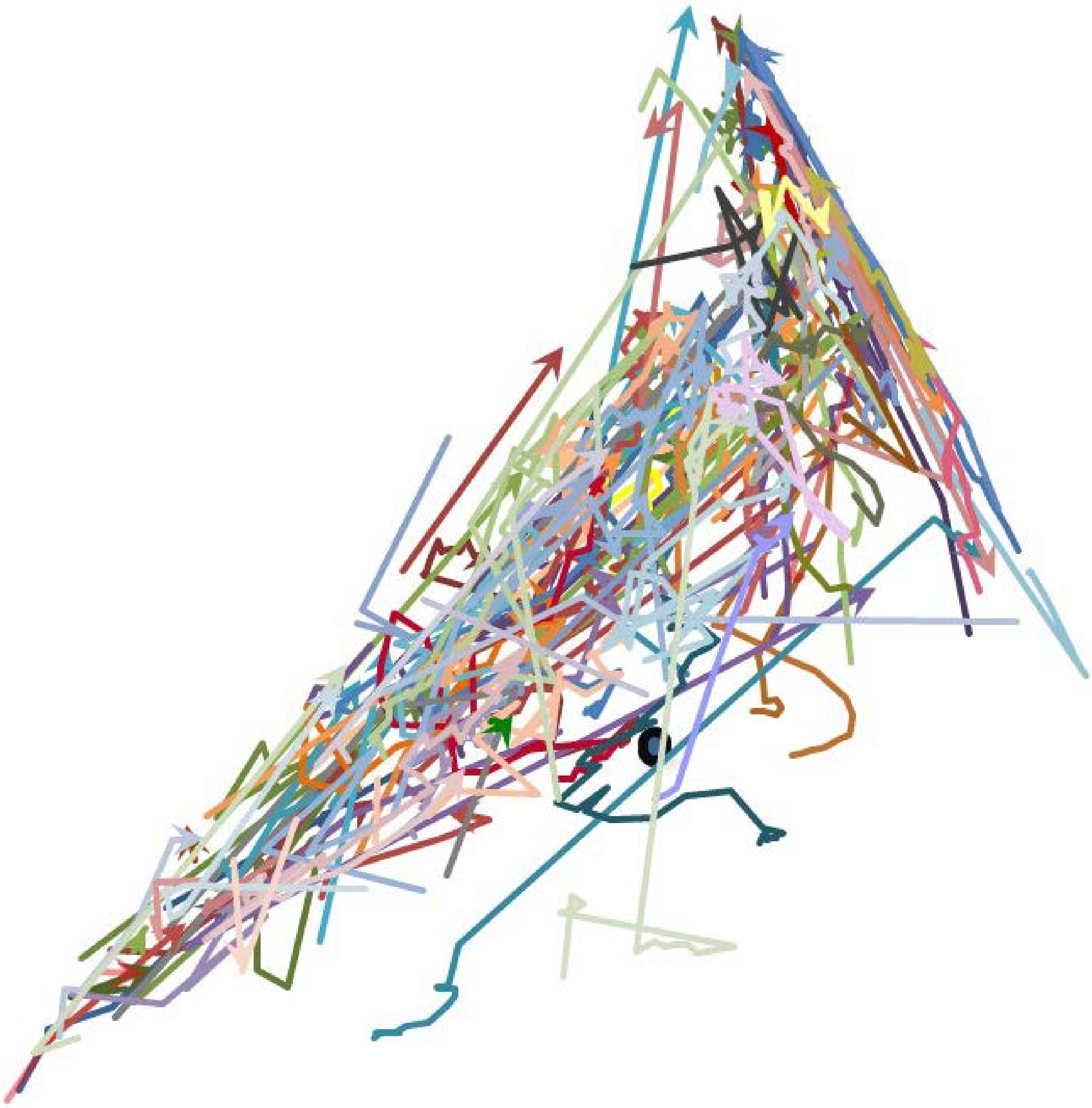


Figure 16

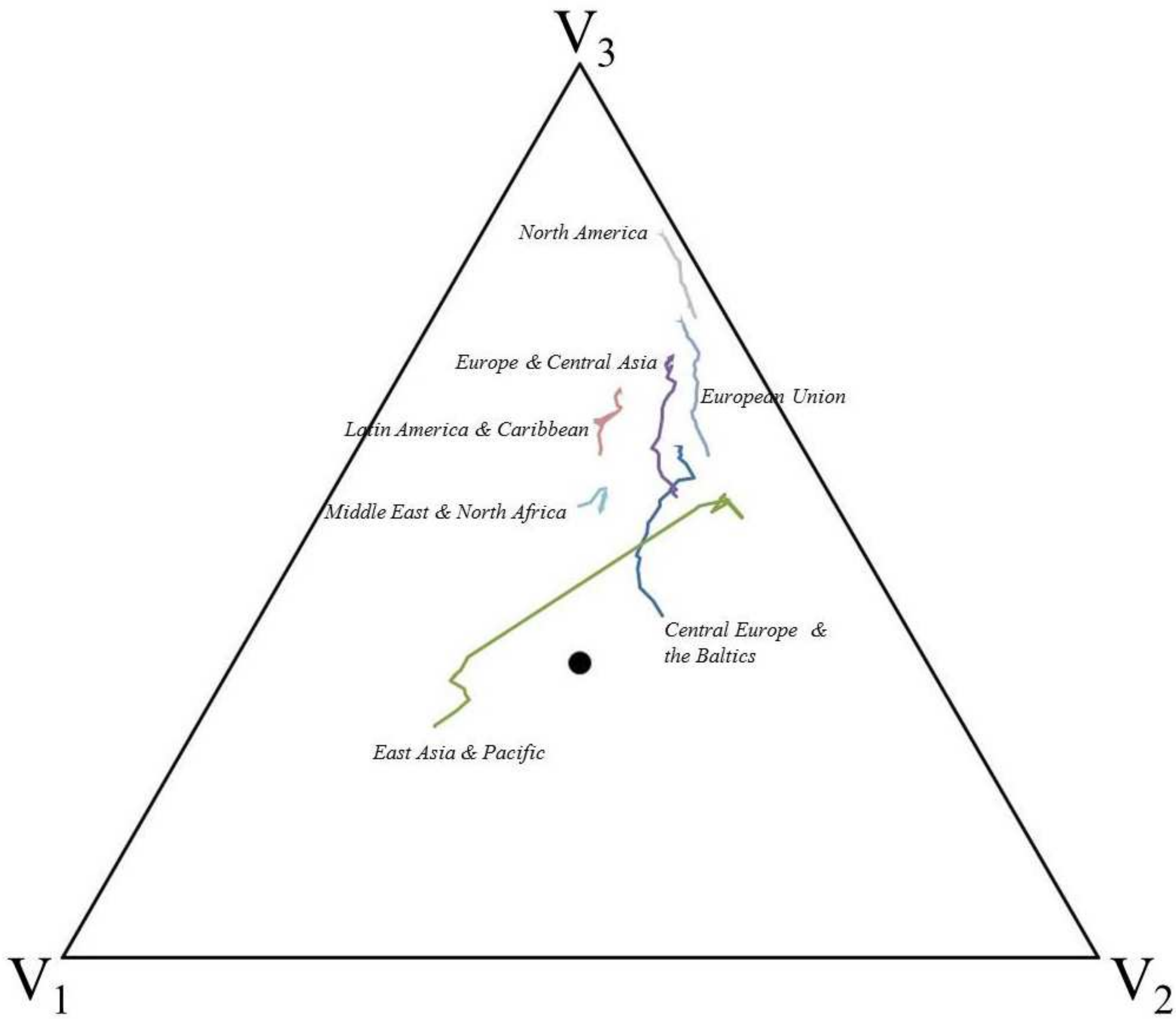
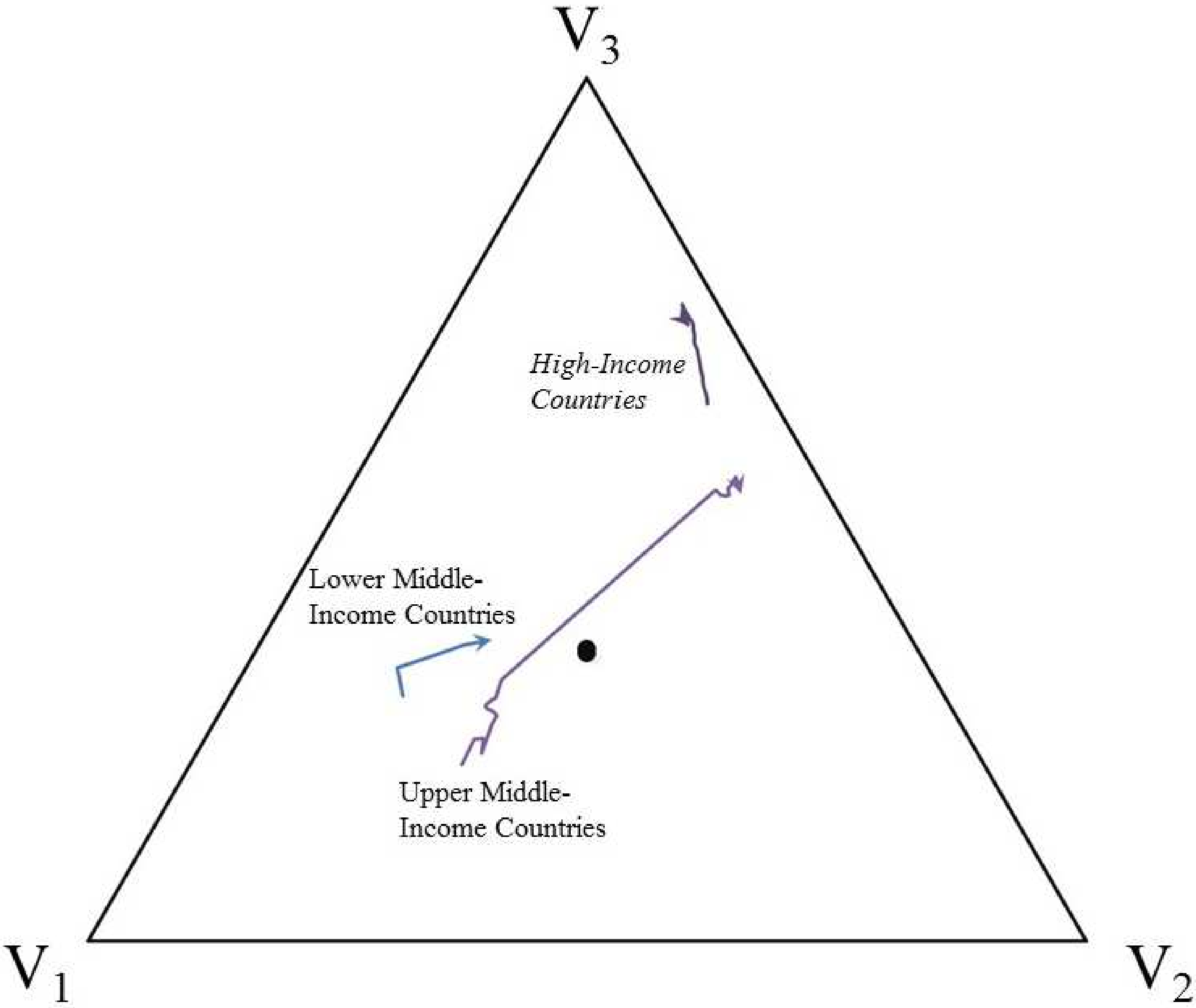

V3
North America
Europe & Central Asia
European Union
Latin America & Caribbean
Middle East & North Africa
Central Europe &
the Baltics
East Asia & Pacific
V1
V2

Figure 17



V_3
High-Income
Countries
Lower Middle-
Income Countries
Upper Middle-
Income Countries
V_1
V_2

Figure 18

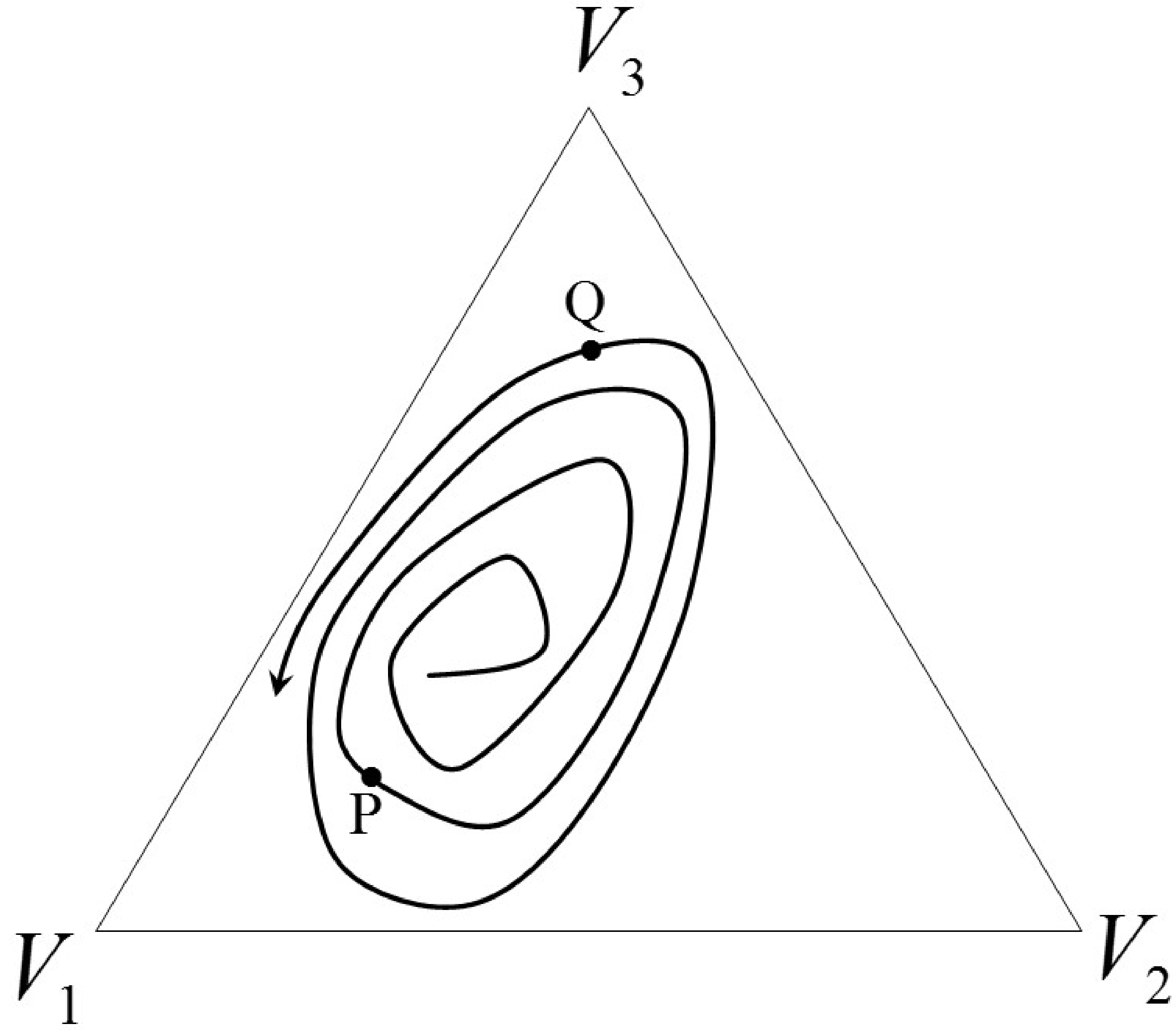


Figure 19

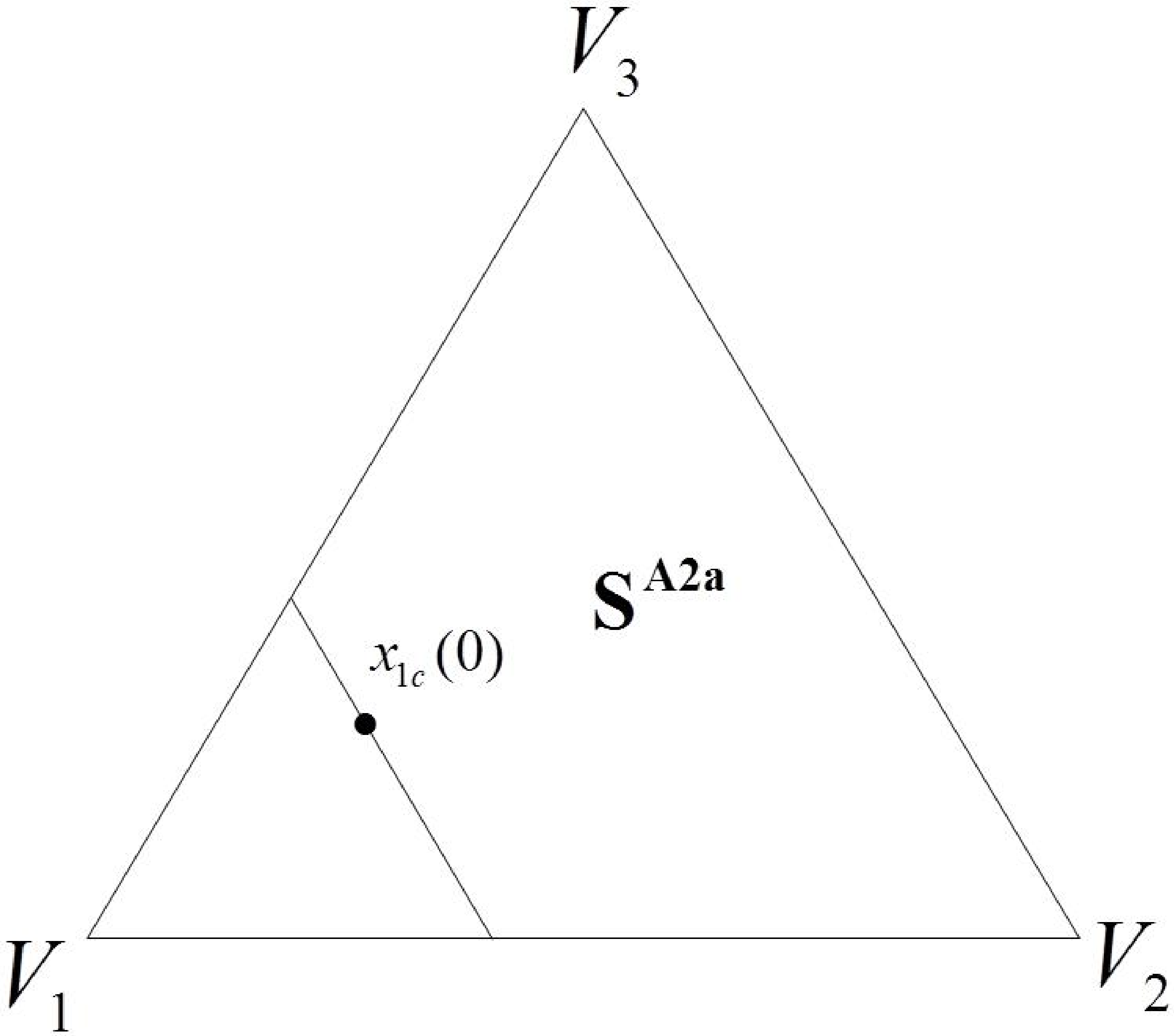


Figure 20

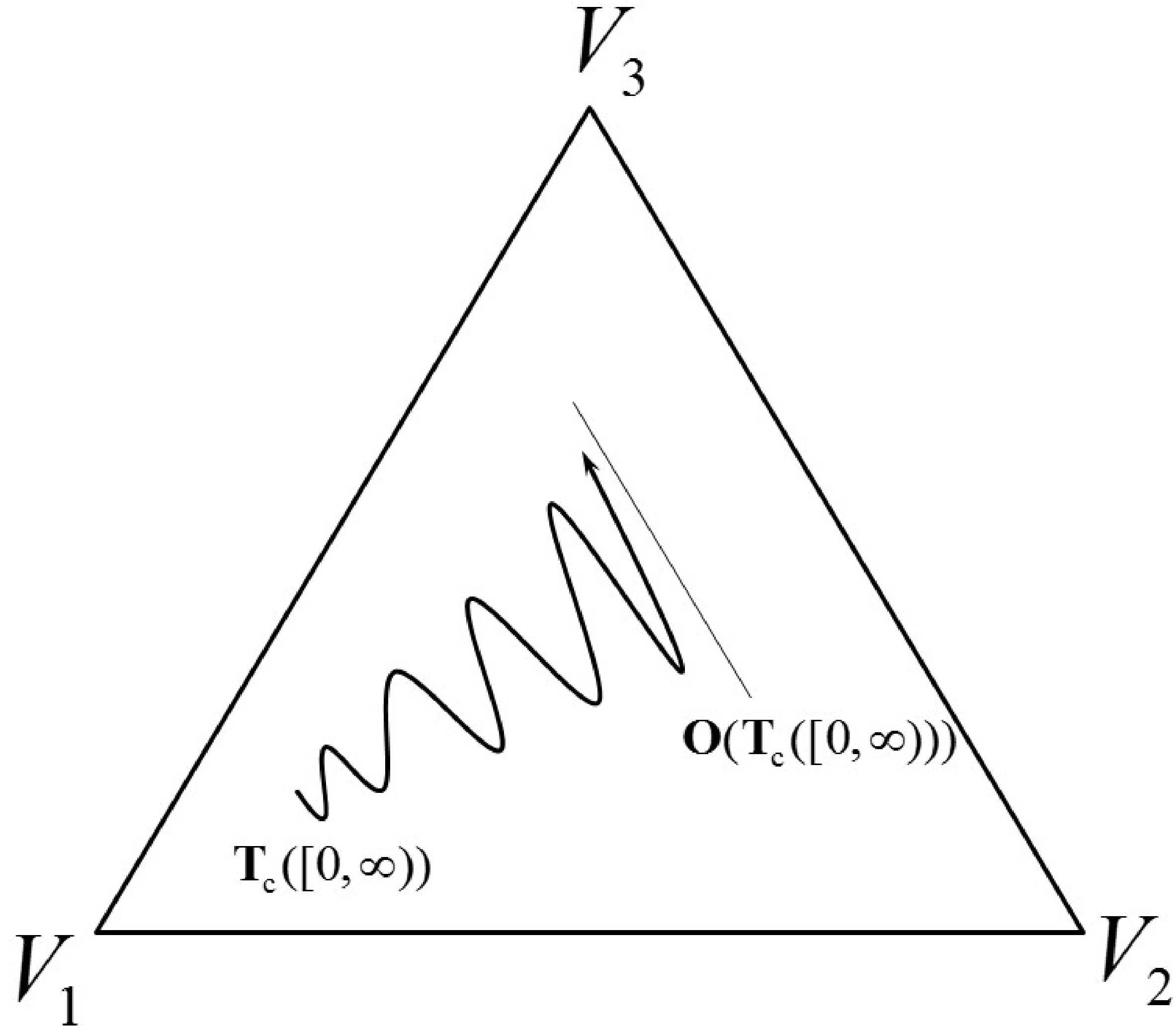


Figure 21

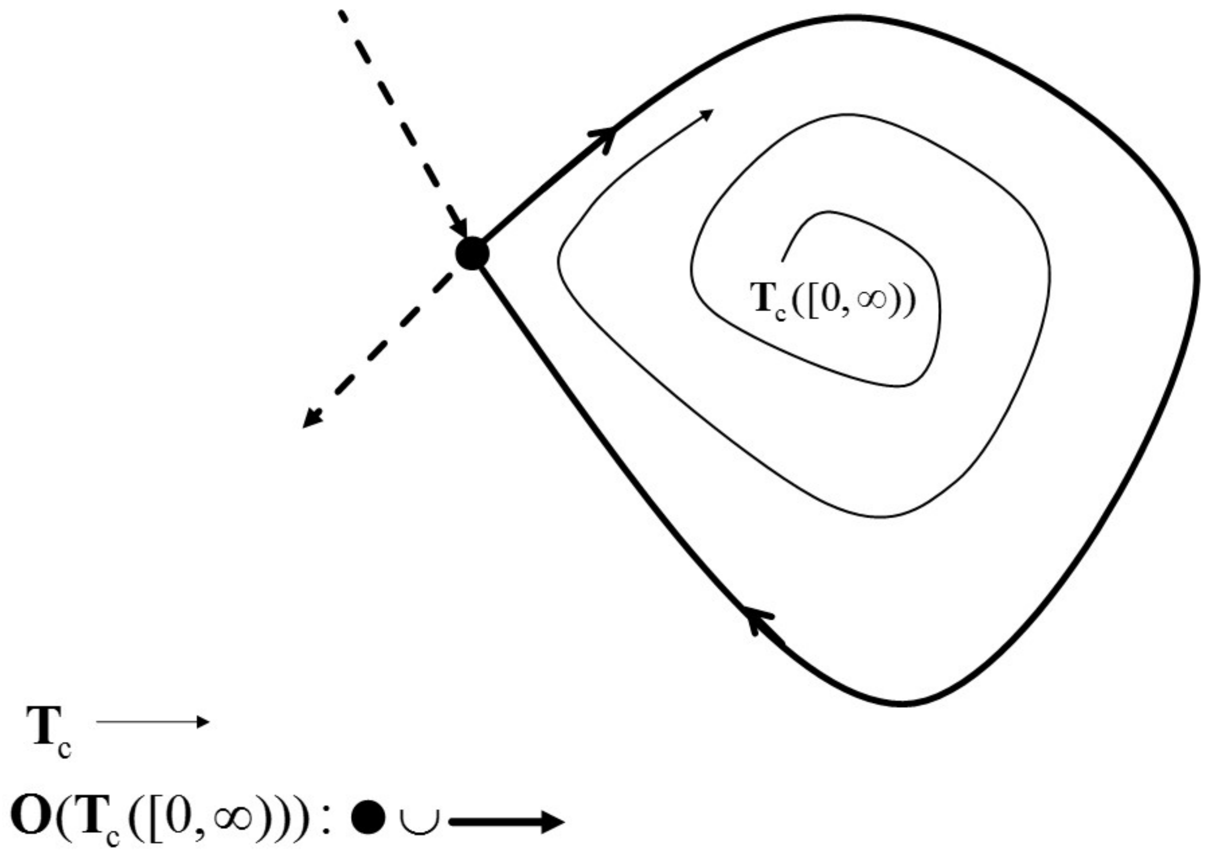
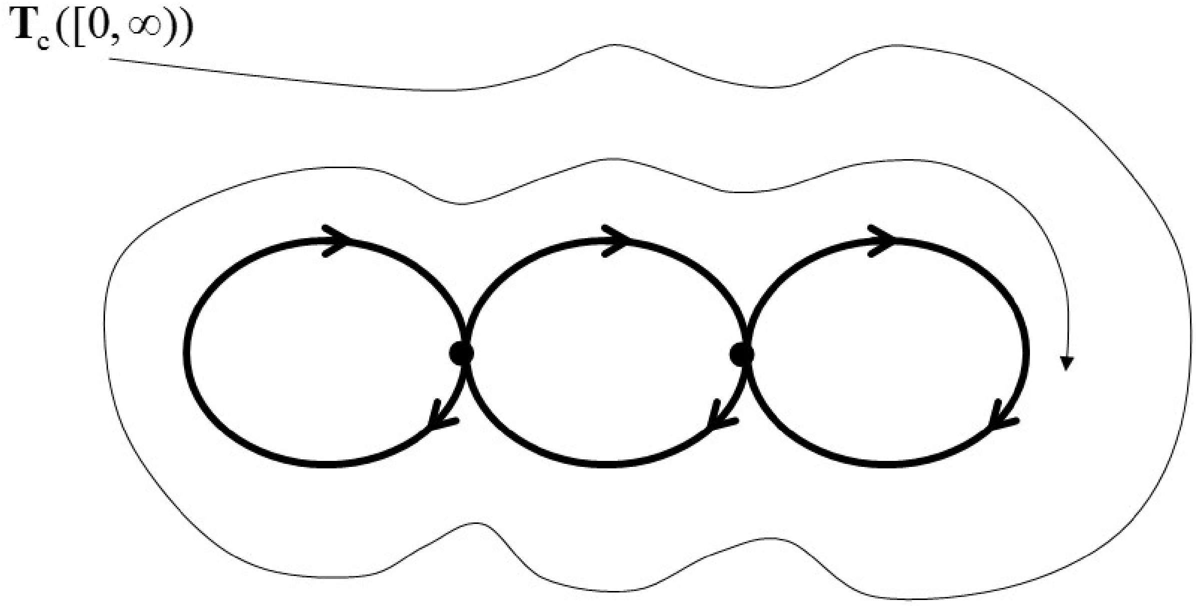


Figure 22



$\mathbf{T}_c \longrightarrow$

$\mathbf{O}(\mathbf{T}_c([0,\infty))) : \bullet \cup \longrightarrow$