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Abstract

KVA represents the extra cost being charged by banks to non collateralized counterparties in order to remunerate banks' shareholders for the mandatory regulatory capital provided by them throughout the life of the deal. Therefore, KVA represents earnings charged to clients that must be retained in the bank's balance sheet and not be immediately paid out as dividends. Since retained earnings are part of core TIER I capital, future KVAs imply a deduction in today's KVA calculation.

Another key component of KVA is the fact that shareholder's returns (dividends and capital gains) are generated after taxes are paid. Therefore, taxes should be reflected in the KVA formula.

By treating KVA as retained earnings, we derive a pricing formula that is consistent with full replication of market, counterparty and funding risks, and that takes the effect of taxes into account.

We provide a numerical example where the KVA obtained under this new formula is compared with other approaches yielding significantly lower adjustments. This numerical example also helps us to assess the relevance of taxes.

1 Introduction

Same as with any other economic activity, derivatives are financed with both equity and debt. In the current environment, regulators have increased and are in the process of increasing capital requirements for banks, that is, the proportion of the derivatives activity to be financed by equity holders. Accordingly, the financial industry is putting more focus on the measurement and management of the return on equity generated by trading activities.

In this new environment, a new adjustment has emerged which reflects the extra cost being charged to banks' non collateralized counterparties in order to compensate equity holders throughout the life of a new deal for the incremental regulatory capital implied by it. This new adjustment is broadly known as KVA (Capital Value Adjustment). Nevertheless, the new adjustment is still not in a mature stage and different questions arise with respect to how it should be measured, how should it be accounted and how should it be managed. In this paper we try to shed some light on all these questions.

KVA is a topic that is currently being discussed in the industry, but most papers devoted to KVA follow the guidelines of [3, 6, 7, 8, 9]. The approach establishes a hedging equation for the full price (including capital costs) of the derivative. Under this approach, it is assumed that the regulatory capital supporting the deal (plus the possibly self imposed cushion) is remunerated at the hurdle rate, no matter whether it has been provided by the shareholder or charged to the client. It is also suggested in [7] that this capital could reduce external funds obtained from debt holders.

The effect of taxes is considered in [9] under unhedgeable counterparty risk and the corresponding taxation of any resultant profit or loss. However, a bank's trading activity is taxed according to the change in derivatives' values and interest expenses, whereas shareholder's return is generated after the bank is taxed. Thus, as we shall see, taxes play a role even under full replication assumptions.

In our approach, we take into consideration the fact that KVA represents a cost charged to bank clients but represents a profit for the bank, since it is money to be paid to the bank's shareholders. Since KVA has been charged to compensate equity holders throughout the life of the deal, it should not be paid out as dividends at the time of closing the deal, but kept in the bank's balance sheet. Once kept in the balance sheet as retained earnings, KVA is a contribution to Core TIER I capital that has not been provided by equity investors' but by clients. As a consequence, banks do not need to ask their non collateralized trading counterparties for the cost of remunerating the whole capital at the hurdle rate. It is enough to ask for the cost of remunerating capital diminished by retained earnings. In [1], this consideration is also made. Nevertheless, they derive a KVA formula using classical finance theory rather than replication arguments. Furthermore, no taxes are taken into account in their approach. These two differences lead to a pricing formula that differs from ours, for which the retained earnings feature, together with the impact of taxes and replication arguments are considered.

2 Contribution of a non collateralized deal to the bank's balance sheet

For clarity of exposition, but without loss of generality, we assume that the bank enters into a new uncollateralized derivative transaction with a positive NPV for the bank after all adjustments have been made (CVA, FVA, TVA, KVA). We do not take DVA into consideration since it must be deducted from capital. Nevertheless, we include FVA since, as shown in this paper, both funding cost and benefit can actually be hedged.

Figure 1 represents the contribution of a particular deal (or set of deals) to the balance sheet of the bank. V_t^F is the value of the derivative taking into account market risk, counterparty credit risk and funding costs but neither considering shareholder's compensation, nor taxes. That is, V_t^F represents the NPV as if it was funded only with debt. V_t represents the value of the derivative taking into consideration every component, including shareholder's remuneration and taxes. We have also included hedges (market and counterparty credit risk) closed with interbank counterparties (which we have assumed to be positive in the picture without losing generality). Notice that due to the fact that market hedges are collateralized, they are funded through the corresponding collateral account. r_t^T represents the tax rate at time t.

In a tax-free world, the value of retained earnings would be $V_t^F - V_t$, since the bank would pay V_t for a derivative whose value would be V_t^F had it been fully financed with debt. Notice that in the presence of taxes, the bank must fund with owners' equity and debt both V_t (outflow paid to the client) and the income tax expense $r_t^T \left(V_t^F - V_t\right)$. This tax expense reduces retained earnings to $RE_t = \left(1 - r_t^T\right)\left(V_t^F - V_t\right)$.

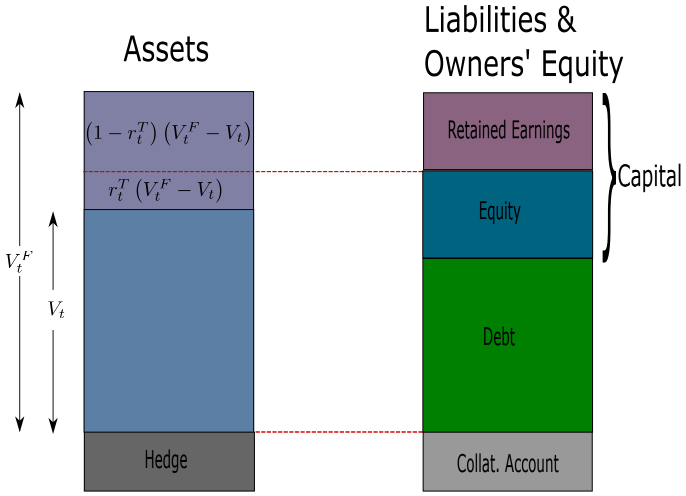

Figure 1: Balance sheet contribution of a new deal taking the effect of taxes into account.

Note that KVA at value date can be higher than the spot capital. This can be the case for long maturity derivatives. This means that at value date we do not need any shareholder's equity. In the rest of the paper we will assume that, in that case, the surplus of capital we get via retained earnings is used to reduce shareholder's equity requirements in other trading activities. A similar consideration will be made with respect to funding, avoiding any non linear term in the pricing equations. This is in line with realistic situations, since banks will always fund their activities with both equity and debt, and a situation in which any of the two terms vanishes is not probable.

3 Obtaining the pricing equation through replication

In this section we will obtain the pricing equation that is consistent with replication of market, counterparty (spread and default) and funding risks. This is done while generating the self imposed return on equity and complying with regulatory capital requirements. In order to do so, we will use the balance sheet contribution of a deal obtained in section 2. By differentiating the corresponding equation, hedging every risk and generating the self imposed return on equity (after interest and tax expenses), we arrive to the PDE followed by the derivative's price. The corresponding pricing equation is obtained after applying Feynman-Kac to the PDE.

We will consider a non-collateralized derivative (extension to portfolio level can easily be done) written on a given underlying asset whose price at time t is S_t. If we assume, for simplicity but without losing generality, one factor dynamics for the different curves involved, the price of the derivative V_t, as seen from the bank's perspective, depends on the following risk factors:

$$V_t := V(t, S_t, h_t, f_t, N_t, N_t^{\text{Bank}})$$

Where

- S_t is the underlying asset. We assume an instantaneous REPO rate r_t^S for the underlying asset and a continuous dividend rate q_t.
- h_t is the non collateralized trading counterparty's overnight CDS premium.
- f_t denotes the short term funding rate at which the trading desk can borrow money from the bank's internal treasury.
- $N_t = \mathbf{1}_{\{\tau \leq t\}}$ and $N_t^{\text{Bank}} = \mathbf{1}_{\{\tau^{\text{Bank}} \leq t\}}$ account for the default indicators of the non collateralized trading counterparty and the bank, respectively.

Under the real measure $\mathbb{P}$, we assume the following dynamics for the different factors:

$$\begin{aligned} dS_t &= \mu_t^S S_t dt + \sigma_t^S S_t dW_t^{S,\mathbb{P}} \\ dh_t &= \mu_t^h dt + \sigma_t^h \, dW_t^{h,\mathbb{P}} \\ df_t &= \mu_t^f dt + \sigma_t^f \, dW_t^{f,\mathbb{P}} \end{aligned} \tag{1}$$

Where μ_t^S, μ_t^h and μ_t^f are the real world drifts; σ_t^S, σ_t^h and σ_t^f the volatilities of the processes and $W_t^{S,\mathbb{P}}$, $W_t^{h,\mathbb{P}}$ and $W_t^{f,\mathbb{P}}$ $\mathbb{P}$-Wienner processes with correlations $\rho_t^{S,h}$, $\rho_t^{S,f}$, $\rho_t^{h,f}$.

At any time t, assets must be equal to liabilities plus owners' equity. Therefore

$$\underbrace{V_t^F + \underbrace{\alpha_t H_t}_{\text{Market Hedge}} + \underbrace{\sum_{j=1}^{2} \epsilon_t^j CDS(t,t_j)}_{\text{CVA Hedge}}}_{\text{Assets}} = \underbrace{\underbrace{\omega_t B^f(t,T) + \beta_t^f}_{\text{Debt}} + \underbrace{\beta_t^C}_{\text{Collateral Account}}}_{\text{Liabilities}} + \underbrace{\underbrace{E_t}_{\text{Equity}} + \underbrace{RE_t}_{\text{Ret. Earnings}}}_{\text{Capital}=K_t} \tag{2}$$

where

- H_t represents the market risk hedging instrument's price (written on S_t) which is closed with an interbank counterparty, thus it is colletaralized.

- $CDS(t, t_j)$ denotes the price of a CDS, with notional ϵ_t^j and maturity t_j, written on the bank's non collateralized trading counterparty. As with any other hedging instruments, we assume the CDS basket to be traded with interbank counterparties and, accordingly, collateralized. Notice that under one factor dynamics, 2 CDSs are needed to hedge spread and jump to default risks.

- β_t^C stands for the collateral account due to collateralized hedges. At time t it will equal to the NPV of the market and CVA hedges

$$\begin{aligned} \beta_t^C &= \alpha_t H_t + \textstyle\sum_{j=1}^{2} \epsilon_t^j CDS(t,t_j) \\ d\beta_t^C &= \left(\alpha_t H_t + \textstyle\sum_{j=1}^{2} \epsilon_t^j CDS(t,t_j)\right) c_t dt \end{aligned} \tag{3}$$

  where c_t represents the collateral accrual rate (OIS rate).

- RE_t represents retained earnings coming from the extra cost charged to the client to remunerate shareholders. Notice that it is fundamental for the retained earnings component to be homogeneous in time. This means that at a future time $u > t$, the retained earnings adjustment should be the same as if the deal was closed at time u. Otherwise, two identical deals (same cash flows at the same dates, counterparty and incremental regulatory capital) closed at different past dates would have different retained earnings metrics and therefore require different shareholders' contributions as of today, which is undesirable. This homogeneity implies that at any time, the following must hold:

$$RE_t = \left(1 - r_t^T\right)\left(V_t^F - V_t\right) \tag{4}$$

- E_t represents the portion of capital provided by the shareholder. Notice that it is the only component in (2) that is not marked to market, since this term is accounted on a historical basis. We assume that the bank top managers have determined a return on equity (aka hurdle rate) r_t^E. Since time t shareholders' contribution must be compensated, a stream of dividends (and/or capital gains) must be paid in the $(t, t+dt)$ interval. Therefore

$$dE_t = r_t^E E_t dt \tag{5}$$

- K_t denotes the incremental spot regulatory capital (plus possible self imposed cushion) associated to the deal and its hedges at time t. We will assume that the top managers have decided to maintain capital as a constant proportion Ω of regulatory risk weighted assets, where the proportion must be obviously greater than that imposed by the regulator. Hence, $K_t = \Omega RWA_t$, where RWA_t represents the deal's incremental regulatory risk weighted assets. Since retained earnings are part of CET1:

$$K_t = E_t + RE_t \tag{6}$$

- β_t^f denotes the un-secured bank account (short term funding) and $B^f(t,T)$ a term bond issued by the bank (long term funding) with notional ω_t. The bank must fund the portion of the derivative plus tax expense that is not funded with equity. Therefore

$$\begin{aligned} \beta_t^f + \omega_t B^f(t,T) &= V_t + r_t^T\left(V_t^F - V_t\right) - E_t \\ d\beta_t^f &= f_t\left(V_t + r_t^T\left(V_t^F - V_t\right) - \omega_t B^f(t,T) - E_t\right)dt \end{aligned} \tag{7}$$

  In line with [4] we assume that the mixture of short and long term funding will be determined so that shareholders become immune to changes in f_t (the bank's funding curve). Notice that the bank does not try to hedge its own default since this source of risk cannot be hedged. Therefore, no DVA component will be reflected in our pricing equation, in line with regulation, where DVA is deducted from CET1.

A key feature of equation (2) is that the real contribution of each term to the balance sheet really depends on its sign. Hence, concepts that are labeled as assets (conversely liabilities) would become liabilities (assets), or a reduction in assets (liabilities), if their value was negative. With respect to K_t, E_t and RE_t, positive values represent an increase and negative values a decrease in each of these concepts. Nevertheless, if we forget about the balance sheet labels in (2), it is valid regardless of the sign of its components.

In the presence of taxes, when we differentiate equation (2), we must add a term that represents the tax expense on the instantaneous income experienced between t and $t+dt$. This instantaneous income before taxes is equal to the change experienced by the assets minus the change of the liabilities

$$dP\&L_t = dV_t^F + \alpha dH_t + \sum_{j=1}^{2} \epsilon_t^j dCDS(t,t_j) - \omega_t dB^f(t,T) - f_t\beta_t^f dt - c_t\beta_t^C dt$$

Thus, the hedging equation in differential form is

$$\begin{aligned}
&\underbrace{dV_t^F + \alpha_t dH_t + \sum_{j=1}^{2} \epsilon_t^j dCDS(t,t_j)}_{\textbf{Assets' Change}} = \underbrace{\omega_t dB^f(t,T)}_{\textbf{Term Debt's Change}} + \underbrace{f_t \beta_t^f dt + c_t \beta_t^C dt}_{\textbf{Interest Expense (Collateral \& Funding)}} \\
&\underbrace{+ r_t^T \left(dV_t^F + \alpha dH_t + \sum_{j=1}^{2} \epsilon_t^j dCDS(t,t_j) - \omega_t dB^f(t,T) - f_t \beta_t^f dt - c_t \beta_t^C dt \right)}_{\textbf{Tax Expense}} \\
&+ \underbrace{\left(1 - r_t^T\right) \left(dV_t^F - dV_t\right)}_{\textbf{Ret. Earnings' Change}} + \underbrace{r_t^E E_t dt}_{\textbf{Equity Dividends \& Capital Gains}}
\end{aligned} \tag{8}$$

When dealing with its own credit risk, the bank's default component is unhedgeable, however the bank will be exposed to changes in its funding spread. Therefore, the resulting pricing formula should be consistent with the hedging of every market variable (including the bank's funding spread) and the trading counterparty's default event, but should not contemplate the bank's own default hedge. This pricing formula would allow replication in every state in which the bank has not defaulted, but the bank's default event would be unhedged and borne by the bank's bond and equity holders. Consequently, we must apply Itô's Lemma for jump diffussion processes to (8) ignoring the bank's default event. This coincides with the approach carried out in [2], under Strategy II: semi-replication with a single bond under non stochastic credit spreads. After applying apply Itô's Lemma and using (3) and (7) we get:

$$\begin{aligned}
&\widetilde{\mathcal{L}}_{Shf} V_t dt + \tfrac{\partial V_t}{\partial S_t} dS_t + \tfrac{\partial V_t}{\partial h_t} dh_t + \tfrac{\partial V_t}{\partial f_t} df_t + \Delta V_t dN_t \\
&+\alpha_t \left(\widetilde{\mathcal{L}}_S H_t dt + \tfrac{\partial H_t}{\partial S_t} dS_t - c_t H_t dt \right) \\
&+ \textstyle\sum_{j=1}^{2} \epsilon_t^j \left(\widetilde{\mathcal{L}}_h CDS(t,t_j) dt + \tfrac{\partial CDS(t,t_j)}{\partial h_t} dh_t + \Delta CDS(t,t_j) dN_t - c_t CDS(t,t_j) dt \right) \\
&= \omega_t \left(\widetilde{\mathcal{L}}_f B^f(t,T) dt + \tfrac{\partial B^f(t,T)}{\partial f_t} df_t - f_t B^f(t,T) dt \right) \\
&+ f_t V_t dt + r_t^T \left(V_t^F - V_t \right) f_t dt - f_t E_t dt + \tfrac{r_t^E}{1 - r_t^T} E_t dt
\end{aligned} \tag{9}$$

Where $\widetilde{\mathcal{L}}_{Shf}$, $\widetilde{\mathcal{L}}_S$, $\widetilde{\mathcal{L}}_h$, $\widetilde{\mathcal{L}}_f$ are defined in appendix A. ΔX_t represents the jump to the non-collateralized counterparty's default of $X_t = \{V_t,\ CDS(t,t_j)\}$. In particular

$$\Delta V_t = \underbrace{R\left(V_t^{\text{Close-Out}}\right)^{+} + \left(V_t^{\text{Close-Out}}\right)^{-}}_{\pi_t} - V_t$$

$V_t^{\text{Close-Out}}$ represents the derivative's close out value to be settled at default of the non collateralized counterparty and π_t the NPV after default. R represents the recovery rate of the non collateralized counterparty.

If we choose α_t, ϵ_t^1, ϵ_t^2 and ω_t so that the different sources of risks -market (dS_t), credit spread (dh_t), default risks (dN_t) and funding (df_t)- are eliminated from (9):

$$\alpha_t = -\frac{\frac{\partial V_t}{\partial S_t}}{\frac{\partial H_t}{\partial S_t}}; \quad \omega_t = \frac{\frac{\partial V_t}{\partial f_t}}{\frac{\partial B^f(t,T)}{\partial f_t}}; \quad \frac{\partial V_t}{\partial h_t} = -\sum_{j=1}^{2} \epsilon_t^j \frac{\partial CDS(t,t_j)}{\partial h_t}; \quad \Delta V_t = -\sum_{j=1}^{2} \epsilon_t^j \Delta CDS(t,t_j) \tag{10}$$

We get

$$\begin{aligned}&\widetilde{\mathcal{L}}_{Shf} V_t - \frac{\frac{\partial V_t}{\partial S_t}}{\frac{\partial H_t}{\partial S_t}} \left(\widetilde{\mathcal{L}}_S H_t - c_t H_t\right) + \textstyle\sum_{j=1}^{2} \epsilon_t^j \left(\widetilde{\mathcal{L}}_h CDS(t,t_j) - c_t CDS(t,t_j)\right) \\ &= \frac{\frac{\partial V_t}{\partial f_t}}{\frac{\partial B^f(t,T)}{\partial f_t}} \left(\widetilde{\mathcal{L}}_f B^f(t,T) - f_t B^f(t,T)\right) + f_t V_t + r_t^T \left(V_t^F - V_t\right) f_t - f_t E_t + \frac{r_t^E}{1-r_t^T} E_t\end{aligned} \tag{11}$$

After applying the PDEs followed by collateralized derivatives and those followed by funding instruments and collateralized credit derivatives -all of these PDEs are included in appendix B and can be inferred from any of the references (a summary is included in both [5, 6])-, we get

$$\begin{aligned}&\widetilde{\mathcal{L}}_{Shf} V_t - \frac{\frac{\partial V_t}{\partial S_t}}{\frac{\partial H_t}{\partial S_t}} \left(-\frac{\partial H_t}{\partial S_t}\left(r_t^S - q_t\right) S_t\right) + \textstyle\sum_{j=1}^{2} \epsilon_t^j \left(-\frac{\partial CDS(t,t_j)}{\partial h_t}\left(\mu_t^h - M_t^h\right) - \frac{h_t}{1-R} \Delta CDS(t,t_j)\right) \\ &= \frac{\frac{\partial V_t}{\partial f_t}}{\frac{\partial B^f(t,T)}{\partial f_t}} \left(-\frac{\partial B(t,T)^f}{\partial f_t}\left(\mu_t^f - M_t^f\right)\right) + f_t V_t + r_t^T \left(V_t^F - V_t\right) f_t - f_t E_t + \frac{r_t^E}{1-r_t^T} E_t\end{aligned} \tag{12}$$

M_t^h represents the market price of credit risk for the non collateralized counterparty and M_t^f the market price of funding risk.

Finally, applying (4) and (6) together with the expressions followed by ϵ_t^j in (10), we get

$$\begin{aligned}&\mathcal{L} V_t + \frac{h_t}{(1-R)} \Delta V_t = V_t\, c_t + \underbrace{V_t\left(f_t - c_t\right)}_{\textbf{FVA Contrib.}} + \underbrace{K_t\, \gamma_t}_{\textbf{KVA Contrib.}} + \underbrace{\left(V_t - V_t^F\right)\gamma_t}_{\textbf{REVA Contrib.}} + \underbrace{r_t^E K_t \frac{r_t^T}{1-r_t^T}}_{\textbf{TVA Contrib.}} \\ &\textbf{s.t} \qquad V(T) = V_T\end{aligned} \tag{13}$$

Where we have defined $\gamma_t = \left(r_t^E - f_t\right)$, V_T represents derivative's terminal condition (we have assumed a derivative with a single cash flow at maturity), TVA stands for taxes value adjustment and $\mathcal{L}$ is again defined in appendix A.

By applying *Feynman-Kac*, the solution to equation (13) can be expressed as,

$$
\begin{aligned}
V_t = {} & \underbrace{E^{\mathbb{Q}}\left[e^{-\int_t^T c(s)ds}\, V_T \middle| \mathcal{F}_t\right]}_{V_t^C:=\textbf{Risk Free Price}} - \underbrace{E^{\mathbb{Q}}\left[e^{-\int_t^\tau c(s)ds}\, \mathbf{1}_{\{\tau<T\}}\, \left(V_\tau^C - \pi_\tau\right) \middle| \mathcal{F}_t\right]}_{\textbf{CVA Contrib.}} \\
& - \underbrace{\int_{s=t}^T E^{\mathbb{Q}}\left[e^{-\int_t^s c(u)du}\, \mathbf{1}_{\{\tau>s\}}\, V_s\, \left(f_s - c_s\right) \middle| \mathcal{F}_t\right] ds}_{\textbf{FVA Contrib.}} - \underbrace{\int_{s=t}^T E^{\mathbb{Q}}\left[e^{-\int_t^s c(u)du}\, \mathbf{1}_{\{\tau>s\}}\, K_s\, \gamma_s \middle| \mathcal{F}_t\right] ds}_{\textbf{KVA Contrib.}} \\
& + \underbrace{\int_{s=t}^T E^{\mathbb{Q}}\left[e^{-\int_t^s c(u)du}\, \mathbf{1}_{\{\tau>s\}}\, \left(V_s^F - V_s\right) \gamma_s \middle| \mathcal{F}_t\right] ds}_{\textbf{Retained Earning VA Contrib.}} - \underbrace{\int_{s=t}^T E^{\mathbb{Q}}\left[e^{-\int_t^s c(u)du}\, \mathbf{1}_{\{\tau>s\}}\, K_s\, r_s^E \frac{r_s^T}{1-r_s^T} \middle| \mathcal{F}_t\right] ds}_{\textbf{Taxes VA Contrib.}}
\end{aligned}
\tag{14}
$$

$\mathbb{Q}$ is the risk-neutral pricing measure under which the default intensity of the counterparty is given by $\frac{h_t}{1-R}$ and the drifts of the different risk factors $S_t,\ h_t$ and f_t by $\left(r_t^S - q_t\right) S_t,\ \mu_t^h - \sigma_t^h M_t^h$ and $\mu_t^f - \sigma_t^f M_t^f$ respectively.

Notice that the full price V_s appears in the right hand side of (14), so that the formula is recursive. In section 5, we show how to solve this issue.

The term K_t is a quantity either obtained with regulatory formulas (standard model approach) or with scenario engines (internal model approach). In both cases, K_t tries to represent a quantile obtained under the real world measure. Therefore, we could be concerned with the fact that equation (14) represents an expected value in a pricing measure, whereas K_t is obtained under the real world measure. However, K_t depends on market variables (spot prices, IR curves, CDS curves, FX rates, ...) at time t, and conditional on these, K_t is deterministic. These market variables are simulated under the risk neutral measure for pricing and K_t acts as a t-filtration deterministic payoff function, so there is no inconsistency in the approach. The same consideration is made in [7, 8].

At this point, it is worth mentioning that the tax rate r_t^T could depend on the banks overall profitability in the time interval $(t, t+dt)$ unless a tax loss carry forward was in place. If this was not the case, r_t^T would be the possibly constant tax rate ($\approx 30\%$) in a profitable time interval and 0 otherwise, although this consideration might be really complex to solve.

Equation (14), when compared to other KVA approaches, exhibits the following differences: the default indicator of the bank is missing, which is consistent with the non hedgeability of the bank's default, and there are two additional terms. One of them (REVA) implies a reduction in the KVA adjustment due to the fact that retained earnings contribute to capital but are not remunerated at r_t^E. The other term (TVA) isolates the effect of taxes and increases the KVA adjustment compared to the tax free case.

To give an economic interpretation to the result, equation (13) can also be written as

$$\mathcal{L}V_t + \frac{h_t}{(1-R)}\,\Delta V_t = \frac{r_t^E}{1-r_t^T}E_t + f_t D_t$$

Where D_t represents the value of debt (overnight and term). The right hand side of the equation represents the fact that both debt and equity holders must be compensated at their corresponding rates, but also that $r_t^E E_t dt$ is paid after having paid taxes.

4 Return on equity management throughout time

In this section we analyze the impact that this hedging strategy has on the dividend policy and also how should the hedging portfolio be rebalanced at time $t + dt$.

With respect to the dividend policy, it is important to remark that to be consistent with the self imposed discipline of generating a given return on equity, the following must hold:

$$r_t^E E_t dt = \underbrace{q_t^E E_t dt}_{\text{Dividends}} + \underbrace{\left(r_t^E - q_t^E\right) E_t dt}_{\text{Equity capital gains}} \tag{15}$$

Where q_t^E represents the bank's dividend rate. So that at $t + dt$ there will be an outflow of dividends equal to $q_t^E E_t dt$ and the equity component in the balance sheet will be increased by $\left(r_t^E - q_t^E\right) E_t dt$. Other dividend policy that was not accompanied with the corresponding capital gains described in (15) would violate the self imposed return on equity.

With respect to rebalancing at time $t + dt$:

- The notionals of the interbank hedging instruments must be updated. This produces no cash flows since they are fully collateralized.
- Risk weighted assets will also be updated. These, multiplied by the self imposed proportion Ω will determine the new capital K_{t+dt}.
- $(1 - r_{t+dt}^T)\left(V_{t+dt}^F - V_{t+dt}\right)$ will determine the new retained earnings metric.
- The bank should either issue new debt to buy back equity or issue equity to buy back new debt such that the following holds:

$$K_{t+dt} = E_{t+dt} + RE_{t+dt}$$

> Notice that the amounts traded in equity and debt must offset each other, so that the rebalancing is self financing:
>
> $$E_{(t+dt)^-} + \omega_t B^f(t+dt,T) + \beta^f_{(t+dt)^-} = E_{t+dt} + \omega_{t+dt} B^f(t+dt,T) + \beta^f_{t+dt}$$
>
> Where the "-" superscript represents magnitudes before rebalancing. ω_t must also be updated to remain hedged with respect to funding curve changes.

It is important to note that equation (13) still holds after the hedging portfolio has been re-balanced, independent of the proportion that is paid as dividends (against equity capital gains) as long as (15) is fulfilled.

5 Calculation in a Monte Carlo framework

In this section we show how to solve the recursive nature of (14). The PDE in (13) can be re-expressed in its equivalent form,

$$\begin{aligned} &\mathcal{L}V_t + \frac{h_t}{(1-R)}\,\Delta V_t = V_t\, r^E_t + \left(K_t - V^F_t\right)\gamma_t + r^E_t K_t \frac{r^T_t}{1-r^T_t} \\ &\text{s.t} \qquad V(T) = V_T \end{aligned} \tag{16}$$

or in terms of expected values, by using r^E_t as discounting rate

$$\begin{aligned} V_t = \; & \underbrace{E^{\mathbb{Q}}\left[e^{-\int_t^T r^E_s ds}\, V_T \middle| \mathcal{F}_t\right]}_{V^E_t := \textbf{ROE Discounted Price}} \underbrace{- E^{\mathbb{Q}}\left[e^{-\int_t^\tau r^E_s ds}\, \mathbf{1}_{\{\tau<T\}}\, \left(V^E_\tau - \pi_\tau\right) \middle| \mathcal{F}_t\right]}_{\textbf{CVA over ROE Discounted Price}} \\ & \underbrace{- \int_{s=t}^T E^{\mathbb{Q}}\left[e^{-\int_t^s r^E_u du}\, \mathbf{1}_{\{\tau>s\}}\, K_s\, \gamma_s \middle| \mathcal{F}_t\right] ds}_{\textbf{ROE discounted KVA Contrib.}} + \underbrace{\int_{s=t}^T E^{\mathbb{Q}}\left[e^{-\int_t^s r^E_u du}\, \mathbf{1}_{\{\tau>s\}}\, V^F_s\, \gamma_s \middle| \mathcal{F}_t\right] ds}_{\textbf{ROE discounted REVA Contrib.}} \\ & \underbrace{- \int_{s=t}^T E^{\mathbb{Q}}\left[e^{-\int_t^s r^E_u du}\, \mathbf{1}_{\{\tau>s\}}\, K_s\, r^E_s \frac{r^T_s}{1-r^T_s} \middle| \mathcal{F}_t\right] ds}_{\textbf{ROE discounted TVA Contrib.}} \end{aligned} \tag{17}$$

with respect to V^F_t, it must fulfill with the following PDE (just plug $\gamma_t = 0$ and $r^T_t = 0$ in (16))

$$\begin{aligned}&\mathcal{L}V_t^F + \frac{h_t^h}{(1-R)}\,\Delta V_t^F = V_t^F\,f_t\\ &\text{s.t} \qquad V^F(T) = V_T\end{aligned} \tag{18}$$

After applying *Feyman-Kac*, equation (18) is equivalent to,

$$V_t^F = \underbrace{E^{\mathbb{Q}}\left[e^{-\int_t^T f_s ds}\; V_T \middle| \mathcal{F}_t\right]}_{V_t^{f_t}} - \underbrace{E^{\mathbb{Q}}\left[e^{-\int_t^\tau f_s ds}\; \mathbf{1}_{\{\tau<T\}}\; \left(V_\tau^{f_t} - \pi_\tau\right) \middle| \mathcal{F}_t\right]}_{\textbf{CVA over price with funding}} \tag{19}$$

In [4], a different expression which implies the same expectation is obtained by using the collateral rate as discounting rate.

Going back to (17), at first sight, the “REVA” term might seem difficult to solve. Let us try to further develop it by making use of (19)

$$\begin{aligned}REVA'_t &= \int_{s=t}^T E^{\mathbb{Q}}\left[e^{-\int_t^s r_u^E du}\; \mathbf{1}_{\{\tau>s\}}\; V_s^F\; \gamma_s \middle| \mathcal{F}_t\right] ds\\ &= \underbrace{\int_{s=t}^T E^{\mathbb{Q}}\left[e^{-\int_t^s r_u^E du}\; \mathbf{1}_{\{\tau>s\}}\; V_s^{f_t}\; \gamma_s \middle| \mathcal{F}_t\right] ds}_{\mathbf{I}}\\ &+ \underbrace{\int_{s=t}^T E^{\mathbb{Q}}\left[e^{-\int_t^s r_u^E du}\; \mathbf{1}_{\{\tau>s\}}\; \left[\int_{u=s}^T E^{\mathbb{Q}}\left[e^{-\int_s^u f_x dx}\mathbf{1}_{\{\tau>u\}}\left(V_u^{f_t} - \pi_u\right) dN_u \middle| \mathcal{F}_s\right]\right]\; \gamma_s \middle| \mathcal{F}_t\right] ds}_{\mathbf{II}}\end{aligned} \tag{20}$$

The first term, **I**, in last equation does not imply further complications with respect to those found to solve for the CVA term. In general terms, we must only provide a pricer that allows us to price the derivative conditional to the state of the economy at future dates.

The second term in the latter equation can be simplified by changing the integration order:

$$\mathbf{II} = \int_{u=t}^T E^{\mathbb{Q}}\left[\left(V_u^{f_t} - \pi_u\right) \mathbf{1}_{\{\tau>u\}}\left(\int_{s=t}^u e^{-\int_t^s r_x^E dx} e^{-\int_s^u f_x dx}\gamma_s\; ds\right) dN_u \middle| \mathcal{F}_t\right] \tag{21}$$

Just by taking into account that $r_t^E = f_t + \gamma_t$,

$$\boxed{\mathbf{II} = \int_{u=t}^T E^{\mathbb{Q}}\left[e^{-\int_t^u f(x)dx}\; A^\gamma(t,u)\; \mathbf{1}_{\{\tau>u\}}\; \left(V_u^{f_t} - \pi_u\right)\; dN_u \middle| \mathcal{F}_t\right]} \tag{22}$$

where $A^{\gamma}(t,u) = \left(1 - e^{-\int_t^u \gamma_s ds}\right)$.

Note that this term is very similar to the CVA over price with funding in (19).

6 Numerical results

In this section we provide numerical results for the price of a FX forward for different maturities and strikes. For illustrative purposes, we have considered counterparty risk capital under SA-CCR and CVA regulatory capital under Basel III. We are neglecting market risk capital.

We assume the counterparty's rating to be BBB. The only stochastic magnitude is the underlying FX , with volatility 10%. The different rates involved are: funding rate = 2%, counterparty credit spread = 2%, domestic rate = 1%, foreign rate = 0.5%, collateral rate = 1%, hurdle rate = 15% and tax rate = 30%. Notional is set to 1 and FX spot is 1. We have not considered the effect of the hedging portfolio in regulatory capital calculations.

In figure 2 we have plotted $V_t^F - V_t$ under 3 approaches (Green's approach with $\phi = \{0, 1\}$ as defined in [8] and ours), with and without taxes. In table 1, we see that the reduction in the amount charged to the bank's counterparty under our approach represents a reduction with respect to the traditional approach, which we assume to be Green's approach with $\phi = 0$, that is similar with and without taxes.

To take taxes into account for the traditional KVA approaches, we have proceeded as in section 3, but changing the equity remuneration and the self financing condition:

$$\begin{gathered} dE_t = K_t r_t^E dt \\ V_t + r_t^T \left(V_t^F - V_t\right) = \phi K_t + \omega_t B^f(t,T) + \beta_t^f \end{gathered} \tag{23}$$

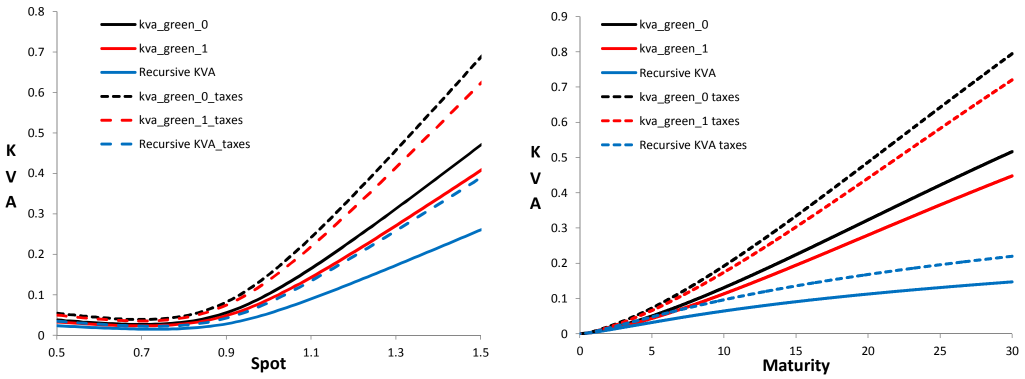

Figure 2: KVA under the three different approaches with and without taxes. Left chart: $V_t^F - V_t$ for a 10 year deal as a function of the spot. Right chart: $V_t^F - V_t$ for an ATM year deal as a function of maturity.

	No Taxes				With Taxes			
Maturity	Trad KVA_0	Trad KVA_1	Our KVA	**Savings**	Trad KVA_0	Trad KVA_1	Our KVA	**Savings**
1y	0.0048	0.0042	0.0039	**18.6%**	0.0069	0.0063	0.0059	**15.0%**
3y	0.0248	0.0215	0.0177	**28.4%**	0.0357	0.0324	0.0265	**25.8%**
5y	0.0502	0.0435	0.0318	**36.6%**	0.0728	0.0660	0.0475	**34.7%**
10y	0.1309	0.1134	0.0644	**50.8%**	0.1925	0.1745	0.0963	**50.0%**
20y	0.3231	0.2800	0.1125	**65.2%**	0.4870	0.4415	0.1681	**65.5%**
30y	0.5169	0.4480	0.1470	**71.6%**	0.7949	0.7207	0.2197	**72.4%**

Table 1: KVA values under the different approaches for significant maturities for an at the money deal, with and without taxes.

7 Conclusions

We have proposed a KVA formula that is consistent with the idea that KVA should be kept in the balance sheet as retained earnings. Since retained earnings are part of CET1 capital, this adjustment charged to clients diminishes funds obtained from equity holders, and therefore implies a lower KVA adjustment. We have also seen that for the formula to be applicable, the dividend policy together with the equity capital gains must be consistent with the self imposed return on equity. With respect to the management, the bank should hedge the price including every adjustment (CVA, FVA, KVA & TVA). This management implies a continuous rebalancing of debt and equity. This new metric also takes the effect of taxes into consideration. This increases the adjustment compared to the tax free case.

8 Acknowledgments

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A Definition of differential operators

In this appendix we define the differential operators appearing in the different PDEs followed by the pricing equations throughout the paper.

$$
\begin{aligned}
\widetilde{\mathcal{L}}_{Shf} &= \tfrac{\partial}{\partial t} + \tfrac{1}{2}S_t^2(\sigma_t^S)^2\tfrac{\partial^2}{\partial S_t^2} + \tfrac{1}{2}(\sigma_t^f)^2\tfrac{\partial^2}{\partial {f_t}^2} + \tfrac{1}{2}(\sigma_t^h)^2\tfrac{\partial^2}{\partial {h_t}^2} \\
&\quad + S_t\sigma_t^S\sigma_t^f\rho_t^{S,f}\tfrac{\partial^2}{\partial S_t\partial f_t} + S_t\sigma_t^S\sigma_t^h\rho_t^{S,h}\tfrac{\partial^2}{\partial S_t\partial h_t} + \sigma_t^h\sigma_t^f\rho_t^{h,f}\tfrac{\partial^2}{\partial h_t\partial f_t} \\
\widetilde{\mathcal{L}}_S &= \tfrac{\partial}{\partial t} + \tfrac{1}{2}(\sigma_t^f)^2\tfrac{\partial^2}{\partial {S_t}^2} \qquad (24) \\
\widetilde{\mathcal{L}}_h &= \tfrac{\partial}{\partial t} + \tfrac{1}{2}(\sigma_t^f)^2\tfrac{\partial^2}{\partial {h_t}^2} \\
\widetilde{\mathcal{L}}_f &= \tfrac{\partial}{\partial t} + \tfrac{1}{2}(\sigma_t^f)^2\tfrac{\partial^2}{\partial {f_t}^2}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L} &= \tfrac{\partial}{\partial t} + (r_t^S - q_t)S_t\tfrac{\partial}{\partial S_t} + (\mu_t^f - M_t^f\sigma_t^f)\tfrac{\partial}{\partial f_t} + (\mu_t^h - M_t^h\sigma_t^h)\tfrac{\partial}{\partial h_t} \\
&\quad + \tfrac{1}{2}S_t^2(\sigma_t^S)^2\tfrac{\partial^2}{\partial S_t^2} + \tfrac{1}{2}(\sigma_t^f)^2\tfrac{\partial^2}{\partial {f_t}^2} + \tfrac{1}{2}(\sigma_t^h)^2\tfrac{\partial^2}{\partial {h_t}^2} \qquad (25) \\
&\quad + S_t\sigma_t^S\sigma_t^f\rho_t^{S,f}\tfrac{\partial^2}{\partial S_t\partial f_t} + S_t\sigma_t^S\sigma_t^h\rho_t^{S,h}\tfrac{\partial^2}{\partial S_t\partial h_t} + \sigma_t^h\sigma_t^f\rho_t^{h,f}\tfrac{\partial^2}{\partial h_t\partial f_t}
\end{aligned}
$$

B PDEs followed by vanilla instruments

This appendix contains the PDEs followed by the price of the different hedging instruments used throughout the paper.

Collateralized derivative written on the underlying asset:

$$
\widetilde{\mathcal{L}}_S H_t + \left(r_t^S - q_t\right) S_t \frac{\partial H_t}{\partial S_t} = c_t H_t \qquad (26)
$$

Collateralized CDS written on the bank's trading counterparty:

$$
\widetilde{\mathcal{L}}_h CDS(t,t_j) + \left(\mu_t^h - \sigma_t^h M_t^h\right) \frac{\partial CDS(t,t_j)}{\partial h_t} + \Delta CDS(t,t_j)\frac{h_t}{1-R} = c_t H_t \qquad (27)
$$

Bonds issued by the bank:

$$
\widetilde{\mathcal{L}}_f B^f(t,T) + \left(\mu_t^f - \sigma_t^f M_t^f\right) \frac{\partial B^f(t,T)}{\partial S_t} = f_t H_t \qquad (28)
$$

References

[1] C. Albanese, S. Caenazzo, S. Crépey
Capital and Funding, Risk, 71-76, May 2016

[2] C. Burgard, M. Kjaer.
Funding Costs, Funding Strategies, Risk, 82-87, Dec 2013.

[3] Y. Elouerkhaoui.
From FVA to KVA: Including the Cost of Capital in the Pricing of Derivatives, Risk in press

[4] Luis M. García Muñoz
CVA, FVA (and DVA?) with stochastic spreads. A feasible replication approach under realistic assumptions. https://mpra.ub.uni-muenchen.de/44252/

[5] Luis M. García Muñoz, Fernando de Lope, Juan Palomar
Pricing Derivatives in the New Framework: OIS Discounting, CVA, DVA & FVA https://mpra.ub.uni-muenchen.de/62086/

[6] A. Green
XVA: Credit, Funding and Capital Valuation Adjustments - Wiley (2015)

[7] C. Kenyon, A. Green, Chris R Dennis KVA
Capital Valuation Adjustment. http://ssrn.com/abstract=2400324.

[8] C. Kenyon, A. Green
Portfolio KVA: I Theory. http://ssrn.com/abstract=2519475

[9] C. Kenyon, A. Green
Warehousing credit risk: pricing, capital and tax. Risk, 1-6, February 2015

[10] V. Piterbarg
Funding beyond discounting: Collateral agreements and derivatives pricing. Risk, February, 97-102, 2010.