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31 October 2016

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October 31, 2016

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ABSTRACT

This paper examines the effect of monetary policy on the liquidity premium, i.e., the market value of the liquidity services that financial assets provide. To guide the empirical analysis, I set up a monetary search model in which bonds provide liquidity services in addition to money. The theory predicts that money supply and nominal interest rates are positively correlated with the liquidity premium, but the premium is negatively correlated with the bond supply. The empirical analysis over the period from 1946 and 2008 confirms the theoretical predictions. This indicates that liquid bonds are substantive substitutes for money and the opportunity cost of holding money plays a key role in asset price determination. Lastly, the theory rationalizes the existence of negative nominal yields in equilibrium, when the cost of holding money is low whereas liquid bonds are scarce, and I present empirical findings to support it.

JEL Classification: E31, E41, E50, E52, G12

Keywords: asset price, monetary search model, liquidity, liquidity premium, money supply, negative nominal yield

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I am grateful to Athanasios Geromichalos, Bonghyun Baek, Jaerim Choi, Andres Carvajal, Federico Guerrero, Espen Henriksen, Oscar Jorda, Jiwon Lee, Ina Simonovska, Yujung Suh, Inmo Sung, and Alan Taylor for useful comments and suggestions. We would also like to thank participants at talks at University of California, Davis, Inha University, and University of Nevada, Reno. All errors are mine.

1 Introduction

Investors value financial assets not only for their intrinsic value, i.e., their expected dividend or payment stream, but also for their liquidity: their ability to help facilitating transactions. For instance, U.S Treasuries are often used as collateral in a secured credit market through repurchase agreements, they are easily sold for cash in secondary asset markets, and, oftentimes, they are used directly as means of payment. Accordingly, many liquid financial assets are priced above their respective fundamental value, and their prices are higher than those of illiquid assets with comparable safety and maturity characteristics; the assets bear liquidity premia. Also, it is well known that the liquidity premia account for a large part of the variation in the liquid asset prices observed in financial markets.

The objective of this paper is to examine the effect of monetary policy on such liquidity premia and to explore the possibility for liquidity based rationale for negative bond yields which have been observed in some countries since the recent financial crisis in 2008. In particular, the paper focuses on the effect of money and bond supplies, which are directly related to open market operations of the central bank. The open market operations, as a primary policy tool to reach its monetary policy objectives, are involved with buying and selling bonds in exchange for money in the "open market", i.e., in the secondary asset trade market. These open market operations allow the monetary authority to affect the supplies of money and bond circulated in an economy, in terms of both their levels and growth rates, and thereby short term interest rates at the end. On the other hand, since the liquidity services provided by liquid bonds such as Treasuries are similar to those of money in the sense that they can help facilitating transactions; otherwise would not occur, bonds partially play a role as a substitute with money. This relationship between money and bonds naturally gives rise to questions: how and under what conditions money and bond supplies affect the liquidity premia, and how bonds' liquidity can cause the negative yields.

First, I set up a monetary search model developed by Lagos and Wright (2005) to answer the questions aforementioned theoretically and use it as a guidance to the empirical exercise. It is worth noticing that the main feature of this type of monetary model is to have microfoundations for monetary exchange in a macro framework: an explicit exchange process is embedded into the model. Due to this characteristic of the model, it can deliver new and different results which are not obvious in other monetary models: money-in-utility (MIU), cash-in-advance (CIA) and cash-credit good (CC) models. Importantly, the monetary search model is more useful in answering the questions addressed in the paper: how the liquidity premia are determined endogenously, under what conditions bonds' liquidity properties can cause negative bond interest rates, and how one-time injection of money or bonds into an economy affects the liquidity premia differently.[1]

Specifically, I extend the baseline framework of Lagos and Wright (2005) by including a risk-free government bond in addition to fiat money. The government bond in the model is liquid in the sense that it is useful in the exchange process. Due to its liquidity, its equilibrium price exceeds the fundamental value, and its supply naturally affects its price (or its yield) through changing the liquidity premium. The empirical exercise with the US data also presents that the bond supply has a negative impact on the liquidity premium. Furthermore, its ability to facilitate transactions in a goods market characterized by frictions (such as anonymity and limited commitment) makes it a substitute for money to some degree because money also performs a similar function. Accordingly, not only bond supply but also money supply affect the bond's liquidity premium and thereby its price. Specifically, the key concept that links money supply and the liquidity premium is the opportunity cost of holding money. An increase in money growth rate raises the inflation rate under fully flexible prices. It implies higher nominal interest rates through the Fisher effect, i.e., the higher cost of holding money in the end.[2] Then, agents substitute more costly fiat money with liquid bonds because they are also useful in the exchange process, which, in turn, leads to greater demand, higher liquidity premia and, ultimately, higher bond prices (lower bond yields). As a result, the money growth rate has a positive influence on the premium. This relationship is also supported empirically.

Interestingly, my model also suggests liquidity based explanation about negative nominal yields as an equilibrium phenomenon. It demonstrates that bonds' liquidity properties can cause the negative yields in the case where there exist transactions where bonds are used as indirect media of exchange, like repurchase agreement contracts (but would not money) and so strong demand on liquid bonds for their liquidity services. In particular, when the money holding cost is low (or short term interest rates are low) and bond supply is scarce, the extra liquidity services from bonds are highly valued, and therefore it can lead to the negative yields. Notice that the negative yields of liquid bonds can emerge even in the case where the short term interest rates are low but positive, i.e., there exist other illiquid assets with nonnegative yields. The empirical findings in Switzerland and the United States confirm this theoretical result. Notice that both Swiss and American government bonds are similar in the sense that there exists strong demand for their liquidity services even internationally unlike other countries' government bonds. We have observed the negative Treasury yields in Switzerland since 2008. In fact, a huge decline in the supply of liquid government bonds was markedly observed in Switzerland around the period of the financial crisis, starting in the last quarter of 2008 and then it has remained historically low until 2015. Moreover, short term nominal interest rates

[1] See Waller (2015) and Lagos, Rocheteau, and Wright (2014) for more general discussion.

[2] One time injection of money into the economy or a change in the level of money supply does not affect the liquidity premium and any other real variables in the model, but its growth rate does. Hence, money is neutral but not superneutral here. However, unlike money, one time injection of bonds has a substantial impact on real variables in the model.

were positive but close to zero during most of periods after 2008.[3] These evidence suggests that the Treasuries' liquidity properties or their higher liquidity premia should cause their negative yields. For the similar reason, we observed the negative Treasury yields even in the US in September, 2015. Last but not least, it should be pointed out that the existence of negative nominal yields is often considered anomalous, because it is hard to reconcile through the lens of traditional monetary models. However, my model of asset liquidity can help rationalizing these observations.[4]

When it comes to the empirical exercise, I test the primary results of the theoretical model. In particular, I test whether money growth rate has a positive impact on the liquidity premia on liquid bonds, and whether bond supply has a negative impact. In addition, I examine empirically whether the existence of liquidity premia can be an important factor in explaining the aforementioned negative yields on liquid bonds. In the empirical tests, I use the US Treasuries as liquid bonds introduced in the model, and measure the liquidity premia of the Treasuries with different maturities for robustness of the empirical analysis. The examples include the spread of AAA-rated corporate bonds against the long term Treasury bonds, the TED spread, and the spreads of AA-rated Commercial Papers and Federal Deposit Insurance Corporation (FDIC) insured Certificates of deposits against Treasury bills.[5] These financial assets are comparably safe but not as liquid as Treasury bonds of similar maturities; therefore one can reasonably argue that the spreads between the yields on these assets and Treasury bonds (of similar maturities) reflect differences in liquidity premia. Next, I use a monetary aggregate, Narrow Money, as a proxy of money because it only includes components which can be used as a direct medium of exchange implied by the theory unlike other broader monetary aggregates such as M1 and M2. Furthermore, its demand is stable against its holding cost, or nominal interest rates in that it displays a downward sloping curve over the sample period as required in the theory later. Using these data, the empirical test results strongly confirm the predictions from the theory: the money growth rate has a positive influence on these measures of the liquidity premium, but bond supply has a negative impact.

From the theoretic point of view, a large money search literature similarly presents that the liquidity premium is a primary factor of variation in the prices of liquid financial assets, and that their supply is negatively correlated with the liquidity premium, whereas money supply is positively. Also, the key mechanism for this relationship is the opportunity cost of holding

[3]The negative Treasury yields were observed in some European countries such as Germany, Italy and France, but they emerged after the European Central Bank lowered its monetary policy target interest rate.

[4]One of the important lessons we've learned from asset liquidity is that it can shed light on existing asset-related puzzles from a new perspective and provide a liquidity-based theory of asset pricing. Examples include Lagos (2010a), Geromichalos, Herrenbrueck, and Salyer (2013), Geromichalos and Simonovska (2011), and Jung and Lee (2015).

[5]I use the yearly measures which Krishnamurthy and Vissing-Jorgensen (2012) use in their paper for comparison as well as an additional measure such as the TED spread. Moreover, the quarterly data are presented here to increase the sample size of the measures.

money. As a pioneer theoretical paper, Geromichalos, Licari, and Suarez-Lledo (2007) set up a Lagos-Wright type of monetary search framework with a real asset, and theoretically present that the money growth rate increases the liquidity premium in the economy where neither money nor the asset are plentiful. They derive this result from the model where the asset is a perfect substitute to money in transactions in a decentralized market, and money growth leads to an increase in the opportunity cost of holding money. Also, several papers with this substitution relationship between money and financial assets deliver the more or less similar results. Examples include Rocheteau and Wright (2005a), Lagos (2010b), Lester, Postlewaite, and Wright (2012), Jacquet and Tan (2012), Williamson (2012), Carapella and Williamson (2015), Geromichalos and Herrenbrueck (2016), Rocheteau, Wright, and Xiao (2016), and Geromichalos, Lee, Lee, and Oikawa (2016). Also, Aruoba, Waller, and Wright (2011) calibrate a money search model to examine how money supply affects capital formation in a similar way. However, to the best of my knowledge, the literature has not tested empirically the impact of money supply on the liquidity premia, and not investigated much how and under what conditions liquidity premia can cause the negative yields on liquid assets, either.

On the other hand, there are a few papers in the finance literature which study the bond supply effect on the liquidity premium, and present that bond supply has a negative impact on the liquidity premium, or the convenience yield.[6] For example, Krishnamurthy and Vissing-Jorgensen (2012) show a strong negative relationship between the U.S. Treasury supply and its convenience yield. Greenwood, Hanson, and Stein (2015) show that the T-bill supply has a negative impact on its liquid premium. Similarly, Longstaff (2004), Vayanos (2004), Brunnermeier (2009) and Krishnamurthy (2010) investigate liquidity premia, but focus on the short time periods such as financial crises. However, they all set up the models without money; therefore, they do not study the effect of money supply on liquidity premia even if liquid bonds can partially play a role as substitutes with money. As a result, the opportunity cost of holding money does not work to account for liquidity premia, either.

Nagel (2014) studies how this substitution relationship between money and liquid bonds affects the liquidity premium through variation in the opportunity cost of holding money, which is represented by the federal funds rate. The paper shows that the federal funds rate is positively correlated with the liquidity premium and that bond supply does not have a 'persistent' effect on the liquidity premium unlike Krishnamurthy and Vissing-Jorgensen (2012) and Greenwood, Hanson, and Stein (2015) present. However, I argue that this result would be derived from the fact that changes in the federal funds rate are involved with those in Treasury and money supplies at the same time, because the open market operations by the Federal Reserve are implemented by buying or selling of the Treasuries with money in the open market. It does not distinguish the effect of bond supply from that of money supply, and the effect of changes

[6]The convenience yield accounts for the premia for both safety and liquidity attributes of financial assets such as the U.S. Treasuries.

in money growth from changes in the level of money supply. Importantly, however, my money search model allows for separating these effects theoretically and empirically.

The rest of the paper is organized as follows. Section 2 lays out a theoretic model to be tested in Section 3. In Section 3, I provide a description of the data which are used in the empirical work and test the results from the theory. In Section 4, I discuss negative yields on liquid bonds with the theoretical and empirical results from the previous sections. Section 5 concludes.

2 Model

2.1 Physical Environment

Time is discrete and continues forever. There is a discount factor between periods $\beta \in (0, 1)$. Each period is divided into two sub-periods. A decentralized market (henceforth DM) with frictions opens in the second sub-period, and a perfectly competitive or centralized market (henceforth CM) follows. The frictions are characterized by anonymity among agents and bilateral bargaining trade. As a result, unsecured credit is not allowed in transactions, and exchange must be $quid\ pro\ quo$ or needs secured credit. There are two divisible and nonstorable consumption goods: goods produced and consumed in the CM (henceforth $CM\ goods$) and special goods in the DM (henceforth $DM\ goods$). There are two types of agents; buyers and sellers, whose measures are normalized to the unit, respectively. They live forever. Their identities are determined by the roles which they play in the DM and permanent. While sellers produce, sell and do not consume DM goods, buyers consume and do not produce. Their preferences in period t are given by

$$\begin{aligned} \text{Buyers}: \quad \mathcal{U}(x_t, h_t, q_t) =& u(q_t) + U(x_t) - h_t \\ \text{Sellers}: \quad \mathcal{V}(x_t, h_t, q_t) =& -c(q_t) + U(x_t) - h_t \end{aligned}$$

where x_t is consumption of CM goods, q_t consumption of DM goods, h_t hours worked to produce CM goods, and $c(q_t)$ a cost of production of q_t. As usual, U and u are twice continuously differentiable with with $U' > 0, u' > 0, U'' < 0, u'' < 0, u(0) = 0, u'(0) = \infty, u'(\infty) = 0$. Also, I assume that $c(q_t) = q_t$. Let $q^* \equiv \{q : u'(q) = 1\}$, i.e., it denotes the optimal consumption level in the DM. Also, assume that there exists $x^* \in (0, \infty)$ such that $U'(x^*) = 1$ and $U(x^*) > x^*$.

There are two types of assets; fiat money and a 1-period real government bond. They are perfectly divisible and storable. Agents can purchase any amount of money and government bonds at the ongoing price ϕ_t and ψ_t in the CM, respectively. Money grows at the rate of μ: $M_{t+1} = (1+\mu)M_t$. I assume that $\mu > \beta - 1$, but also consider the limit case where $\mu \to \beta - 1$, i.e., the case where the money growth rate approaches closely the Friedman rule. If μ is positive, it implies that new money is injected, but if μ negative, withdrawn through lump-sump transfers to buyers in the CM. A government bond issued in period t delivers one unit of CM good in period $t+1$, and its supply in period t is A_t. Since we focus on stationarity equilibria, A_t is fixed at A. The government (a consolidate authority) budget constraint is

$$G_t + T_t - \phi_t \mu M_t + A(1 - \psi_t) = 0,$$

where G_t is government expenditure, T_t is a lump-sum transfer or tax, $\phi_t \mu M_t$ is seigniorage of new money injection, and $A(1 - \psi_t)$ is government debt service.

Now, I describe more details about activities which occur in each sub-period. First, I start with the description of the second sub-period, where a CM opens. Both buyers and sellers consume and produce a CM good. They work or use their assets, money (m) and government bonds (a), which they are holding from the previous period in order to consume, so as to pay back the credit made in the previous period and to adjust their portfolios which they will bring to the next DM. They have access to technology that turns one unit of labor into one unit of general goods. Also, they trade money, and bonds among all agents to re-balance their portfolio they will bring to the next period.

Next, a DM opens in the first sub-period. All of the buyers are matched with a seller in a bilateral fashion and vice versa. Buyers make a take-it–or-leave-it (henceforth $TIOLI$) offer to a seller to determine the terms of trade.[7] Since buyers are anonymous and have limited commitment, a medium of exchange (henceforth MOE) is required in their transactions. Both money and government bonds can serve as media of exchange. Specifically, the DM is divided into two sub-markets, DM1 and DM2, depending on what type of medium of exchange can be used. In the DM1, sellers accept only a direct medium of exchange. Both assets are used as a direct medium of exchange, but, unlike money, only a fraction $g \in (0, 1)$ of bonds can serve as a direct medium of exchange. g is an illiquidity parameter of government bonds, and reflects the fact that the government bonds are not as liquid as money as a direct medium of exchange. Intuitively speaking, it can take time and cost in playing a role as money do in exchange in the DM1. On the other hand, in the DM2, sellers accept only collateralized credit (or loans), i.e., secured credit as a MOE, and bonds are used as collateral for credit. The credit is repaid back in CM goods in the forthcoming CM. Also, a portion $h \in (0, 1)$ of bonds can only be used as collateral; therefore, buyers always have incentives to pay back their credits. This is so-called the Loan to Value (henceforth LTV) ratio, and also is related to the haircut since it is defined by 1 minus the LTV, following the standard approach in finance. The model will focus on cases of incentive compatible contracts. All buyers and sellers visit $DM1$ and $DM2$ with probabilities θ

[7] I could assume that they negotiate the terms of trade through a Kalai bargaining protocol, where the buyers' bargaining power is less than one. However, since the bargaining protocol is not critical to derive most of interesting results of the paper, I use the simplest setup here by assuming that buyers make a TIOLI offer to their trading partners.

and $1-\theta$, respectively. Figure 1 summarizes the events within each period.

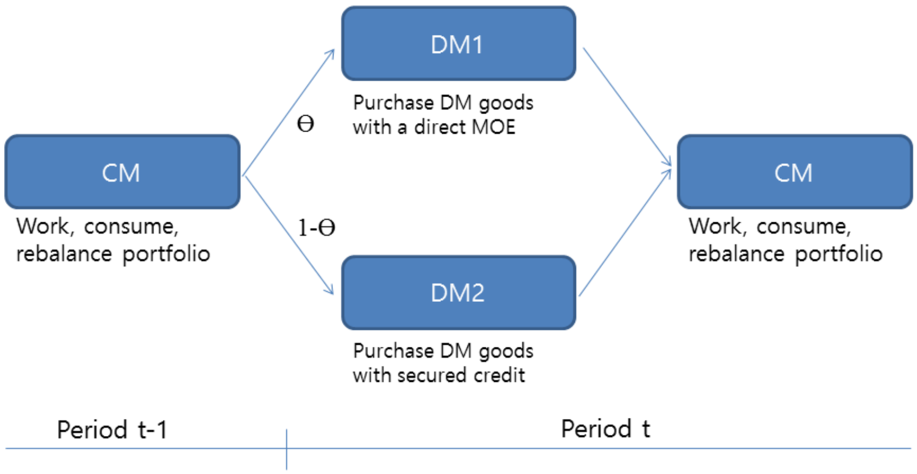

Figure 1: Market Timing

2.2 Value Functions

First, I describe the value function of a representative buyer who enters the CM with money (m), bonds (a) and the collateralized credit (ℓ) made last period, since it is the buyer that makes primary decisions for interesting results from the model. The value function of the buyer is

$$W^B(\mathbf{w}_t, \ell_t) = \max_{x_t, h_t, \mathbf{w}_{t+1}} \left\{ U(x_t) - h_t + \beta \mathbb{E}\left[V^B(\mathbf{w}_{t+1})\right] \right\} \quad (1)$$
$$\text{s.t. } x_t + \boldsymbol{\phi}_t' \mathbf{w}_{t+1} = h_t + \boldsymbol{\phi}_t \mathbf{w}_t - \ell_t + T$$

where $\mathbf{w}_t = (m_t, a_t)$, $\boldsymbol{\phi}_t' = (\phi_t, \psi_t)$, and $\boldsymbol{\phi}_t = (\phi_t, 1)$. ℓ_t stands for the collateralized loan which is made last sub-period, and so must be paid back in the form of general goods. T_t is a lump-sum transfer to the buyer. V^B represents the buyer's value function in the next period DM. It can be easily verified that $x_t = x^*$ at the optimum. Substituting h_t in the budget constraint into the value function W^B yields

$$W^B(\mathbf{w}_t, \ell_t) = \boldsymbol{\phi}_t \mathbf{w}_t - \ell_t + \Lambda_t^B \quad (2)$$

where $\Lambda_t^B \equiv U(x^*) - x^* + T_t + \max_{x_t, \mathbf{w}_{t+1}} \{-\boldsymbol{\phi}_t' \mathbf{w}_{t+1} + \beta \mathbb{E}\left[V^B(\mathbf{w}_{t+1})\right]\}$. Notice that the value function in the CM is linear in the choice variables due to the quasi-linearity of $\mathcal{U}$, as in the standard models which are based on Lagos and Wright (2005). Consequently, the optimal choices of the buyer do not depend on the current state variables.

Next, consider a representative seller with money, bonds, and the collateralized loan who enters the CM. The loan is paid back by the counterpart buyer who she met in the previous DM.

$$W^S(\mathbf{w}_t, \ell_t) = \max_{x_t, h_t} \left\{ U(x_t) - h_t + \beta \mathbb{E}\left[V^S(\mathbf{0})\right] \right\}$$
$$\text{s.t. } x_t = h_t + \boldsymbol{\phi}_t \mathbf{w}_t + \ell_t$$

where V^S denotes the seller's value function in the DM. Notice that $\mathbf{w}_{t+1} = \mathbf{0}$ for the seller. Since the seller does not consume any good in the DM, there is no incentive to bring money and bonds to the next period DM, when the money holding cost is strictly positive due to $\mu_t > \beta - 1$.[8] It is also easily verified that $x_t = x^*$ at the optimum as in the case of the buyer. Replacing h_t into the value function yields

$$W^S(\mathbf{w}_t, \ell_t) = \boldsymbol{\phi}_t \mathbf{w}_t + \ell_t + \Lambda_t^S \tag{3}$$

where $\Lambda_t^S \equiv U(x^*) - x^* + \beta \mathbb{E}[V^S(0)]$.

Next, the DM opens. Buyers visit the DM1 with the probability θ and the DM2 with the probability $1 - \theta$. Also, all agents match in each DM. Hence, the expected value function of a buyer with portfolio $\mathbf{w}_t$ in the DM is given by

$$V^B(\mathbf{w}_t) = \theta \left[u(q_t^1) + W^B(\mathbf{w}_t - \boldsymbol{p}_t, 0)\right] + (1 - \theta)\left[u(q_t^2) + W^B(\mathbf{w}_t, \ell_t)\right] \tag{4}$$

where $\boldsymbol{p}_t = (p_t^m, p_t^a)$ is a portfolio exchanged for DM goods in a meeting with a seller in the DM1, and ℓ_t is the collateralized loan made in the DM2. q_t^1 (q_t^2) represents the quantity that are traded in the DM1 (DM2). The terms of trades in each market are determined by bargaining in pairwise meetings which Section 2.3 describes.

The value function of a seller is similar except for the fact that the seller does not bring any money and bonds to the DM for transactions.

$$V^S(\mathbf{0}) = \theta\left[-q_t^1 + W^S(\boldsymbol{p}_t, 0)\right] + (1 - \theta)\left[-q_t^2 + W^S(\mathbf{0}, \ell_t)\right]$$

2.3 Bargaining Problems in the DM

There are two sub-markets in the DM: DM1 and DM2, depending on what type of means of payment can be used in transactions. First, consider a meeting in the DM1 where a buyer with portfolio $\mathbf{w}_t$ meets with a seller. Sellers accept both money and bonds as a medium of exchange. However, a fraction g of bonds can only be accepted. The terms of trade is determined by the proportional bargaining over the quantity of DM goods, and a total payment of money and bonds exchanged between them. A buyer makes a take-it-or-leave-it offer to a seller to

[8] See Rocheteau and Wright (2005b) for the precise and careful proof.

maximize her surplus under the seller's participation constraint and her budget constraint. Then, the bargaining problem is expressed by

$$\max_{q_t^1, \boldsymbol{p}_t} \left\{ u(q_t^1) + W^B(\mathbf{w}_t - \boldsymbol{p}_t, 0) - W^B(\mathbf{w}_t, 0) \right\} \tag{5}$$
$$\text{s.t.} - q_t^1 + W^S(\boldsymbol{p}_t, 0) - W^S(\mathbf{0}, 0) = 0,$$

and the effective budget constraint $\boldsymbol{p}_t \leq \widetilde{\mathbf{w}}_t$, and $\widetilde{\mathbf{w}}_t = (m_t, g \cdot a_t)$. Notice that, since bonds are not as liquid as money in the DM1, the effective budget is less than the total budget. Only a fraction $g \in (0, 1)$ of bonds can be used as a MOE here. Of course, I will consider an extreme case where $g \to 1$ later to discuss how negative interest yields emerge. Substituting (2) and (3) into (5) simplifies the above problem as follows.

$$\max_{q_t^1, \boldsymbol{p}_t} \left\{ u(q_t^1) - \boldsymbol{\phi}_t \boldsymbol{p}_t \right\} \tag{6}$$
$$\text{s.t.} - q_t^1 + \boldsymbol{\phi}_t \boldsymbol{p}_t = 0,$$

and $\boldsymbol{p}_t \leq \widetilde{\mathbf{w}}_t$, and $\widetilde{\mathbf{w}}_t = (m_t, g \cdot a_t)$. The following lemma summarizes the terms of trade which are determined by the solutions to bargaining problem.

Lemma 1. *The real balances of a representative buyer are denoted as $z(\mathbf{w}_t) \equiv \boldsymbol{\phi}_t \mathbf{w}_t$. Define $q^* = \{q : u'(q_t) = 1\}$, and z^* as the real balances of the portfolio (m_t, a_t) such that $\phi_t m_t + g a_t = q^*$. Also, $\mathbf{p}^*$ is the pairs of (m_t, a_t) in z^*. Then, the terms of trade are given by*

$$q_t^1(\mathbf{w}_t) = \begin{cases} q^*, & \text{if } z(\mathbf{w}) \geq z^*, \\ z(\widetilde{\mathbf{w}}_t), & \text{if } z(\mathbf{w}_t) < z^*. \end{cases} \quad \boldsymbol{p}_t(\mathbf{w}_t) = \begin{cases} \boldsymbol{p}^*, & \text{if } z(\mathbf{w}_t) \geq z^*, \\ \mathbf{w}_t, & \text{if } z(\mathbf{w}_t) < z^*. \end{cases} \tag{7}$$

Proof. See the appendix ☐

Similarly, in the DM2, a buyer makes a take-it-or-leave-it offer to a seller as in the DM1. However, she maximize her surplus subject to a different constraint, which is the credit limit constraint, unlike the effective budget constraint in DM1. Then, the bargaining problem is described as follows.

$$\max_{q_t^2, \ell_t} \left\{ u(q_t^2) + W^B(\mathbf{w}_t, \ell_t) - W^B(\mathbf{w}_t, 0) \right\} \tag{8}$$
$$\text{s.t.} - q_t^2 + W^S(\mathbf{0}, \ell_t) - W^S(\mathbf{0}, 0) = 0,$$

and the credit limit constraint $\ell_t \leq h a_t$. Substituting (2) and (3) into (8) yields the following expression.

$$\max_{q_t^2,\ell_t} \left\{ u(q_t^2) - \ell_t \right\} \tag{9}$$

$$-q_t^2 + \ell_t = 0, \tag{10}$$

and $\ell_t \leq ha_t$. The solution to the bargaining problem is described by the following lemma.

Lemma 2. *Define the total real value of a buyer's bond holdings as $z^a(\mathbf{w}_t) \equiv ha_t$. Also, define $z^{a*} \equiv q^*$. The terms of trade are given by*

$$q_t^2(\mathbf{w}_t) = \begin{cases} q^*, & \text{if } z^a(\mathbf{w}_t) \geq z^{a*}, \\ z^a(\mathbf{w}_t), & \text{if } z^a(\mathbf{w}_t) < z^{a*}, \end{cases} \quad \boldsymbol{\ell}(\mathbf{w}) = \begin{cases} z^{a*}, & \text{if } z^a(\mathbf{w}_t) \geq z^{a*}, \\ z^a(\mathbf{w}_t), & \text{if } z^a(\mathbf{w}_t) < z^{a*}. \end{cases} \tag{11}$$

Proof. See the appendix □

Since buyers make a TIOLI offer, i.e., they take all the bargaining power, the solution is straightforward. The main variables to determine the level of DM goods produced are the real balances, or the bond holdings of buyers in each transaction. For example, if the real balances are enough to get the optimal consumption level q^*, i.e., if $z(\mathbf{w}_t) \geq z^*$, then the optimal q^* level will be exchanged with the corresponding payment, z^*, which can be less than $z(\mathbf{w}_t)$. On the other hand, if the real balances are not enough in the same sense, then the buyers will hand over all of their real balances to sellers to purchase as many DM goods as possible. The sellers will produce the quantity that her participation constraint implies. The similar interpretation can be applied to the DM2.

2.4 Buyers' Optimal Choices

Now, I describe the objective function which a buyer maximizes by choosing money and bonds (m_{t+1}, a_{t+1}) in the DM. Substituting (4) into the inside of the maximization operator in (1) and using linearity of the value functions yield the following objective function J.

$$J = -\boldsymbol{\phi}_t'\mathbf{w}_{t+1} + \beta\Big\{\theta\big[u(q_{t+1}^1) + \boldsymbol{\phi}_{t+1}(\mathbf{w}_{t+1} - \boldsymbol{p}_{t+1})\big] + (1-\theta)\big[u(q_{t+1}^2) + \boldsymbol{\phi}_{t+1}\mathbf{w}_{t+1} - \ell_{t+1}\big]\Big\} \tag{12}$$

The first term stands for the cost of choosing money (m_{t+1}) and bonds (m_{t+1}) which buyers bring to the forthcoming DM. The terms in the curly bracket present the benefits they can obtain from transactions in the DM subject to their portfolios. Then, the Euler equations are given by

$$\phi_t = \beta \left[(1-\theta) + \theta u' \left(min\{\boldsymbol{\phi}_{t+1}\widetilde{\mathbf{w}}_{t+1}, q^*\}\right)\right] \phi_{t+1}, \tag{13}$$

$$\psi_t = \beta \Big\{ \theta \big[(1-g) + gu' \left(min\{\boldsymbol{\phi}_{t+1}\widetilde{\mathbf{w}}_{t+1}, q^*\}\right)\big] + (1-\theta)\big[(1-h) + hu' \left(min\{ha_{t+1}, q^*\}\right)\big] \Big\}, \tag{14}$$

The left-hand side on each Euler equation presents the marginal cost of buying a unit of money or government bond. It is equal to its price to be paid when a buyer purchase it in the CM. On the other hand, the right-hand side is the marginal benefit from holding it in the DM. Buyers can use them as a medium of exchange to purchase DM goods produced by sellers. If they are used in the DM, i.e., if u' is zero on the right-hand side, its price should be equal to its fundamental value, $\beta\phi_{t+1}$ or β, respectively.

Figure 2 presents the continuous and decreasing money demand against the cost of holding money captured by $\phi_t/(\phi_{t+1}\beta)$, which comes from equation (13). Similarly, inserting equation (13) into (14) shows the inverse bond demand curve against its price. Their inverse relationship makes sense because the bond price implies the cost of holding the bonds, given the fixed dividend in the forthcoming CM. Also, the bond demand curve depends on the cost of holding money. It is easily found that the curve shifts out (in) as the money holding cost increases (decreases) as in Figure 3. This relationship is intuitively straightforward to understand. If the money holding cost increases, agents become less willing to hold money, i.e., the money demand will decrease. However, since the government bonds can also play a role in relaxing the liquidity constraint in the DM to some extent as money does, even if not perfectly, the demand on the government bonds will increase. Notice that both demand curves are flat in the regions where $m \geq m^*$ and $ha_t \geq q^*$, respectively. This is because one extra unit of money or government bond is not useful in the DM transactions any more. In these territories, buyers already hold money or bonds enough to purchase q^*.

2.5 Equilibrium and Characterization

I focus on stationarity equilibria, in which both real money and bond balances are constant over time. It implies that $\phi_t M_t = \phi_{t+1} M_{t+1}$ and $A_t = A$. Then, the money growth rate is equal to the inflation rate in the CM, i.e., $1 + \mu = \phi_t/\phi_{t+1} = 1 + \pi$.

Definition 1. A steady state equilibrium is a list of real balances of buyers, $\tilde{z}_t = \phi_t M_t + gA$, and bond holdings $\tilde{z}^a = hA$, money and bond prices $\boldsymbol{\phi}'_t$, bilateral terms of trade in $DM1$: $q(\mathbf{w}_t)$ and $\boldsymbol{p}(\mathbf{w}_t)$ which are given by Lemma 1, and bilateral terms of trade in $DM2$: $q(\mathbf{w}_t)$ and $\ell(\mathbf{w}_t)$ which are given by Lemma 2 such that:

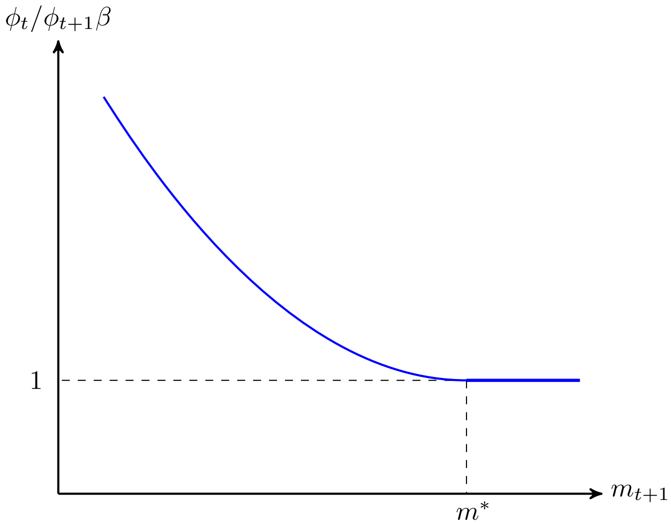

Figure 2: Money Demand

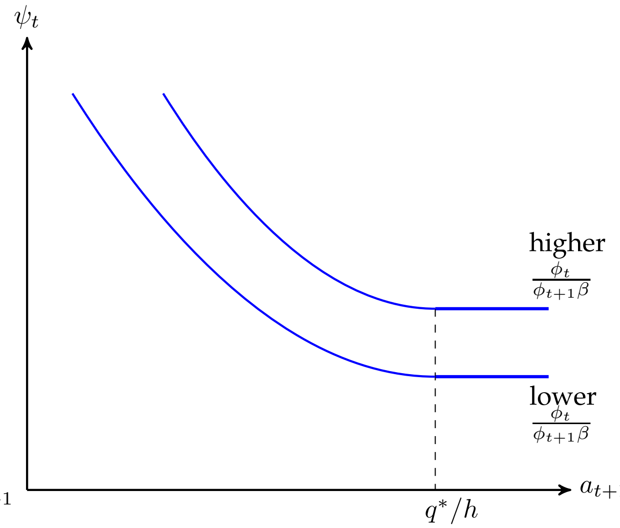

Figure 3: Asset Demand

(i) the decision rule of a representative buyer solves the individual optimization problem (1), taking prices ϕ_t' and $\phi_t/\phi_{t+1} = 1 + \mu$ as given;

(ii) the terms of trade in the DM satisfy (7) and (11);

(iii) prices are such that the CM clears, i.e., $\mathrm{w}_{t+1} = [\mu M_t, A]$ for buyers.

Then, the following lemma summarizes the equilibrium objects.

Lemma 3. *There exists a unique steady state equilibrium with four different cases.*

(i) If $\tilde{z}_t \geq z^*$ and $\tilde{z}^a \geq z^*$, then, $q_t^1 = q_t^2 = q^*$, $\phi_t = (z^* - gA)/M_t$, and $\psi_t = \beta$;

(ii) If $\tilde{z}_t \geq z^*$ and $\tilde{z}^a < z^*$, then, $q_t^1 = q^*$, $q_t^2 = \tilde{z}_t^a$, $\phi_t = (z^* - gA)/M_t$, and $\psi_t = \beta\{\theta + (1-\theta)\big[(1-h) + hu'(q_t^2)\big]\}$;

(iii) If $\tilde{z}_t < z^*$ and $\tilde{z}^a \geq z^*$, then, $q_t^1 = \tilde{z}_t$, $q_t^2 = q^*$, $\phi_t = (q_t^1 - gA)/M_t$, and $\psi_t = \beta\big\{\theta\big[(1-g) + gu'\left(q_t^1\right)\big] + (1-\theta)\big\}$;

(iv) If $\tilde{z}_t < z^*$ and $\tilde{z}^a < z^*$, then, $q_t^1 = \tilde{z}_t$, $q_t^2 = \tilde{z}_t^a$, $\phi_t = (q_t^1 - gA)/M_t$, and $\psi_t = \beta\big\{\theta\big[(1-g) + gu'\left(q_t^1\right)\big] + (1-\theta)\big[(1-h) + hu'\left(q_t^2\right)\big]\big\}$.

Proof. See the appendix. □

It is straightforward to understand the definition of equilibrium. The fact that the real money balances and the bond supply are constant over time in the steady state implies that both $\tilde{z}_t$ and $\tilde{z}^a$ are constant. Then, given the market clearing condition, $\tilde{z}_t$ and $\tilde{z}^a$ determine the quantities and real money and bond balances exchanged in the DM, following Lemma 1 and 2.

Now, the Euler equations, (13) and (14), for the optimal money and bond holdings with the above definition can be reexpressed as follows.

$$\phi_t = \beta\Big\{1 + \theta\left[u'\left(min\{\tilde{z}_{t+1}, q^*\}\right) - 1\right]\Big\}\phi_{t+1} \tag{15}$$

$$\psi_t = \beta\Big\{1 + \theta \cdot g\left[u'\left(min\{\tilde{z}_{t+1}, q^*\}\right) - 1\right] + (1-\theta)h\left[u'\left(min\{\tilde{z}^a, q^*\}\right) - 1\right]\Big\} \tag{16}$$

These two equations present how the equilibrium prices of money and bonds are determined. First, let's look at equation (16) for the real bond price which we are interested in. The current bond price (ψ_t on the left hand side) is equal to the sum of its fundamental value (β) and the weighted average of its marginal benefits in the DM1 and DM2. Each of these marginal benefits is called the $liquidity\ premium$. Similarly, the real money price is also determined by its fundamental value and liquidity premium. Now, in order to examine how the equilibrium bond price responds to changes in money and bond supply, let's plug (15) into (16), then the price is as follows.

$$\psi = \beta\Big\{1 + g(\frac{1+\mu}{\beta} - 1) + (1-\theta)h\left[u'\left(\tilde{z}^a\right) - 1\right]\Big\} \tag{17}$$

$$= \beta\Big\{1 + gi + (1-\theta)h\left[u'\left(\tilde{z}^a\right) - 1\right]\Big\} \tag{18}$$

where $i \equiv (1+\mu)/\beta - 1$. There are several interesting points to notice here. First of all, the last equation is obtained by the Fisher equation, because $\mu = \pi$ in the stationary equilibrium and $1/\beta = 1 + r$, where r stands for the yield on a real bond which is not useful in the DM exchange in the sense that it is not accepted by sellers. Hence, i represents a nominal interest rate of a totally illiquid real bond. Its real price (β), which is the inverse of the real interest rate $1/\beta$, is exactly equal to asset prices which are derived in the traditional asset pricing models where assets are only used as a store of value and their supply does not affect their prices: the asset prices equal the present discount value of their future stream of consumption dividends. It is worth noticing that both of money growth rate (μ) and the nominal interest rate (i) represent the opportunity cost of holding money at the end. Second, the price of a liquid bond (ψ) is always higher than that of a illiquid bond (β) when the asset supply is not enough in the sense that $hA < q^*$, i.e., $u'(\tilde{z}^a) > 1$. In other words, the rate of return on a liquid bond is lower than that on an illiquid, i.e., $\rho < i$.[9] Hence, it implies that the difference between them can be used to measure the market price of the liquidity service that the liquid bond provides, i.e., the liquidity premium. Lastly, the zero net nominal interest rate ($i = 0$) implies that the money growth equals the Friedman rule, i.e., $\mu = \beta - 1$ and thereby the money growth rate does not affect the bond price any more.

[9] To distinguish nominal yields between an illiquid bond and an liquid bond, let ρ denote the latter.

More importantly, equations (17) and (18) present that not only bond supply but also money growth rate determine the equilibrium bond price together with the bond demand when $0 < g < 1$, i.e., only when they are substitutes to some extent in the sense that bonds help to relax the liquidity constraint in the DM1 as money does. This implies that the demand on money and bonds are interconnected because both of them are useful in exchange process, to a greater and lesser extent, as in the papers in the money search literature such as Geromichalos, Licari, and Suarez-Lledo (2007) in which money and real assets are perfect substitutes, and Lester, Postlewaite, and Wright (2012) in which the illiquid parameter g is endogenized. It is important to notice that the price of the illiquid bond is fixed at β over time, whereas the price of the liquid bond varies with both money and bond supply. Hence, the price variation of the liquid bond is equal to the variation of the liquidity premium, and the two words can be used interchangeably when the bond price exceed the fundamental value.

Next, it is necessary to think about the existence of non-monetary equilibria. Not all $\mu \in (\beta - 1, \infty)$ are consistent with monetary equilibria. The next corollary describes the range of money growth rate for the monetary equilibria when $gA < q^*$.

Corollary 1. *A monetary equilibrium is supported on the range of* $(\beta - 1, \bar{\mu})$, *where* $\bar{\mu} \equiv \{\mu : \mu = \beta[1 + \theta[u'(gA) - 1]] - 1\}$.

Proof. See the appendix. □

First, if $gA = q^*$, $\bar{\mu} = 0$. In this economy, money does not play any role in providing liquidity services, and therefore there only exist the non-monetary equilibria. As μ increases, buyers reduce their money balances to optimize their choices. However, if the money growth rate is higher than the upper bond, $\bar{\mu}$, they do not hold any money for the transaction in the DM, because it lead to less consumption. On the other hand, if we allow for the case where $\mu = \beta - 1$, it implies $\tilde{z} \geq q^*$, and so will be the lower bound for a monetary equilibrium. In this case, the marginal change in money supply never affect the liquidity premium of a liquid bond. On the other hand, the upper bound $\bar{\mu}$ decreases in bond supply A. It implies that agents are less patient with high inflation, so that less willing to hold money given the supply of money, as the supply of bond increases.

The following proposition summarizes how the real equilibrium bond price, or the liquidity premium, is associated with money and bond supply. As mentioned in the introduction, we focus more on the monetary equilibria, where $\mu \in (\beta - 1, \bar{\mu})$.

Proposition 1. *The real bond price exceeds the fundamental value, i.e,* $\psi > \beta$, *as long as* $\mu > \beta - 1$. *Also, it is increasing in* μ *but decreasing in* A: $\partial\psi/\partial\mu > 0$ *and* $\partial\psi/\partial A < 0$.

The proof is straightforward. Notice in Proposition 1 that μ is also replaced with i because they are linear by the definition as in equation (18), and both of them stand for the money holding cost in the model. The real bond price exceeds its fundamental value because a liquid bond plays a role in facilitating transactions in the DM; otherwise would not occur. Hence, the liquid bond bears a liquidity premium. Money growth rate positively affects the liquidity premium. For example, high μ implies the high opportunity cost of holding money, so that it leads to an increase in the demand on bonds and ends up with the high bond price. On the other hand, bond supply negatively affects the liquidity premium, because the bond supply has an less impact on relaxing the liquidity constraint in the DM exchange as it increases. The marginal benefit decreases. In addition, if $\tilde{z}^a \geq q^*$, i.e., the bond supply is plentiful in that it allows agents to purchase the first best quantity, q^*, in the DM2 even though the probability of visiting DM2 $1-\theta$ is positive. The marginal increase in the bond supply does not allow buyers to purchase additional goods in the DM2 any more. As a result, changing the bond supply does not affect transactions in the DM2, but only in DM1; therefore it leads to the zero liquidity premium through DM2.

Now, consider now some extreme cases where money and bonds are perfect substitutes or not substitutes at all so as to understand intuitively how the parameters, g and $1-\theta$, can affect bond prices, or the liquidity premium. Moreover, this will help for better understanding about empirical tests, and allow us to check which case can be well supported by the U.S. data in Section 3. As mentioned in subsection 2.1, g is an illiquid parameter, implying how liquid bonds are, comparing with money in DM transactions. This parameter can be also interpreted as a parameter of how developed or how liquid a secondary asset market is, where bonds are exchanged for money. For instance, less friction in the secondary market implies higher g because it means that bonds are more easily converted to money, vice versa. In addition, if there are more investors or buyers for bonds due to the developed institution, including high-quality trading platform technology of secondary markets, bonds can be liquidated more easily, and so to provide liquidity services better. On the other hand, the parameter $1-\theta$ can be interpreted as how well the collateralized credit market functions, where money would not work, or how strong demand on liquid and high quality of bonds is in repo contracts. More collateralized transactions in the credit market can lead to higher $1-\theta$.

Now, consider the four cases as follows, depending on the different combinations of g and θ (or 1-θ). All the results are delivered by equations (17) and (18).

(*Case 1: Perfectly illiquid bonds*) Bonds are totally illiquid in the sense that they are useless in the DM exchange, i.e., $g \to 0$ and $\theta \to 1$. This is the case where the bonds only function as a store of value. Hence, the real bond price ψ is equal to the fundamental value, β, i.e., the present value of the dividend that the bonds deliver next period, and so they do not carry the liquidity premium at all. As a result, it is not affected by money and bond supply at all.

(*Case 2: Perfect substitutes to money*) Bonds are perfect substitutes to money and the DM2 does not exists, i.e., $g \to 1$ and $\theta \to 1$. The bond prices are equal to $1+\mu$. Then its nominal yield

ρ is give by

$$1 + \rho = (1 + \pi)\frac{1}{\psi} = \phi_t/\phi_{t+1} \times \phi_{t+1}/\phi_t = 1. \tag{19}$$

The gross nominal interest rate $(1 + \rho)$ equals 1 and the net nominal interest rate equals *zero* all the times. In this case, since the bonds are identical with money in terms of ability of facilitating transactions in the DM and additionally deliver dividends, their real prices are higher than the fundamental value β. Moreover, a high (a low) money growth rate causes high (low) real prices, or the liquidity premia.

(*Case 3: Liquid bonds but not substitutes to money*) Bonds are liquid in the DM, but not substitutes to money at all, i.e., $g \to 0$ and $0 < \theta < 1$. This is the case where the bonds are perfectly illiquid in the DM1, but liquid in the DM2. This implies that the two decentralized markets are totally separated. Hence, the supply of each does not affect each other, so that the liquidity premia which the bonds carry is only affected by their supply, not by the money growth rate.

(*Case 4: Liquid bonds and perfect substitutes*) Bonds are liquid in the DM2 and also perfect substitutes to money in the DM1, i.e., $g \to 1$ and $0 < \theta < 1$. Bonds carry extra values in exchange process. Then, equation (17) yield the net nominal interest rate as follows.

$$\rho = \frac{-\beta(1-\theta)h\left[u'(hA)) - 1)\right]}{(1+\mu) + \beta(1-\theta)h\left[u'(hA)) - 1)\right]} \tag{20}$$

$$= \frac{(1-\theta)h\left[u'(hA)) - 1)\right]}{i + (1-\theta)h\left[u'(hA)) - 1)\right]} < 0 \tag{21}$$

In this case, the numerator is always negative only if $u'(hA) > 1$, i.e., only if bond supply is scarce. Also, the liquidity premium is affected by both money and money supply. The nominal rate of return on a liquid bond is negative, irrespective of money and bond supply, or the nominal rate of return on a illiquid bond i. It implies that lenders are willing to pay interests to buyers even though it is lenders who take default risk on lending, because the bonds provide extra liquidity services in transactions. I will discuss more details about whether the liquidity properties of bonds can cause the negative yield or not in reality, and under what conditions the negative yields emerge in a generic case, unlike this extreme case, in Section 4.

3 Data and Empirical Results

The theory in the previous section predicts that the real price of a liquid bond, or the liquidity premium is affected by both money and bond supply in a general case where $0 < g, \theta < 1$, whereas that of an illiquid bond is not affected, as summarized in Proposition 1. In this section, I empirically test the predictions with the U.S. data: whether the liquidity premium is positively associated with the money growth rate, but negatively with bond supply in reality. Also, I will choose a nominal interest rate which can reflect the opportunity cost of holding money as the money growth rate does, and test whether it is positively related with the liquidity premium. Notice that I focus on the liquidity premium rather than the real price, because it does not seem to reach a consensus on how to measure the real price of a liquid and default-free bond in the literature, but when it comes to the liquidity premium, there exist proxies which are widely used in the literature. For this reason, Equations (22) and (23) described below are used as a guide for the empirical analysis.

3.1 Data

First of all, it is important to discuss how to measure all of the variables mentioned in the theory such as the liquidity premium, money growth rate (or the nominal interest rate of an illiquid bond), and bond supply from the data before we move on to the empirical results. As well known, the liquidity premia of liquid bonds are not observed in reality. Also, there exist several types of monetary aggregates, such as Monetary Base, Narrow Money, M1, and M2 which we can use to measure money supply, and a variety of liquid government bonds are traded in the financial market.

First, I describe how to measure the liquidity premium. The problem is that neither the real yield nor the liquidity premium can be obtained as officially published indexes. Only the nominal yield is observable. However, the yield spreads between two different bonds only in terms of liquidity can be used as a proxy of the liquidity premium as widely used in the relevant literature. Of course, they should have the same, or at least similar maturities and default risks. These measures are not only computational, but also appropriate to empirically test the predictions from the theory, because the real prices of liquid bonds are affected by money and bond supply through their liquidity premium, whereas the real prices of illiquid bonds are not. Also, this is why changes in the real yield on a liquid bond are equivalent to changes in the liquidity premium which the bond carries in the theory. For this reason, I choose pairs of liquid and illiquid bonds of the same or similar maturity and safety.

The theory presents the nominal yield of an illiquid bond (i), a liquid bond (ρ), and the spread (s) between them as follows.

$$1+i = (1+\pi)(1+r) = (1+\pi)\frac{1}{\beta}$$
$$1+\rho = (1+\pi)\frac{1}{\psi}$$
$$s = i-\rho \approx \psi-\beta = \left[1+g(\frac{1+\mu}{\beta}-1)\right] + (1-\theta)h\left[u'\left(\tilde{z}^a\right)-1\right] \quad (22)$$
$$= 1+gi+(1-\theta)h\left[u'\left(\tilde{z}^a\right)-1\right], \quad (23)$$

where r stands for the real yield on an illiquid bond. The first two equations present the nominal yields of an illiquid bond and a liquid bond by the Fisher equation, respectively. Then, the log difference between them is approximately equal to the yield spread between them, which is given by Equations (22) and (23). Notice that this difference eliminates the effect of inflation on the nominal yields of both bonds at the same time, and therefore delivers the real yield differential in two different bonds in terms of liquidity, i.e., the liquidity premium. Notice that I attempt to eliminate other components such as the risk premium and the term premium in bond prices by choosing the pairs of bonds of same or similar maturities and default risks.

Then, what types of bonds can represent liquid and illiquid bonds in reality? First, when it comes to a liquid bond, we define a liquid bond in the model as a bond which is useful in exchange process. In reality, it implies that the liquid bond should be easy to be sold for cash in secondary asset markets, to be accepted directly as a medium of exchange, or to be used for credit (or loans) in the credit markets such as the Repurchase Agreement market (or REPO in short) and the collateralized federal funds market. Moreover, it should be safe in the sense that it is sure to deliver its dividend at maturity, i.e., there is no probability to default before maturity. For this reason, here, I use the yields of U.S. Treasuries such as Treasury bonds, notes and bills, as the nominal yields of the liquid bond meant by the model. They all represent liquid bonds with different maturities.

On the other hand, an illiquid bond in the model implies a bond which can not be used in exchange, and its holder should keep it in his hand until maturity for cash unless she accepts a huge discount for the secondary trade. Hence, it is inferior to the liquid bond only in terms of being liquidated or relaxing her liquidity constraint. Of course, they should be exactly or, at least, similarly as safe as the liquid bond to avoid the case where their yield difference includes the risk premium. In the case where there is only a small difference in terms of the default risk, it could be controlled in regressions by adding variables to explain it, even though it is not perfect. Similarly, the maturities should be matched for both bonds.[10] Taking all of these

[10] In reality, since there exist secondary trade markets for almost all of bonds only if sellers of bonds accept more or less, or even considerable losses, there would not exist a perfectly illiquid bond. In other words, it is hard to find totally illiquid bonds, even if they are almost as safe as Treasuries and have the same or similar maturities. However, it cannot be denied that the yield spread reflects the liquidity premium, even if we consider it.

into account, I use the yields of the long term Aaa-rated corporate bonds, 3-month Commercial Papers, Federal Deposit Insurance Corporation (FDIC) insured Certificates of deposits (henceforth FDIC CDs) as measures of the yields of illiquid bonds. When it comes to these data, I almost use the data set provided by Krishnamurthy and Vissing-Jorgensen (2012)[11]. I match each of those yields with Treasuries with consideration for maturity in order to compute the liquidity premium on each maturity.

Lastly, I also use the TED spread as a measure of the liquidity premium on three-month Treasury bills, which is the difference between three-month LIBOR based on US dollars and three-month Treasury bill. This spread is used frequently as a measure for the liquidity premium in the literature. It is true that 3-month LIBOR based on US dollars bears the risk premium, because the contracts between banks are not default-free. However, it is also true that it is small. I try to control the default risk by adding a variable to represent default risk in the regression.

Next, let's consider which of monetary aggregates should be used to measure money supply in the data. As mentioned above, there are several monetary aggregates which are complied by the Fed: Monetary Base, Narrow Money, M1, M2, and M3. However, there are two criteria for which one among them is appropriate. First, what the theory regards as money is perfectly liquid in exchange process, or it is a perfect medium of exchange, comparing to bonds. Hence, the monetary aggregate meant by the theory should not include any type of illiquid financial assets such as savings deposits including money market deposit accounts and small-denomination time deposits, which is precisely defined as time deposits in amounts of less than \$100,000.[12] Then, this criterion excludes M2, and also broader monetary aggregates such as M3. Secondly, its demand against the opportunity cost of holding it, which can be represented by nominal interest rates (or the inflation rate), should be stable. They should have a stable negative relationship. Otherwise, the mechanism through which the theory works could not be applied to explain the relationship between money supply and the liquidity premium. Specifically, it should be true that, when the opportunity cost of holding money rises up, the demand declines. If it is not true, the high money holding cost will never lead to an increase in the demand on liquid bonds as a substitute to money, and neither does the liquidity premium the bonds bear in the end. It turns out that the demand on M1 against the nominal interest rate has not been stable, so that M1 is excluded in the regressions shown later.[13]

As a result, I use Narrow Money as a measure of money, based on these criteria. Narrow Money is well suited to the theory in the sense that it is absolutely used as a medium of ex-

[11]I updated the yields of Aaa-rated corporate bonds (series AAA), and 3-month Commercial Papers (series $CP3M$ and $CP3M$) in their dataset with the data FRED Economic Data (https://fred.stlouisfed.org/) provides because some values are revised. See the appendix in Krishnamurthy and Vissing-Jorgensen (2012) for more details.

[12]Source: http://www.federalreserve.gov/releases/h6/current/default.htm

[13]See Lucas Jr. and Nicolini (2015) for details about the stability of M1. The paper investigates why monetary aggregates become unstable over time.

change in transactions.[14] Also, it presents a stable demand curve over the sample period, i.e., a unambiguously negative relationship with the nominal interest rate over the period from 1946 to 2008. Notice that I use the federal funds rate as a proxy of the opportunity cost of holding money, which is the nominal interest rate on an illiquid bond in the theory. I could use other interest rates such as 3 month commercial paper rate, but they deliver the same relationship because all the short term interest rates, including 3 month commercial paper rate, show the strong co-movement historically. Figure 4 displays the ratio of Narrow Money to nominal GDP against its holding cost (i), which implies $L = M/PY$ in order to look at the real demand on money or real money balances proportional to Y implied by Equation (15). In the case where bond supply is not plentiful to achieve the first best outcome in the DM, it can be re-expressed to present the money demand in the equilibrium as follows.

$$\frac{\phi_t}{\phi_{t+1}\beta} = 1 + \theta \left[u'(\tilde{z}) - 1 \right]$$
$$\Leftrightarrow 1 + i = 1 + \theta \left[u'(\tilde{z}) - 1 \right],$$

where $\frac{\phi_t}{\phi_{t+1}\beta} = 1 + i$.[15] Obviously, the graph on the left panel presents that the real balances $\tilde{z}$, or money demand $\hat{m}$ is negatively associated with a nominal interest rate, or the opportunity cost of holding money, which is represented by the federal funds rate.

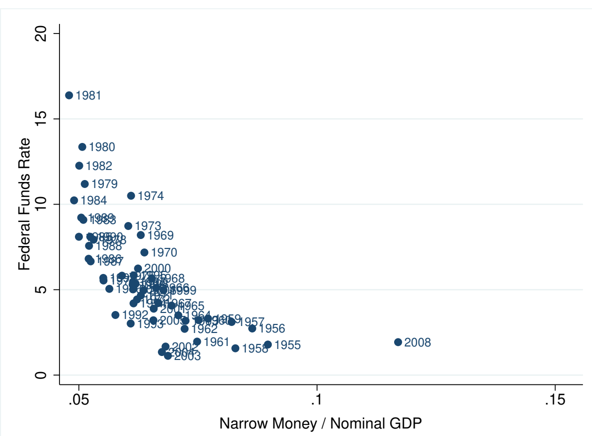

Figure 4: Money Demand

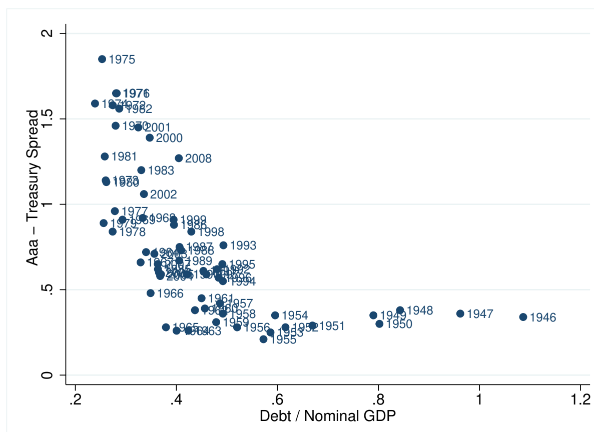

Figure 5: Bond Demand

Also, notice that money growth rate has a one to one relationship with the nominal yield of an illiquid bond in the stationary equilibrium, which is implied by the Fisher equation, i.e., $1+i = (1+\pi)(1+r) = (1+\mu)/\beta$, as seen in Equations (22) and (23). They theoretically stand for the same economic notion: the opportunity cost of holding money. The theory does not distin-

[14]Narrow Money includes nonbank public currency used as a medium of exchange, deposits held at Federal Reserves Banks, and reserve adjustment magnitude, which is"adjustments made to the monetary base due to changes in the statutory reserve requirements". See http://research.stlouisfed.org/publications/review/03/09/0309ra.xls for more details.

[15]Since $\tilde{z}_t = \tilde{z}_{t+1}$ in stationary equilibria, the time subscript is omitted.

guish between them. The empirical result can show which variable is more suitable to explain changes in the liquidity premium. The nominal interest rate as an index of the opportunity cost of holding money has a positive impact on the liquidity premium through the conceptually same mechanism in which the money growth rate works. In reality, there are a variety of interest rates in the financial market, and it is also difficult to find the yields of totally illiquid bonds. However, as well known, they have the strong co-movement relationship among them. Here, I use the federal funds rate as a proxy of the nominal interest rate which the model presents. The federal funds rate can reflect the money holding cost better than any other, because it is highly correlated with the short term interest rates of other financial assets which agents in an economy can consider as substitutes for cash, even if they are not perfectly substitutes. Moreover, since it is the policy interest rate which the Fed uses, it is comparable to money supply as a policy variable.

When it comes to bond supply, I use the ratio of the outstanding stock of the public debt to the nominal GDP for the liquid real bond supply meant by the theory, as in Krishnamurthy and Vissing-Jorgensen (2012). The ratio of debt to GDP is measured as the market value of the public debt at the end of a fiscal year divided by the GDP of the same year.[16] Using the same data as in Krishnamurthy and Vissing-Jorgensen (2012) and Nagel (2014) allows for comparison of my empirical results with the results in the literature.

Last but not least, Figure 5[17] looks at the bond demand against the yield spread between the Aaa-rated corporate bond and the long term Treasury bond. Notice that, as Krishnamurthy and Vissing-Jorgensen (2012) points out, it is the bond demand for not only liquidity but also safety, but it is mainly driven by the demand for the liquidity services which Treasuries provide. It had been stable up until 2008 in the sense that it is an unambiguous downward sloping curve. However, after 2009, the demand curve seems to shift out after the recent financial crisis. This is similar to the money demand curve. For this reason, I only use the data over the period from 1945 up to 2008.

3.2 Empirical Results

Now, I present the empirical test results for the theoretical predictions. As mentioned in the previous section, I first compute several different yield spreads between illiquid bonds and Treasuries, matching them according to their maturities: the yield spreads between Aaa-rated corporate bonds and the long term Treasuries, between Aa-rated commercial papers and 3-month Treasury bills, and between 6-month FDIC insured Certificates of Deposit (henceforth CD) and 6-month Treasury bills. The last two Treasuries are short-term bonds in the sense that

[16]See Henning Bohn's website for more details: http://econ.ucsb.edu/ bohn/data.html

[17]Source: Krishnamurthy and Vissing-Jorgensen (2012)

their maturities are shorter than one year.[18] Each of the spreads stands for the liquidity premium which each of different maturities of Treasuries bears. It implies that all of these spreads reflect the market values for the liquidity services the Treasuries provide, but different in the sense that the former is the liquidity premium on the long term Treasuries and the latter on the short term Treasuries.[19]

Next, it is important that I use the growth rate of Narrow Money to represent money supply in the regressions. As seen in Equation (15), what affects liquidity premia is the growth rate of money supply, not the absolute level of money supply. it implies that a one-time change in money supply does not affect the liquidity premia and also other real variables such as real balances, quantities traded in the DM, but its growth rate does; money supply is not super-neutral but neutral. A one-time injection or withdraw of money to an economy is ineffective because its relative price is adjusted to keep the real variables unchanged. On the other hand, I use the level of the debt to the nominal GDP ratio for bond supply in the regressions, because it is not neutral unlike money. Even a one-time change in bond supply affects the real variables.

Also, in order to control the risk premium, associated with default risks, which can be included in the yield spreads calculated above, a stock market volatility index is used as a default control variable which is used in Krishnamurthy and Vissing-Jorgensen (2012).[20] Including this measure in the regressions attempts to control default risks so that changes in the yield spreads are driven mainly by changes in the liquidity premia, given the low default rate on Aaa-rated corporate bonds and Aa-rated Commercial Papers.[21] However, unlike these two securities, FDIC insured CDs are as safe as Treasuries, given FDIC insurance, so that its spread against the same maturities of Treasuries can be used as a good proxy of the liquidity premium.

Before the regressions, I graphically look at how money growth rate had evolved with the different measures of the liquidity premium over the sample period from Figures 6 and 7. It seems to be consistent with the predictions from the theory. The figures show that the movement of money growth is almost positively related to the liquidity premium, and their variations are also similar in terms of frequency and width.

[18] I use the spreads which are used in Krishnamurthy and Vissing-Jorgensen (2012) but slightly different because they are updated from the original source.

[19] Notice that there are two indexes which represents the opportunity cost of holding money meant by the theory: money growth rate and nominal interest rates. Even though they are theoretically considered equivalently by the Fisher equation and the definition of the stationary equilibrium, I will show that there can be cases where only one of them significantly affect long or short term liquidity premia in the data. In fact, Nagel (2014) only presents the cases where the federal funds rate has a positive effect on the liquidity premia, which are measured monthly only by some short term bonds. However, here I present not only that the liquidity premia are also affected by money growth, but also how it is robust to the liquidity premium of the long term bond such as the long term Treasuries.

[20] See Krishnamurthy and Vissing-Jorgensen (2012) for the details about why this measure can be a proxy for default risk. In short, they argues that this measure have a high correlation with another default risk measure such as the median expected default frequency credit measure from Moody's Analytics.

[21] See Krishnamurthy and Vissing-Jorgensen (2012) for details. According to them, "there have never been a default on high-grade CP." Also, they use the spread of Aaa-rated bonds against Treasuries to estimate the market value of the liquidity convenience, assuming that the default risk of the Aaa-rated bonds is low.

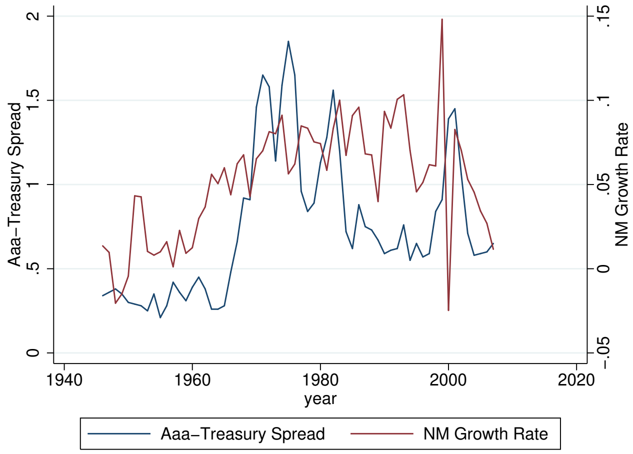

Figure 6: Aaa - Treasury Spread

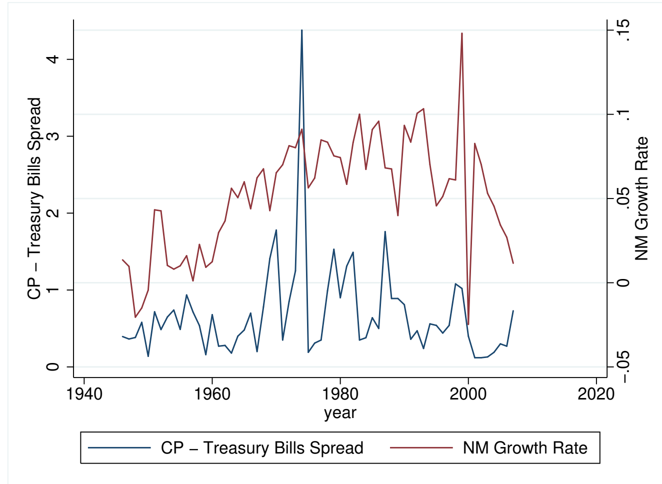

Figure 7: CP - T-Bill Spread

Now, let's move onto the empirical test results from the regressions. Table 1 presents the impacts of money growth, bond supply, and the federal funds rate on the liquidity premium, which is measured by the yield spreads between the Aaa-rated corporate bond and the long term Treasury bond, and between FDIC CDs and 6-month Treasury bills. Specifically, Regressions (1) to (3) look at the impact of money growth and bond supply on the liquidity premium which the long-term Treasuries carry. First, the regressions test whether Treasury bonds are substitutes with money or not. The theory predicts that if they are not substitutes, money growth does not affect the liquidity premium of Treasuries at all, because the money-traded market are totally separate from the bond-collaterlized market. In fact, the regression results deliver that money growth has a significant and positive impact on the liquidity premium. In particular, its coefficient is 0.56 in Regression (3). Also, bond supply is negatively correlated with the liquidity premium, and its coefficient is -1.33. An increase in bond supply reduces its market price for the liquidity services which the bonds provide.[22] Importantly, all of these results present evidence that Treasury bonds are substantive substitutes with money to some degree in that they provide liquidity services like money does. Moreover, the results are robust to default risk. A control variable for default risk is included as an explanatory variable in Regression (3), and the coefficients of money growth and bond supply are still significant. It is meant to be statistically robust that the liquidity premium is significantly affected by both of them. The coefficient of the default risk control variable also appears significant. On the other hand, Regression (4) to (5) show the impact of the federal funds rate on the liquidity premium. Unlike money growth, its impact on the liquidity premium appears insignificant in the regressions with bond supply, even though the federal funds rate as a proxy of the nominal interest rate is equivalent theoretically to the money growth, because both of them stand for the opportunity cost of holding

[22]The negative effect of bond supply is consistent with the results in Krishnamurthy and Vissing-Jorgensen (2012) even without the log specification. They use the log specification in their regressions because it provides a good fit and there is only one parameter they are interested in.

money.

Regressions (6) to (9) also support the previous test result. The FDIC CDs as an illiquid financial security are as safe as Treasuries, and thereby its spread with Treasuries reflects the difference in their liquidity premia. Since the FDIC CDs do not have default risk, given FDIC insurance, this yield spread is a more precise measure of the liquidity premium than the other measures. The result confirms the existence of the liquidity premium, even in regression (8) with a default risk control variable: money growth has a positive impact on the liquidity premium and the Treasury supply has a negative impact. Regression (8) presents that a one percentage point in the money growth rate is associated with an increase of 2.39 bps in the liquidity premium in 6-month Treasury bills, whereas a one percentage point in bond supply with a decrease of 4.49 bps. However, the effect of the federal funds rate turns out to be insignificant as in Regression (5).

Table 1: Impact on the Liquidity Premium of Long-Term and 6-Month Treasuries

Dependent Vars	AAA Corp. - Treasury					FDIC CDs - T-Bills			
	(1) 1946-2008	(2) 1946-2008	(3) 1946-2008	(4) 1955-2008	(5) 1955-2008	(6) 1984-2008	(7) 1984-2008	(8) 1984-2008	(9) 1984-2008
NM Growth	1.648**	0.917***	0.557***			2.378***	2.535***	2.389***	
	(0.735)	(0.244)	(0.160)			(0.249)	(0.231)	(0.136)	
Debt to GDP		-1.496***	-1.326***	-3.795***	-3.188***		-5.471***	-4.494**	-5.074***
		(0.421)	(0.295)	(0.710)	(0.745)		(1.584)	(1.768)	(1.777)
Federal Funds Rates				0.0219	-0.253				3.657
				(1.621)	(1.522)				(4.626)
Volatility			4.495***		3.484***			2.219	
			(0.893)		(0.903)			(1.704)	
Constant	0.654***	1.348***	0.690***	2.258***	1.567***	-0.0569	2.151***	1.446	2.026**
	(0.0833)	(0.205)	(0.211)	(0.366)	(0.434)	(0.134)	(0.633)	(0.892)	(0.785)
Observations	63	63	63	54	54	25	25	25	25
Adjusted R-squared	0.092	0.417	0.610	0.542	0.661	0.272	0.551	0.645	0.206

Notes: Coefficients are estimated by Newey-West estimators with lag(1) and its standard errors are presented in parenthesis. The dependent variables are the yield spreads between private and Treasury bonds, which are measured in a percentage unit. Explanatory variables are the growth rate of Narrow Money and the ratio of the market value of Treasury debt outstanding to nominal GDP. A control variable for the default risk on private assets is $Volatility$, which is measured by annualized standard deviation of weekly log stock returns on the $S\&P$ 500 index (Source: KVJ 2012). *** p<0.01, ** p<0.05, * p<0.1.

Next, Table 2 uses different measures of the liquidity premia: the yield spreads of Aa-rated Commercial Papers against Treasury bills, and the TED spread. All the regressions present that the impact of money growth is significantly positive and is robust with bond supply and default risk controls. Also, these results strongly support the theoretical prediction that Treasury bills are substitutes with money like Treasury bonds, again.[23] A one percentage point rise in the money growth rate causes an increase of around 1.2 bps in the liquidity premium of 3 month Treasuy Bills, according to Regressions (3) and (7). On the other hand, Regressions (4), and (8) present whether federal funds rate affects the liquidity premium in the case where bonds are substitutes with money as in Regressions (3), and (7). The federal funds rate has a significantly

[23]The regressions with the quarterly data over the same period present the similar result. See Appendix B for the regression results.

positive impact on the liquidity premia, whereas bond supply does not have a significantly negative effect.[24]

To summarize, money growth has a significant and positive impact on the liquidity premia, whereas bond supply does negative as the theory predicts. Also, the federal funds rate has a significantly positive effect on it in most of the regressions. Even if money growth and the nominal interest rate which the federal funds rate stands for are theoretically equivalent in the sense that both of them represent the opportunity cost of holding money, they seem not to be equivalent empirically all the times. We can find that the effect of the federal funds rate appears insignificant in the regressions where the liquidity premia of the long-term Treasuries and the 6-month Treasuries are used as the dependent variables. These results may come from the fact that different maturities of Treasuries are traded in different markets, by different agents, or under different regulations, even though they provide similar liquidity services. Hence, it would not be a surprise that they can differently respond to various variables which have the same economic meaning. However, notice that this does not change the economic mechanism of how money or nominal interest rates affect the liquidity premia which liquid bonds carry. All the results provide strong support the theoretical predictions: bond prices bear the liquidity premium, bonds are substantive substitutes with money even if not perfect, and the money holding cost is a primary factor in the mechanism which deliver the aforementioned results.

Table 2: Impact on the Liquidity Premium of 3-Month Treasury Bills

Dependent Vars	AA CP - T-Bills				TED spread			
	(1) 1946-2008	(2) 1946-2008	(3) 1946-2008	(4) 1955-2008	(5) 1986-2008	(6) 1986-2008	(7) 1986-2008	(8) 1986-2008
NM Growth	1.681***	1.256***	1.168***		1.341***	1.343***	1.209***	
	(0.543)	(0.247)	(0.298)		(0.194)	(0.187)	(0.224)	
Debt to GDP		-0.871*	-0.829*	-0.564		-0.0543	0.980	0.212
		(0.471)	(0.430)	(1.037)		(1.535)	(1.665)	(1.127)
Volatility			1.097	0.271			2.010	1.608
			(1.722)	(1.344)			(1.464)	(1.521)
Federal Funds Rate				9.472***				9.23*
				(3.230)				(4.680)
Constant	0.586***	0.990***	0.829***	0.362	0.565***	0.588	-0.120	-0.082
	(0.0784)	(0.272)	(0.254)	(0.435)	(0.107)	(0.668)	(0.808)	(0.447)
Observations	63	63	63	54	23	23	23	23
Adjusted R-squared	0.053	0.107	0.111	0.254	0.180	0.139	0.147	0.233

Notes: Coefficients are estimated by Newey-West estimators with lag(1) and its standard errors are presented in parenthesis. The dependent variables are the yield spreads between private and Treasury bonds, which are measured in a percentage unit. Explanatory variables are the growth rate of Narrow Money and the ratio of the market value of Treasury debt outstanding to nominal GDP. A control variable for the default risk on private assets is $Volatility$, which is measured by annualized standard deviation of weekly log stock returns on the $S\&P$ 500 index (Source: KVJ 2012). *** p<0.01, ** p<0.05, * p<0.1.

[24]In fact, this result is consistent the results which Nagel (2014) delivers. The paper argues that there is no significant impact of the bond supply on the time-varying liquidity premium. However, it examines different measures of the liquidity premium which are not used here.

4 Discussion on Negative Interest Rates

Negative Treasury yields have been observed in some countries such as Switzerland, the United States, Japan and Germany for the recent years. In particular, since the European Central Bank lowered its target rate to below zero, not a few European countries have been experiencing the negative yields on their government bonds, which includes both short and long term bonds. Since the negative target rate accompanies with the negative interest rate on the excess reserves deposited in their central banks, the government bonds can be as a safe store value to investors who hold a large denomination of excess money to their hands, if the target interest rate is negative. This is the case for most of countries which have been observing the negative yields on their governments bonds. However, we could also observe two countries where the negative Treasury yields were observed, even though their monetary policy rates were positive: Switzerland and the United States. In Switzerland, the yields of almost all the government bonds have been negative after 2008. Also, the secondary market yield of 3 month Treasury bills in the United States had been negative for several days in September, 2015, even if the federal fund rates were slightly positive. In reality, these two governments bonds are treated similarly in the sense that investors regard them as safe assets, and thereby as liquid assets domestically and internationally,[25] unlike other government bonds.[26] Hence, I use the theory and the data to examine whether the asset (or bond) liquidity can cause the negative yields, and if so, under what conditions it can do.

Intuitively, a negative bond yield implies that a bond buyer or lender pays a bond issuer or borrower interests on her lending. It wouldn't make sense if the interest rate were considered as the risk premium which is compensated for the borrower' default risk, because it is the lender that takes the default risk of the loan. On the other hand, money is also a liability issued by a government (a consolidated authority) as government bonds, and therefore money is unambiguously as safe as the government bonds. The existence of money implies that the lender could hold cash, as an perfect alternative or substitute to bonds, whose interest rate is 0%, i.e., non-negative. Or, if the lender were a financial institution, it could hold money as a reserve deposit its central bank, because it pays a positive interest. Hence, I argue here that this negative Treasury yields can be caused by liquidity properties of government bonds, rather than by safety properties, and so I use the theory set up in the previous section and the relevant data to explore the possibility of asset liquidity based rationale of the observed negative yields.

[25]According to Aleks Berentsen, Swiss government bonds can be used as collateral in some markets outside of Switzerland but where the Swiss franc cannot like American government bonds.

[26]The yields of both Swiss and American government bonds fell down after the 2008 financial crisis started, whereas the yields of other government bonds rose up a lot because of their default risk.

As shown in the previous section, the yield of a liquid bond from the model is given by

$$\rho = \frac{(1-g)\left[\frac{1+\mu}{\beta}-1\right]-(1-\theta)h\left[u'(z^a)-1\right]}{(1-g)+g\frac{1+\mu}{\beta}+(1-\theta)h\left[u'(z^a)-1\right]} \tag{24}$$

$$= \frac{(1-g)i-(1-\theta)h\left[u'(z^a)-1\right]}{1+gi+(1-\theta)h\left[u'(z^a)-1\right]}. \tag{25}$$

The following proposition summarizes under what conditions a bond's liquidity can cause the negative yield.

Proposition 2. *The yield on a liquid bond falls below zero under the following conditions, and it is caused by bond liquidity.*

(i) If $1-\theta = 0$, the yield on a liquid bond is negative, only when $i < 0$.

(ii) If $1-\theta \neq 0$, the yield on a liquid bond can be negative, even when $i \geq 0$,

- *a) if $i = 0$ and $u'(z^a) > 1$, i.e., the nominal yield on an illiquid bond is zero and the liquidity premium is positive, or*
- *b) if $i > 0$ and $i < (1-\theta)h\left[u'(z^a)-1\right]/(1-g)$., i.e., a certain portion of the marginal liquidity service benefit provided by a liquid bond is greater than the marginal cost of holding money.*

The proof is straightforward from equation (25) and thereby is omitted here. The theory predicts the following interesting results. First of all, if there do not exist financial markets such as Repo markets where there exists strong demand on liquid bonds for their liquidity services, called DM2 in the model, negative yields on liquid bonds (or, government bonds) can not be negative even when i, as a proxy of the money holding cost, is positive. Specifically, the DM2 is the market where government bonds are used in Repo contracts as collateral, but would not money. If we think about i as a monetary policy rate or other short term interest rates, the cases of Switzerland and the U.S. would not happen, because the monetary policy target range in Switzerland had been positive until December in 2014[27] and the federal funds rate has never been negative so far. However, if there exist DM2, i.e., $1-\theta \neq 0$, the negative yield can emerge, in particular, when bond supply is scarce and the monetary policy rate is low: money is redundant and therefore the opportunity cost of holding money is low. In other words, the negative yield can appear even in the situation where the policy rate is positive. This implies that the liquidity services which liquid bonds provide are valued high relatively to the money holding cost. As a result, bond buyers become willing to hold the bonds with negative yields in their portfolios and to use in transactions.

Interestingly, the case of Switzerland is exactly consistent with the predictions of the theory. As mentioned above, we could observe the negative Treasury yields even during period

[27] It is fixed at 0 - 1.00% on 12/11/2008, 0 - 0.75% on 3/12/2009, 0 - 0.25% on 8/3/2011, -0.75 - 0.25% on 12/18/2014, and -1.25 - -0.25% on 1/15/2015.

when its monetary policy target was positive. As shown in Figure 8, the negative yields on government bonds emerged around 2010 for the first time. Also, it presents huge changes in money and the government bond supply around the 2007-2008. For example, the ratio of the government bond supply relative to GDP was higher than 50%, but fell down to around 30%, whereas the money supply, measured by M1[28], relatively increased more than twice during the same period, and also it grew at a higher rate. Moreover, the interest target range of the Swiss National Bank was 0-1.00% at then end of 2008, and then, continued to hover around zero. According to both theoretical and empirical results, all of these data strongly suggest the idea that these two changes would increase the liquidity premia, and also cause the negative Treasury yields. In other words, the relative scarcity of the liquid government bonds against money in the market would have led to the negative yields through an increase in the liquidity premia on the government bonds.

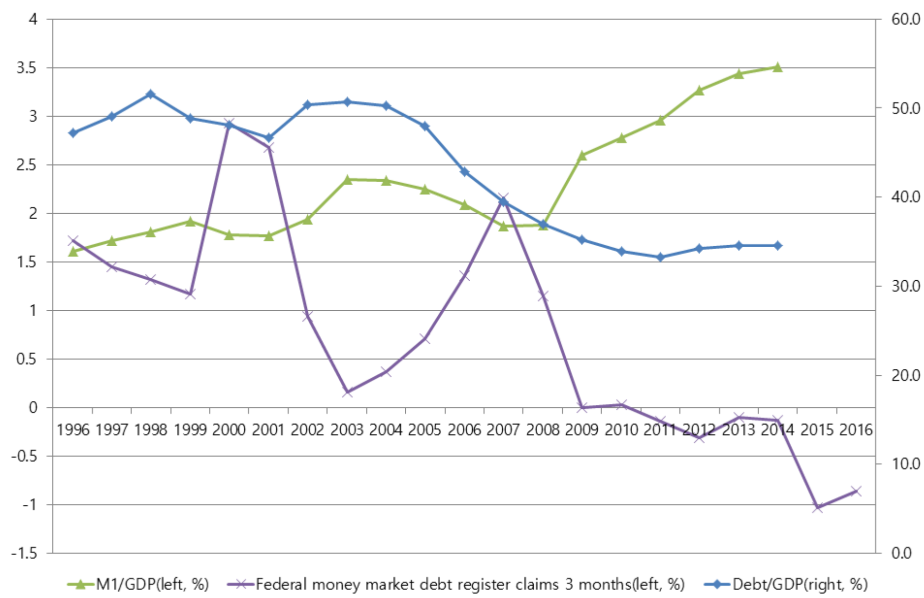

Figure 8: Interest Rates, Money and Gov't Bond Supply in Switzerland

Moreover, we can find another example even in the United States. In particular, the theory is also consistent with the comments from the market participants during the periods when negative nominal yields on Treasuries were observed in the US. For example, according to *Bloomberg* (September 25, 2015), Kenneth Silliman, head of US short-term rates trading in New York at TD Securities unit, one of 22 primary dealers that trade with the Fed said,

"Yields on U.S. Treasury bills fell below zero as an influx of cash and pent-up appetite for safe assets led investors to accept negative returns after the Federal Reserve decided not to raise its short-term

[28]It includes currency in circulation, sight deposits and deposits in transaction accounts.

interest rate. ...... Investors will have additional funds totaling about $100 billion returned to them in the next month as the government cuts bill supply heading into negotiations with Congress about the statutory debt limit", [29][30]

In brief, he mentions that the main factors which drove down the nominal bond yields to below zero were 'a cut in bill supply' and the low short term interest rate. Notice that the policy rate of the Fed, has been hovering around zero for more than 7 years since 2008. In other words, when the negative bond yields occurred, the money holding cost was low, and also the bill supply was expected to decrease and to be low relatively. However, there have existed strong demand on the government bonds. Both factors were at work together in the direction to raise up the liquidity premium of liquid bonds such as Treasury bills, so that the nominal bond yields seem to fall down to the negative territory even during the short time period.

To summarize, the observed data in Switzerland and the United States strongly suggest that the low money holding cost and the scarcity of liquid bonds could cause the negative Treasury yields through the liquidity channel in both countries.

5 Conclusion

This paper explores the effects of monetary policy on the liquidity premia counted in the prices of liquid financial assets such as government bonds. The theory delivers elaborate predictions about under which conditions and through what mechanism money growth can affect the liquidity premia. For example, money growth rate, not its level, has a positive relationship with them because high money growth rate implies the high opportunity cost of holding money and thereby the high demand on liquid bonds only when liquid bond supply are scarce. Also, the theory provides a rationale that this mechanism works because liquid bonds play a role as substitutes to money in part. On the other hand, the level of liquid bond supply also directly affects the liquidity premia by changing relative scarcity of the bonds. The empirical analysis with the U.S. data provides a strong support for the theoretical predictions: the money growth rate, not its level, is positively correlated with the liquidity premia, whereas bond supply negatively. Last but not least, the paper explains how the liquidity premia are associated with the negative nominal yields on liquid bonds which were observed in the US and Switzerland. The theory

[29]Source: http://www.bloomberg.com/news/articles/2015-09-17/treasury-bill-yields-turn-negative-as-fed-leaves-rates-unchanged

[30]See the following comment in *The Wall Street Journal* (Sept 23, 2014) for another example: "Short-term debt trading at negative yields was essentially unheard of before the 2008 financial crisis. But since then, the condition has cropped up at times of market stress, reflecting extraordinarily expansive central-bank policy and anemic growth in much of the world. Yields on some U.S. bills traded below zero at the end of each of the past three years amid strong demand for liquid assets, according to analysts." Source: http://www.wsj.com/articles/treasury-bill-yield-tips-into-negative-territory-1411516748

and the relevant data strongly suggest that those negative yields would be caused by scarcity of liquid bond supply and so high liquidity premia, when the money holding cost is low.

References

ARUOBA, S. B., C. J. WALLER, AND R. WRIGHT (2011): "Money and capital," *Journal of Monetary Economics*, 58(2), 98 – 116.

BRUNNERMEIER, M. K. (2009): "Deciphering the Liquidity and Credit Crunch 2007-2008," *Journal of Economic Perspectives*, 23(1), 77–100.

CARAPELLA, F., AND S. WILLIAMSON (2015): "Credit Markets, Limited Commitment, and Government Debt*," *The Review of Economic Studies*.

GEROMICHALOS, A., AND L. HERRENBRUECK (2016): "Monetary Policy, Asset Prices, and Liquidity in Over-the-Counter Markets," *Journal of Money, Credit and Banking*, 48(1), 35–79.

GEROMICHALOS, A., L. HERRENBRUECK, AND K. SALYER (2013): "A Search-Theoretic Model of the Term Premium," Discussion paper, UC Davis.

GEROMICHALOS, A., J. LEE, S. LEE, AND K. OIKAWA (2016): "Over-the-counter trade and the value of assets as collateral," *Economic Theory*, 62(3), 443–475.

GEROMICHALOS, A., J. M. LICARI, AND J. SUAREZ-LLEDO (2007): "Monetary Policy and Asset Prices," *Review of Economic Dynamics*, 10(4), 761–779.

GEROMICHALOS, A., AND I. SIMONOVSKA (2011): "Asset Liquidity and International Portfolio Choice," Discussion paper, National Bureau of Economic Research.

GREENWOOD, R., S. G. HANSON, AND J. C. STEIN (2015): "A Comparative-Advantage Approach to Government Debt Maturity," *The Journal of Finance*, 70(4), 1683–1722.

JACQUET, N. L., AND S. TAN (2012): "Money and asset prices with uninsurable risks," *Journal of Monetary Economics*, 59(8), 784–797.

JUNG, K. M., AND S. LEE (2015): "A Liquidity-Based Resolution of the Uncovered Interest Parity Puzzle," MPRA Paper 64164, University Library of Munich, Germany.

KRISHNAMURTHY, A. (2010): "How Debt Markets Have Malfunctioned in the Crisis," *Journal of Economic Perspectives*, 24(1), 3–28.

KRISHNAMURTHY, A., AND A. VISSING-JORGENSEN (2012): "The aggregate demand for treasury debt," *Journal of Political Economy*, 120(2), 233–267.

Lagos, R. (2010a): "Asset prices and liquidity in an exchange economy," *Journal of Monetary Economics*, 57(8), 913–930.

——— (2010b): "Asset prices and liquidity in an exchange economy," *Journal of Monetary Economics*, 57(8), 913–930.

Lagos, R., G. Rocheteau, and R. Wright (2014): "The art of monetary theory: A new monetarist perspective," *forthcoming, Journal of Economic Literature*.

Lagos, R., and R. Wright (2005): "A Unified Framework for Monetary Theory and Policy Analysis," *Journal of Political Economy*, 113(3), 463–484.

Lester, B., A. Postlewaite, and R. Wright (2012): "Information, liquidity, asset prices, and monetary policy," *The Review of Economic Studies*, 79(3), 1209–1238.

Longstaff, F. (2004): "The Flight-to-Liquidity Premium in U.S. Treasury Bond Prices," *The Journal of Business*, 77(3), 511–526.

Lucas Jr., R. E., and J. P. Nicolini (2015): "On the stability of money demand," *Journal of Monetary Economics*, 73, 48 – 65, Carnegie-Rochester-NYU Conference Series on Public Policy monetary Policy: An Unprecedented Predicament held at the Tepper School of Business, Carnegie Mellon University, November 14-15, 2014.

Nagel, S. (2014): "The Liquidity Premium of Near-Money Assets," Working Paper 20265, National Bureau of Economic Research.

Rocheteau, G., and R. Wright (2005a): "Money in search equilibrium, in competitive equilibrium, and in competitive search equilibrium," *Econometrica*, 73(1), 175–202.

——— (2005b): "Money in search equilibrium, in competitive equilibrium, and in competitive search equilibrium," *Econometrica*, 73(1), 175–202.

Rocheteau, G., R. Wright, and S. X. Xiao (2016): "Open Market Operations," *Working paper*.

Vayanos, D. (2004): "Flight to Quality, Flight to Liquidity, and the Pricing of Risk," Working Paper 10327, National Bureau of Economic Research.

Waller, C. J. (2015): "Microfoundations of Money: Why They Matter," *Federal Reserve Bank of St. Louis Review*, Fourth Quarter, pp. 289–301.

Williamson, S. D. (2012): "Liquidity, Monetary Policy, and the Financial Crisis: A New Monetarist Approach," *American Economic Review*, 102(6), 2570–2605.

A Appendix

Proof. Proof of Lemmas 1 and 2.

First, consider Lemma 1. Substituting $\boldsymbol{\phi}_t \boldsymbol{p}_t$ into the objective function in Equation 6 re-express the bargaining problem as

$$\max_{q_t^1} \left\{ u(q_t^1) - q_t^1 \right\}$$

subject to $q_t^1 = \boldsymbol{\phi}_t \boldsymbol{p}_t$, and $\boldsymbol{p}_t \leq \widetilde{\mathbf{w}}_t$. If $\boldsymbol{\phi}_t \widetilde{\mathbf{w}}_t \geq q^*$, the optimal choice of q_t^1 will be the first best quantity q^*, i.e., $q_t^1 = q^*$. Then, $\boldsymbol{p}_t = (p_t^m, p_t^a)$ such that $\phi_t p_t^m + g p_t^a = q^*$. However, if $\boldsymbol{\phi}_t \widetilde{\mathbf{w}}_t < q^*$, the effective budget constraint is binding. Accordingly, the buyer will give up all her real balances in order to purchase as many as possible. Then, the optimal choice of q_t^1 will be the same as her real balances $\boldsymbol{\phi}_t \widetilde{\mathbf{w}}_t$. Also, $\boldsymbol{p}_t = (m_t, a_t)$. When it comes to Lemma 2, the same steps above can be taken for proof. Since it is straightforward, it is omitted. □

Proof. Proof of Lemma 3

First, consider whether the real balances are as a direct medium of exchange or as collateral to borrow credit enough to obtain the optimal quantity q^* in each of the two DM markets. If $\tilde{z}_t \geq q^*$ or $\tilde{z}_t^a \geq q^*$, $q_t^1 = q^*$ or $q_t^2 = q^*$; otherwise, $q_t^1 = \tilde{z}_t$ or $q_t^2 = \tilde{z}_t^a$ by lemmas 1 or 2. Then, plugging these results into the first order conditions (13) and (14) for the maximum of the objective function will yield the equilibrium prices ϕ_t and ψ_t. Also, the marginal utility function u' is monotonically decreasing in its argument, so that the equilibrium is uniquely determined. □

Proof. Proof of Corollary 1

The quantity which buyers can consume with their money holdings in the DM is determined by equation (15) when the real balances are not enough to obtain q^*.

$$\frac{1+\mu}{\beta} = 1 + \theta \left[u'(\phi M + gA) - 1 \right]$$

Then, if we plug zero into ϕM and solve this equation for μ, we can obtain the upper bound of μ for the monetary equilibria as follows.

$$\bar{\mu} \equiv \{\mu : \mu = \beta[1 + \theta[u'(gA) - 1]]\} - 1$$

□

B Impact of money growth rate on the liquidity premia of Treasuries: Quarterly Data

Variables	AA CP - T-Bills			TED Spread		
	(1)	(2)	(3)	(4)	(5)	(6)
	1971Q2-2008Q4	1971Q2-2008Q4	1971Q2-2008Q4	1986Q1-2008Q4	1986Q1-2008Q4	1986Q1-2008Q4
NM Growth	1.144***	1.740***		3.116***	3.863***	
	(0.170)	(0.230)		(0.211)	(0.353)	
Dept to GDP		-1.742***	-0.0345		-2.940***	-0.0337
		(0.418)	(0.308)		(0.922)	(1.321)
Federal Funds Rate			0.0926***			0.0914***
			(0.0122)			(0.0193)
Constant	0.587***	1.424***	0.0312	0.616***	2.314***	0.264
	(0.0538)	(0.236)	(0.185)	(0.0565)	(0.539)	(0.731)
Observations	151	151	151	92	92	92
Adjusted R-squared	0.005	0.209	0.422	0.184	0.329	0.201

Variables	FDIC CDs - T-Bills			AAA Cor. - Treasury Bond		
	(7)	(8)	(9)	(10)	(11)	(12)
	1998Q2-2008Q4	1998Q2-2008Q4	1998Q2-2008Q4	1946Q1-2008Q41	1966Q1-2008Q4	1966Q1-2008Q4
NM Growth	3.182***	2.757**		3.060***	2.353***	
	(0.352)	(1.167)		(0.999)	(0.222)	
Debt to GDP		1.734	4.705		-1.708***	-1.969***
		(4.417)	(3.789)		(0.250)	(0.367)
Federal Funds Rate			-0.0845*			-0.0235*
			(0.0451)			(0.0136)
Constant	0.0835	-0.938	-2.332	0.709***	1.716***	2.038***
	(0.102)	(2.558)	(2.191)	(0.0406)	(0.135)	(0.254)
Observations	43	43	43	252	172	172
Adjusted R-squared	0.218	0.207	0.159	0.064	0.320	0.276

Notes: Coefficients are estimated by Newey-West estimators with lag(1) and its standard errors are presented in parenthesis. The dependent variables are the yield spreads between private financial assets and Treasuries. They are measured in a percentage unit. Explanatory variables are the growth rate of Narrow Money and the ratio of the market value of Treasury debt outstanding to nominal GDP. *** p<0.01, ** p<0.05, * p<0.1.