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Fiscal Requirements for Price Stability in Economies with Private Provision of Liquidity and Unemployment

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Abstract

We study the impact of fiscal policies on inflation, unemployment and interest rate spreads dynamics in an environment where firms provide liquidity. Firms link labor and asset markets by hiring workers and issuing claims to their future profits. Matching frictions in the labor market drastically alters the effect of fiscal policy as the tax base increases with the number of jobs filled. As a result, labor market conditions directly affect the return on private assets and inflation dynamics. Moreover, since frictions in decentralized financial markets exist, public and private assets are also used as collateral. These features in the labor and financial markets drastically change the nature of monetary equilibria. In particular, monetary steady states are generically not unique and endogenous fluctuations are observed. Furthermore, characteristics of the labor market affect the demand for private and public assets, making the interaction between monetary and fiscal policies more intricate. Finally, traditional stabilization policies based on frictionless financial and labor markets are not robust to this frictional environment.

JEL Codes: D82, D83, E40, E50.
Keywords: decentralized financial markets, unemployment, liquidity, fiscal rules.

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1 Introduction

After the global financial crisis, a certain degree of consensus emerged that a better understanding of how private provision of liquidity and its subsequent trading in decentralized financial markets impacts the economy would be beneficial.\(^1\) Since the crisis the U.S. Federal Reserve System has closely followed developments in the labor market and has been concerned how unemployment pressures affect financial markets.\(^2\) These decentralized markets and policy features have not been fully explored by the literature. It therefore seems appropriate that underlying frameworks used to inform policy should be further explored. These frameworks would naturally have agents trading in centralized and decentralized financial markets with multiple assets (both private and public) while workers face unemployment. Herein we consider such an environment where agents face anonymity, search and bargaining frictions and study the impact of fiscal policies on inflation, unemployment and interest rate spread dynamics.

Firms, other than hiring workers, also supply private assets (claims to their future profits) that are useful as a store of value but also as collateral when agents trade in frictional decentralized financial markets. Relative to environments with a fixed supply of private assets, the channels through which monetary and fiscal policy interact are more intricate. Once the provision of private assets is endogenous, government policies that affect firms’ profits have a direct impact on the supply of private assets and unemployment. This is the case as firms link both the labor and asset markets. Thus, fiscal and monetary policies affect the relative price between private and public assets and equilibrium interest rates. Moreover, because of the decentralized labor market, taxes affect the entry decision of firms. When firms expect future taxes to be high, they reduce recruitment. Since the tax base increases with the number of jobs filled, government budget balancing requires a higher tax level, in accordance with employers’ beliefs.

To better understand the extent to which fiscal policy affects inflation, unemployment and interest rate spreads, this paper builds on Rocheteau and Rodriguez-Lopez (2014). To generate unemployment, the environment has a frictional labor market \textit{a la} Mortensen and Pissarides (1994) where matched workers and firms produce the numéraire good. Firms also supply private assets by issuing claims to their future profits. The demand for both private and public assets arise from a \textit{liquidity block} \textit{a la} Shi (1995) and Trejos and Wright (1995).\(^3\) Shi-Trejos-Wright traders can

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\(^1\)See Brunnermeier (2009) and Gorton and Metrick (2012a) for discussions on the main contributors to the GFC.

\(^2\)The Federal Open Market Committee was, and is currently, providing guidance about the conduct of monetary policy in relation to the evolution of unemployment. \textit{“In setting monetary policy, the Committee seeks to mitigate deviations of inflation from its longer-run goal and deviations of employment from the Committee’s assessments of its maximum level. These objectives are generally complementary. However, under circumstances in which the Committee judges that the objectives are not complementary, it follows a balanced approach in promoting them, taking into account the magnitude of the deviations and the potentially different time horizons over which employment and inflation are projected to return to levels judged consistent with its mandate.”} Statements from the Federal Open Market Committee, January 2012, 2013, and 2014.

\(^3\)These authors emphasize the role of assets (money) as media of exchange. Recently, search and bargaining frictions have been also emphasized by Duffie, Garleanu and Pedersen (2005) when describing over the counter...
produce the numéraire good and have access to a frictionless financial market where they rebalance their portfolio. Moreover, these traders have stochastic trading opportunities in decentralized frictional financial markets where they obtain utility from financial services. Traders in these markets are anonymous and face asset recognizability problems. These frictions can be overcome by collaterized loans.

Since firms employ workers and supply private assets that are valued by traders, this environment allows us to explicitly analyze the inherent links between interest rate spreads and unemployment. This is the case as firms connect the frictional asset and labor markets. As a result, interest rate spreads between government bonds and private assets will depend on labor market conditions. Moreover, as in Friedman (1956), Tobin (1961) and Brunner and Meltzer (1972), the equilibrium price level in this environment is determined by the joint valuation of all assets. However, in contrast to Tobin (1961), interest rate spreads are not driven by the risk properties of assets but rather by market incompleteness.

The environment we consider has the feature that labor market characteristics impact the value of private and public assets. Moreover, non ad-valorem taxes on firms affect their entry decisions. Thus the tax capacity of the economy is crucial for inflation dynamics. To have a better understanding of the role of liquidity and the economy’s taxing capacity, we first analyze an environment where only Shi-Trejos-Wright traders are taxed. We then explore and compare the previous monetary equilibria to those resulting from an environment where all agents face non ad-valorem taxes.

When only Shi-Trejos-Wright traders face non ad-valorem taxes and there are insufficient government liabilities to meet their collateral needs, interest rate spreads that depend on real bonds exist. As a result, the government bond repayment is non-linear so multiple steady states exist, thus sunspot equilibria can be easily constructed. When all agents (workers, firms and traders) face non ad-valorem taxes, fiscal policies affect the firms’ decision to participate in the labor market. In particular, when employers expect future taxes to be high, they reduce recruitment. Since the tax base increases with the number of jobs filled, government budget balancing requires a higher tax level, in accordance with employers’ beliefs. This fiscal channel can then potentially generate multiple steady states regardless of the collateral needs of traders. Steady states and their associated dynamic properties depend on both monetary and fiscal policy parameters as well as labor market characteristics. Furthermore, monetary equilibria will typically display endogenous volatility. Thus, traditional fiscal requirements to stabilize prices that are based on frictionless and

(OTC) financial markets. Trejos and Wright (2015) discuss the similarities and differences between the monetary economics used by Shi (1995) or Trejos and Wright (1995), with the applications in finance used by Duffie, Garleanu and Pedersen (2005). The authors integrate the two approaches and generate endogenous transaction patterns and belief-based dynamics.

4In this framework we can interpret the liquidity block as OTC financial markets where trades by private firms are collateralized with private and public assets.
centralized environments are going to be ineffective.

By considering frictional and decentralized labor and financial markets, this paper attempts to improve our understanding of how monetary and fiscal policy interact when agents face search, bargaining and informational frictions. We show that the properties of monetary equilibria fundamentally change once the provision of private assets and unemployment are equilibrium outcomes.

2 Related Literature

Even though substantial progress has been made in understanding how financial frictions affect the economy, much less attention has been devoted to the study of monetary and fiscal policy interactions in economies with unemployment.\textsuperscript{5} A notable exception is the pioneering work of Cooley and Quadrini (2004) who study the optimal monetary policy in a framework that integrates the search theory of unemployment with firms facing cash-in-advance constraints to purchase intermediate inputs. These authors show that when the economy is subject to productivity shocks, the optimal policy is procyclical, and with commitment, the optimal inflation rate is inversely related to the bargaining power of workers. Within the same spirit and using the New Keynesian cashless framework, Ravenna and Walsh (2012) consider a frictional labor market and show that when wages are rigid and fixed, the optimal tax correcting for inefficient hiring is small but very volatile over the business cycle. Gains made by deviating from price stability are larger in economies with more volatile labor flows. Building on the cashless framework, Arsenneau and Chugh (2012) consider a calibrated matching model that generates empirically relevant labor-market fluctuations conditional on exogenous fiscal policy. The authors find that tax volatility induces dramatically smaller, albeit efficient, fluctuations of labor markets by keeping distortions constant over the business cycle.

Once financial markets are incomplete, so that fiat money has a role as a medium of exchange, Berentsen et al. (2007) find that the same frictions that give fiat money a positive value generate an inefficient quantity of goods in each trade and an inefficient number of trades in decentralized and frictional markets. The authors show that the Friedman rule eliminates the first inefficiency and the Hosios rule, the second. Within the same framework, Gomis-Porqueras et al (2013) show how a production subsidy in a frictional goods market and a vacancy subsidy, financed by a dividend tax, can achieve efficiency even when the Hosios condition does not hold.

The previous body of work has explored the link between inflation and unemployment in a variety of environments where different frictions have been emphasized. However, much less work has analyzed how the private provision of liquidity can affect the equilibrium relationship between inflation and unemployment. One of the few papers that explore such an important issue is the

\textsuperscript{5}We refer to Quadrini (2011) and Brunnermeier, Eisenbach and Sannikov (2013) for a recent survey of the literature that deals with the impact of financial frictions on the macroeconomy.
work by Rocheteau and Rodriguez-Lopez (2014). These authors find that an increase in the supply of real government bonds raises the real interest rate, crowding out private liquidity and increasing unemployment. If unemployment is inefficiently high, the authors show that keeping liquidity scarce can be socially optimal. Rocheteau et al (2014) also show that a liquidity crisis (affecting the acceptability of private assets as collateral) widens the spreads between private and public liquidity. Subsequently unemployment increases. This paper complements this latter work by exploring the consequences for inflation, unemployment and interest rate spreads when alternative fiscal arrangements and fiscal policy rules are considered.

3 Environment

We build on the framework by Rocheteau and Rodriguez-Lopez (2014). Time is continuous where three types of private agents (workers, firms and traders) participate in goods, labor and financial markets. There is also a government that needs to finance some exogenous expenditures through non ad-valorem taxes and the issuance of fiat money and nominal bonds.

Workers and firms participate in a frictional labor market a la Mortensen and Pissarides (1994). In this market workers sell their time in exchange for a wage when producing a perishable numéraire good. This perishable good is consumed by all private agents and the government. Other than producing goods, firms also issue and sell their securities to financial market participants. Traders, on the other hand, can produce and consume the numéraire good and have access to a frictionless financial market where they can rebalance their portfolio. These traders are also able to produce and consume a perishable financial service which only they value. This service is traded in a frictional market characterized by anonymity, search and bargaining frictions as in Shi (1995) and Trejos and Wright (1995).\(^6\)

3.1 Government

The government can issue fiat money and nominal bonds in order to finance some exogenous government expenditures. Money is a pure fiat object. Nominal bonds, on the other hand, are pure discount bonds that yield one unit of fiat money at a Poisson rate equal to one. The government can also levy non ad-valorem taxes. In the benchmark model the government only taxes Shi-Trejos-Wright traders so that

\[ G + \phi_m B = T + \phi_m M + \phi_b B \]

where \( G \) are exogenous government expenditures, \( M \) is the monetary base, \( B \) represents nominal bonds, \( \phi_m \) (\( \phi_b \)) denotes the real value of a unit of fiat money (a nominal bond) in terms of the

\(^6\)Search and bargaining frictions have also been emphasized by Duffie, Garleanu and Pedersen (2005) when describing OTC financial markets.
numéaire good and $T$ represents the non ad-valorem tax levied to Shi-Trejos-Wright traders.

It is convenient to write the government budget constraint in terms of real government liabilities which is given by

$$
\dot{m} + \dot{b} = G + \frac{\phi_m}{\phi_m} m + \left( \frac{\phi_m}{\phi_b} + \frac{\phi_b}{\phi_b} \right) b - T
$$

where $m = \phi_m M$ represents real balances and $b = \phi_b B$ denotes real bonds.

To describe the particulars of fiscal policy, we consider a fiscal rule where taxes depend on the quantity of real government debt outstanding.\(^7\) This fiscal rule has been extensively analyzed by the proponents of the Fiscal Theory of the Price Level (FTPL).\(^8\) This rule allows taxes to be a function of real government debt which is given by

$$
T = \eta_0 + \eta_1 b,
$$

where $\eta_0(\eta_1)$ are constant policy parameters.\(^9\) For the operating procedure for monetary policy, we consider a constant money growth rate, $\zeta$, rule so that the money supply evolves according to $\dot{M} = \zeta M$.\(^{10}\)

### 3.2 Workers and Firms

Workers are endowed with one indivisible unit of labor per unit of time. They are risk-neutral and they discount future consumption at rate $\rho > 0$. Thus, their lifetime expected utility is given by

$$
E \int_0^\infty e^{-\rho t} dC(t)
$$

where $C(t)$ is their cumulative net consumption of the numéaire good and $E$ is the expectation operator.

Each firm can considered as a technology that produces the numéaire good using a worker’s indivisible labor as input. As in Mortensen and Pissarides (1994), workers and firms face search frictions and are matched bilaterally. The flow of hires is equal to $h(u, v)$, where $u$ denotes the measure of unemployed workers and $v$ represents the measure of vacancies. The matching function, $h(\cdot, \cdot)$, has constant returns to scale, is strictly concave with respect to each of its arguments, and satisfies Inada conditions. Then, the job finding rate of a worker is $p(\theta) \equiv h(u, v)/u = h(1, \theta)$ where $\theta = v/u$ represents the labor market tightness. Similarly, the vacancy filling rate is $q(\theta) \equiv \ldots$

\(^7\)The use of a debt sustainability framework (see Ghosh et al (2013) and the references therein) is commonly used for policy analysis as emphasized by IMF Article IV country reports.

\(^8\)This literature emphasizes that bonds are denominated in nominal terms so that they may be fully backed by real resources or backed only by nominal cash flows as initially argued by Leeper (1991), Sims (1994), Woodford (1994) and Cochrane (2001). For more details, we refer to a recent survey by Canzoneri et al (2011).

\(^9\)We abstract from enforcement issues and solely focus on the effects that a fiscal rule would have if the government could commit to enforcing it.

\(^{10}\)Here we sidestep the interesting and relevant issue of studying the implications of considering various interest rate spreads in the Taylor rule.
\( h(u, v)/v = h(\theta^{-1}, 1) \).

Upon each successful match of a worker and a firm, a constant flow of numéraire output equal to \( \Phi \) is produced. A match is exogenously destroyed with a Poisson arrival rate of \( \delta > 0 \). The wage of an employed worker is a constant fraction of output which we denote by \( w \).\(^{11}\)

In order to fill a job, a firm must open a vacancy which has associated a cost flow, in terms of the numéraire good, equal to \( \gamma > 0 \). The recruiting expenses of firms are paid by Shi-Trejos-Wright traders in exchange for claims to their future profits (numéraire output) which they can then securitize.\(^{12}\) Alternatively, we can think of firm’s recruiting expenses being paid by households which then sell claims to Shi-Trejos-Wright traders in a competitive asset market. The critical aspect to consider is that the Shi-Trejos-Wright traders have a demand for private liquidity when trading in the frictional decentralized market. The equilibrium allocation is independent of who finances the firms’ vacancy costs, as the ultimate agents that demand these private assets are just traders.

New firms are financed as long as the flow cost of opening a vacancy, \( \gamma \), is no greater than the flow expected value of a vacancy. Free entry then implies that

\[
\gamma = q(\theta)V_F
\]

and the total supply of private assets corresponds to the total capitalization of firms which is given by

\[
L^p = nV_F.
\]

As in Mortensen and Pissarides (1994), the resulting value of a filled job solves the following Bellman equation

\[
rV_F = \Phi - w - \delta V_F + \dot{V}_F
\]

where \( r \) denotes the rate of return of a firm’s share, while the law of motion for employment is given by

\[
\dot{n} = p(\theta)(1 - n) - \delta n.
\]

Relative to Mortensen and Pissarides (1994), the key difference is that \( r \) is endogenous and not necessarily equal to the time discount rate. This is the case as assets, both private and public, can be used as collateral objects when trading in frictional and decentralized markets.

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\(^{11}\)This type of wage formation is consistent with proportional bargaining. In this paper we assume that the government does not provide unemployment benefits.

\(^{12}\)Securitization is a process in which different assets or portfolios of cash flow generating securities are pooled together and then sold to third parties.
3.3 Shi-Trejos-Wright Traders

Traders derive linear utility from the numéraire good and from a perishable financial service, which they can both consume and produce. The lifetime expected utility of a trader is then given by

\[ E \left[ \sum_{n=1}^{\infty} e^{-\rho T_n} [u(y(T_n)) - x(T_n)] + \int_0^{\infty} e^{-\rho t} dC(t) \right] \]

where the first term represents the utility associated with financial services, while the second term denotes the utility from net consumption of the numéraire good. \( T_n \) represents a Poisson process with arrival rate \( \beta > 0 \), indicating the times at which traders are bilaterally matched. These traders can not produce the numéraire when trading in this frictional market; i.e., \( t \in \{ T_n \}_{n=1}^{\infty} \).

As we can see, traders only have stochastic trading opportunities to obtain utility from financial services. The market where these services are traded is characterized by anonymity, search frictions, bilateral matching and bargaining as in Shi (1995) and Trejos and Wright (1995). Traders in this market are anonymous so unsecured loans are not possible. Moreover, since traders can not always produce the numéraire good to pay for the financial services, a medium of exchange is required. One potential arrangement to facilitate exchange is a secured loan.\(^{13}\) These features of the environment result in demand for public assets (fiat money, \( M \), and nominal bonds, \( B \)) and private assets (\( a \)) by these traders.\(^{14}\) After trading has occurred in the decentralized market, Shi-Trejos-Wright agents can rebalance their portfolio in a frictionless financial market.

Upon a successful bilateral match, a trader is chosen at random, with equal probability, to be either a supplier or a user of financial services. The utility from consuming \( y \) units of financial services is \( u(y) \) while the disutility from producing \( x \) units of financial services is \( x \).\(^{15}\) The exact terms of trade in this decentralized financial market are determined by a buyer take-it-or-leave-it offer. A contract is then a pair, \( (y, d) \), that specifies a production of services, \( y \), in exchange for a payment, \( d \). Such contract is consistent with a collateralized loan where the buyer promises to repay \( d \) units of numéraire as soon as he exits the decentralized financial market. The repayment of this loan is secured by the deposit of \( d \) units of acceptable assets.\(^{16}\)

Traders face collateral differences among the public and private assets they hold in their portfolio. This is the case as not all traders can equally authenticate assets (which buyers can costlessly counterfeit) as fraudulent.\(^{17}\) As a result, in a fraction \( \mu_m \) of bilateral matches only fiat money is

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\(^{13}\)When buying assets from firms, traders can fully diversify their portfolio of different equities via securitization of large pools of assets which can turn these private claims into safe and liquid assets.

\(^{14}\)It is important to note that since workers are risk-neutral (no need for consumption smoothing), the demand for liquid assets is entirely driven by Shi-Trejos-Wright traders.

\(^{15}\)As in Lagos and Rocheteau (2009), one could interpret \( u(y) \) as a reduced-form utility function that stands in for the various reasons why investors may want to hold different quantities of the asset. Differences in collateral needs, financing or financial-distress costs, correlation of asset returns with endowments (hedging needs) or relative tax disadvantages.

\(^{16}\)Alternatively, the contract can also be viewed as the buyer paying directly with assets.

\(^{17}\)This could also be interpreted as reflecting the *complexity* in some financial products.
acceptable as collateral because the other two potentially fraudulent assets can not be recognized.\textsuperscript{18} Similarly, in a fraction $\mu_p$ of matches, fiat money and government bonds are useful as collateral.\textsuperscript{19} For the remaining fraction of matches, $\mu_p = 1 - \mu_m - \mu_g$, all public and private assets can be used as collateral to secure the loan.\textsuperscript{20}

Let $W(A_0)$ denote the lifetime expected discounted utility of Shi-Trejos-Wright trader that is holding $A(t)$ liquid assets (i.e. claims on firms’ profits, real government bonds and real balances). The trader needs to decide his asset holdings, $A(t) = (a(t), b(t), m(t))$, consumption path, $c(t)$, and discrete jumps, $\Delta_j$, in order to maximize her discounted cumulative consumption and the present continuation value of a trading opportunity in the frictional market. Formally, the trader’s problem is given by

\[
W(A_0) = \max_{a,b,m,c,k;\{\Delta_j,t_j\}} \left\{ \mathbb{E} \int_0^{T_1} e^{-\rho t} c(t) dt - \sum_{j=1}^k e^{-\rho t_j} \Delta_j \mathbb{1}_{\{t_j \leq T_1\}} + e^{-\rho T_1} Z[A(T_1)] \right\}
\]

s.t. \quad \dot{a} + \dot{m} + \dot{b} = r^m m + r^g b + ra - c - T \quad \text{for all } t \neq t_j
\]

\[
\Delta_j \equiv C(t_j^+) - C(t_j^-) \quad \text{for all } j = 1, \ldots, k
\]

where $A_0 = (a_0, b_0, m_0)$ are the initial asset conditions, $T_1$ is the random time at which the trader is bilaterally matched, $r^m(r^g)$ represents the return on real balances (real bonds) and $\mathbb{1}_{\{t_j \leq T_1\}}$ is an indicator function that is equal to one if $t_j \leq T_1$ (and zero otherwise). The second term of equation (6) represents lumpy consumption (production if $\Delta_j < 0$) financed by discrete jumps in asset holdings. The trader chooses both the sizes of these discrete adjustments, $\Delta_j$, and their timing, $t_j$, where $k$ denotes the number of adjustments. Finally, $Z[A(T_1)]$ denotes the value of trading in the decentralized financial market.

Using the fact that $T_1$ is exponentially distributed with arrival rate $\beta$ and results from Seierstad and Sydsæter (1987), equations (6) through (8) are equivalent to that of a portfolio problem with initial real money balances, real bonds and private assets $(m_0, g_0, a_0)$. More precisely, we have that

\[
\max_{\dot{a},m,b,a,m,c} \mathbb{E} \left[ \int_0^\infty e^{-(\beta + \rho) t} \{ c(t) + \beta Z[A(t)] \} dt \right] \quad \text{s.t. } \dot{a} + \dot{m} + \dot{b} = r^m m + r^g b + ra - c - T
\]

where the continuation value of a Shi-Trejos-Wright trader once matched, $Z(A)$, is given by

\[
Z(A) = \frac{\mu_p}{2} \max_{y_p \leq m+b+a} [u(y^p) - y^p] + \frac{\mu_g}{2} \max_{y_g \leq m+b} [u(y^g) - y^g] + \frac{\mu_m}{2} \max_{y_m \leq m} [u(y^m) - y^m];
\]

which captures that the terms of trade are given by a buyer take-it-or-leave-it offer, that a trader has equal probability to be a buyer or a seller and that not all assets can be used as collateral in

\textsuperscript{18}One can endogenize these fractions by introducing a costly technology to authenticate assets (e.g Lester et al, 2012), or an informational asymmetry regarding the terminal value of the asset through an adverse selection problem (e.g Rocheteau, 2011) or a moral hazard problem (e.g Li et al, 2012).

\textsuperscript{19}As in Canzoneri and Diba (2005), government bonds can also provide liquidity services. In this environment, private assets also provide such services.

\textsuperscript{20}Financial regulation, such as the Dodd-Frank Act, could also impact the values of $\mu_m$ and $\mu_g$. 
As we can see from (9), the trader has access to a frictionless centralized financial market to rebalance his portfolio. Moreover, public and private assets serve as a store of value and collateral. The trader’s optimal portfolio links the return of the various assets with the collateral needs of Shi-Trejos-Wright traders which are given by

\[
\frac{\rho - r}{\sigma} = \mu_p[u'(y^p) - 1];
\]
\[
\frac{\rho - r^g}{\sigma} = \mu_g[u'(y^g) - 1];
\]
\[
\frac{\rho - r^m}{\sigma} = \mu_m[u'(y^m) - 1];
\]

where \(y^p = \min\{m + b + L^p, y^*\}, \ y^g = \min\{m + b, y^*\}, \ y^m = \min\{m, y^*\}, \ u'(y^*) = 1\) and \(1/\sigma\) denotes the expected time before the trader receives an opportunity to purchase financial services.

Since fiat money yields no dividend, its rate of return is given by

\[
r^m = \frac{\dot{\phi}_m}{\phi_m};
\]

while the price of bonds solves the following asset pricing condition,

\[
r^g \phi_b = \phi_m - \phi_b + \phi_b;
\]

which rules out any arbitrage opportunity.

Private assets dominate government bonds in their rate of return \((r > r^g)\) whenever \(\mu_g > 0\) and \(y^g < y^*\). Similarly, if \(\mu_m > 0\) and \(y^m < y^*\) government bonds have a greater return than fiat money \((r^g > r^m)\). Spreads depend on government policies and the frictions in decentralized labor and financial markets.

As in Friedman (1956), Tobin (1961) and Brunner and Meltzer (1972), the equilibrium price level is determined by the valuation of all assets jointly. However, in contrast to Tobin (1961), interest rate spreads are not driven by the risk properties of assets but rather by market frictions. The different uses that assets can have, as store of value and/or collateral objects, and how they are traded have a direct impact on their value. Understanding inflation dynamics then requires an explicit description of how agents trade and settle transactions in frictional markets as assets can provide more than a store of value role.

4 Monetary Equilibrium

In this section we study the equilibrium properties of our frictional economy.

**Definition 1** Given some exogenous government expenditures, a constant money growth rate and a fiscal policy rule that links taxes with government debt, a monetary equilibrium is an allocation of
real assets, \{m, b, a\}, labor market outcomes, \{\theta, n\}, goods and services, \{C, y\}, as well as interest rates, \{r^m, rg, r\}, that satisfies the optimality conditions of workers, firms and traders while labor and financial markets clear.

After imposing market clearing and private agents’ optimality conditions, the resulting monetary dynamic equilibrium is given by a system of non-linear differential equations that specify the evolution of the real government liabilities, firms’ value and employed workers. More precisely, we have that

\[ \dot{m} = m \left( \zeta + \frac{\dot{\phi}_m}{\phi_m} \right); \]  
\[ \dot{m} + \dot{b} = G + \frac{\dot{\phi}_m}{\phi_m} m + \left[ \frac{\dot{\phi}_m}{\phi_b} + \frac{\dot{\phi}_b}{\phi_b} \right] b - (\eta_0 + \eta_1 b); \]  
\[ \dot{V}_F = rV_F - (\Phi - w - \delta V_F); \]  
\[ \dot{n} = h(1, \theta)(1 - n) - \delta n; \]

where \( V_F = \frac{\gamma}{h(\theta - 1)} \) and the interest rate spreads are given by

\[ r - r^g = \mu_g \sigma (u'(y^g) - 1); \]  
\[ r^g - r^m = \mu_m \sigma (u'(y^m) - 1); \]  
\[ r = \rho - \sigma \mu_p [u'(y^p) - 1]. \]

Depending on the underlying fundamentals and government policies, different monetary equilibria can emerge. These arise depending on which of the various collateral constraints bind. Note that inflation dynamics critically depend on the collateral services of fiat money, relative to other assets. This is the case as inflation is intimately linked to the return on fiat money which is given by

\[ \frac{\dot{\phi}_m}{\phi_m} = \rho - \sigma \mu_p [u'(y^p) - 1] - \sigma \mu_g [u'(y^g) - 1] - \sigma \mu_m [u'(y^m) - 1]. \]

From now on, to simplify exposition, we assume specific functional forms so that \( h(1, \theta) = A\theta^{1-\alpha} \) and \( u(x) = Dx^\nu \). In the next sections we characterize monetary equilibria that deliver the first best level of financial services in some states of the world. We first characterize the equilibria when total government liabilities are plentiful to satisfy the collateral needs of traders. Finally, we study the case where all assets (private and public) are needed to meet the collateral needs of Shi-Trejos-Wright traders.
4.1 Plentiful Government Liabilities

Fiat money alone is not sufficient to satisfy the collateral needs of traders, however, all government liabilities can. This situation is consistent with economies that have positive nominal interest rates. However, since all government liabilities (fiat money and government bonds) meet the collateral needs of traders, there is no interest rate spread between private claims and government bonds so that \( r^g = r = \rho \). It is easy to show that the corresponding dynamic monetary equilibrium is then given by

\[
\dot{m} = m \left( \zeta + \rho + \mu_m \sigma - \mu_m \sigma \nu Dm^{\nu - 1} \right); \\
\dot{b} = G - m \zeta + (1 + \rho - \eta_1) b - \eta_0; \\
\dot{\theta} = \frac{\theta}{\alpha} \left[ (\rho + \delta) - (\Phi - w) \frac{A^\theta - \alpha}{\gamma} \right]; \\
\dot{n} = A^{\theta - \alpha} (1 - n) - \delta n;
\]

where \( m < y^* \) and \( m + b \geq y^* \) which reflects that private assets are not required to satisfy the collateral needs of Shi-Trejos-Wright traders.

Steady States

Let us consider a situation where \( \dot{m} = \dot{b} = \dot{n} = \dot{\theta} = 0 \). It is easy to show that a unique monetary steady state exists where real balances and labor market tightness are given by

\[
m = \left( \frac{D \nu \mu_m \sigma}{\rho + \zeta + \mu_m \sigma} \right)^{\frac{1}{\sigma}}; \quad \theta = \left( \frac{A(\Phi - w)}{\gamma (\rho + \delta)} \right)^{\frac{1}{\sigma}};
\]

while real bonds and employment are given by

\[
b = \frac{\zeta m + \eta_0 - G}{1 + \rho - \eta_1}; \quad n = \frac{A^\theta - \alpha}{A^\theta - \alpha + \delta}.
\]

Finally, the resulting steady state inflation is exactly equal to the money growth rate.

Lemma 2 Monetary (\( \zeta \)) and fiscal policies (\( \eta_0, \eta_1 \)) do not affect the steady state unemployment rate. Moreover, changes in job separation rates (\( \delta \)), matching efficiency (\( A \)) and vacancy costs (\( \gamma \)) do not affect real balances nor real bonds.

Since Shi-Trejos-Wright traders can produce the numéraire good and have enough public assets to satisfy their collateral needs, the demand of private assets issued by the firm are not of first order importance for traders. As a result, the labor market characteristics are inconsequential for the portfolio allocation of Shi-Trejos-Wright traders.
Local Dynamics

The dynamic monetary equilibria displays a dichotomy between asset and labor markets. Moreover, the evolution of real balances is independent of the evolution of government debt. The corresponding characteristic polynomial is given by

\[ p(\lambda) = (a_{11} - \lambda)(a_{22} - \lambda)(a_{33} - \lambda)(a_{44} - \lambda); \]

where \( a_{ij} \) denotes the \((i, j)\) element of the Jacobian. Note that the eigenvalues associated with the asset market (i.e. the first two rows of the Jacobian) are independent of each other and from those associated with the labor market (i.e. the third and fourth rows of the Jacobian). The resulting asset market eigenvalues are given by

\[ \lambda_1 = (1 - \nu)(\mu_m\sigma + \rho + \zeta); \quad \lambda_2 = 1 + \rho - \eta_1; \]

while the labor market eigenvalues are

\[ \lambda_3 = \rho + \delta, \quad \lambda_4 = -\left(\delta + A\frac{(A(\Phi - w))^{1\gamma}}{(\rho + \delta)^\gamma}\right). \]

**Lemma 3** Monetary policy \((\zeta)\) can not alter the local determinacy properties.

This is the case, as when we approach to the zero lower bound \((\zeta \to -\rho)\), the monetary eigenvalue, \(\lambda_1\), can not change sign. Monetary policy can only then affect the rate of convergence/divergence towards the steady state. Similar properties are obtained in environments with frictionless financial and labor markets where the central bank follows a money growth rate rule.

**Lemma 4** The monetary equilibria is dynamically determinate whenever fiscal policy is such that \(\eta_1 > 1 + \rho\).

When government assets provide enough collateral to Shi-Trejos-Wright traders, we recover the standard stabilization policy prescriptions obtained in frictionless labor and financial markets under a money growth rate rule.\(^{21}\) In particular, the unique monetary equilibrium is dynamically determinate (indeterminate) when the fiscal is such that \(\eta_1 > 1 + \rho\) (\(\eta_1 < 1 + \rho\)), unsurprisingly given that taxes are pure lump sum and there are no interest rate spreads between bonds and private assets.

To summarize, when the government provides enough government liabilities to satisfy the collateral needs of Shi-Trejos-Wright traders, neither the particulars of the financial architecture (centralized and decentralized) nor the details of the labor market (frictional or frictionless) change the traditional fiscal requirements for price stability.

\(^{21}\)It is easy to check that in Leeper (1991) money in the utility function model or Woodford’s (1998) cashless economy, the same stabilization prescription is obtained.
4.1.1 Numerical Exercise

A numerical analysis is required to determine the quantitative implications. We can then evaluate the responsiveness of traditional stabilization fiscal policies relative to labor market characteristics. Parameters are chosen to represent an economy where government liabilities are enough to cover the collateral needs of traders. To discipline parameter values for the labor market, we closely follow Shimer (2005) who considers data for the U.S. from 1951 to 2003.\textsuperscript{22} The unit of time represents one quarter, thus we set $\rho = 0.012$ so that the annual real interest rates is approximately 5%. The job destruction rate is set such that $\delta = 0.1$ which is consistent with Shimer’s (2005) observation that jobs last approximately two and a half years on average. He also estimates that job finding rates are 0.45 per month. Following Shimer (2005), we normalize labor market tightness, $\theta = 1$, so that the worker-finding rate is equal to the job-finding rate. Therefore, we have that $\alpha = 0.72$ and $A = 1.35$. Arseneau and Chugh (2012) find that the cost of advertising a vacancy is 3% of total firm output, which yields a $\gamma = 0.03$ when we normalize firm output to one so that $\Phi = 1$. We then set $w = 0.6$ to be consistent with the labor share and set $G = 0.21$ so that government expenditures to GDP over the period equals 13%.

For the decentralized financial market, we normalize $D = 1$. We follow Chiu and Koppel (2016) and assume that traders are active in the market at Poisson rate $\sigma = 2.27$ which is consistent with observed turnover rates.\textsuperscript{23} Finally, given the average annual inflation rate over this period is 3.5%, we try to reproduce the observed base money to GDP ratio of 6.3% over the same period. To do so we set $\mu_m = 0.016$ and $\nu = 0.5$.

Under this benchmark parametrization, the labor market eigenvalues are $\lambda_3 = 0.112$ and $\lambda_4 = -7.309$. In order for fiscal policy to have a greater role in stabilizing the economy, fiscal policy has to be such that $\lambda_2 < \lambda_4$, which implies that $\eta_1 > 8.309$. Fiscal policy has to be exceptionally responsive to bond issuance.

4.2 Insufficient Government Liabilities

This situation shows the importance of considering the endogenous private provision of assets which are useful in decentralized frictional financial markets. Relative to the plentiful government liabilities, the evolution of market tightness and employment are the same. However, the dynamics for government liabilities differ. The resulting dynamic equilibrium is given by equations (19), (20) and

\begin{equation}
\dot{m} = m \left( \zeta + \rho + \mu_m \sigma - \mu_m \sigma \nu D m^{\nu-1} - \mu_g \sigma (\nu D (m + b)^{\nu-1} - 1) \right) 
\end{equation}

\textsuperscript{22}We refer to Hornstein, Krusell and Violante (2005) for a detailed discussion of the calibration.

\textsuperscript{23}Bao, Pan and Wang (2008) give turnover rates between one and two years for corporate bonds, while Goldstein, Hotchkiss and Sirri (2007) give the annual rate in the range of 0.8-1.2. See also Edwards, Harris and Piwowar (2007). Data for structured products are not readily available.
\[ \dot{\beta} = G - m\zeta + (1 + \rho + \mu_g\sigma - \eta_1 - \mu_g\sigma\nu D(m + b)^{\nu - 1})b - \eta_0 \]  

(22)

where \( m + b < y^* \) and \( m + b + \frac{n\gamma\theta}{A} \geq y^* \) reflecting that private assets are relevant to satisfy the collateral needs of traders.\(^{24}\)

Notice that the return on government bonds is now lower than that which would have been obtained with private assets; i.e, \( r = \rho > r^g \). The spread explicitly depends on the characteristics of the decentralized financial market which is given by

\[ r^b = \rho - \mu_g\sigma(\nu D(m + b)^{\nu - 1} - 1). \]

Private assets and government bonds are not perfect substitutes; neither as a store of value nor as collateral objects.\(^{25}\) This spread reflects that Shi-Trejos-Wright traders are willing to buy government bonds at a higher price because of the additional collateral services they provide.

As we can see from equation (21), the evolution of real balances now depends on government debt. Thus the fiscal position of the government has a direct impact on inflation dynamics. In particular, the return on fiat money is such that

\[ \frac{\dot{\phi}_m}{\phi_m} = \rho - \mu_m\sigma(\nu Dm^{\nu - 1} - 1) - \mu_g\sigma(\nu D(m + b)^{\nu - 1} - 1) \]

which depends on all government liabilities.

**Steady States**

After imposing the steady state conditions on equations, real balances are implicitly given by

\[ \zeta m + \rho - \mu_m\sigma(Dm^{\nu - 1} - 1) = \mu_g(D\nu(m + b(m))^{\nu - 1} - 1); \]

(23)

where \( b(m) = \frac{G - m\zeta - \eta_0}{\zeta + \eta_1 - \eta_0 D\nu m^{\nu - 1} - 1} \) and the steady state labor market observables are given by

\[ \theta = \left( A(\Phi - w) \right) \gamma (\rho + \delta); \quad n = \frac{A\theta^{1-\alpha}}{A\theta^{1-\alpha} + \delta}. \]

After having characterized steady state equilibria, we can now establish certain properties. All proofs can be found in the Appendix.

**Proposition 5** The monetary steady state is generically not unique.

This result is not surprising as interest rate spreads explicitly depend on real bonds. Now the government faces an interest payment that is non-linear in bonds outstanding. This non-linearity allows the possibility of two monetary steady states to exist. This situation reflects the bond seignorage revenue. In this situation part of government revenues respond to fiscal policy which

\(^{24}\)Recall that the private provision of liquidity is carried out by active firms so that \( L^p = n\frac{\gamma\theta}{A}. \)

\(^{25}\)Private assets and government bonds are not perfect substitutes as collateral objects as typically we have that \( \mu_g \neq 1 - \mu_m - \mu_g. \)
drastically changes inflation expectations as current fiscal choices are necessarily linked to future fiscal and monetary policy actions. It is important to highlight that this Laffer curve is purely driven by the collateral needs of Shi-Trejos-Wright traders.

We can conclude then that different asset combinations that deliver plentiful collateral are critically important in generating multiple steady states. Moreover, they also change how inflation expectations are formed. Finally, given the possibility of multiple steady states, sunspot equilibria can be easily constructed which further complicates the design of stabilization policies.\footnote{This feature resulting from the liquidity properties of bonds, is not observed in frameworks with frictionless financial markets.}

**Local Dynamics**

The characteristic polynomial associated with an economy where all assets are required to meet the collateral needs of traders is given by

\[
p(\lambda) = (a_{33} - \lambda)(a_{44} - \lambda)[(a_{11} - \lambda)(a_{22} - \lambda) - a_{12}a_{21}].
\]

As we can see, the eigenvalues of the labor block are independent of those corresponding to the asset market. Moreover, now the eigenvalues associated with the asset market are not independent of each other. In particular, the asset market eigenvalues are given by

\[
\lambda_1 = \frac{a_{11} + a_{22} + \sqrt{(a_{11} + a_{22})^2 - 4(a_{11}a_{22} - a_{12}a_{21})}}{2};
\]
\[
\lambda_2 = \frac{a_{11} + a_{22} - \sqrt{(a_{11} + a_{22})^2 - 4(a_{11}a_{22} - a_{12}a_{21})}}{2};
\]

while the labor market eigenvalues are

\[
\lambda_3 = -\left(\delta + A \left(\frac{A(\Phi - w)}{(\rho + \delta)\gamma}\right)\right); \quad \lambda_4 = \rho + \delta;
\]

where the different elements of the Jacobian are given by

\[
a_{11} = \sigma \nu (1 - \nu) D [\mu_{m} m^{\nu - 1} + \mu_{g} m (m + b(m))^{\nu - 2}]; \quad a_{12} = \sigma \nu (1 - \nu) D \mu_{g} m (m + b(m))^{\nu - 2};
\]
\[
a_{21} = -\zeta + b(m) \sigma \nu (1 - \nu) D \mu_{g} (m + b(m))^{\nu - 2};
\]
\[
a_{22} = 1 + \rho + \mu_{g} \sigma - \mu_{g} \sigma \nu D (m + b(m))^{\nu - 1} - \eta_{1} + \mu_{g} \sigma \nu (1 - \nu) D b(m) (m + b(m))^{\nu - 2};
\]

where \( m \) is the solution to equation (23).

**Lemma 6** The eigenvalues associated with the labor market are not affected by monetary nor fiscal policies.

As in the previous section, there is a decoupling between the labor and financial markets. Such is the case as government policies are not directly affecting the supply of private assets. Moreover,
the return on private assets is the natural interest rate in the economy. Thus, neither monetary nor fiscal policies can further increase the attractiveness of private assets as useful collateral or as a store of value.

**Lemma 7** The eigenvalues associated with the asset market depend on both monetary and fiscal policy parameters.

When private assets are required to provide the desired collateral needs by traders, the effects of all government policies jointly affect the stability properties of the eigenvalues associated with the asset market. This is not surprising as all policies alter the spread between private and public assets. This is in sharp contrast with frictionless labor and financial markets where the eigenvalues associated with the asset market are decoupled, so that one eigenvalue only depends on monetary policy parameters and the other depends only on fiscal parameters.

4.2.1 Numerical Exercise

To quantify the monetary equilibria the previous benchmark calibration needs to be modified, in particular, the parameters reflecting the demand for financial services. To do so we consider a one time *permanent shock* so that the level of utility per unit per financial service, \( D \), goes from 1 to 2. This captures disruptions in the financial market relative to normal times. As a result, agents require more assets to pledge as collateral.

With this later parametrization, Table 1 reports some monetary equilibria with the corresponding steady state market tightness, employment, real balances, real bonds and interest rate spreads as well as their eigenvalues.

| \( \eta_0 \) | 0.2 | 0.2 | 0.2 | 0.2 |
| \( \eta_1 \) | 1.02 | 1.02 | 1.01 | 1.01 |
| \( \mu_m \) | 0.016 | 0.016 | 0.01 | 0.01 |
| \( \mu_g \) | 0.7 | 0.7 | 0.7 | 0.7 |
| \( \zeta \) | 0.008 | 0.008 | 0.009 | 0.009 |
| \( \theta \) | 1158.68 | 1158.68 | 1158.68 | 1158.68 |
| \( n \) | 0.989 | 0.989 | 0.989 | 0.989 |
| \( m \) | 0.544 | 0.853 | 0.645 | 0.817 |
| \( b \) | 0.451 | 0.13 | 0.34 | 0.165 |
| \( \rho - r^g \) | 0.004 | 0.016 | 0.015 | 0.018 |
| \( \lambda_1 \) | 0.01 | -0.11 | 0.003 | -0.004 |
| \( \lambda_2 \) | 0.961 | 0.971 | 0.96 | 0.964 |
| \( \lambda_3 \) | 0.112 | 0.112 | 0.112 | 0.112 |

Table 1: Equilibria when only traders are taxed and government assets are scarce.

As we can see from Table 1, multiple steady states exist. This is the case irrespective of a fiscal stance; i.e. \( 1 + \rho < \eta_1 \) or \( 1 + \rho > \eta_1 \). In particular, the steady state associated with a larger interest
rate spread is dynamically determinate and is associated with the largest issuance of government bonds in steady state. On the other hand, the steady state with the lower spread is dynamically indeterminate and has associated the smallest government bonds in steady state. This is in sharp contrast to the case where there are no interest rate spreads.

Numerical results also show that the steady state level of tax revenues \((\eta_0, \eta_1)\) are critically important in determining interest rate spreads and the rate of convergence to the steady state. This is in sharp contrast to models with frictionless financial and labor markets, where tax revenues in the long run do not affect the nature of stabilization policies. Finally, the qualitative properties highlighted in Table 1 are robust to various departures from the benchmark parametrization.\(^{27}\)

Summarizing, the composition of assets that deliver plentiful assets for collateral purposes is key in determining the properties of monetary equilibria. This is the case as different combinations of assets meeting the collateral needs of traders can imply interest rate spreads between government bonds and private assets. The existence of these spreads allows for all government policies to affect the degree of substitution between private and public assets as store of value through changes in the interest rate spread. Moreover, it can deliver multiple steady states. Thus it follows that conventional stabilization policies obtained in frictionless environments are not going to be effective.

5 Taxing Capacity

To better understand how the varying collateral needs of traders interact with the fiscal capacity of an economy, we now consider an alternative tax scheme where all agents are taxed. Herein, active firms and workers also face non ad-valorem taxes. This alternative taxing structure is not as innocuous as it may seem. Other than increasing the fiscal backing of bonds, this new tax scheme impacts firms' hiring decisions, as taxes now affect the value of the firm.\(^{28}\) As a result, part of the total tax base (the one corresponding active firms) responds to fiscal policies as it affects entry decisions. This holds true even when taxes are non ad-valorem. When firms expect future taxes to be high, they reduce recruitment. Since the tax base increases with the number of jobs filled, government budget balancing requires a higher tax level, in accordance with employers' beliefs about future policies. This fiscal channel arising from the frictional labor market, drastically changes inflation expectations relative to environments frictionless labor markets. In frictionless environments, the total amount of revenue impacts inflation expectations and not the specifics of who pays taxes as they are pure lump sum. Once the labor market is frictional, this property does

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\(^{27}\) Similar results are obtained when fiat money is not important relative to nominal bonds and private assets as collateral (\(\mu_m\) is small). We consider different curvatures to the payoff of financial services and the expected time before the trader receives an opportunity to purchase financial services is reduced (increased) (which translates to an increase (decrease) in \(\sigma\)).

\(^{28}\) The fact that fiscal policy affects the supply of private assets that competes with public assets in traders portfolio captures Tobin's (1981) view of policy: "The obvious point that taxation lowers expected yields is not the whole story, because it changes the entire distribution of (uncertain) returns".
not hold true anymore. This additional expectational channel has important implications for the evolution of inflation, debt, unemployment and interest rate spreads. We analyze these differences in the next sections.

5.1 Plentiful Government Liabilities

Relative to the benchmark model where only Shi-Trejos-Wright traders are taxed, the evolution of real balances and employment remain the same. However, the dynamics for real bonds and market tightness are different. The new dynamic monetary equilibria is given by equations (17), (20) and the new evolution for government bonds and market tightness are given by

\[ \dot{b} = G - m\zeta + \left(1 + \rho - (1 + 2n)\eta_1\right)b - (1 + 2n)\eta_0; \]

\[ \dot{\theta} = \frac{\theta}{\alpha} \left[ (\rho + \delta) - (\Phi - w - (\eta_0 + \eta_1b)) \frac{A\theta^{-\alpha}}{\gamma} \right]; \]

where \( m < y^* \) and \( m + b \geq y^* \).

In this new taxing scheme, the dynamics of labor market tightness, \( \theta \), (which are related to firms’ profits) explicitly depends on the evolution of real bonds. This is not surprising given that the fiscal rule links the evolution of taxes to real bonds. Unemployment, on the other hand, is indirectly affected by fiscal and monetary policies through the dynamics of labor market tightness. This is the case as taxes affect the entry decisions of firms. Once entry costs are covered, the level of taxes influence the firm’s ability to issue private claims. The value of these private assets is further affected by the government’s ability to issue fiat money and bonds that compete with private assets both as a store of value and as collateral.

Labor market characteristics and outcomes also affect asset market dynamics. This is the case as a number of factors directly affect the value of the firm, including labor market conditions, the costs of posting vacancies and the severity of the labor market frictions. The resulting profits, in turn, affect the firm’s ability to provide private liquidity to financial markets.\(^{29}\)

Steady States

After imposing the steady state conditions and substituting equations (25) and (19) into equation (26), we have that the steady state market tightness is implicitly given by

\[ \frac{(\rho + \delta)\gamma^\alpha}{A} = \Phi - w - \eta_0 - \eta_1 b(\theta); \]

where \( m = \left( \frac{D_\nu \mu_{\nu \sigma}}{\rho + \zeta + \mu_\sigma} \right)^{-1} \), \( n(\theta) = \frac{A^{\theta^{-\alpha}}}{\alpha A^{\theta^{-\alpha + \theta}}} \) and \( b(\theta) = \frac{(1 + 2n(\theta))\eta_0 + m\zeta - G}{1 + \rho - \eta_1(1 + 2n(\theta))}. \)

**Proposition 8** The monetary steady state equilibria is generically not unique.

\(^{29}\)Gatti et al (2012) show that for 18 OECD countries over the pre-crisis period, 1980-2004, the impact of financial variables depend strongly on the labor market context while the impact of labor market characteristics on financial markets appears to be less significant.
In contrast to the case where only traders were taxed, this equilibrium has the property that multiple steady states can not be ruled out. This result is not surprising as non-ad-valorem taxes affect the entry decisions of firms, impacting their hiring decisions and the supply of private liquidity. As a result, the tax base stemming from firms’ activities will respond to changes in taxes delivering a “Laffer curve”. Similar results in the pure labor search context are found in Rocheteau (1999a, 1999b).30

Local Dynamics

The evolution of labor market outcomes are not independent of the real value of government debt. The corresponding characteristic polynomial associated to this monetary equilibria is given by

\[ p(\lambda) = (a_{11} - \lambda)(a_{22} - \lambda)(a_{33} - \lambda)(a_{44} - \lambda) + a_{34}a_{43}a_{24}; \]

where the different elements of the Jacobian are given by

\[ a_{11} = (1 - \nu)(\rho + \zeta + \mu_m \sigma); \quad a_{22} = 1 + \rho - (1 + 2n(\theta))\eta_1; \quad a_{24} = -2(\eta_0 + \eta_1 b(\theta)); \quad a_{32} = \frac{\eta_1 \theta^{1-\alpha} A}{\alpha \gamma}; \]
\[ a_{33} = A\theta^{-\alpha}(\Phi - w - \eta_0 - \eta_1 b(\theta)); \quad a_{43} = A(1 - \alpha)(1 - n)\theta^{-\alpha}; \quad a_{44} = -(A\theta^{1-\alpha} + \delta); \]

where \( \theta \) is the solution to equation (26). The eigenvalues corresponding to the asset and labor market are not completely decoupled. One of the asset market eigenvalues is given by

\[ \lambda_1 = (1 - \nu)(\rho + \zeta + \mu_m \sigma); \]

while the rest are roots to the following cubic polynomial

\[ p_3(\lambda) = (a_{22} - \lambda)(a_{33} - \lambda)(a_{44} - \lambda) + a_{32}a_{43}a_{24}. \]

Lemma 9 A monetary equilibria will typically display endogenous volatility.

Endogenous volatility is likely as the generic solution to a cubic polynomial has two complex roots. As a result, unemployment and inflation will fluctuate even when the environment has no exogenous shocks. This endogenous volatility is consistent with the results found in Rocheteau (1999a) in the pure labor market context.31

Lemma 10 Eigenvalues depend on both monetary and fiscal policy parameters as well as the characteristics of the labor market.

The steady state tax revenue and the money growth rate affect the magnitude of the eigenvalues. This is not surprising as part of the tax base can respond to changes in the tax policy which agents

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30The author finds that under a balanced-budget rule, the existence of multiple equilibria is a generic property. The government can lead the economy to a high equilibrium by fixing the rate of tax on wages and then setting the replacement ratio so that its expenditure matches its receipts.

31Rocheteau (1999a) shows that a balanced-budget rule is able to generate endogenous cycles.
take as given. Thus, traditional fiscal policy prescriptions to stabilize prices are unlikely to be effective. This is the case as multiple equilibria can be observed and the stability properties of the eigenvalues associated with the asset market depend on both monetary and fiscal policy parameters.

This holds even when there are no interest rate spreads. These different policy implications arise from labor experiencing market search frictions. Taxing these productive activities, even when these are non ad-valorem, change the firm’s entry decision which ultimately affects the fiscal backing of bonds. The exact composition of tax revenues matters when frictional labor markets are present.

5.1.1 Numerical Exercise

We consider the same baseline calibration used in Section 4.1.1 where no interest rate spreads are observed. Table 2 reports some monetary equilibria.

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<td>0.108</td>
<td>-0.347+0.602 i</td>
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<td>( \lambda_4 )</td>
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<td>0.108</td>
<td>-0.347+0.602 i</td>
</tr>
</tbody>
</table>

Table 2: Equilibria when all agents are taxed and \( \mu^m=0.016 \).

As we can see, the multiplicity of steady states are observed regardless whether the fiscal authority follows policy, such that \( \eta_1>1+\rho \) or \( \eta_1<1+\rho \). The monetary steady state associated with the largest government debt is dynamically indeterminate and exhibits endogenous volatility. In contrast, the steady state with the smallest government debt is dynamically determinate and does not exhibit damped oscillations. Irrespective as to whether fiscal policy is aggressive or passive, the tax revenue in the steady state critically affects the speed of convergence (divergence) and the size of the endogenous fluctuations.

These equilibrium properties are in sharp contrast to the case where traders are the only private agents being taxed. This difference highlights the importance of taxation in environments with frictional labor markets. Not considering the labor market a la Mortensen and Pissarides (1994) is not as innocuous as it may seem as it drastically changes the nature of monetary equilibria. It not only yields different predictions regarding the dynamic properties, but also in terms of the multiplicity of equilibria. These differences are observed even when there are no interest rate spreads. This a direct consequence of the composition of the total tax revenues. In frictionless
environments, the total amount of revenue impacts inflation expectations and not the specifics of who pays the taxes as they are pure lump sum. Once the labor market is frictional, this is not the case, as non ad-valorem taxes affect marginal decisions, ultimately impacting the taxable base. Finally, the qualitative properties highlighted in Table 2 are robust to various departures from the benchmark parametrization.

5.2 Insufficient Government Liabilities

The evolution of real balances and employment remain the same relative to the benchmark model where only Shi-Trejos-Wright traders are taxed. However, the dynamics of real bonds and market tightness change as to reflect the new taxing environment. When all government liabilities are sufficient to meet the collateral needs, it is easy to show that the dynamic monetary equilibrium is given by equations (20), (21) and

\[ \dot{b} = G - m\zeta + (1 + \rho + \mu_g \sigma - (1 + 2n)\eta_1 - \mu_g \sigma \nu (m + b)^{\nu - 1}) b - (1 + 2n)\eta_1; \] (27)

\[ \dot{\theta} = \frac{\theta}{\alpha} \left[ (\rho + \delta) - (\Phi - w - (\eta_0 + \eta_1 b)) \frac{A^{\theta - \alpha}}{\gamma} \right]; \] (28)

where \( m + b < y^* \) and \( m + b + n A^{\theta m} \geq y^* \).

As in the case where government liabilities provide enough liquidity provided, the dynamics of labor market tightness depend on the evolution of real bonds. Similarly, unemployment dynamics are indirectly affected by the evolution of real bonds through labor market tightness. Finally, the spread between public and private assets is given by

\[ r^b = \rho - \mu_g \sigma (\nu D(m + b)^{\nu - 1} - 1) \]

and the evolution of inflation depends on the dynamics of all government liabilities.

Steady States

After imposing the steady state conditions, the stationary monetary equilibria solves the following system of equations for real balances (m) and real bonds (b) are given by

\[ \zeta + \rho - \mu m \sigma (D \nu m^{\nu - 1} - 1) = \sigma \mu_g (D \nu (m + b)^{\nu - 1} - 1); \] (29)

\[ G - m\zeta - (1 + 2n(b)) \eta_0 + (1 + \rho - (1 + 2n(b)) \eta_1) b = b \sigma \mu_g (D \nu (m + b)^{\nu - 1} - 1); \] (30)

where labor market observables are such that \( \theta(b) = \left( \frac{A(\Phi - w - m - m b)}{\gamma(\rho + \delta)} \right)^{\frac{1}{\alpha}} \) and \( n(b) = \frac{A \theta(b)^{1 - \alpha}}{A \theta(b)^{1 - \alpha} + \delta}. \)

Note that in this monetary equilibrium, agents acquire government bonds because of their collateral value in decentralized financial markets, thus exhibiting a liquidity premium.
Local Dynamics

The corresponding characteristic polynomial associated to the Jacobian of this monetary equilibria is given by

\[ p(\lambda) = (a_{11} - \lambda)(a_{22} - \lambda)(a_{33} - \lambda)(a_{44} - \lambda) + a_{32}a_{43}a_{24} - a_{12}a_{21}(a_{33} - \lambda)(a_{44} - \lambda). \]

As we can see, there is no decoupling of the asset market and the labor market eigenvalues. These eigenvalues depend on asset and labor market characteristics. This is sharp contrast to what is typically found in the literature of centralized and frictionless financial and labor markets. The eigenvalues are roots to quartic polynomial given by

\[ p(\lambda) = A_4\lambda^4 + A_3\lambda^3 + A_2\lambda^2 + A_1\lambda + A_0 \]

where \( A_i \) are non-linear functions of the different elements of the Jacobian which are given by

\begin{align*}
a_{11} &= \sigma \nu (1 - \nu) D[\mu_m m^{\nu-1} + \mu_g m (m + \beta)^{\nu-2}]; \\
a_{12} &= \sigma \nu (1 - \nu) D\mu_g m (m + \beta)^{\nu-2}; \\
a_{21} &= -\zeta + b \sigma \nu (1 - \nu) D\mu_g (m + \beta)^{\nu-2}; \\
a_{22} &= 1 + \rho + \mu_g \sigma - \mu_g \sigma \nu D(m + \beta)^{\nu-1} - \eta_1 (1 + 2n(b)) + \mu_g \sigma \nu (1 - \nu) D b (m + \beta)^{\nu-2}; \\
a_{24} &= -2(\eta_0 + \eta_1 b); \\
a_{32} &= \frac{A \theta(b)^{1-\alpha} \eta_1}{\alpha}; \\
a_{33} &= \frac{A \theta(b)^{-\alpha}(\Phi - w - \eta_0 - \eta_1 b)}{\gamma}; \\
a_{43} &= A(1 - \alpha)(1 - n) \theta(b)^{-\alpha}; \\
a_{44} &= -(A \theta(b)^{1-\alpha} + \delta); \\
\end{align*}

where \( m \) is the solution to equation (23).

Lemma 11 Asset and labor market eigenvalues will typically depend on the specifics of monetary and fiscal policies and the characteristics of the frictional labor and financial market.

With this new tax scheme, all policies impact firm’s profits which intimately link labor and asset markets by hiring workers and issuing equity. Since fiscal policies affect the entry decision of firms, government policies directly impact the tax base, ultimately changing the fiscal backing of bonds. Moreover, since the tax base increases with the number of jobs filled, government budget balancing requires a higher tax level, in accordance with employers’ beliefs about future policies. This fiscal channel (arising from the frictional labor market) drastically affects inflation expectations. Thus, traditional stabilization policies are not expected to be effective.
Lemma 12  A monetary equilibria will typically display endogenous volatility.

This is the case as the solution to a quartic polynomial generically has complex roots. As a result, unemployment and inflation will fluctuate even when the environment has no exogenous shocks.

5.2.1 Numerical Exercise

A numerical analysis is required to determine the impact of traditional stabilization fiscal policies on this frictional environment. The underlying parameter values are the same as in Section 4.2.1. Table 3 reports some monetary equilibria.

<table>
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<th>η₀</th>
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<th>η₃</th>
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<td>0.111</td>
<td>2.626-4.96 i</td>
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<tr>
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<td>3.13+5.767 i</td>
<td>0.539</td>
<td>2.626+4.96 i</td>
</tr>
</tbody>
</table>

Table 3: Equilibria when all agents are taxed.

In contrast to traditional frictionless labor and financial market frameworks, multiplicity of equilibria are observed regardless as to whether fiscal policy is aggressive or passive. The equilibrium associated with the steady state with largest unemployment is dynamically indeterminate. Locally, inflation and unemployment will converge to the steady steady with fluctuations that grow over time when η₁ < 1 + ρ is followed. On the other hand, the equilibrium associated with the steady state with lowest unemployment is dynamically determinate. In contrast, when the fiscal authorities follow η₁ > 1 + ρ policy, endogenous volatility is always observed regardless if the unemployment is high or low.

6 Conclusions

The global financial crisis has highlighted a new set of imperfections associated with financial and labor markets that should be taken into account when thinking about policy. In order to better inform policymakers, frameworks should have agents trading various financial markets (frictionless and centralized as well as frictional and decentralized) with multiple assets (both private and
public), where workers face unemployment and firms issue private assets that can be used as a store of value and collateral.

The properties of monetary equilibria fundamentally change once firms (other than hiring workers in decentralized labor markets) also supply private assets that are valued by traders because of their collateral and store of value functions. This is the case as firms connect the frictional asset and labor markets. As a result, interest rate spreads between government bonds and private assets will depend on labor market conditions. Moreover, as in Tobin (1961), the equilibrium price level is determined by the joint valuation of all assets. However, in contrast to this author, interest rate spreads are not driven by the risk properties of assets but rather by market incompleteness. Thus the specific nature of the financial frictions are going to be key in shaping inflation dynamics.

In this frictional environment fiscal policies not only affect the equilibrium interest rate spreads but also the tax base. These features allow multiple steady states to be generically observed. This multiplicity can be generated by two channels. One is characterized by the collateral value of assets in decentralized frictional financial markets. These imperfections generate a liquidity premium so that a bond seignorage Laffer curve is observed. The other source of multiplicity is a direct consequence of the composition of the tax revenues. In this frictional environment taxes on firms directly affect their entry decision, making them distortionary. This then generates the traditional Laffer curve.

Finally, this frictional environment has an additional channel through which inflation expectations are modified which is drastically different relative to environments with frictionless labor markets. More precisely, firms reduce recruitment when they expect future taxes to be high. Since the tax base increases with the number of jobs filled, government budget balancing requires a higher tax level, in accordance with employers’ beliefs about future policies. It is then not surprising that monetary equilibria will typically exhibit endogenous volatility. Thus, because of the existence of multiple steady states and endogenous volatility, traditional stabilization policies obtained in frictionless labor and financial environments are not effective. We can conclude then that ignoring the frictional features in financial and labor markets is not as innocuous as it may seem a priori as they predict drastically different equilibrium properties.

References


Appendix

Proof Proposition 5

Let us define the LHS as the left hand side and RHS as the right hand side of equation (23), respectively. It is easy to show that LHS is increasing in $m$ while the RHS is non-monotonic. Thus, we can rule out the existence of multiple steady states.

Proof Proposition 8  The left hand side of equation (26) is always increasing in $\theta$ while the right hand side is non-monotonic. Thus, we can rule out the existence of multiple steady states.