Paths of economic development: modelling factors of endogenous growth

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Introduction

Economic theory originated essentially under the impact of economic development, and more specifically just as the major Western nations witnessed the rise of an endogenous, self-sustaining mechanism of cumulative economic growth. This mechanism hinges on entrepreneurial initiative which, chiefly through innovative decisions, harnesses the resources of technology, in the broadest sense, to the service of the economy, making them one main basis of profitability and competition. Nevertheless, the basic features of entrepreneurship and innovation, and their linkages to the rise of uncertainty have not received adequate treatment by economists, especially by macro-economists. The studies on endogenous growth that followed Kaldor's (1960) function of technical progress, Arrow's (1962) idea of learning by doing and Shell's (1967) specification on the inventive sector devoted to produce knowledge represent the most advanced answer to some of these weaknesses. Romer (1990), Grossman and Helpman (1991), Aghion and Howitt (1992) have enriched Shell's intuition by linking the appearance of new intermediate products and quality based innovation to the development of knowledge.

All the above models (and those centred on the quality of human capital, as in Lucas, 1988) explain endogenous growth through the addition of some particular factors in the production function. Hence, they consider production simply as the transformation of given inputs into output, ignoring that modern dynamic economies are characterized markedly by repeated shifts of production functions due to innovation, as well as by uncertainty and the entrepreneurs' discovery role.

Kaldor's (1960) openness to Schumpeter's (1954; 1977) teaching and the references to Schumpeterian creative destruction by some followers of the mainstream economics are remote to fill this gap. As a matter of fact, Schumpeter, while he insists on the role of innovative entrepreneur, practically forgets the associated phenomenon of uncertainty. This prevents him enucleating some major features of entrepreneurship and representing satisfactorily the evolutionary mechanism of modern economy, in particular,

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the basic interaction between innovation and uncertainty, and specifying some supply functions not merely derived by the production function.

A useful tool to remedy the above drawbacks is von Mises' (1976) and Hayek's (1989) teaching on the incompleteness of knowledge, the role of unintentional events and uncertainty, the discovery process by trial and error through markets, the co-ordination function of these and the meaning of entrepreneurship that emerges in this context, as systematized for the economy by Kirzner (1973; 1989).

A central aspect of this essay is in fact a notion of dynamic competition resulting from the interaction between innovative and adaptive entrepreneurship, i.e. the combination of the Schumpeterian notion of creative destruction and Kirzner's treatment of entrepreneurial alertness, market process and co-ordination. This imposes a redefinition of the theory of the decisions to produce that rejects the traditional theory of firm in favour of an alternative approach consistent with the notion of dynamic competition, that permits to point out some explanatory factors of output different from the availability of inputs, often ignored.

Another key point of this analysis is the role it ascribes to the variance of profit rates across firms, considered as a proxy of the degree of uncertainty and disequilibria characterizing the economy, that allows some clarification on the theory of accumulation and to fuse adaptive and innovative entrepreneurship in a unitarian process. This leads to an explanation of business cycle and an extension of the theory to social development and cycle, evidentiating an unambiguous direction of the evolutionary process.

To better express the mechanism of evolution that this interpretation sets out, a concise application to successive phases of economic development is also performed by introducing, in the basic model, some alternative assumptions, mainly on entrepreneurship, technological progress and, more broadly, the performance of dynamic competition, on income distribution and the forms of markets organization. This provides a comprehensive interpretative tool and pictures of different growth processes and the crucial factors on which they depend.

The model hypothesizes a closed economy, excludes money and does not consider the public sector, but these shortcomings might easily be remedied.

The model

A dynamic economy, i.e. characterized by incessant and unpredictable changes of technology, consumers' preferences and the availability of natural resources, hence by innovation and uncertainty, is hardly representable by traditional economics. A more suitable theoretical framework must be centred on the specification of the basic mechanism propelling such a dynamism. This cannot be fed and driven efficiently by a centralized bureaucracy. So, the explanation of economic growth and development must give a central role to the entrepreneur and a realistic interpretation of markets. Besides, it must endogenize innovation, uncertainty and adaptation, representing the main features of the evolutionary process of the economy and society. The fulfilment of these requirements can take advantage from the specification of an appropriate notion of dynamic competition. But a preliminary definition is indispensable for a better understanding of the analysis that will follow.

This essay expresses the dimensions of disequilibria (accompanying the development process), and the existing set of opportunities for profit, through the differences among the profit rates of firms, as measured by the variance of profit rates across firms. It also considers that this measure offers an important proxy of the degree of uncertainty of economic life, both because the residual nature of profits fully expresses the ultimate impact of stochastic elements affecting the economic process, and because the differences among the profit rates of firms gives a faithful picture of the incompleteness of knowledge; in fact, a zero difference (variance) of profit rates across firms would require a perfect knowledge, with omniscience of the entrepreneur.

Economic competition is driven by the search for profit that takes two forms: first, the creation of new opportunities for profit through innovation, which can be called innovative entrepreneurship and competition or, following Schumpeter (1954), creative destruction. (Schumpeter says: "This kind of competition is as much more effective than the other (based on prices) as a bombardment is in comparison with forcing a door"); second, the systematic exploitation of existing opportunities for profit (synthetized by the variance of profit rate) due to market disequilibria, which can be called adaptive entrepreneurship and competition – it is well expressed by the Kirzner's notions of market process and entrepreneurial alertness.

Innovative entrepreneurship causes disequilibria, obsolescence, the amplification of the variance of profit rates, and uncertainty, while adaptive entrepreneurship is the leading force of the adjustment processes towards new equilibria, thus implying the reduction of disequilibria and uncertainty. Innovative entrepreneurship is embodied by innovative investment, while adaptive entrepreneurship is expressed by the incessant revision of productive choices stimulated by the variability of profit rates and expectations.

The notion of dynamic competition that this essay sets out results from the combination of the two above forms of competition. There exist some important interactions between them, that may be described as follows.

In the presence of high disequilibria and a large variance of profit rates across firms and uncertainty, adaptive entrepreneurship (i.e. the revision of productive choices directed to take profit of the existing disequilibria or, more precisely, pushed by the variance of profit rate) prevails, at the expense of innovative entrepreneurship. This is also compressed by the fact that investment, mainly innovative, involving long run expectations, sunk costs and irreversibilities, is discouraged by high uncertainty that reduces the reliability of information and increases its cost (furthermore innovations have to meet initially various unexpected drawbacks, that imply lower return, high uncertainty and a large use of entrepreneurial skills). But the predominance of adaptive entrepreneurship resulting in the reduction of disequilibria and the variance of profit rate, squeezes out profit opportunities; so that such a predominance erodes its own basis. To overcome the depressive impact of this squeeze on profit rate, entrepreneurs will be encouraged to intensify innovation, which again amplifies the variance of profit rate, hence the set of existing opportunities for profit. Such a shift towards innovative competition is also stimulated by the fact that the decrease in uncertainty due to the equilibrating nature of adaptive competition favours and encourages substitution of plants, previously postponed, and hence innovation. But the diffusion of innovations causes a rise in obsolescence, disequilibria, the variance of profit rate and uncertainty, thus recreating the basis of adaptive competition, and the fall of the profits achieved by the pioneers; this opens the door to the recovery of adaptive entrepreneurship, while innovative projects are put in the drawer, waiting for some clearer perspectives and a further recovery of innovative competition. In sum, disequilibria and uncertainty intensify adaptation (i.e. the revision of productive choices aimed at taking advantage of the existing opportunities for profit due to disequilibria), and depress innovation; this pushes the economy to adjust towards equilibrium (thus squeezing adaptive opportunities for profit) that, in turn, stimulates innovation both to recreate profit opportunities and owing to the improvement of long r un expectations due to the decrease in uncertainty; and so on, with a cyclical interaction of the above two forms of competition.

This innovation-adaptation process is at the heart of the mechanism of economic growth and development and represents a specification, for the economy, of the more general succession "innovation-structural organization" giving the basic engine of social development. Instead, it is almost irrelevant for
the explanation of the development process to investigate the extremely various
ways inventions take place, as these produce effects on the economy through innovation. If a competitive mechanism stimulating innovative investment operates, the knowledge and inventions required to feed it will certainly be produced, in one way or the other. A subtle deepening of this matter is in Scott (1992). A complete formalization of the process innovation-adaptation would require a micro analysis. A first step of that formalization, useful for macro analysis, may consist in the explanation of innovation and uncertainty-variance of profit rate through a Lotka-Volterra predator-prey system (with uncertainty acting as the predator and innovation as the prey) that describes the cyclical interaction between these two variables:

$$DI_I = \beta_1 I_I - \beta_2 u I_I \quad (1)$$

$$Du = -\beta_3 u + \beta_4 I_I u + c_1 \xi \quad (2)$$

I_I = innovative investment

u = variance of profit rate across firms, which also is a proxy of uncertainty per unit of output

ξ = exogenous factor of uncertainty

D = differential operator d/d_t.

Of course, the variance of profit rate can be explained only at micro level. Nevertheless, the simple explanation given by equation (2) seems to be helpful for macro-economic investigation.

The parameter β_1 is a constant exponential rate of growth of innovation, expressing the autonomous push to innovate due to entrepreneurial aggressiveness; its impact on *DI* is reduced by the rise in the variance of profit rate across firms and uncertainty. β_3 is an exponential rate of variation of the variance of profit rate and uncertainty; its negative sign expresses the compressing effect on *u* arising from adaptive competition.

The above formalization implies a prominence of equations (1) and (2) on the remaining relations of the model. These are influenced by the Volterra system but do not influence it. A more detailed representation of reality would remove such a prominence and establish a bidirectional linkage.

It may be assumed that the "reproduction" hypothesis, typical of Volterra's study on populations, operates only in the equation of innovation in that each innovation is strongly influenced by the state of knowledge due to previous innovations. In the equation of the variance of profit rates, however, it may operate only backwards as uncertainty stimulates adaptation. This means that in (2) the cross-product term of Volterra, the encounter between predator and prey, will be replaced by the prey (innovation) only.

The parameter of the above differential system gives an important picture of dynamic competition and the economic development process characterizing various countries and different sectors of the same country (as well as an explanation of the difference in the rate of growth among countries).

Various studies have measured the degree of dynamic competition in the economy (Mueller, 1990; Odagiri, 1994) by the rapidity of reduction in the differential (hence variance) of profit rates. Such a procedure only considers the adaptive aspect of dynamic competition, so substantially it limits itself to the term β_3 of equation (2). This is a poor draft of the forces of competition and economic dynamism. The dynamic competition process consists in a disequalibrating-equilibrating movement. To understand its meaning, intensity and implications it is necessary to consider all the parameters of equations (1) and (2), taking present that parameters β_1 and β_2 express respectively the innovative push and its brake, and parameters β_3 and β_4 synthetize respectively the adaptive push and its brake. Thus β_1 and β_4 represent the disequilibrating forces while β_3 and β_2 synthetize the equilibrating forces.

To complete the formalization of the process of dynamic competition, we need a theory of entrepreneurial decisions to produce consistent with such a process, which permits specification of an explanatory function of output. The two basic assumptions of the mainstream theory of firm, i.e. the hypothesis that entrepreneurs know technology perfectly (the constraint of the optimum problem in such a theory) and the notion of perfect competition based on prices, are at all inconsistent with dynamic competition, hence unable to represent reality. In the modern economies dominated by innovative competition and uncertainty, technical coefficients are known only after the accomplishment of the productive process. This study substitutes to the mainstream theory of the firm based on the maximization of profit under the constraint of the available technology and productive resources, a theory of the entrepreneurial choice postulating the maximization of a function expressing the "attractiveness" of each productive choice, under the constraint of the available entrepreneurial skill. More precisely, the hypothesis is that the entrepreneur distributes his skills among the sectors lying in the area of his interests and choice, with the purpose of maximizing the total benefit which can be derived from productive skills. This approach is suggested, among other things, both by the elementary consideration (first pointed out by Schumpeter) that the only scarce resource for an entrepreneur is his skill as all other resources can easily be provided at the market prices by a successful businessman, and by the fact that the entrepreneur's knowledge of his skill (the constraint of our maximum problem) is much better than that of the variable technologies. Of course, entrepreneurial skills have a different content with reference to an individual or a managerial firm; they are largely represented by the decisional routines typical of each firm.

The maximum problem for each entrepreneur may be formulated as follows:

$$\text{Max } \Sigma_i \, r^{\circ}_i \, f(X_i)$$
$$\Sigma_i \, g_i u_i X_i \leq E^{\,p}$$
$$r^{\circ}_i X_i \geq M_i$$

i refers to each activity:

- X = level of output;
- u = degree of uncertainty;
- g = entrepreneurial skills required per unit of output, in the presence of a given degree of uncertainty (say u = 1);
- $E^{\,p}$ = available entrepreneurial skill (entrepreneur engaged in some activities should not refer the first constraint to $E^{\,p}$ but reallocated entrepreneurial skill resulting from the difference between his total available skill and that absorbed by the level of profitable activities in which he operates);
- M = minimum expected advantage required to operate in sector i.

r° is a measure of "attractiveness", for each entrepreneur, of the various activities. It may be represented as a function of the actual observed profit rate (r), uncertainty (u) and the excess demand (X^d/X) influencing expectations and τ (the non-monetary benefits connected to the entrepreneurial role), i.e.:

$$r^{\circ} = f(r, u, X^d/X, \tau)$$

with $\vartheta r^{\circ}/\vartheta u<0$. r is a sectoral profit rate and X (in the term X^d/X) refers to sectoral output, not to output of each entrepreneur that would imply circularity in the optimization problem. (More precisely, the function r° should put $\dot{r}_i$, indicating the highest profit rate that entrepreneur knows, in the place of r_i. For its part, r'_i should be explained through an equation expressing the search for profit, which is obviously promoted by the variance of profit rate. Of course, these developments would imply a micro specification of the theory)

The constraints of the maximization problem are either linear, with g and u being given for the entrepreneur, or convex, as the scale of production may at first fall and later increase the skills required per unit of output; while the objective function is concave owing to the increasing effect of output on profitability due to economies of scale, and the decreasing one due to organizational limits and increasing risk. (Thus the Kuhn-Tucker conditions are necessary and sufficient for the existence of a global maximum.) The above problem expresses the entrepreneurs' tireless revision of choices, directed to exploit the best existing market opportunities. The entrepreneur is obliged to search for the best profit opportunities by uncertainty, i.e. by the fear that otherwise he might make losses in the competition with maximizing entrepreneurs, and be forced out of business. Of course, the theoretical foundation of the above optimization approach does not require that entrepreneurs effectively solve maximization problems, but only that such an approach well represents the basic behaviour of entrepreneurs. The central position it attributes to skills and uncertainty is consistent with bounded rationality in a world characterized by imperfect information so that learning processes and any associated non-linearities (Day and Chen, 1993), as well as with Nelson's and Winter's (1982) teaching on decisional routines which represent an important expression of entrepreneurial skills. Moreover, the hypothesized entrepreneur's behaviour is consistent with a large variety of firm organization and decisional routines, which determine the degree of success of each firm. The maximum problem has a unique solution, implying the functional relationship $X_i = f(r°_i, E^p, g_i, u_i, M_i)$. At the macro level g and M disappear but not $r°$ (the aggregate attractiveness on entrepreneurs to produce) as its value influences output, both through entry and exit (hence E^p) and because of the inequality constraint on E^p and M. The substitution of $r°$ with its explanatory variables (i.e. r, X^d/X, u, τ) gives the following aggregate function for output:

$$X = f(r, u, E^p, X^d/X, \tau)$$

with $\vartheta(X)/\vartheta u<0$.

X^d/X = excess demand;

r = actual observable profit rate.

If $u = 0$, implying a perfectly repetitive economy, no entrepreneurship is required, as such an economy can be directed efficiently by a bureaucratic management and by computers. Of course, the operator f in the above function is influenced largely by the lack of entrepreneurial knowledge; as a matter of fact, if a productive opportunity is unknown, it will be non-influential on the entrepreneur's decision and output.

In the presence of market power, the above supply function is characterized by smooth variations of r, due to the relative invariance of the markup margin. But the parameter of X^d/X is, in the case of an oligopolistic market, higher than in a market regulated by demand and supply, since the defence of the price imputed needs that price makers promptly adjust supply to demand, by varying the degree of capacity utilization.

Solving the expression of X for r, gives:

$$r^* = f(X, u, E^p, X^d/X, \tau) \tag{3}$$

with $\vartheta r^*/\vartheta E^p < 0$; $\vartheta r^*/\vartheta (X^d/X) < 0$; $\vartheta r^*/\vartheta\tau < 0$.

r^* is a desired or partial equilibrium profit rate, required to produce X unit of output for given values of u, E^p, X^d/X, τ.

An expression for the growth of income could be derived by optimization of an intertemporal objective function. In this paper, however, the equation of r^* is used to provide an expression for the rate of growth of income:

$$DX/X = \alpha(r - r^*) \quad (4)$$

Equation (4) explains the variation of output through the adjustment of actual profit rate towards the desired profit rate. If $r > r^*$, i.e. the actual profit rate exceeds the profit rate required to produce the current level of output, output grows, while it decreases if $r < r^*$. If $r = r^*$, the rate of growth is nil (stationary equilibrium) as the entrepreneur obtains just the profit rate required to produce the current level of output.

Parameter α in (4) indicates the entrepreneurial alertness in taking advantage of the market opportunities; the lower the entrepreneurs' degree of knowledge of such opportunities, the lower is α. It may be important to point out that the above equation of output embodies all factors influencing the decision to produce, precisely both the conditions of profitability expressed by r (including the effect of income distribution, demand, technology, prices), and those of entrepreneurship, expressed by the variables on which r^* depends. This prevents the one sidedness characterizing other theories of growth.

Equations (1), (2) and (4) give the representation of the whole process of dynamic competition and the basic engine of growth.

The explanation of repetitive investment may be derived by substituting capital to output in (4). But there is an important difference with respect to production. We have previously seen that decisions to invest, involving long run expectations, are much more influenced by uncertainty than those to produce, owing to sunk costs, irreversibilities and rapid growth, with uncertainty, of the costs of information necessary to support long run forecasting. This suggests that the equation of the variation of capital should be of the following form:

$$DK/K = \beta(r - r^*) - a_1(u) \quad (5)$$

K stands for the stock of capital, while $a_1 u$ indicates the above additional impact of uncertainty on investment. Substituting equation (4) into equation (5), gets:

$$DK/K = \beta/\alpha(DX/X) - a_1(u). \quad (5')$$

As we can see, in the presence of innovation and uncertainty, the variation of capital tends, on the one hand, to exceed the variation of X owing to the innovative push, but on the other hand is slowed by uncertainty. This means that when uncertainty is low, the capital-output ratio tends to increase, while it tends to decrease in the opposite situation. The constancy of the capital-output in the long run is probably the result of the cyclical interaction between innovative and adaptive entrepreneurship and competition.

It is crucial for an explanation of economic growth and development to combine the analyses of Schumpeter and Kirzner on entrepreneurship and competition. Their separate use is misleading. Schumpeter's theory neglects the importance of uncertainty and, more generally, of the adaptive process; but, if competition were only based on creative destruction, it would cause an excessive destruction, i.e. obsolescence. Imitation (of innovators) does not promote, *per se*, the return to the circular flow; on the contrary, it increases the impact of innovation on the economy, hence obsolescence, disruption and disequilibria, notwithstanding it favours the return to routine in the specific sector. Therefore, it explains the cycle of products, but is unable to explain that of the whole economy. For its part, the Kirzner's analysis centred on market adjustment process as a result of the entrepreneurs' alertness does not take care of explaining economic development, "seen merely as a special case" (Kirzner, 1973, p. 81). But the Kirznerian equilibrating process is incomplete without the disequilibrating one, which is Schumpeterian, that creates the basis for market adjustment process.

A succinct exposition of the remaining equations of the model is given below. Labour demand is explained by the inverse of a production function, as follows:

$$L \quad X^{\frac{1}{5}} K^{-\frac{6}{5}} e^{-\frac{7}{5} I_I} e^{\frac{8}{5} Du} \tag{6}$$

L is employment and e indicates exponential.

The above production function includes, among other things: technical progress, expressed by innovative investment (I_I); the variation of uncertainty, that influences the efficient combination of productive factors. The last two factors reflect the cyclical behaviour of innovation and uncertainty; their opposite effect does not cancel out as the variation of u oscillates around a value tendentially constant of this variable, while is $I_I > 0$. Of course, equation (6) gives by implication labour productivity, output being explained by equation (4). Note that here the production function does not explain output directly, as the models of endogenous growth do; it influences output indirectly, through the presence of L in the profit identity below. Output depends on productive decisions of the entrepreneur and the mechanism of dynamic competition.

The relevance that this model ascribes to the distinction between actual and partial equilibrium profit rate makes it necessary to give an accurate description also of the first. It may be expressed by the following identity:

$$r \quad \frac{X}{K} - \frac{L\,w}{K\,p} - i - \text{tax} \tag{7}$$

Where w/p is the real wage, i is the real interest rate and tax stays for the coefficient of taxation on capital. r is the profit rate, taken in real terms. Note that the profit rate differs from the real interest rate which is simply considered an exogenous cost, but it could easily be made endogenous. The return on capital is $r + i$; it is equal to i only in the exceptional case that $r = 0$.

Price can be expressed by the adjustment of demand and supply, i.e.

$$Dp/p = \mu(X^d - X) \tag{8}$$

or, in the presence of market power, by:

$$P = (1 + \pi) w \vartheta L / \vartheta X \tag{8'}$$

where π stands for the degree of monopoly and could be indicated as an increasing function of the variation rate of innovative investment (DI_I/DI_I); ϑ is a symbol of partial derivative.

Various hypotheses will be formulated on income distribution, in connection with some assumptions on the form of markets and the evolution of economy towards successive phases of development, discussed later.

Real consumption (C) is expressed as:

$$C = cX \tag{9}$$

c is an average propensity to consume.

Aggregate demand is specified through the identity:

$$X^d = C + DK + Z \tag{10}$$

where Z is the exogenous factor of demand.

Finally, an explanatory equation for the availability of entrepreneurial skills is postulated:

$$E^p = X^{\eta} \quad \text{with} \quad 0 < \eta < 1. \tag{11}$$

This equation supposes that, at the macro level, the availability of entrepreneurial skills (E^p) grows with aggregate output, as a result of the entry of new entrepreneurs as the market expands. $0 < \eta < 1$ (i.e. E^p grows at a decreasing rate) owing to: first, the physical and organizational limits to the skills of firms, mainly the reduction, with the concentration process, of adaptive skills (this limitation gives a main explanation of the limit to the size of firm); and second, the entry of less skilful entrepreneurs when X grows, previously kept outside the market.

The interaction innovation-adaptation and evolution. Business cycle versus social development cycle

In this model, growth is driven by dynamic competition that generates innovation and reduces, through adaptive action, the consequent disequilibria and uncertainty. Innovation stimulates growth owing to its impact on the actual profit rate (r) via the production function (i.e. labour productivity) and investment hence demand, while adaptation stimulates growth through the reduction in uncertainty, hence in r^* and the increase in labour productivity. The core equations of the model are those for innovation (1), uncertainty (2), and output (4). Important roles are also played by the equation for the stock of capital (5) and the production function (6), as well as equation (11) for the availability of entrepreneurial skill, owing to the impact of this variable on r^*, and hence on the rate of growth of income.

Equations (1) and (2) of innovation and uncertainty display a dominant role in the model, as they influence the rest of the system without any feedback. This aspect of the model is unrealistic, but this can easily be remedied through the introduction in the two equations of some other explanatory variable.

The aggregate nature of the model impedes the role of sectoral disequilibria in generating actual profit rate (r) and the partial equilibrium profit rate (r^*). In the absence of innovation and the resulting disequilibria and uncertainty, r and r^* disappear and the stationary state dominates the scene. In reality, disequilibrium prevails (a disaggregated model would show this clearly) as a consequence of the entrepreneurial innovative competition, with positive or negative rates of growth according to the conditions of profitability (influencing r) and those of entrepreneurship (influencing r^*) making $r > r^*$ or vice versa (in an economy without entrepreneurs, growth could only be pushed by some autonomous decision to invest, carried out by the political sphere or bureaucracy. But bureaucratic behaviour tends to avoid innovations that undermine the established roles, thus preventing development). For its part, accumulation fluctuates around the rate of growth of income, owing to the higher impact that uncertainty plays on investment than on output. As a consequence, the output-capital ratio fluctuates around a tendentially constant value.

At the heart of economic dynamics there is the entrepreneurial action or, more precisely, the interaction between innovative and adaptive entrepreneurship and competition that, as just seen, pushes endogenous growth. Such growth has a cyclical behaviour, due to the alternation, over time, of a phase of innovative push, characterized by the rise in innovation, and a phase of structural organization, devoted to establishing some new equilibria starting from the previous innovations and distinguished by a squeeze of innovation and the prevalence of adaptation. If these two phases (innovation and structural organization) were not reciprocally lagged or were separated by very short lags, the cycle would disappear or would be very smooth. But this is not the case. The innovative dash requires a well ordered system, i.e. coasting some equilibrium positions. As soon as this happens, innovation projects are introduced; their diffusion induces further innovations along with increasing disequilibria, that stimulate the need for structural reorganization and adaptation. The cycle innovation-uncertainty affects output both directly and owing to labour productivity, investment and demand.

The interaction innovation-adaptation may explain cycles of different periods simply by considering various kinds of innovations. When the large set of innovative possibilities due to basic innovations has been exploited, along intermediate cycles innovation-adaptation, new technological paradigms and new basic innovations (even in institutions), implying long waves, will take place, that will allow the continuation of the dynamic competition process. These cycles of innovations are strengthened by the cyclical behaviour of inventions underlined by Kuhn (alternation in the development of ordinary and extraordinary knowledge).

The mechanism described here seems to provide also a useful tool for the interpretation of social and historical development. This may be represented through the alternation of a breaking phase, characterized by the prevalence of fundamental discoveries, and an adjustment phase aimed at developing, exploiting and systematizing the implications of the main discoveries. This shows some similarities to the Kuhn theory of the str ucture of scientific revolutions. But Kuhn describes the evolution of sciences as due to the exhaustion of paradigm, marked by the appearance of “anomalies”. This is not convincing. In a stationary system, the paradigm perseveres immortally, anomalies do not appear. It is the propensity to innovate that gives the basic push to development, thus breaking up conventional knowledge and the strength of tradition. Such a propensity depends on the form of civilization (degree of openness to the novelties or the existence of some mechanism forcing innovation, as the search for profit based on innovative competition). It is not the wasting of paradigms that opens the road to innovation, rather the contrary happens.

The cycle discussed here expresses the simple evidence that innovative push is the true engine of the movement of societies (its absence would have left mankind in the stone age). But even adaptation plays a central role, as it provides the diffusion over the whole system of such a push and the consistency among the various comp onents of the system. The interaction betwe en innovation and structural organization may be seen as a basic law of social evolution, with a role similar to that played by the Darwinian succession “accidental mutation-selection” for the natural world evolution. In the present age, distinguished by the centrality of the economic subsystem, the above engine of development is mainly activated by entrepreneurial competition. But some other subsystem and evolutionary mechanism may prevail in the future.

The discrete extraordinary events represented by innovations, with their impact on environment expressed by the continuous adaptive process, produce irrevocable changes. The direction of evolution, marked by the increasing variety of goods and knowledge, hence the increase in social complexity, gives the histo rical time. Such an evolutionary process does not app ear at the agg regate level, where only the g rowth in productivity due to technical progress is visible. Its representation needs disaggregation.

Econometric estimation

The lack of observations has prevented a simultaneous estimation of the complete model. This has been divided in two submodels for estimation: one concer ning Volter ra’s system fo r innovation and uncertainty; the other represented by the equations of output (and desired profit rate), employment and actual profit rate.

In the absence of data series on innovative investment, the estimate of innovation uses the data on patent applications (giving a reliable expression of the intention to innovate) published by the US Department of Commerce, while the data on the interquartile variation of profit rates (instead of the variance of profit rates) come from a sample of about 8,000 manufacturing firms, performed for the period 1982-1992 by the Italian Centrale dei Bilanci. The estimation results must be judged in the light of the deficiencies of appropriate data series and the shortness of the observation interval.

A full information maximum likelihood estimator was used to preserve the interactions among equations. The estimation of the differential system for innovation and the variance of profit rates (equations (1) and (2), with the term I_l in the right hand side of equation (2) instead of $I_l u$, has given the following results shown in Tables I and II.

It is considered that the shortness of the sample (only 12 observations) and the use of interquartile variation of profit rates (instead of variance) are the reasons why the asymptotic standard errors are relatively high. However, the values of the parameters are reasonable and always have the correct sign and also the estimated standard deviations are reasonable.

Table I. Estimation of the system (1), (2)

	Estimate of parameters	Asymptotic standard error	t values
β_1	0.164	0.274	0.66
β_2	0.150	0.291	0.52
β_3	0.688	0.649	1.06
β_4	0.324	0.290	1.11

Table II. Mean and standard deviation

	Observed		Estimated	
	Mean	Standard deviation	Mean	Standard deviation
$I_l u$	1.9102	0.1614	1.9100	0.1523
u	0.8568	0.0535	0.8560	0.0564

Now we come to a simultaneous FIML estimation of the following model:

$$D\log VA = \alpha_1(r - r^*)$$

with

$$r^* = \log\gamma + a_0\log VA - a_2(\log E^p - \lambda t + \log KU)$$

$$D\log L = \alpha_2 \log \frac{L^*}{L}$$

with

$$\log L^* = \log A + \beta_5 \log VA - (1 - \beta_5)\log K$$

$$r = \frac{VA.P - wL - iKP}{KP}$$

$$\log \mathrm{PROD} = \log VA - \log L.$$

Where VA indicates real value added in industry; L stays for employment in industry and w for wage rate; r is the actual profit rate in industry; and r^* the desired profit rate; PROD stays for industrial productivity of labour; P is the deflator of industrial value added; K is the stock of capital in industry and KP its expression in nominal terms; i is a nominal interest rate and KU the degree of utilization of plants. Log stays for natural logarithms.

The conditions of entrepreneurship in the equation for r^* are represented by the term $E^p/e^{\lambda t}$ giving the deviation of the number of firms from their trend. r^* being unobservable, for estimation it needs to substitute its equation in the equation for $D\log VA$.

Estimates have used a sample of 56 quarterly observations on the Italian economy from 1980 to 1993, coming from ISTAT. The estimated parameters are shown in Table III.

Parameters	Enter equation number	Estimate of parameters	Asymptotic standard error	t values
α_1	(1)	2.410	0.871	2.77
α_2	(2)	0.090	0.026	3.49
a_0	(1)	0.448	0.085	5.27
a_2	(1)	0.213	0.039	5.44
β_5	(2)	1.302	0.205	6.37
λ	(1)	–0.01	0.0022	4.80
$\log\gamma$	(1)	–1.657	0.756	2.19
$\log A$	(2)	–14.147	1.147	12.49

Note: All parameters are significantly different from zero around 1 per cent level, have the correct signs and assume some quite plausible values except the intercept of equation (2) for employment

Table III. Estimated elasticities and intercepts

The implications of the model for subsequent phases of economic development. Some simulation experiments

This section, devoted to the model's capability to interpret various historical situations, is an indicator of its degree of generality. It also accounts for the succession of some stages of economic development simply by making some suitable alternative assumptions on the availability of entrepreneurial skills, technology, income distribution, the market forms and, more broadly, the performance of dynamic competition, i.e. the basic engine of evolution.

It may be convenient to postulate an initial phase of development. This will probably be distinguished by: weak entrepreneurial skills and aggressiveness (especially in the presence of a civilization hostile to entrepreneurs and market institution) hence a substantial absence of dynamic comp etition; low productivity, i.e. high labour coefficient; real wages close to the subsistence level, hence incompressible downward. Such an economy produces a low surplus, which is largely paid to the owners of scarce resources. In consequence and according to equation (7), the profit rate r is low and decreasing. Besides, the lack in entrepreneurial skill will stimulate the desired profit rate (r^*).

Therefore the rate of growth of output will decrease towards zero, according to equation (4). This is the only equilibrium possible, as the stagnation stops the increase in the rent, hence the decrease in profit, and the increase in r^*. This situation is illustrated partly by the model in formulation of a simplified version in the Appendix, showing that the term $X^{\eta-1}$ with $0 < \eta < 1$ (that postulates a decreasing availability of entrepreneurial skills per unit of output) implies the existence of a stable stationary equilibrium. This "trap of underdevelopment" expresses a situation similar to that described by the Ricardian theory of growth and stagnation, correlated to the absence of the evolutive push conferred by dynamic competition.

If the economy succeeds in avoiding the trap, it can start the development of a dynamic sector. To do that, it requires a primary accumulation in the form of infrastructure, education, investment in the new sector, and to promote entrepreneurship, necessary to invest the available resources and increase productivity through dynamic competition. In this regard, it may be important to influence the ethic system, so as to increase τ (non-monetary benefit attached to the entrepreneurial function), that may compensate the disincentive to produce due to low profit rate. In particular, an entrepreneurial Stakhanovism may be promoted, as well as state entrepreneurship (which is not necessarily influenced by profit rate but only requires the consideration of relative profit rates of firms to evaluate the convenience to finance them). In this way, the absorption of the resources for accumulation is warranted. Another possibility is to build a command economy, i.e. without entrepreneurs. This short cut can initially be successful but, in the long run, it pushes the economy into a blind road, as is well known. This model, with a dynamic sector expressing a strong dynamic competition, may produce two different evolutions.

First, prices and wages are governed by demand and supply. In this case, if there does not exist a lack of entrepreneurship, i.e. if this grows linearly with production (as assumed in formulations for markets regulated by demand and supply and oligopolistic markets in the Appendix), a Lewisian process of growth can take place. The excess of labour present in the backward sector squeezes wages in the dynamic sector towards the subsistence level. (More precisely, a low differential of wages with the backward sector is enough to convey towards the dynamic sector the labour force it needs.) This, together with a sectoral productivity much higher than that of the backward sector, implies a high profit rate, that promotes a high accumulation rate. A cyclical expansion (due to the interaction between innovation and uncertainty) of the dynamic sector around an increasing trend leads to a gradual absorption of the excess labour in the backward sector, ending with its disappearance.

Second, the growth in the size of firms and the bargaining power of trade unions creates an oligopolistic market for goods and labour. In this case real wages in the dynamic sector go beyond the subsistence level, under the influence of the increase in labour productivity. This behaviour is promoted both by the demands by trade unions for productivity increases distribution and the interest of firms to wage increases that warrant the constancy of unit labour cost hence, for a given margin of mark up, the downward rigidity of oligopolistic prices, as well as an increasing demand of sectoral production. Profits are now much lower than in the first case; consequently, the sectoral rate of growth slows down. Furthermore, the increasing wage differential between the two sectors provokes an explosive exodus from the backward sector, that exceeds the labour force needs of the advanced sector. This stimulates the birth of a refuge sector and an increasing inflationary potential, due to social tensions and bottlenecks in the immigration areas and the proliferation of assisted areas. As long as firms have the control of income distribution through market power, these inefficiencies and costs are charged (via inflation) on labou r that negotiates money wages, without affecting profit rate. But the rate of growth is now lower than in the first case, therefore the disappearance of the backward sector requires much more time than in the first case.

Over time, real wages may cease to be a residue, through their indexation or in the presence of a fixed exchange rate. As a consequence, the costs and inefficiencies accompanying the excess exodus squeeze profits and cause a cyclical involution towards stagnation (trap of dualism). (An estimation for Italy of this phenomenon is in Fusari, 1986.) In this situation the strength of dynamic competition in the advanced sector may imply, together with the deepening of dualism, the strengthening of the trap. This may be formalized by postulating a modified system of Lotka-Volterra equations, expressing the interaction between the rate of growth of labour productivity in the dynamic secto r, as the prey that fe eds (through an incre asing difference in the intersectoral standards of life) the excess exodus, and the latter as the predator, that squeezes the exponential rate of growth of productivity in the dynamic sector (presumably pushed by an autonomous catching up mechanism). Note that, differently from the basic Volterra model, where uncertainty fluctuates around some values tendentially invariant in the long run, i.e. towards a limit cycle, here the excess exodus tends to cumulate. More precisely, the Volterra system has to be expressed now in a degenerate form, i.e. without the negative constant in the equation of the excess exodus (predator). This causes an uninterrupted growth of the predator and probably the disappearance of the prey, i.e. the involution of the cycle towards a zero growth equilibrium.

Another way to express this involutive process is to specify, in the basic model: one equation for the excess exodus; one representing the inflationary effect of this exodus; one equation for re al wages in the dynamic secto r depending on the sectoral increase in labour productivity; the impossibility for this sector (due to international competition) to charge the costs caused by the excess exodus to domestic inflation. So, the rate of profit in the dynamic sector is squeezed, hence the accumulation process.

To prevent this, income policies, the control of migration and of disequilibria able to avoid excess exodus are required. We can se e, therefore, that the promotion of the take-off needs instr uments that markedly differ with the lateness of the development process, as pointed out by the famous storiographic research of Gerschenkron (1962).

This kind of involution is very frequent today, especially in Latin America, South East Asia, and the Middle East; it is mainly expressed by the abnormal growth of many towns in the third world; it also represents a great menace for Easter n Europe countries in transition towards market institution, mainly Poland with its overcrowded agriculture.

Now suppose that both the trap of underdevelopment and dualism are avoided. Two basic situations can arise:

(1) An economy characterized by a well-oiled form of dynamic competition, flexible money wages determined by excess demand, real wages fluctuating with the marginal productivity of labour. Such an economy satisfies the conditions for the achievement of a high and stable rate of growth if dynamic competition manages to avoid, through innovation, bottlenecks in natural resources and to promote a high increase in productivity.

(2) An economy distinguished by a diffuse market power, hence money wage rigidity, its share of income distribution determined by the degree of monopoly of firms and therefore with the real wage as a residue. This model has been analysed in the Appendix under the formulation with market power (and abstracting from dynamic competition mechanism), that shows it can yield a stable equilibrium. But the rigidity of money wages precludes full employment.

Finally, in a mature and consumeristic economy, high inflation destroys monetary illusion, hence the assumption of real wages as a residue, that has been proved in the simplified formulation in the Appendix to be a basic condition for the achievement of a long run equilibrium. More precisely, wage bargaining, public sector inefficiencies and expenditure, formal or informal indexation mechanisms strongly influence, in the end, profit rate, hence the path of growth. The control of income distribution is usually restored by high unemployment. This tendency towards stagnation may be overcome through some institutional changes that limit income distribution conflict to the political and social sphere, to restore its irrelevance for firms. We can see, therefore, that history and economic theory go hand in hand even at the aggregate level.

It may be useful to perform some simulation experiment through a model formed by the two groups of equations estimated separately in the previous section. Differently from that estimation, now uncertainty is added in the equation for output, while innovation and the variation of uncertainty are added in the equation for employment.

Four phases of economic development have been considered:

(1) Phase I, representing an initial stage of development, that assumes the following values of the parameters of the Volterra system (1-2): β_1 = 0.262; β_2 = 0.517; β_3 = 0.112; β_4 = 0.663. These values imply a languishing dynamic competition. Besides, the parameter of uncertainty in the equation of output is 0.08 and those of innovation and uncertainty in the equation for employment are respectively 0.1 and 3.8. α_1 has been reduced to 2.14, to express a reduction in entrepreneurial alertness; also the parameter indicating the impact of the conditions of entrepreneurship on output has been reduced from 0.213 estimated to 0.203. The initial number of patent applications is 300 and the initial interquartile variation of profit rates is 0.65.

(2) Phase II presumes that a Lewisian mechanism of development is at work. The parameters of the system of Volterra are now as follows: β_1 = 0.38;

β_2 = 0.49; β_3 = 0.232; β_4 = 0.53, expressing a substantial increase , with respect to phase I, of the parameters β_1 and β_3 concerning innovative and adaptive push, and a remarkable decrease in β_2 and β_4 representing the brake to innovation and adaptation. The initial number of p atent applications is now 350 and the initial interquartile variance of profit rate is 0.75. The parameter of uncertainty in the equation of output is 0.05 and those of innovation and uncertainty in the equation for employment are 0.14 and 2.8. Moreover, an hyp othesis that re al wage remains constant over time is assumed, as in Lewis' mechanism.

(3) Phase III presumes that wages grow less than labour productivity. An impact of the conditions of entrepreneurship on output higher than the previous phases is hypothesized (0.2184 against 0.2134 estimated). The parameters of the equations of innovation and uncertainty are now as follows: β_1 = 0.415; β_2 = 0.455; β_3 = 0.275; β_4 = 0.48, that presume a further considerable increase in the strength of dynamic competition. The initial number of patent applications is 425 and the initial interquartile variance of profit rates 0.82.

(4) Phase IV refers to a mature consumeristic economy, distinguished by a strong conflict for income distribution, hence by a severe reduction in profit rate. All parameters are identical to phase III.

The results of simulations are expressed in Figures 1-6. They are not discussed for space reasons, but the different paths, in the various phases, of each one of the key variables considered, as well as the different performance of the whole economy, are immediately evident.

Conclusion

This essay shows that the explanation of economic growth and development requires an accurate deepening of the crucial phenomena of entrepreneurship,

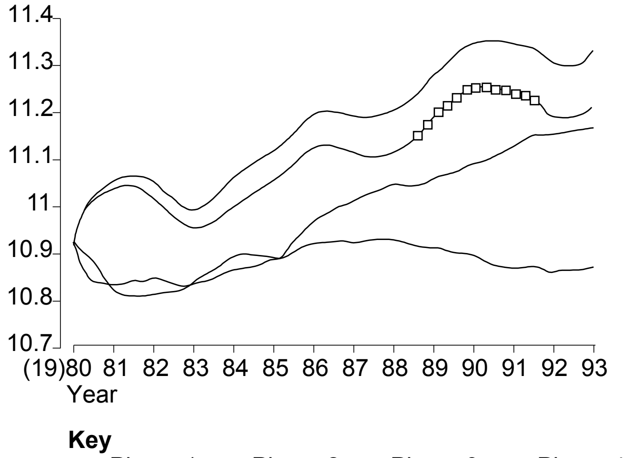


Figure 1.
Real value added in industry (natural logorithms)

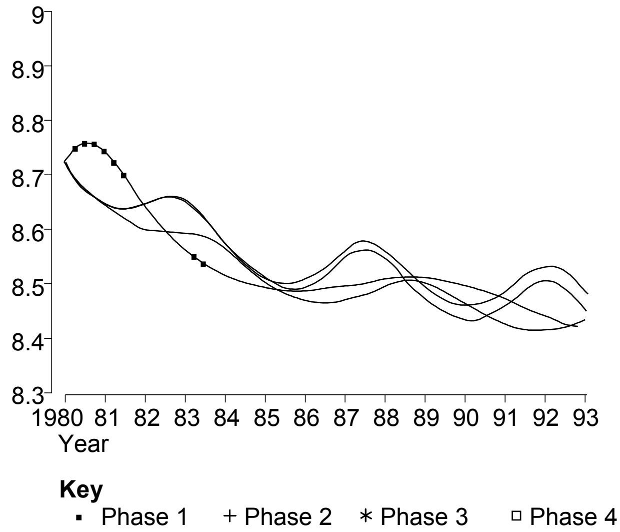


Figure 2.
Employment in industry (natural logorithms)

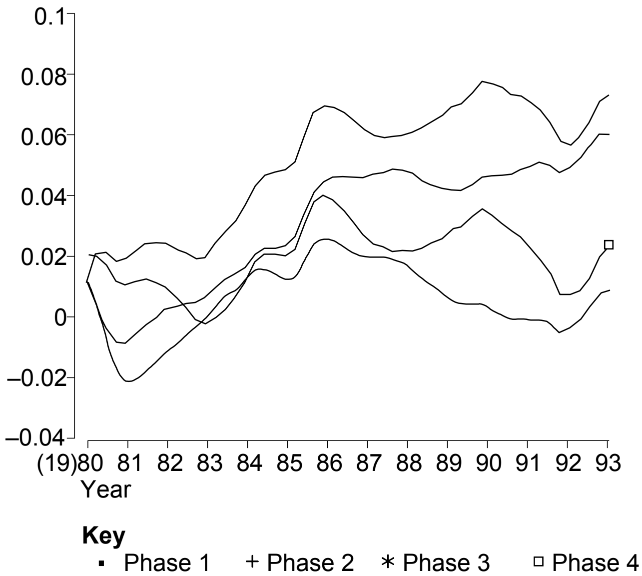


Figure 3.
Actual profit rate in industry

innovation, uncertainty of economic life and their interrelationships. Such a de epening can be centred efficaciously on a particular notion of dynamic comp etition that results from the combination of two main kinds of entrepreneurship and their interaction, the innovative and the adaptive, and made active by the struggle for profit. This competition explains the succession disequilibria-equilibria in the economy and the basic feature of the engine of development and growth.

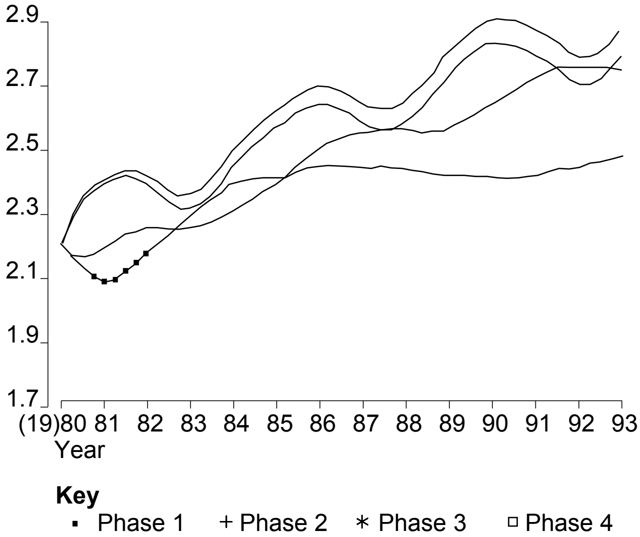


Figure 4.
Real labour productivity in industry (natural logorithms)

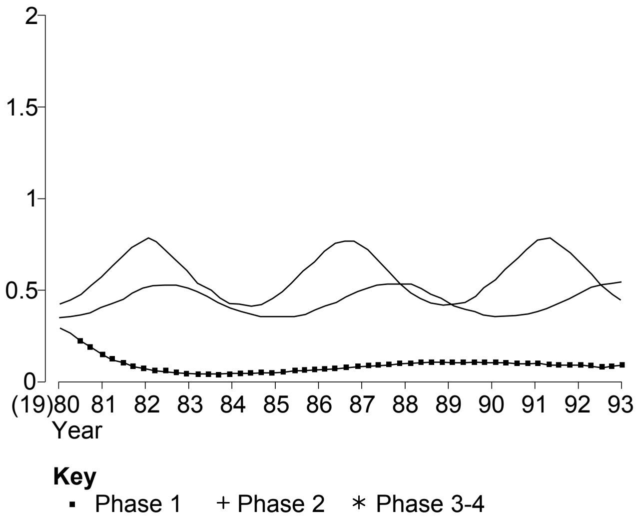
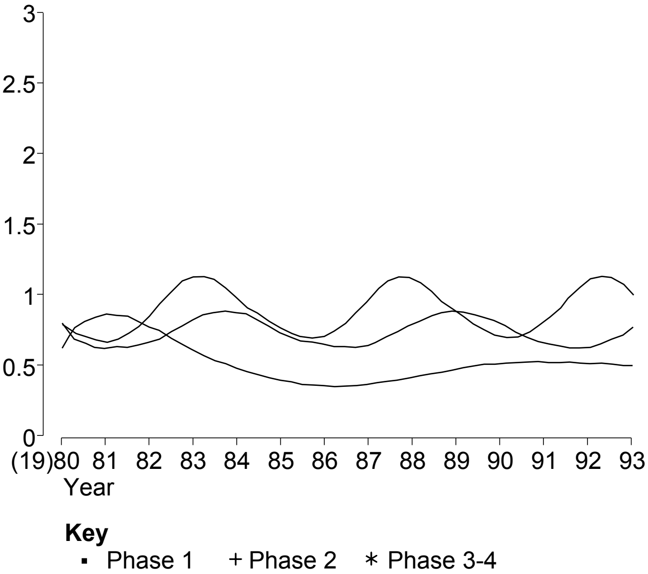


Figure 5.
Patent applications

All indications today are that in modern society, with its immense technological p otential, the re al limit to economic development is its endowment of entrepreneurship. This influences decisively both partial equilibrium and actual profit rates, hence the pace of activity, investment and cycle behaviour. An application of the model to successive phases of historical development gives a proof of the generality and flexibility of the theory proposed and of its explanatory power in regard to economic evolution, and shows some different paths of economic development.

Figure 6.
Variance of profit rates in industry



Further more, the model outlines the importance, in the social life, of the binomial innovation-str uctu ral o rganization, i.e. the disequilibrating-equilibrating motion, and shows succinctly that the theory presented can flow into a more general theory of social development. This is a major task for social sciences. It is no accident that the "propensity" for development varies greatly between civilizations and historical eras. The presence and responsibility of innovative entre preneu rship imply, as preconditions, certain premisses concerning values and institutions typical of decentralized economic orders: the market, profit, openness to innovation and change, to the unorthodox and the non-conformist, the critical sense, individualism (not necessarily the acquisitive self-interest), the competitive spirit, and so on. Adam Smith took pains to stress the decisive influence of the size of the market in intensifying the division of labour, and hence in generating the rise in productivity. Actually, though, the imp o rtance of the market for development is antecedent even to this. In particular, it stems from the fact that the market, together with its related ethic values and behaviour patterns, gives the division of labour an evolutionary character, preventing it from becoming the reflection and the seal of more or less rigid social stratification if not the actual division of society into castes. But these topics go beyond the object of this essay.

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Appendix. Qualitative analysis of the model

The central mechanism of development and growth that the model in this essay points out, that is the incessant disequilibrating-equilibrating motion resulting from the interaction between innovative and adaptive entre preneu rship, cle arly implies the imp ossibility of a stable equilibrium at the micro level. It also legitimates some serious doubt on the existence of a stable equilibrium at the sectoral level.

This appendix investigates the existence of a stable equilibrium solution at the macro level, i.e. the level to which analyses on the stability of equilibrium usually refer. We shall see that to get a macro stable equilibrium it needs to eliminate from the model the Volterra system for innovation and uncertainty (giving an aggregate expression of the dynamic competition process). Such a result indirectly proves the impossibility of a stable equilibrium even at the sectoral level, if the mechanism of dynamic competition is preserved. This implies that the stability properties of some economic model must be considered with great caution, as they always depend on the removal from the model of some crucial part of reality.

Now we come to provide some formulation of the model able to get a stable equilibrium solution.

Markets regulated by demand and supply

First we consider a market regulated by the excess demand, also making the hypothesis that real wage is equal to the marginal productivity of labour, that implies the irrelevance for firms of the conflict for income distribution. In order to obtain a stable equilibrium, the original model must be expressed as follows:

$$r^* = a_0 + a_1 u - a_2\gamma - a_3(X^d/X) - a_4\tau \qquad \text{(A1)}$$

$$u = b_1 DK/K + b_3\xi \qquad \text{(A2)}$$

$$DX/X = \alpha\,(r - r^*) \qquad \text{(A3)}$$

$$DK/K = a - du + b_2(X^d/X) \qquad \text{(A4)}$$

$$r = X/K - L/K(w/p) - i - \text{tax} \qquad \text{(A5)}$$

$$L/K = (X/K)^{1/\beta 5} K^{-(\beta 6 + \beta 5 - 1)/\beta 5} e^{-(\beta 7/\beta 5) DK/K} e^{(\beta 8/\beta 5) u} \qquad \text{(A6)}$$

$$Dp/p = \mu\,(X^d/X) \qquad \text{(A7)}$$

$$X^d/X = C/X + DK/K(K/X) + Z/X \qquad \text{(A8)}$$

$$C = cX \qquad \text{(A9)}$$

$$w/p = \vartheta X/\vartheta L. \qquad \text{(A10)}$$

Endogenous

r^* = partial equilibrium or desired real profit rate;

u = uncertainty per unit of output;

X = output;

K = stock of capital;

r = real profit rate;

L = employment;

p = price;

X^d = real aggregate demand;

C = real consumption;

w/p = real wage;

E^p = availability of entrepreneurial skills, replaced in equation (A1) by γ (see below).

Exogenous

τ = non-monetary benefits attached to entrepreneurial role;

ξ = exogenous factor of uncertainty;

i = real interest rate;

tax = taxation per unit of capital;

Z = exogenous final demand;

D = differential operator d/d_t, ϑ indicates partial derivative.

E^p in equation (A1) has been expressed as a linear function of X, i.e. $E^p = \gamma X$, and divided by the scale factor X. Later this expression will be substituted by $E^p = X^\eta$ and the implications of this more realistic formulation discussed.

Another important assumption is represented by equation (A10), stating the equality between real wage and marginal productivity of labour ($w/p = \vartheta X/\vartheta L$). Such an hypothesis on real wages yields a decisive simplification for the model reduction, as it permits to eliminate equation (A6) and p in identity (A5). Later the assumption expressed by equation (A10) will be released, in relation to the hypothesis of oligopolistic market, and it will be shown that such new formulation does not affect the stability of equilibrium.

Equation (A6) has been divided through by K; so it expresses the labour – capital ratio, and equation (A8) has been divided through by X. This facilitates the reduction of the model without modifying its content. The share of the endogenous demand (Z/X) has been considered as exogenous and indicated with z. Equation (A7) implies that price variations cannot be negative or zero; but this equation is irrelevant for the model reduction, as Dp/p does not appear in any other equation while p in identity (A5) is eliminated as above.

A zero order equation for uncertainty (equation A2) has been substituted to the corresponding differential equation in the original model. This implies a substantial dynamic irrelevance of uncertainty, that in fact will be eliminated by substitution. Also the differential equation for innovation has been excluded from the model. The consequent elimination of the Volterra system permits the reduction of the model to one differential equation, that g reatly simplifies the qualitative analysis. This elimination is essential to achieve the stable equilibrium solution discussed below and represents a drastic modification of the original model, as it cancels out the crucial dynamic mechanism of an entrepreneurial economy.

The elimination of the Volterra system, hence of the cycle of uncertainty, implies in the original equation for capital a prior assumption that $DK/K \neq DX/X$ and therefore that a stable solution requires $DX = 0$ and $DK = 0$. To avoid this implication, the equation for capital has been reformulated as in equation (A4).

The reduction of this new model may start from the substitution of equations (A6) and (A10) into equation (A5). Multiply L/L to the expression of marginal productivity of labour, hence substitute to L its expression. So we get $\vartheta X/\vartheta L = \beta_5(X/L)$, implying $\vartheta X/\vartheta L(L/K) = \beta_5 X/L(L/K) = \beta_5 X/K$. As $\vartheta X/\vartheta L = w/p$, it follows: $L/K(w/p) = \beta_5 X/K$. Substitute this in identity equation (A5), obtaining:

$$r = (1 - \beta_5)X/K - i - \text{tax}. \tag{A5a}$$

Now substitute equation (A9) into equation (A8). Also equation (A2) can be substituted in equations (A1) and (A4), and equation (A8) into equations (A1) and (A7). After elimination of all irrelevant variables, we get a system of two differential equations in X and K that, considering the:

$$D\frac{X}{K} \quad \frac{X}{K}\frac{DX}{X} - \frac{X}{K}\frac{DK}{K}$$

can be reduced to one differential equation in the output – capital ratio, as follows:

$$D\frac{X}{K} \quad \frac{X}{K} \quad a_0 \quad a_1\frac{X}{K} \quad \frac{[(a_2 - 1)\frac{X}{K} \quad a_3](a \quad b_2 c \quad b_2 z - db_3 \quad)}{(1 \quad db_1)\frac{X}{K} - b_2}$$

with:

$a'_1 = \alpha(1 - \beta_5)$; $a'_2 = -\alpha a_1 b_1$; $a'_3 = \alpha a_3$; $a'_0 = \alpha(-i - \text{tax} - a_0 + a_2\gamma + a_3 c + a_3 z + a_4\tau - db_3\xi)$;
$z = Z/X$.

This equation has a particular solution $X/K = 0$ (trivial solution) and one obtainable by equating to zero the expression multiplying the first X/K in the right-hand side, provided that $X/K \neq b_2/(1 + db_1)$. It is a second degree equation, i.e.:

$C_0 x^2 + C_1 x + C_2 = 0$ with $x = X/K$ and:

$C_0 = \alpha(1 - \beta_5)(1 + db_1)$

$C_1 = + a'_0(1 + db_1) - \alpha(1 - \beta_5)b_2 + (-\alpha a_1 b_1 - 1)(a + b_2 c + b_2 z - db_3\xi)$

$C_2 = - a'_0 b_2 + \alpha a_3(a + b_2 c + b_2 z - db_3\xi)$.

Dividing C_1 and C_2 by C_0 gets:

$$A_1 \quad \frac{a_0(1 \quad db_1) - \quad (1 - \quad _5)b_2 - \quad _1 \; _2}{(1 - \quad _5)(1 \quad db_1)}$$

$$A_2 \quad \frac{-a_0 b_2 \quad a_3 \; _2}{(1 - \quad _5)(1 \quad db_1)}$$

where: $\mu_1 = 1 + \alpha a_1 b_1$ and $\mu_2 = a + b_2 c + b_2 z - db_3\xi$.

$A_1 < 0$ as all terms in the numerator are non-negative, while the denominator is positive; true to tell, the first term in the numerator could be positive, but it is certainly lower than the negative sum of the other two since $a_0'(1 + db_1)$ tends to be lower than $\mu_1\mu_2$, especially if $\alpha < 1$, as it stands out if we consider the parameters involved and take into account the inequality below, to be satisfied to get a real solution.

$A_1 < 0$ implies the positivity (i.e. the economic relevance) of at least one solution and, if $A_2 < 0$, that the solutions are real. But it is very probable that A_2 is positive. This means that, to verify if the solutions are real, it is necessary to analyse the following expression:

$$A_1^2 - 4A_2 \quad \frac{[a_0(1 \quad db_1) - \quad (1 - \quad _5)b_2 - \quad _1 \; _2]^2 - 4 \quad (1 - \quad _5)(1 \quad db_1)(-a_0 b_2 \quad a_3 \; _2)}{[\quad (1 - \quad _5)(1 \quad db_1)]^2}.$$

Develop the square term in the numerator, collect terms, add and subtract $2\alpha\mu_1\mu_2(1 - \beta_5)b_2$, getting:

$$A_1^2 - 4A_2 \quad \frac{[a_0(1 \quad db_1) - \quad (1 - \quad _5)b_2 - \quad _1 \; _2]^2 \quad 4 \quad _2(1 - \quad _5)[\; _1 b_2 - \quad a_3(1 \quad db_1)]}{2}.$$

$$[\quad (1 - \quad_5)(1 \quad db_1 \,)]$$

Of course,the non-negativity of the above expression (implying real solutions) depends on $[\mu_1 b_2 - \alpha a_3(1 + db_1)]$. This must be non-negative or, if negative, must not exceed in absolute value the square term in the numerator. Take the condition $[\mu_1 b_2 - \alpha a_3(1 + db_1)] \geq 0$, i.e. substituting to μ_1 its expression, $b_2/a_3 \geq \alpha(1 + db_1)/(1 + \alpha a_1 b_1)$. $d > a_1$, the influence of uncertainty on investment being higher than on production. Therefore, if $\alpha = 1$, the right hand side term of the above inequality is higher than one; so, a real solution is warranted if the parameter b_2 of demand in the equation of investment exceeds the parameter a_3 of demand in the equation of output at least of the amount $(1 + db_1)/(1 + a_1 b_1)$. Values of α higher than unity reinforce the necessity of such prevalence, while $\alpha < 1$ tends to reverse it. But real solutions may exist even if $[\mu_1 b_2 - \alpha a_3(1 + db_1)]$ is negative, provided that the square term in the numerator be at least equal to it. Unfortunately, this is unlikely to happen, as some factors (α, μ_2, β_5) on which the positivity of the above square
term depends also are coefficients (multiplied by four) of the negative value of $[\mu_1 b_2 - \alpha a_3(1 + db_1)]$.

Being $\sqrt{A_1^2 - 4A_2}$ lower than the absolute value of A_1 and $A_1 < 0$, we get the two following positive (real) solutions:

$$X^*/K^* = (-A_1 + g)/2 \text{ and } (-A_1 - g)/2, \text{ with } g = \sqrt{A^2_1 - 4A_2}.$$

The substitution of these two equilibrium values in the reduced form equations of output and capital gets the equilibrium rate of growth of these two variables.

Now consider if this equilibrium is stable. Take:

$$Dx = C_0 x^2 + C_1 x + C_2$$

with

$$x = X/K.$$

To the equilibrium value x^* it corresponds:

$$C_0 x^*_2 + C_1 x^* + C_2 = 0.$$

The deviation from the equilibrium may be expressed as:

$$D(x - x^*) = C_0(x - x^*)^2 - C_0 x^{*2} + (2C_0 x^* + C_1)x + C_2$$

that, after some transformations, becomes:

$$D(x - x^*) = C_0(x - x^*)^2 + (2C_0 x^* + C_1)(x - x^*).$$

Therefore, the stability condition is $(2C_0 x^* + C_1) < 0$ or, dividing by C_0, $(2x^* + A_1) < 0$. Another way to obtain the stability condition is the following: Define $x = X/Kx^* (= e^{\log X - \log K - \log x^*})$, with $x^* = X/K$. Therefore: $x^* Dx = x^{*2}x^2 + A_1 x^* x + A_2$ is In $x = 1$ equilibrium. Linearize the above expression, getting: $Dx = 2x^* x + A_1 x + 1/x^* [R(x)]$, where R is the remainder of the Taylor expansion. The stability condition is $2x^* + A_1 < 0$. Substitute in this stability condition the value of x^*, i.e. $(-A_1 + g)/2$ and $(-A_1 - g)/2$, we get: $2x^* + A_1 = \pm g$. Therefore, we have a stable solution in correspondence of $-g$ (provided that $g > 0$), to which it corresponds the lower equilibrium output-capital ratio.

It must be useful to explore the possibility to push the equilibrium growth rate of output and capital using some parameters as instruments. Of course, this would need the substitution of X^*/K^* in the reduced expressions of those variables. But that gives some very intricate formulae. Alternatively, some parallel variations of parameters in the non-equilibrium reduced expressions of DX/X and DK/K can be considered, making the hypothesis that the effects on X^*/K^* of such parallel variations tend to compensate. This shows that the steady-state trajectory can be pushed through parallel reductions in parameters a_1 and d, i.e. by stimulating entrepreneurial aggressivity, as well as through some contemporary increases in a_3 and b_2, concerning the impact
of demand on output and capital. Also appropriate variations of the parameters giving the term μ_2, and the instrumental parameters present in the expression of a'_0, stimulate equilibrium growth. Particularly important, in this regard, is the role of γ and τ, i.e. of policies aimed at increasing the availability of entrepreneurial skills (E^p), or of ethic-ideologic motivations pushing entrepreneurial stakanovism. In conclusion, equilibrium growth of both output and capital may be stimulated through high values of a_3, b_2, a, c, z and a'_0, and low values of ξ, d, a_1.

If the availability of labour (L^s) is included in the model, together with the assumption that real wages depend on demand and supply of labour with its self-correcting tendency towards full employment, the equilibrium converges towards a rate of growth resulting from the sum of the variation rates of average labour productivity and of the availability of labour, $D\log(X/L) + D\log L^s$, i.e. the so called "natural rate of growth", where the labour productivity results from the factors influencing X and L, that largely differ from those determining growth through the production function in the traditional theory.

Oligopolistic markets

In this case equation (A10) is eliminated and equation (A7) is substituted by the following:

$$p = (1 + \pi) w \vartheta L / \vartheta X$$

implying that the real wage is a residue, as the share of labour on income is determined by the price leader, while money wage is considered exogenous. So, the irrelevance for firms of income distribution conflict holds. Take equation (A6) of the original model, i.e.:

$$L = X^{1/\beta 5} K^{-(\beta 6 + \beta 5 - 1)/\beta 5} e^{-(\beta 7/\beta 5) DK/K} e^{\beta 8/\beta 5 u}.$$

Multiply and divide by L/X its derivative with respect to X, getting:

$$\vartheta L / \vartheta X = 1/\beta_5 L/X.$$

Substitute the above expression in that of oligopolistic price and the resulting equation in identity equation (A5). This gets:

$$r = [1 - \beta_5/(1 + \pi)] X/K - i - \text{tax} \qquad \text{(A5a)}$$

$\beta_5/(1 + \pi)$ replaces β_5 in equation (A5b) in the previous Appendix section "Markets regulated by demand and supply". Therefore, the stable equilibrium achieved there still holds. But there exists an important difference with respect to the case concerning markets regulated by demand and supply. The assumption that money wages are now exogenous, as in the Keynesian approach, means that the equilibrium rate of growth may imply unemployment, i.e. it does not converge towards the "natural" rate of growth ($D\log X/L + D\log L^s$).

A simplified version of the model including a non-linear function for the availability of entrepreneurship

Now introduce the non-linear term X^{η} for the availability of entrepreneurial skill in the equation for output and the assumption that investment is proportional to output, implying the substitution of the equation for the stock of capital by $K = vX$; v is the capital output ratio. Besides, a prod- uction function with constant coefficient ($L = lX$, with l representing the labour coefficient) and a markup relation fo r price are used. This simplified model is appropriate to clarify some implication of the relation $E^p = X^{\eta}$.

Substituting K with vX, we get the expression for the profit rate: $r = \pi/v(1 + i) - i - \text{tax}$.

The reduced form of the model is:

$$DX/X = \alpha\, [\pi/v(1 + \pi) - i - \text{tax} + a_2 X^{\eta-1} + a_3(c + vDX/X + z) - a_0 + a_1 \xi \chi i + a_4 \tau] \qquad \text{(A11)}$$

that, putting $a'_0 = \pi/v(1 + \pi) - i - \text{tax} - a_0 + a_1\xi + a_3(c + z) + a_4\tau$, becomes:

$$DX/X = a'_0/(1 - a_3 v) + a_2 X^{\eta-1}/(1 - a_3 v). \qquad \text{(A11a)}$$

Put $a'_0/(1 - a_3 v) = a$; $a_2/(1 - a_3 v) = b$ and $\eta - 1 = H$, hence equation (A11a) takes the form $D\log X = a + be^{H\log X}$. Equating this expression to zero and dividing by b, yields: $H\log X = \log(-a/b)$. We can see that the existence of a stationary equilibrium ($X^* = (-a/b)^{1/H}$) requires that a and b have opposite signs, so that $\log(-a/b)$ is positive. a_2 (in the expression of b) and $1 - a_3 v$ being positive, the existence of the equilibrium requires that $a'_0 < 0$.

As is well known, the stability analysis starts from the deviation from equilibrium $y = \log X - \log X^*$. The constancy of X^* implies that $Dy = D\log X$. Now express the model in terms of deviations from the equilibrium:

$$Dy = a + [be^{H(\log X - \log X^*)}] e^{H \log X^*}$$

X^* being equal to $(1/H)\log(-a/b)$, $Dy = a + be^{Hy}e^{\log(-a/b)}$. Therefore

$$Dy = a - ae^{Hy}, \text{ i.e. } Dy = a - a - aHy - aH^2y^2/2! \ldots$$

This expression tends to zero, i.e. the equilibrium is stable, if a and H have the same sign. As previously seen, the existence of equilibrium requires $a'_0 < 0$ hence $a < 0$; it follows that the condition for stability is $\eta < 1$ so that $H < 0$, i.e. the availability of entrepreneurial skill must vary less than output.

If we put $\eta = 1$, the expression (A11a) of DX/X becomes constant. This equilibrium expressed by a constant is unstable, but it is possible to achieve it through variations of some control parameters.

The existence of a stable equilibrium solution can be proved immediately in the following oversimplified model:

$$DX = \alpha(X^d - X) \tag{A12}$$

$$P = (1 + \pi)lw \tag{A13}$$

$$C = c(Lw)/P \tag{A14}$$

$$Xd = C + G \tag{A15}$$

$$L = lX. \tag{A16}$$

Where G indicates public expenditure and c is the propension to consume.

The reduced form is:

$$DX = \alpha\ [cX/(1 + \pi) + G - X]. \tag{A12a}$$

Therefore, the equilibrium solution is: $X^* = G[(1 + \pi)/(1 + \pi - c)]$. The coefficient of G is the Keynesian multiplier. Now rewrite the equation for DX as follows: $DX = \alpha[(-1 + c - \pi)/(1 + \pi)]X$
$+ \alpha G$. The negativity of the coefficient of X proves that the solution implies a stable equilibrium.

The complete model

The model developed in this paper, that is including the Lotka-Volterra equations and the term X^{η} (for the availability of entrepreneurial skills) in the equation of supply does not collapse to one equation. If we preserve the hypotheses that real wage is equal to marginal productivity of labour or, in the presence of oligopolistic markets, is a residual, and assume that the variance of profit rates across firms coincides with uncertainty, the reduced form of the model is as follows:

$$\frac{DI_I}{I_I} = b_1 - b_2 u \tag{A17}$$

$$\frac{Du}{u} = -b_3 + b_4 I_I + c_1 x \tag{A18}$$

$$\frac{DX}{X} = a\left[(1 - b_5)\frac{X}{K} - i - tax - a_1 u + a_2 X^{\eta - 1} + a_3 \frac{DK}{K}\frac{K}{X} - a_0 + a_3(c + z) + a_4 t\right] \tag{A19}$$

$$\frac{DK}{K} = \frac{b}{a}\frac{DX}{X} - a_5 u. \tag{A20}$$

In the hypothesis of oligopolistic markets, β_5 in equation (A19) above is replaced by $\beta_5/(1 + \pi)$. This model describes a cycle depending on the values of the parameters β_1, β_2, β_3, β_4. Only for some particular and quite improbable values of those parameters do we get a limit cycle.