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Recurrent Explosive Behaviour of Debt-to-GDP Ratio

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Abstract

In this paper the recurrent explosive behaviour of debt-to-GDP ratio is tested in three countries with a long fiscal record: Sweden, the UK and the US. The testing is based on the method developed by Phillips et al. (2015) which is new in this context. The method allows us to avoid the size distortion problem of the traditional tests of fiscal sustainability and makes it possible to examine potential unsustainability as a transitory rather than permanent phenomenon. It has been demonstrated that in the economies analyzed, long periods of fiscal sustainability were interrupted by relatively short periods when the debt-to-GDP ratio had explosive dynamics.

Keywords: Public debt, Sustainability, Unit root tests, Explosiveness

JEL classification: C12, C22, E62, H63

1 Introduction

Studies of fiscal sustainability have gained momentum recently, especially in the context of monetary policy rates reaching the zero lower bound in many countries. Numerous analyses based on the fiscal theory of price level suggest that active (or unsustainable) fiscal policy can be viewed as a nominal anchor that is alternative to active monetary policy (see Davig and Leeper 2011). Thus, in order to assess the consistency of fiscal policy with intertemporal budget constraint, many authors have investigated integration and cointegration properties of the key fiscal variables: revenues, expenditures and debt (see, inter alia, Hamilton and Flavin 1986;

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Trehan and Walsh 1991; Paparas et al. 2015 provide a comprehensive review of the recent studies). Bohn (2007, 2008) shows that in a dynamically efficient economy, where the long run interest rate r is greater than the long run GDP growth rate g ($r - g > 0$), the debt-to-GDP ratio b_t being integrated of any finite order n ($b_t \sim I(n)$) satisfies the transversality condition

$$\lim_{s \rightarrow \infty} e^{-(r-g)s} E_t b_{t+s} = 0, \quad (1)$$

which is equivalent to fiscal sustainability.

In order to violate the condition (1), the process $\{b_t\}$ has to be explosive and growing at a rate of at least $(r - g)$. In terms of the usual ADF-type equation,

$$\Delta b_t = \mu + \rho b_{t-1} + \sum_{j=1}^p \phi_j \Delta b_{t-j} + \varepsilon_t, \quad (2)$$

the cases $\rho < 0$ or $\rho = 0$ imply that the transversality condition is satisfied, b_t being $I(0)$ and $I(1)$, respectively. An interesting case is $\rho > 0$, which means an explosive behaviour of b_t . It is not a sufficient condition for insolvency, since the transversality condition (1) is violated only if $\rho > (r - g)$. However, positive values of ρ can be viewed as a strong case for insolvency, making it economically much more insightful to test the hypothesis $H_0 : \rho = 0$ (sufficient solvency condition) against an explosive alternative $H_1 : \rho > 0$, instead of the usual $H'_1 : \rho < 0$.

In the context of stock markets Evans (1991) demonstrates that the standard unit root tests fail to detect explosive behaviour in the presence of recurrent explosive bubbles. In order to deal with this problem, Phillips et al. (2011) have developed a new recursive procedure that proved to have a good power against mildly explosive alternatives. Yoon (2012) applies this method to test the fiscal sustainability in the US. The analysis conducted therein is inconclusive, as the null of unit root is rejected only for a subset of model specifications. In addition, this testing procedure has been designed to detect a single episode of explosive behaviour over the whole sample and can lack a power if there are shorter recurrent episodes of explosive behaviour in a time series.

Fiscal unsustainability is likely to happen repeatedly, over shorter episodes, rather than being an all-sample phenomenon. It can be either an effect of extraordinary events, such as wars, or it can result from actions taken by irresponsible governments, which are soon replaced by those that are more fiscally sound. Phillips et al. (2015) propose a new recursive flexible

window method, which has a good power to detect multiple, recurrent episodes of explosive behaviour. In this paper, the method developed by Phillips et al. (2015) is applied to detect and time-stamp periods of fiscal profligacy in three countries with long recorded history of public debt: Sweden, the UK and the US.¹

2 Methodology and Data

The testing procedure is based on the ADF equation (2) which is estimated using a backward expanding sample sequence. For a time series of length T , the smallest sub-sample is selected to be of length $T_0 = \lfloor r_0 T \rfloor$ where $r_0 \in (0, 1)$. The endpoint of each sub-sample is fixed at $T_2 = \lfloor r_2 T \rfloor$ where $r_2 \in [r_0, 1]$, and the start point of each sub-sample, $T_1 = \lceil r_1 T \rceil$ varies from 1 to $T_2 - T_0 + 1$ ($r_1 \in [0, r_2 - r_0]$). The corresponding ADF statistic sequence is $\{ADF_{r_1}^{r_2}\}_{r_1 \in [0, r_2 - r_0]}$. The backward sup ADF (BSADF) statistic is then defined as the sup value of the ADF statistic sequence over interval $[0, r_2 - r_0]$,

$$BSADF_{r_2}(r_0) = \sup_{r_1 \in [0, r_2 - r_0]} \{ADF_{r_1}^{r_2}\}. \quad (3)$$

The statistic (3) can be used to time stamp the episodes of explosive behavior. The generalised sequential ADF statistic (GSADF), based on the sup value of the BSADF,

$$GSADF(r_0) = \sup_{r_2 \in [r_0, 1]} \{BSADF_{r_1}(r_2)\},$$

is used for testing the null of unit root against the alternative of recurrent explosive behaviour.

The procedure is applied to the annual debt-to-GDP ratios in Sweden, the UK and the US over the period from 1792 to 2012. The minimal length of a sub-sample is fixed at 30 years corresponding to $r_0 = 0.137$. (For a robustness check, the analysis was also carried out for $r_0 = 0.190$; however, the change had no significant effect on the results). In each sub-sample the lag length in the ADF equation (2) is selected using the Bayesian information criterion (BIC) with the maximum lag length set to $\lceil \log(T)^{1.05} \rceil$.

¹The literature includes other ways of tackling the time variability of fiscal reaction functions similar to (2). Afonso and Rault (2010) allow for a structural break within the sample. Daving and Leeper (2011) use the Markov switching framework to account for possible nonlinearities.

3 Results

The test results are reported in Table 1. Along with the results of the GSADF test we report the results of the standard (left-side) unit root tests. We also report the results of the SADF test of Phillips et al. (2011). The latter was applied by Yoon (2012) to test for the explosive behaviour of the US debt-to-GDP ratio.

The standard (left-side) unit root tests produce mixed evidence about the presence of unit roots in the series of debt-to-GDP ratios. The ADF test with trend rejects the null of unit root in favour of trend-stationary process in the case of Sweden. The KPSS test does not reject the null of stationary or trend-stationary process in the UK and the US. The right-side SADF test (Phillips et al. 2011) rejects the unit root in favour of the explosive alternative, but only in the case of the US. However, in all three cases, the GSADF test (Phillips et al. 2015) rejects the unit root in favour of the explosive alternative.

Table 1: Unit Root Tests

Type of Test	Critical Value	Sweden	Country UK	US
ADF (Const, BIC)	-2.880	-2.781	-2.021	-1.713
ADF (Trend, BIC)	-3.430	-4.068*	-2.186	-2.992
KPSS (Const)	0.463	0.770*	0.458	0.815*
KPSS (Trend)	0.146	0.200*	0.133	0.103
SADF ($r_0 = 0.137$)	1.400	0.597	0.396	1.865*
GSADF ($r_0 = 0.137$)	2.010	3.013*	3.573*	2.321*

Critical values for SADF and GSADF are from Phillips et al. (2015).

Significance level equals 5%. The asterisk * marks the rejection of null hypothesis.

Figures 1-3 show the time series plots of debt-to-GDP ratios (upper panel of each figure) and the plots of BSADF statistics, together with the corresponding critical values (lower panel of each figure). The Swedish debt-to-GDP ratio shows explosive behaviour in several episodes. The first episode (1864-1870) is associated with poor harvests and famine. The subsequent period of explosive debt growth (1878-1894) correlates with the economic reforms pursued to modernise the economy of Sweden. In the twentieth century, the episodes of explosive debt growth were in 1943-1949 (World War II) and 1992-1994 (the Swedish Banking Rescue). In the US, the explosive behaviour of debt-to-GDP ratio is detected in 1865 (the American Civil War), 1918 (the World War I), 1921 (the recession of 1920-1921) and in 1943-1950 (the World

War II). The time-stamping procedure, based on the BSADF statistic, does not perform in an intuitively-appealing way in the UK. The explosive behaviour is detected in the periods 1864-1878, 1899-1901, and in 1922. However, it is not detected during the World War II and its immediate aftermath, when the debt-to-GDP ratio in the UK reached its historical maximum.

4 Conclusions

The GSADF test applied to debt-to-GDP ratio is a viable alternative to the traditional, left-sided sustainability tests. It also allows relaxation of the overly restrictive assumption that fiscal processes are time-invariant. In this paper, the GSADF test is applied to detect episodes of explosive debt dynamics in three countries with a long fiscal record. However, it is worth noting that this test has been primarily developed to detect stock market bubbles which have an extended ascending trajectory followed by a collapse. The explosive behaviour of debt-to-GDP ratio does not always follow this pattern. Exogenous factors, such as wars and natural disasters may lead to a sudden increase in the public debt followed by its reduction over several years. As a result, the time-stamping mechanism developed in Phillips et al. (2015) might be suboptimal in this application. Refining the GSADF test so that it would account for the specific characteristics of fiscal variables is a promising venue for further research.

Figure 1: Backward Sequential ADF, Sweden

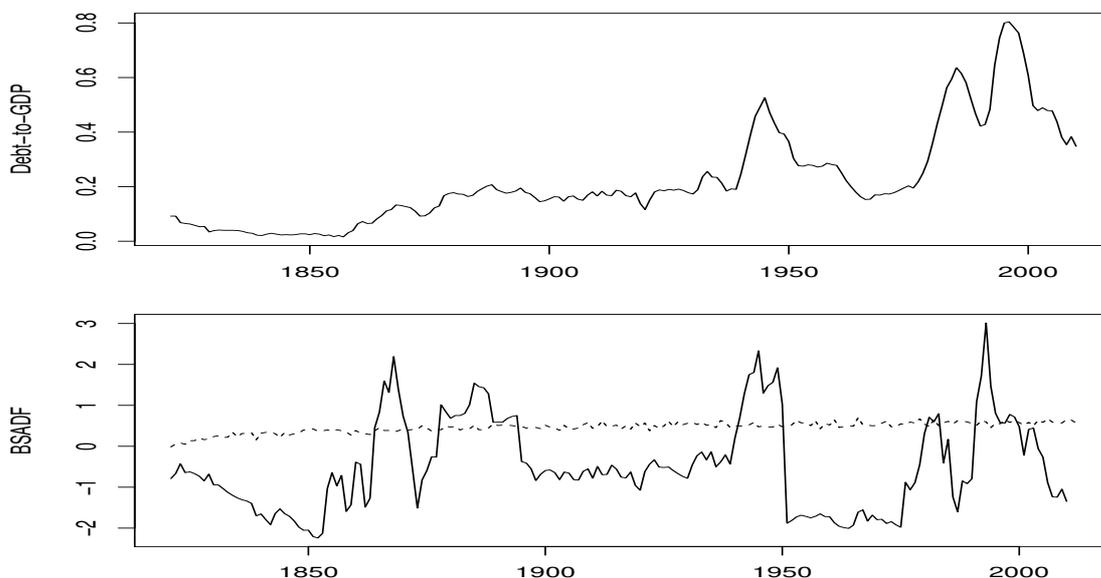


Figure 2: Backward Sequential ADF, UK

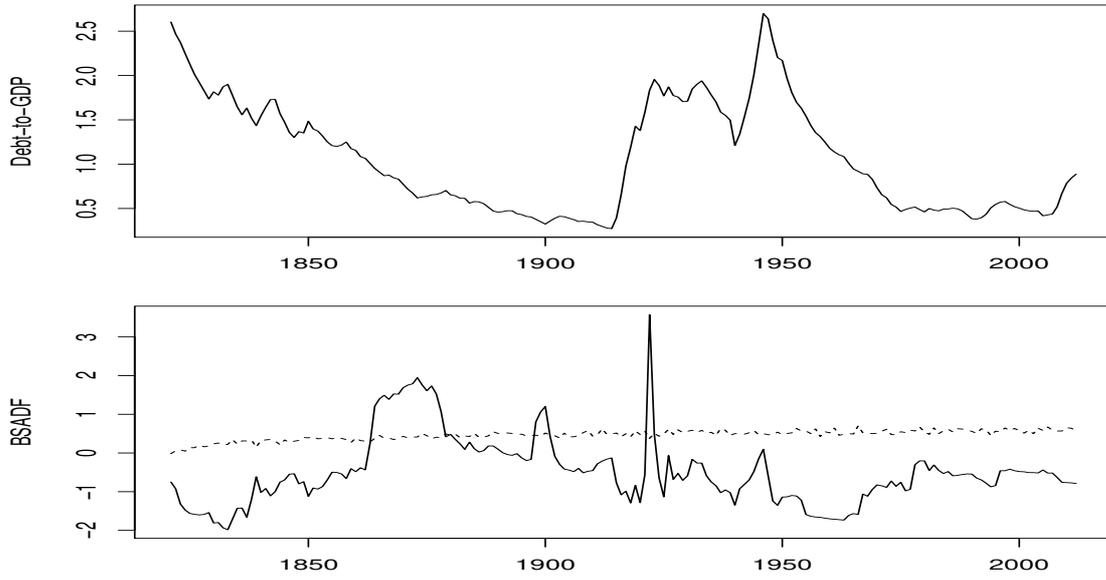
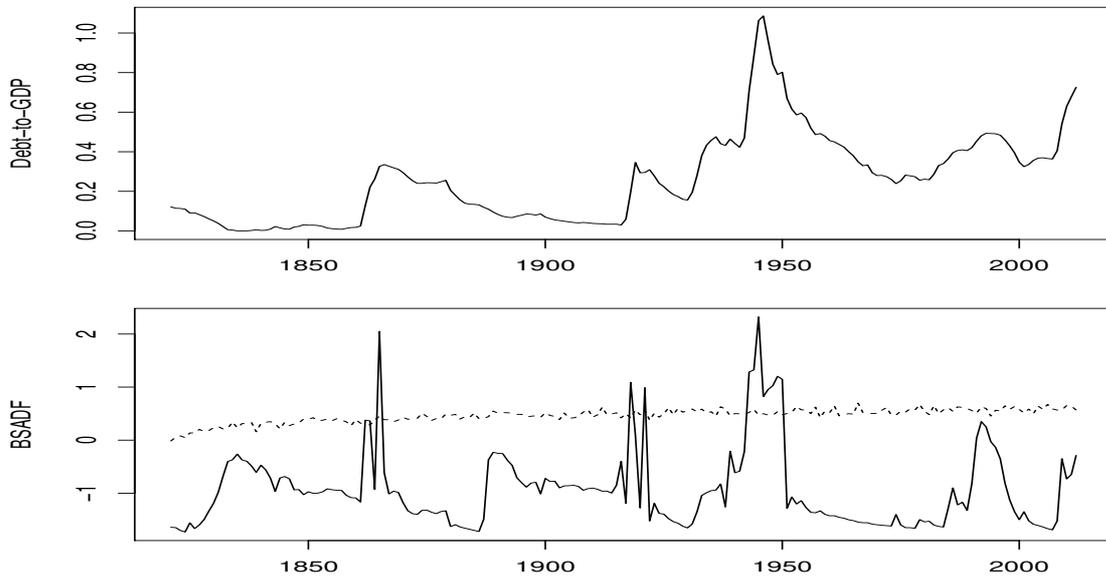


Figure 3: Backward Sequential ADF, US



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