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On the Sources of the Feldstein–Horioka Puzzle across Time and Frequencies *

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Abstract

There are several candidate explanations for the Feldstein–Horioka puzzle. This paper provides a quantitative assessment of the relative role of the existing explanations for saving–investment comovement performing time-frequency domain analysis for nine countries from 1885 to 2010. The main findings are summarized as follows. First, in large economies, such as the US and Italy, to a greater extent high correlations between saving and investment are observed than in middle-sized and small countries. Second, we obtain two groups of countries in terms of time-changing patterns of correlations: U-shaped and decreasing patterns of capital mobility. Third, fiscal balance seems to be most related to positive saving-investment correlations in many countries. Fourth, a global common factor also plays an important role in explaining the Feldstein-Horioka puzzle.

Keywords: Feldstein–Horioka puzzle; Saving–investment correlation; International capital mobility; Wavelets
JEL classification: C32; F21; F32

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1 Introduction

Feldstein and Horioka (1980) claim the absence of perfect capital mobility based on a regression of investment on saving for 21 OECD countries for the period 1960 to 1974. Feldstein and Horioka (1980) show the slope parameter on saving to be significantly different from zero but not from one, which implies that domestic investment mainly depends on domestic saving. Feldstein and Horioka (1980) interpret these results in terms of long-run immobility of international capital and imperfect integration of international capital markets. This so-called Feldstein-Horioka puzzle stimulates voluminous debates on international capital mobility.\(^1\)

In this paper, we revisit how the pattern of comovements between saving and investment has changed throughout history in the short run and the long run. Specifically, we measure comovements in both time and frequency space by resorting to wavelet analysis. Most of the existing literature of the Feldstein-Horioka puzzle focusing on the cross-section or time-series relationship relies on time-domain analysis.\(^2\) One of the few exceptions is Levy (2000), who uses the frequency-domain framework to simultaneously examine the long-run, cyclical, and short-run saving-investment relationship. In this paper, following Aguiar-Conraria et al. (2008) and Rua (2010), we use a wavelet coherency analysis that enables us to simultaneously estimate the periods and frequencies as which saving and investment are strongly correlated. In other words, we can detect the changing pattern of the Feldstein-Horioka puzzle and reveal whether the close link between saving and investment is a short-run or long-run phenomenon. We use annual data of nine countries ranging from the year 1885 to the year 2010.\(^3\)

Three main findings based on wavelet coherency analysis are as follows. First, our results indicate that high correlations between saving and investment are more observable in large economies such as Italy and the US while this positive link is less observable in small-sized countries such as Argentina and Sweden. Second, we have two groups of countries in terms of the time changing pattern of the Feldstein-Horioka puzzle. Larger countries including Italy, Spain, the UK, and the US exhibit the inverted U-shaped pattern of the saving-investment correlations. In other words, for these countries, the correlation between saving and investment was low in the former sample period until the 1920s and 1930s. Since then, it has increased substantially in the middle periods and finally decreased to its low value since during the 1960s and 1970s. Our evidence supports the view of Obstfeld and Taylor (2002) who insist on a U-shaped pattern for capital mobility. On the other hand, for other countries such as Argentina, Australia, Canada, Finland, and Sweden, however, we find increasing coherence in the latter period. In other words, we observe a decreasing pattern for capital mobility. Third, frequency is important in explaining the pattern of the Feldstein-Horioka puzzle for each country. For Italy and the UK, this little-large-little pattern is mostly explained by the coherency at low frequencies while, for Spain and the US, large coherency at all frequencies is important in explaining the Feldstein-Horioka puzzle. For Argentina and Australia, the increasing pattern of

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\(^1\)For the extensive survey, see Apergis and Tsoumas (2009) among others.

\(^2\)For example, see Evans, Kim, and Oh (2008), Fouquau, Hurlin, and Rabaud (2008), Georgopoulos and Hejazi (2005), and Hussein (1998) among others. Other papers, including Feldstein and Horioka (1980), rely on cross-section analysis.

\(^3\)The nine countries are the US, the UK, Italy, Canada, Spain, Argentina, Australia, Sweden, and Finland in the order of country size.
the coherent relationship between saving and investment is explained by the increasing coherency at all frequencies. For Canada, the large coherency at high frequencies since the 1970s can explain the increasing pattern. For Sweden and Finland, the positive link between saving and investment becomes prevalent during the 1970s at business cycle frequencies.

Our second purpose in this paper is to investigate what causes this relationship to change. A number of interpretations of a high coefficient in saving-investment regressions have been emphasized in the existing literature. Omitted factors may drive both saving and investment in the same direction and if this is the case, saving and investment would be positively correlated even under full capital mobility.

Given this, we shed new light on capital flows by performing partial coherency analysis controlling for the omitted explanatory variables. Partial coherency developed by Aguiar-Conraria and Soares (2014) is useful in estimating the interdependence between saving and investment after eliminating the effect of other variables. Our five candidate variables are population growth, productivity growth, fiscal budget, trade costs, and global common factors. In general, we find that the coherency region shrinks once we control for the candidate variables. Additional findings are as follows. First, primary balances seem to be the main source of lower coherency between saving and investment in many countries. For Australia, Canada, the UK, and the US, the coherency between saving and investment is mostly lowered once we control for the primary balances. Second, the second major source appears to be the global common factor. In particular, high coherency in Argentina, Australia, Canada, Finland, Italy, and the UK can be explained by the global common factor. Third, among six countries with available data, trade costs play an important role in the four countries Australia, Canada, Sweden, and the US. Fourth, population and output growth appear to be less important in explaining the Feldstein-Horioka puzzle except for Argentina and Sweden in the post-Bretton Woods period.

The remainder of the paper is organized as follows. Section 2 reviews the literature. Section 3 describes the wavelet analysis. Section 4 explains the data. Sections 5 and 6 present the results on the wavelet coherency and partial coherency, respectively. Section 7 concludes the paper.

2 Literature review

There is a strand of literature that investigates how the correlation between national saving and domestic investment has changed throughout history. Obstfeld and Taylor (2002) assert that the integration of capital markets from the second half of the nineteenth century to the late twentieth century has followed a U-shaped pattern. After capital market globalization deepened remarkably up until the year 1914, the global economy was destroyed in World War I. In the Bretton Woods era (1945 to 1971), there was an attempt to rebuild the global economy. The post-Bretton Woods floating rate era is characterized by a strong upsurge in capital mobility. Ohanian and Wright (2010) find that flows during the golden era are consistent with standard theory as capital flows

\footnote{Two common factors are extracted from the principal analysis. For details, see Section 6.}

\footnote{Bayomi (1990) claims that this was the only historical period that came closest to the paradigm of perfect capital mobility.}
from low to high return countries, but this relationship breaks down during the interwar period. Adalet and Eichengreen (2007) find that current account reversals are smaller in the global standard period but the interwar period has frequent reversals with high output costs. Giannone and Lenza (2010) find that the correlation between saving and investment decreases over time and became small in the last two decades.

Another second strand of literature investigates whether the Feldstein-Horioka puzzle is a short-run or long-run phenomenon. Sarno and Taylor (1998) show that the short-run correlation is significantly higher than the long-run correlation for the UK economy. Hoffmann (2004) also finds that for the UK and the US, variations in capital mobility over the century have largely been reflected in changes in the short-run saving retention coefficient, whereas long-run capital mobility has been remarkably stable and fairly high throughout the century. Reporting that cointegration is rejected in the flexible exchange rates period for a number of countries including Argentina, Canada, Sweden, and the US, Corbin (2004) insists that there is no long-run relationship between saving and investment.

On the other hand, based on frequency domain analysis, Levy (2000) finds that saving and investment in the US economy are highly correlated at the frequencies corresponding to the business cycle and the long run. However, Levy (2000) also finds that the squared coherence is not statistically significant at the high frequency band corresponding to two- to three-year cycles.

The size of the economy also matters in explaining the Feldstein-Horioka puzzle. Baxter and Crucini (1993) show that the retention parameter in a larger country is higher than the retention parameter in a smaller country. This is because a large country with a substantial share of the world’s total investment and saving can influence the world interest rate. For example, an increase in the national saving of a large country lowers the world interest rate and, hence, increases the investment. Therefore, the retention parameter of a large country tends to be high. Estimating an error-correction model, Jansen (1996) suggests that the saving-investment relationship may be subject to a large-country effect and the saving-retention coefficient is large in large countries. Ho (2003) shows that the saving-retention coefficient increases as the relative GNP share becomes larger, which substantially supports the country size argument. Bahmani-Oskooee and Chakrabarti (2005) also find systematic effects of country size on the saving-investment relationship.

Economic growth might drive up saving, leading, in turn, to higher investment. According to the life-cycle theory, emphasized by Feldstein and Horioka (1980), the long-run drivers such as population and productivity growth lead to both increased saving and investment. Using an overlapping-generation model, Obstfeld (1986) shows that the high coefficient in saving-investment regression could be attributed to the effect of omitted factors such as population and economic growth. For example, if an economy experiences growth shocks such as population growth and labor augmenting technology growth, then both saving and investment increase. Obstfeld and Rogoff (1996) show that in the life-cycle theory of consumption, sustained demographic and productivity changes that increase a country’s long-term investment rate may also increase its saving rate. Using

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7Corbin’s (2004) interpretation is that the existence of a cointegration would be considered evidence of capital immobility.

8In contrast, Kim (2001) finds little evidence on the effects of country size on the retention coefficient.
an infinite-horizon general equilibrium model, Mendoza (1991) and Baxter and Crucini (1993) demonstrate that positively correlated and temporary productivity shocks can produce a positive saving-investment correlation even under complete financial markets. Using a VAR model, Asdrubali and Kim (2009) also find that income growth endogenously increase both saving and investment rates, while an exogenous investment boom without income growth does not increase saving contemporaneously. Attanasio, Picci, and Scorcu (2000) investigate Granger causality among saving, investment, and growth rates.

Fiscal deficit lowering national saving could also crowd out domestic investment. Levy (1995) also finds that an endogenous fiscal policy could move saving and investment in the same direction. Roubini (1988) shows that optimal tax smoothing implies a one-to-one relationship between the current account and the fiscal deficit. Roubini (1988) includes fiscal and other cyclical determinants of saving and investment in the regression and finds that the estimated saving-retention coefficient is lowered. Summers (1988) and Feldstein and Bacchetta (1991) also find that budget deficits are inversely related to the difference between private saving and private investment. Levy (1995) shows that a positive saving-investment correlation can occur in a world with endogenous fiscal policy. Bussiere, Fratzscher, and Muller (2010) find that a positive primary government budget balance drives up investment.

Moreover, if a country experiences a certain global shock, the country cannot lend or borrow capital from abroad because other countries are concurrently confronting the same situation. The international real business cycle literature, including Mendoza (1991), Backus, Kehoe, and Kidland (1992), Baxter and Crucini (1993), and Stockman and Tesar (1995), find high comovements of saving and investment even though financial capital is perfectly mobile. Glick and Rogoff (1995) assert that the distinction between global and country-specific shocks are important in explaining international borrowing and lending. Ventura (2003) finds that controlling for global shocks has little or no effect on the estimation of the saving-investment coefficient.

Recently, the relationship between saving and investment has been investigated in the factor-augmented regression literature. Employing this methodology, we can separate comovements originating from the country-specific level from comovements at the global level. Byrne, Fazio, and Fiess (2009) find no evidence of a long-run relationship in the idiosyncratic components of saving and investment. The authors find that the global components in saving and investments comove. Giannone and Lenza (2010) find that the correlation between saving and investment decreases over time, becoming small in the last two decades. Costantini and Gutierrez (2013) find that the estimated saving-retention coefficient is close to zero when global shocks are considered. While all these works focus only on the period after 1970, they commonly find that idiosyncratic components between saving and investment do not comove.

Obstfeld and Rogoff (2001) emphasize the role of trade costs to solve the six international puzzles including Feldstein and Horioka (1980).

9In the stochastic general equilibrium model, for example, Backus, Kehoe, and Kidland (1992), Baxter and Crucini (1993), and Stockman and Tesar (1995) among others, shocks originating in one country are diffused to another country.
3 Methodology

This section outlines the wavelet tools that we use to analyze how saving and investment are related. The wavelet analysis enables us to investigate the changing relationship between investment and saving at various frequencies in a unified framework.

3.1 Wavelet transform

Our empirical method builds on complex wavelets and the continuous wavelet transform in line with recent studies that apply the wavelet analysis to economics (e.g., Aguiar-Conraria et al., 2012; Rua, 2010, 2012, 2013; Aguiar-Conraria and Soares, 2014).

Given a time series \( x(t) \), its continuous wavelet transform for a mother wavelet \( \psi \) is expressed as follows:

\[
W_x(\tau, s) = \int_{-\infty}^{\infty} x(t) \tilde{\psi}_{\tau,s}(t) dt, \quad s, \tau \in \mathbb{R}, s \neq 0,
\]

where

\[
\tilde{\psi}_{\tau,s}(t) = \frac{1}{\sqrt{|s|}} \psi \left( \frac{t - \tau}{s} \right),
\]

and asterisk denotes complex conjugation. At this point, there are two key factors. The first is the scaling factor, \( s \), which determines the length of the wavelet and concerns frequency. If the absolute value of \( s \) is less than 1, a wavelet is compressed. Conversely, if the absolute value of \( s \) is greater than 1, it is stretched. The second is the translation parameter, \( \tau \), which represents the location of a wavelet in time.

All of the wavelet measures described below are based on the characteristics of this transform. To confirm this aspect, it is useful to draw a comparison between the Fourier transform and the present transform described by (1). In other words, the Fourier transform depends only on frequencies, whereas \( W_x \) depends on time as well as frequencies. This means that the wavelet transform is more flexible by allowing each periodic component to vary over time.

The difference between these two transforms can be better understood by specifying the form of the mother wavelet \( \psi \) in (1). Following our predecessors— Aguiar-Conraria et al. (2012), Rua (2010, 2012, 2013), Aguiar-Conraria and Soares (2014), and many others—we assume the Morlet wavelet with which optimal joint time-frequency concentration can be attained in the sense that the Heisenberg box area is the minimum value.\(^{10}\) The Morlet wavelet is considered one of the most widely used mother wavelet of the form

\[
\psi_{\omega_0}(t) = \pi^{-1/4} \left( e^{i\omega_0 t} - e^{-\omega_0^2/2} \right) e^{-t^2/2}.
\]

If \( \omega_0 \geq 5 \), then it is approximately equal to

\[
\pi^{-1/4} e^{i\omega_0 t} e^{-t^2/2}.
\]

\(^{10}\)For more details of the association with the Heisenberg uncertain principle, see, for example Aguiar-Conraria and Soares (2014).
From this representation, unlike the case of the Fourier transform in which any time series is assumed to consist of trigonometric waves, the present wavelet in (2.2) is enveloped by a Gaussian function. While Fourier’s basic wave is a permanent length of the form
\[ e^{i\omega t} = \cos (\omega t) + i \sin (\omega t), \]
where \( i \) is an imaginary unit, the corresponding waves in (2.2), \( \tilde{\psi} \{(t - \tau)/s\} \) are localized around \( \tau \) with the length depending on \( s \).

In the Morlet wavelet, when \( \omega_0 = 6 \simeq 2\pi \), we obtain an inverse relationship between frequencies \( f \) and the scaling factor \( s \) such that \( f \simeq 1/s \). Because of this very tractable relationship, we assume \( \omega_0 = 6 \) as in Aguiar-Conraria et al. (2012) and many others.

Based on the above wavelet transform, \( \mathcal{W}_x(\tau, s) = |\mathcal{W}_x(\tau, s)| \cos \phi_x + i \sin \phi_x \), valuable information is extracted. Using the amplitude of the wavelet transform \( \mathcal{W}_x(\tau, s) \) we can calculate the wavelet power spectrum, which is defined as
\[ WPS_x(\tau, s) = |\mathcal{W}_x(\tau, s)|^2, \]
and differs from the classic power spectrum of the Fourier transform and indicates how the strength of the time series \( x(t) \) is distributed in the frequency domain and the time domain.

### 3.2 Bivariate analysis of investment-saving comovements

To examine the interdependence between investment and saving, we need to consider the bivariate framework. Then, in general, we let \( x(t) \) and \( y(t) \) denote two time series of interest, respectively. To measure these correlations, we use the wavelet transform and employ the cross wavelet transform, which is defined as
\[ \mathcal{W}_{xy}(\tau, s) = \mathcal{W}_x(\tau, s)\mathcal{W}_y^*(\tau, s). \]

From this, complex wavelet coherency, which represents the normalized covariance between \( x(t) \) and \( y(t) \), is given by:
\[ \Gamma_{xy}(\tau, s) = \frac{S(\mathcal{W}_{xy}(\tau, s))}{\sqrt{S(\mathcal{W}_{xx}(\tau, s))S(\mathcal{W}_{yy}(\tau, s))}}, \]
where \( S \) is a smoothing operator in time and frequencies. Note that the complex wavelet coherency is normalized by the wavelet power spectra, \( \mathcal{W}_{xx} = WPS_x(\tau, s) = |\mathcal{W}_x(\tau, s)|^2 \) and \( \mathcal{W}_{yy} = WPS_y(\tau, s) = |\mathcal{W}_y(\tau, s)|^2 \). Moreover, from the absolute value of \( \Gamma_{xy} \), we can obtain the wavelet coherency:
\[ R_{xy}(\tau, s) = \frac{|S(\mathcal{W}_{xy}(\tau, s))|}{\sqrt{S(\mathcal{W}_{xx}(\tau, s))S(\mathcal{W}_{yy}(\tau, s))}}. \]

The wavelet coherency is calculated as the absolute value and, consequently, the wavelet coherency is not less than 0 and not more than 1 (i.e., \( 0 \leq R_{xy}(\tau, s) \leq 1 \)). Therefore, the wavelet coherency \( R_{xy} \) provides us with no information concerning whether the correlation between \( x(t) \) and \( y(t) \) is positive or negative although the wavelet coherency is often used (e.g., Aguiar-Conraria et al., 2012; Rua, 2012; Aguiar-Conraria and Soares, 2012).
Because the Feldstein-Horioka puzzle is not negative but has a positive correlation between saving and investment, it is essential to identify the sign of the relationship in the time-frequency space.

Hence, following Rua (2010, 2013), we utilize the modified wavelet coherency

\[
\tilde{R}_{xy}(\tau, s) = \frac{\text{Re}[S(W_{xy}(\tau, s))]}{\sqrt{S(W_{xx}(\tau, s))S(W_{yy}(\tau, s))}},
\]

with \(-1 \leq \tilde{R}_{xy}(\tau, s) \leq 1\), where \(\text{Re}[\cdot]\) denotes the real part. The modified wavelet coherency \(\tilde{R}_{xy}(\tau, s)\) is akin to correlation coefficients, and we interpret it as a localized correlation coefficient over time reflected by \(\tau\) and across frequencies reflected by \(s\). Unlike standard wavelet coherency \(R_{xy}(\tau, s)\), the modified measure \(\tilde{R}_{xy}(\tau, s)\) can take a negative value, which provides information on the sign of the correlation.

### 3.3 Multivariate analysis: controlling the other factors

Previous works attempt to trace the causes of the Feldstein-Horioka puzzle to multiple factors (i.e., population growth, labor-augmenting productivity growth, fiscal budget, and global common factors). In other words, it is possible that these hidden factors commonly affect both investment and saving and, consequently, the correlation between them becomes spuriously high. Hence, to eliminate these effects on the correlations between saving and investment, we consider multivariate frameworks beyond the above bivariate analysis.

To evaluate the direct relationship between two series after controlling the other factors, the partial wavelet coherency proposed by Aguiar-Conraria and Soares (2014) is useful. To see the usefulness in our analysis, we begin by considering the case where there are three variables, \(x(t), y(t)\) and \(z(t)\), and our interest is in the correlation between \(x(t)\) and \(y(t)\), but \(z(t)\) have relationships with both \(x(t)\) and \(y(t)\). Analogous to partial correlation, the original measure provided by Aguiar-Conraria and Soares (2014), the partial wavelet coherency of \(x(t)\) and \(y(t)\) after controlling for the effects of \(z(t)\), is given by

\[
R_{xy, z}(\tau, s) = \frac{|\Gamma_{xy} - \Gamma_{xz} \Gamma_{yz}^*|}{\sqrt{(1 - R_{xz}^2)(1 - R_{yz}^2)}}.
\]

To identify the sign of partial correlation as before, the modified partial wavelet coherency that we use is defined as

\[
\tilde{R}_{xy, z}(\tau, s) = \frac{\text{Re} [\Gamma_{xy} - \Gamma_{xz} \Gamma_{yz}^*]}{\sqrt{(1 - R_{xz}^2)(1 - R_{yz}^2)}}.
\]

We interpret this as a localized partial correlation over time and across frequencies.

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11 To cover the shortcomings, many authors additionally use the phase difference defined as

\[
\rho_{xy}(\tau, s) = \tan^{-1} \left( \frac{\text{Im}[W_{xy}(\tau, s)]}{\text{Re}[W_{xy}(\tau, s)]} \right),
\]

where \(\text{Re}[W_{xy}]\) and \(\text{Im}[W_{xy}]\) denote the real part and the imaginary part of the cross wavelet \(W_{xy}\), respectively. For the meanings of phase difference, see for example Aguiar-Conraria and Soares (2014).
We now turn to the general case where there are \( n \geq 3 \) time series, \( x_1, x_2, \ldots, x_n \). For \( \forall l, m \in n = \{1, 2, \ldots, n\} \) we let \( W_l \) denote the wavelet transform of series \( x_l \), \( W_{lm} \) the cross-wavelet transform of \( x_l \) and \( x_m \), and \( \Omega_{lm} \) the smoothed \( W_{lm} \) in time and frequencies (i.e., \( \Omega_{lm} = S(W_{lm}(\tau, s)) \)). We let \( \Omega \) denote the \((n \times n)\) Hermitian matrix of \( \Omega_{lm} \). The partial wavelet coherency of \( x_l \) and \( x_m \) can then be written as

\[
R_{lm-n_{lm}}(\tau, s) = \frac{|\Omega_{ml}^d|}{\sqrt{\Omega_{ll}^d \Omega_{mm}^d}},
\]

where \( n_{lm} \) is all the indexes excluding the indexes \( l \) and \( m \), and \( \Omega_{ml}^d \) is the cofactor of the element in position \((l, m)\) of \( \Omega \) for \( \forall l, m \in n \). As before, the modified partial wavelet coherency that we use is of the form

\[
\tilde{R}_{lm-n_{lm}}(\tau, s) = \frac{\text{Re} [\Omega_{ml}^d]}{\sqrt{\Omega_{ll}^d \Omega_{mm}^d}}. \tag{5}
\]

In our wavelets calculation presented in subsequent sections, we used ASToolbox developed by Luís Aguiar-Conraria and Maria Joana Soares. Following Aguiar-Conraria et al. (2012), Aguiar-Conraria and Soares (2014), and others, we perform Monte Carlo simulations to assess the statistical significance of the wavelet coherency and partial wavelet coherency. We use the surrogate series generated by a fitted ARMA(1, 1) model with errors from a Gaussian distribution.

In our calculation, the problem of border distortions arises at the beginning and the end of the sample periods due to the finite length of the time series. Such edge regions of the time-frequency space are called the cone of influence (COI), and the results are unreliable.

4 Data

Data are annual observations covering the period 1885 to 2010. Saving and investment data up to 1992 come from Taylor (1992). We select nine countries including Argentina, Australia, Canada, Finland, Italy, Spain, Sweden, the UK, and the US to obtain long and consecutive data. We extend the data using World Economic Outlook (WEO) database. Population and GDP per capita are taken from Maddison. We use GDP per capita as a productivity measure because we cannot obtain the long data for total factor productivity or labor productivity. Primary fiscal balance data is taken from Mauro et al. (2015). When we investigate the effect of primary balance, Spain is not included because of missing data for the years 1936 to 1939.

We take trade cost measures for six countries from Jacks et al. (2011) that define bilateral trade costs as follows:

\[
TC_{ij,t} = \left(\frac{DT_{ii,t} DT_{jj,t}}{IT_{ij,t} IT_{ji,t}}\right)^{-\frac{1}{(\sigma-1)}} - 1,
\]

where \( TC_{ij,t} \) denotes geometric average trade costs between country \( i \) and \( j \), \( DT_{ii,t} \) denotes the domestic trade flows of country \( i \), \( IT_{ij,t} \) denotes international trade flows from country

\[\text{Argentina, Finland, and Italy are not included because of missing data.}\]
$i$ to country $j$, and $\sigma$ denotes elasticity of substitution between goods. Following Jacks et al. (2011), we set $\sigma = 8$. Novy (2013) finds that this trade cost measure is consistent with a broad range of trade models including Ricardian and heterogeneous firm models. In this paper, trade costs in country $j$ are defined as an average cost of country $j$ conducting international trades with five remaining countries.\(^{13}\)

Population, output, and primary fiscal balance data are transformed in log-difference forms so that the relationship is consistent with theory. Trade costs are transformed in a logged form.

5 Timings, frequencies, and sizes of the Feldstein-Horioka puzzle

In this section, we perform a coherency analysis between saving and investment for nine countries. Utilizing time- and frequency-domain analysis, we investigate when the Feldstein-Horioka puzzle is distinguishable throughout the sample period and assess the relative role of each frequency level that attributes to the puzzle. In particular, we explain our results with wavelet coherency in three dimensions: in terms of country size, a short-run versus long-run perspective, and a historical evolution view.

5.1 Country size and frequency

Figure 1 displays the size of the coherency region for the total sample period. The size of coherency is calculated as a share of regions at the five percent significance level, excluding regions affected by the COI. We start by discussing the result based on country size, ordering each country from the left based on its economy size. Following Levy (2000), we divide the significant coherency region into three frequencies: high frequency corresponding to one-to three-year cycles, business cycle frequency corresponding to three- to eight-year cycles, and low frequency corresponding to cycles longer than eight years. Blue, green, and brown colors indicate high, business cycle, and low frequencies, respectively.

Generally, our result is consistent with the existing literature of the Feldstein-Horioka context concerning country size. Large economies, such as Italy and the US, reveal much broader significant regions compared to smaller economies. However, the degree of coherency does not decrease monotonically as country size becomes smaller. For example, the UK shows relatively low coherency irrespective of its large country size. Additionally, we observe an inconsistent result for Finland: although it is the smallest country in our sample, it shows a higher correlation between saving and investment than Argentina and Sweden.

Next, we discuss the results based on the long-run and the short-run relationships between the time series of saving and investment. The most important finding is that large coherency is observed at low frequencies for most countries. In particular, this large coherency is significant in many periods for relatively large economies such as Italy, the UK, and the US. At business cycle frequencies corresponding to three- to eight-year cycles, we also observe large coherency in Canada, Finland, Italy, Spain, Sweden, and

\(^{13}\)We use average rather than aggregate measures because there are many missing data during the two world-war periods.
the US. Large coherency at high frequencies is observed only in some countries such as Italy, Spain, and the US.

Figure 1 is inserted here.

5.2 Century of correlation dynamics

Obstfeld and Taylor (2002) and Adalet and Eichengreen (2007) argue that the time path of the capital mobility measure would have described a U-shape over the course of the twentieth century. We investigate this, and explain our results by dividing the sample period into four subsample periods: 1885-1913, 1914-1945, 1946-1971, and 1971-2010. The first period is the pre-World War I period of the classical gold standard. The second period is the interwar period when capital flows were increasingly restricted following World War I and the Great Depression. The third period is the post-war period up to the breakdown of the Bretton Woods system. The fourth period is the post-Bretton Woods period up to the end of the sample.

Figure 2 exhibits wavelet coherency between saving and investment revealing the time-varying evolution at different frequency levels in nine countries. In each panel, the time (horizontal axis) and frequency (vertical axis) with large (small) coherency are associated with the regions with positive large power and described in red colors. We arrange coherency for each country according to the share of the world output. High wavelet coherency with red colors implies high retention parameters in the Feldstein-Horioka context. In Figure 3, we divide the results into four subsample periods graphically showing the relative importance of high, business-cycle, and low frequencies.

We observe a U-shaped pattern for large economies such as Italy, Spain, the UK, and the US. For the US, we observe a U-shaped time path of coherency, which is consistent with Obstfeld and Taylor (2002). We supplement the authors’ conjecture by offering empirical evidence. In the first subsample period, increasing capital mobility up to the end of World War I is mostly observed at four- to fourteen-year frequencies. Then, a period of low capital mobility is observed during the interwar period with a remarkably strong relationship between saving and investment at all frequencies. In the third subsample period, many insignificant coherency regions start to be observed at high frequencies. In the fourth period, the strong relationship between savings and investments disappears at all frequencies.

The UK, compared to the US, exhibits relatively wide regions with green and blue colors throughout the total sample period. In the Feldstein-Horioka context, this implies that the capital market in the UK economy is more open than the capital market in the US economy. For Italy, this U-shaped pattern is prominent at business cycle frequencies. Our result is also in line with Sarno and Taylor’s (1998) finding that the short-run correlation is significantly higher than the long-run correlation. Employing a Chow test for structural stability, Sarno and Taylor (1998) find that a structural break occurred around 1979 when the exchange rate control was abolished ending a long period of restrictions on capital flows between the UK and the international economy. We support their finding with a big island at two- to four-year cycles in the 1970s that disappears around 1979.

14Relying on the literature such as Kim (2001), large countries include Italy, the UK, and the US in our sample. Medium-sized countries are Argentina, Australia, Canada, and Spain. Small countries are Finland and Sweden.
For Spain, we observe the inverted U-shaped pattern with high coherency from the 1930s to early 1970s. Interestingly, this overlaps with the period of Francisco Franco’s military dictatorship.

The second group is composed of the countries with increasing coherency in the latter period. Argentina, Canada, Finland, and Sweden are in this group. This result is hard to reconcile considering that many countries have liberalized their financial markets since the 1970s and 1980s.\(^\text{15}\)

Figures 2 and 3 are inserted here.

Overall, our wavelet coherency analysis indicates how strong the relationship between saving and investment is for each frequency over time. However, high correlations between saving and investment do not necessarily imply imperfect international capital mobility because some omitted factors may exist that move these two variables in the same direction. According to life-cycle theory, for example, long-run drivers such as population and productivity growth increase both saving and investment. We presume that the large coherency of the second group observed in the latter period may capture this effect.

6 Partial coherency

Saving and investment can be correlated even if capital flows are perfectly mobile. In this section, therefore, we investigate in what way omitted factors neglected in the previous section contribute to a positive correlation between saving and investment. Comparing partial coherency with wavelet coherency in the previous section, we detect the propagation mechanism throughout the controlled variables. Our prior is that once we eliminate the relationship of the candidate variables with saving and investment, the degree of the saving-investment correlation lessens. Inspecting when and where the coherency disappears, we can detect how the candidate variables influence the saving-investment correlation.

There are three domestic factors that could move saving and investment in the same direction. High rates of economic growth due to increasing population or productivity might drive up saving, leading, in turn, to higher investment. In our estimation, we use GDP per capita as a measure of productivity because we do not have the long-term data for productivity measures such as total factor productivity and labor productivity.\(^\text{16}\)

Fiscal deficit is another important factor that may move national saving and domestic investment in the same direction. In cases where budget deficits crowd out private investment, fiscal deficit would generate a direct decrease in national savings and a decrease in investment. This implies that in response to positive fiscal balance shocks, with

\(^\text{15}\)Kaminsky and Schmukler (2008) offer a new chronology of financial liberalization for emerging and mature markets with indexes capturing the deregulation of the domestic financial sector, opening of the capital account, and opening of stock markets. Dates of complete account liberalization for our sample countries are as follows: 1978 to 1982 and from 1989 in Argentina, from 1975 in Canada, from 1987 in Finland, from 1992 in Italy, from 1975 in Spain, from 1989 in Sweden, from 1979 in the UK, and from 1973 in the US.

\(^\text{16}\)Feldstein and Horioka (1980) and Blanchard and Giavazzi (2002) use the dependency ratio rather than population growth rate, which is more relevant in the traditional life-cycle model. However, we use population growth data as in Summers (1985) and Obstfeld (1986) because the long-term data for the dependency ratio is not available.
an increase in tax revenue or a decrease in government expenditure, both saving and investment increase and vice versa.

There are also international components that induce the Feldstein-Horioka puzzle. Trade costs have long been considered one of the most important determinants of international investment and trade patterns. Using an open economic macro model, Obstfeld and Rogoff (2001) emphasize the role of trade costs to explain their six puzzles including the Feldstein-Horioka puzzle. Our final candidate is a global factor that induces positive correlations between national saving and domestic investment. Giannone and Lenza (2010) find that global shocks with heterogenous effects can create imbalance in world capital markets. Byrne, Fazio, and Fiess (2009) find that the global components in saving and investment commove. Costantini and Gutierrez (2013) find that the estimated saving-retention coefficient is close to zero for a panel of 21 OECD countries when global shocks are considered through common factors.

We further investigate whether omitted factors can explain the saving-investment correlation during this period in the subsequent section. Our five possible candidates are population growth, productivity growth, the primary balance, trade costs, and global common factors. Based on Bai and Ng (2002) criteria, we extract two common factors from 18 variables for saving and investment.

In particular, to investigate the effect of common factors, we proceed with two steps. First, we extract common factors based on the principal components approach following Bai and Ng (2002). Second, we perform partial coherency analysis controlling for these common factors. Through this procedure, we can reassess the defactorized relationship between saving and investment for each country.

### 6.1 Partial coherency for the total sample period

Figure 4 displays partial coherency between saving and investment after controlling for possible candidate variables. For comparison, we show the benchmark result on the left. First, in general, we find that the coherency region shrinks once we control for the candidate variables. Second, primary balances appear to be the main source of a lower coherency between saving and investment in many countries. For example, the share of the significant coherency regions of Australia is decreased from approximately 24 to eight percent. In Canada, the UK, and the US, if we control for the primary balance, the coherency decreases from approximately 30 to 15 percent, 28 to nine percent, and 71 to 53 percent, respectively. Third, the second major source seems to be the first global factor. Coherency between saving and investment is considerably lowered for many countries including Argentina, Australia, Canada, Finland, Italy, and the UK. Fourth, among six countries with available data, trade costs play an important role in four countries including Australia, Canada, Sweden, and the US. Fifth, population and output growth appear to be less important in explaining the Feldstein-Horioka puzzle. Productivity growth is the most important source only in Sweden. Sixth and surprisingly, Spain has greater coherency once we control for the candidate variables.

Figure 4 is inserted here.
6.2 Partial coherency for U-shaped countries

In this subsection, we investigate the source of the inverted U-shape of coherency. In Figure 3 of the previous section, the little-large-little pattern of coherency for Italy and the UK is mostly explained by the coherency at low frequencies while, for Spain and the US, large coherency at all frequencies is important in explaining the Feldstein-Horioka puzzle.

Figure 5 displays partial coherency for these U-shaped countries. In particular, we focus on the second and third periods when the positive link between saving and investment is largest. In the second subsample period (1914 to 1944), the primary balance and the first global factor are the major sources of lower the coherence in Italy and the UK. For Spain, productivity seems to be the only force to lower the coherency during this period. For the US, trade costs are the most important factor in explaining the Feldstein and Horioka puzzle. In the third subsample period (1945 to 1971), the primary balance and global factors are still dominant forces in explaining the UK’s coherency at low frequencies. The primary balance is important for the US. For Italy, productivity plays a major role in lowering coherency.

Figure 5 is inserted here.

6.3 Partial coherency for other countries

In the previous section, we find that frequencies asymmetrically explain increasing coherency for five countries. For Canada, the large coherency at high frequencies since the 1970s can explain the increasing pattern. For Sweden and Finland, the positive link between saving and investment has been prevalent since the 1970s at business cycle frequencies. On the other hand, for Argentina and Australia, this increasing pattern in the coherent relationship between saving and investment is explained by increasing coherency at all frequencies.

Figure 6 displays partial coherency for countries with increasing coherency in the third and fourth periods. In the third period, the primary balance seems extremely important to the coherency between saving and investment for all countries. For Australia, Canada, and Sweden, large coherency at low frequencies and business cycle frequencies is mostly lowered when we control for the primary balance. For Australia, population growth and the second global factor also significantly lower the coherency at low frequencies. In the fourth period, the primary balance again explains the considerable decrease in coherency in many countries including Australia, Canada, and Finland. Global factors also play a non-negligible role in many countries. Population growth seems to be the major source of the Feldstein-Horioka puzzle in Argentina while productivity growth is the determining factor in Sweden. Trade costs play a non-negligible role in many countries.

Figure 6 is inserted here.

7 Conclusion

In this paper we apply a continuous wavelet approach to investigate the Feldstein-Horioka puzzle in nine countries for the period from 1885 to 2010. We adopt the wavelet coherency
analysis to detect at which time and frequency the correlations between saving and investment decreased or increased. In particular, we use multivariate partial coherency to examine which candidates contribute to the correlations.

Our main results based on bivariate coherency analysis are as follows. First, we find that large countries such as the US and Italy exhibit high correlations compared to medium-sized and small countries. Second, we have two groups of countries with a time changing pattern in the Feldstein-Horioka puzzle. We find a U-shaped pattern of the time path of capital mobility in many countries such as Italy, Spain, the UK, and the US. In other words, we find high mobility in the pre-World I period and in recent decades. For the other countries such as Argentina, Australia, Canada, Finland, and Sweden, however, we find increasing coherency in the latter period.

We also perform a wavelet partial coherency analysis to examine the role of omitted factors. First, primary balances are the main source of lower coherency between saving and investment in many countries. Second, the second major source appears to be the global factor. Third, among six countries with available data, trade costs play an important role in the four countries Australia, Canada, Sweden, and the US. Fourth, population and output growth appear to be less important in explaining the Feldstein-Horioka puzzle, except for Argentina and Sweden in the post-Bretton Woods period.

References


Fig. 1. Size of coherency for each frequency.
Fig. 2. Coherency between saving and investment. The black contour designates the five percent significance level estimated from Monte Carlo simulations. The white line indicates the COI. The region with positive (negative) coherency is described in red (blue) colors.
Fig. 3. Coherency for each subsample period. I, II, III, and IV denote four subsample periods: 1885-1913, 1914-1945, 1946-1971, and 1971-2010, respectively.
Fig. 4. Partial coherency for the total sample period. POP, LP, PB, TC, CF1, and CF2 denote population, labor productivity, primary balance, trade costs, the first common factor, and the second common factor, respectively.
**Fig. 5.** Partial coherency for U-shaped countries. POP, LP, PB, TC, CF1, and CF2 denote population, labor productivity, primary balance, trade costs, the first common factor, and the second common factor, respectively.
Fig. 6. Partial coherency for non-U-shaped countries. POP, LP, PB, TC, CF1, and CF2 denote population, labor productivity, primary balance, trade costs, the first common factor, and the second common factor, respectively.