Public Education and Child-Care Policies with Pay-As-You-Go Pension

Miyake, Atsushi and Yasuoka, Masaya

Kobe Gakuin University, Kwansei Gakuin University

20 July 2016

Public Education and Child-Care Policies with Pay-As-You-Go Pension*

Atsushi Miyake† Masaya Yasuoka‡

July 20, 2016

Abstract

This paper presents consideration of the effects of child allowances and subsidies for private education investment on fertility and private education investment. The level of public education expenditure plays an important role in the effects of child care policies. To raise fertility, although child allowances are effective in an economy with low public education investment, subsidies for education investment are effective in an economy for which public education investment is high. The results presented in this paper are helpful for reconciling the conflicting results reported from previous studies. In addition, this paper presents an examination of the effects of those child care policies on pension benefits. A subsidy for private education can raise both fertility and pension benefits.

Key words: Fertility, Child allowance, Subsidy for education investment

JEL Classifications: J13, I2, D91, H52, H55

*Research for this paper was financially supported in part by JSPS KAKENHI Grant Numbers 23730283, 26380253 to the second author. The authors are thankful to Tomomi Miyazaki, Yasuyuki Osumi, and seminar participants of University of Hyogo, Doshisha University and the 70th Annual Conference of International Institute of Public Finance for helpful comments. Any remaining errors are the authors' responsibility.

†Faculty of Economics, Kobe Gakuin University. 518, Arise, Ikawadani, Nishi, Kobe, Hyogo, 651-2180, Japan, Tel.: +81-78-974-4285. E-mail: miyake@eb.kobegakuin.ac.jp

‡Corresponding author: School of Economics, Kwansei Gakuin University. 1-155 Uegahara Ichiban-cho, Nishinomiya, Hyogo 662-8501, Japan. Tel.: +81-798-54-6993. E-mail: yasuoka@kwansei.ac.jp

1 Introduction

This paper presents an examination of the effects of child allowances and education subsidies on fertility and education investment for children. In some economically developed countries, aging societies with fewer children are progressing. [1] If fertility and human capital increase, then the effective labor supply increases.

Fertility depends on the level of child care. As demonstrated by the experiences of U.K., France, and Sweden, a high level of child care can bring about a high level of fertility. Education policy should therefore be considered along with child care. If education costs are low, then parents can provide education investment funds for their children. Consequently, human capital is accumulated to a greater degree. Thereby, more wage income is obtainable. By virtue of their low education expenses, the parents can have more children. Then, in general, a policy to decrease education costs such as education subsidies and public education investment might have the same effect as a child allowance. The figure shows the public education level, child care policy level, and fertility in some OECD countries. Results show that high public education levels and high child care policy levels can bring about high fertility.

[Insert Fig. 1. around here.]

This paper presents a demonstration that the effectiveness of child care policies depends on the level of public education investment. In the economy with low public education investment, the child allowances can raise fertility. In contrast, subsidies for private education investment are effective for raising fertility in an economy with high public education investment. With respect to the effects of child allowances and education subsidies on fertility, as noted below, conflicting results have been obtained from related studies. The model of this paper can explain the conflicting results reported in the relevant literature.

Many studies in this field examine child-care policies using an endogenous fertility model. van Groezen, Leers and Meijdam (2003), Yasuoka and Goto (2011), and others report that child allowances can raise fertility. These studies emphasize the positive effect of child allowances on fertility.[2] However, Fanti and Gori (2009) show a negative effect of a child allowance.[3]

In the model of quality and quantity of children as an education investment for children, Becker

[1] The share of older people (over 65 years old) among all people is 24.1% in Japan, 20.4% in Italy, 20.4% in Germany, 18.2% in Sweden, and 16.8% in France (Data: United Nations "World Population Prospects The 2010 Revision Population Database," Ministry of Internal Affairs and Communications in Japan "Population Statics," Cabinet Office, Government of Japan "Decreasing Birthrate White Paper").

[2] Sleebos (2003) reports not only that a child allowance as direct financial aid in cash but also child care services available by avoiding quitting work to rear children are important to increase fertility. Apps and Rees (2004) and Ferrero and Iza (2004) derive a positive correlation between fertility and the labor participation rate.

[3] Fanti and Gori (2009) assess taxation for children, which is negative child allowances and derives that the child tax can raise the fertility. Substantially, this result demonstrates that child allowance policies reduce fertility.

(1960), Becker, Murphy and Tamura (1990), Tamura (1994) and de la Croix and Doepke (2003, 2004) examine the mechanism involved in the tradeoff between the quality and quantity of children. The tradeoff between the quality and quantity of children shows that if the relative education cost decreases, then the parents raise the level of education investment for children. Then fertility decreases, and vice versa.

Zhang (1997), Zhang and Casagrande (1998), and Yasuoka and Miyake (2014) use an endogenous fertility model with human capital accumulation to assess effects of child care policies and education subsidies. These studies consider the child care policy as a child allowance and education subsidy. These policies affect the costs of quality and quantity of children and bring about a substitution between the quality and quantity of children.

Zhang (1997) reports that an education subsidy raises education investment, but decreases fertility. Chen (2014) sets an endogenous fertility model with occupational choice. In the model of Chen (2014), education subsidy that decreases the cost to work as skilled labor increases not only the income level but also the fertility. Yasuoka and Miyake (2014) demonstrate that an education subsidy can raise both education investment and fertility because of rising pension benefits by virtue of the education subsidy. As described in this paper, however, we examine whether the effects of education subsidies on fertility depend on the level of public education investment.

Omori (2009) and Fanti and Gori (2011) show that it is not a policy to decrease child-care costs, such as child allowances, but rather public education investment that can raise fertility and human capital accumulation. However, the analyses described in this paper demonstrate that the effects of child care policy depend on the level of public education investment. This aspect was not examined in earlier studies.

Our paper includes the results obtained by the related researches. Zhang (1997) shows the positive effect of child allowance on the fertility, however, Fanti and Gori (2009) shows the negative effect. Zhang (1997) shows the negative effect of the subsidy for education on the fertility, however, Fanti and Gori (2011) shows the positive effect. Our paper includes these results. That is, our model setting can derive the positive and negative effect of child allowance and the subsidy for education on the fertility and the education investment.

In addition, our paper derives whether the policy can raise the social welfare or not. The result depends on the discount factor for each generation's utility.

The remainder of this paper is organized as follows. Section 2 sets the model. Section 3 derives the equilibrium and assesses the effect of child allowances and an educational subsidy. Section 4 presents an

examination of how policies affect the pension benefit and social welfare. The final section concludes this paper.

2 The Model

The model economy is based on a two-period (young and old) overlapping generations model with small open. Households experience two periods: young and old. As assumed by Zhang (1997), de la Croix and Doepke (2003) and others, households' utility function u_t is given as

$$u_t = \ln n_t h_{t+1} + \alpha \ln c_{t+1},\ 0 < \alpha. \tag{1}$$

Households care for both the number n_t and the quality h_{t+1} of their children as well as consumption c_{t+1}. In the equation, t represents the period.

During the young period, each household raises children and supplies labor to earn labor income. This analysis assumes that it is necessary for households (parents) to input child-care services to have children. Households have one unit of time and supply labor inelastically. The government provides a pay-as-you-go pension system. In addition, the government imposes taxation on young households to subsidize education or provide child allowances. Each household distributes its labor income into child-care services, education for children, and consumption during the old period. Consequently, we obtain the following budget constraint.

$$(z_t - q_t)n_t + (1 - x_t)e_t n_t + \frac{c_{t+1}}{1+r} = (1 - \gamma - \tau - \theta)wh_t + \frac{p_{t+1}}{1+r} \tag{2}$$

The model economy is assumed to be a small open economy. Given $f(k)$ as neoclassical product function, where $f(k)$ and k denote output per capita and capital-labor ratio and assuming $f' > 0$ and $f'' < 0$, the interest rate and wage rate are determined exogenously in competitive market, where r and w respectively denote the interest rate and wage rate per unit of human capital h_t. Moreover z_t denotes the price of child-care service; q_t is the child allowance for one child. Therefore, parents with n_t children receive child allowances that are equal to $q_t n_t$. In addition, e_t denotes private education investment. The rate of educational subsidy is given as x_t. In addition, γ is the tax rate to subsidize child-care support policies.[4] For these analyses, τ is the tax rate or contribution rate to provide pension benefits for older people. The pension benefit for older people is p_{t+1}. The tax rate to provide public education investment is θ.

Next, we consider human capital accumulation. The children's human capital h_{t+1} depends on private education investment e_t and public education investment v_t according to the following accumulation

[4]This tax revenue is proportional to a wage income. However, this tax finance does not distort household decisions. Fundamentally, this assumption is the same as that made by Zhang (1997).

equation,[5]

$$h_{t+1} = \beta e_t^{\epsilon} v_t^{1-\epsilon},\ 0 < \epsilon < 1,\ \beta > 0. \tag{3}$$

An individual chooses the level of consumption, number of children, and educational level to maximize lifetime utility (1) subject to the lifetime budget constraint (2) and the equation of human capital accumulation (3). The optimal allocations are determined as presented below

$$c_{t+1} = \frac{\alpha(1+r)}{1+\alpha}\left[(1-\tau-\theta-\gamma)wh_t + \frac{p_{t+1}}{1+r}\right], \tag{4}$$

$$n_t = \frac{1-\epsilon}{(z_t-q_t)(1+\alpha)}\left[(1-\tau-\theta-\gamma)wh_t + \frac{p_{t+1}}{1+r}\right], \tag{5}$$

$$e_t = \frac{\epsilon}{1-\epsilon}\frac{z_t-q_t}{1-x_t}. \tag{6}$$

Next, we consider the child-care services market. Aggregate child-care services Y_t are produced by $Y_t = \rho L_t^c$; also, $\rho > 0$ and L_t^c denote the effective labor input for child-care services.[6] Denoting w_t^c as the wage rate of child-care service, the profit function π_t is

$$\pi_t = z_t \rho L_t^c - w_t^c L_t^c. \tag{7}$$

Profit maximization reduces to $z_t = \frac{w_t^c}{\rho}$ inperfect competitive market. Assuming free labor mobility between the final goods sector and the child-care service sector, the wage rate w_t^c is given as $w_t^c = wh_t$. Consequently, we obtain $z_t = \hat{z} wh_t$, where $\hat{z} = \frac{1}{\rho}$.

We assume that each policy is financed by each taxation. The government imposes taxation at tax rate γ to provide child allowances or to subsidize educational investment, τ to provide pension benefits for older people, and θ to provide public education investment. Then, the government's budget constraint of child-care support policies is presented as

$$\gamma wh_t = \hat{q} wh_t n_t + x e_t n_t, \tag{8}$$

$$p_t = \tau w n_{t-1} h_t, \tag{9}$$

$$v_t = \frac{\theta wh_t}{n_t}. \tag{10}$$

This model economy assumes $q_t = \hat{q} w_t h_t$ and $x_t = x$. The child allowance $\hat{q} w_t h_t$ is income-proportional to provide a sufficient child allowance with income growth. Subsidy rates of a child allowance and education

[5]Some cases include the setting of human capital accumulation, as shown by Glomm and Ravikumar (1992) and others. Glomm and Kaganovich (2003) consider both public and private education investment, which are mutual substitutes. Omori (2009) sets education investments of the two types and assumes private education investment as the parental education time for their children. We can consider human capital accumulation (3). For instance, public education is regarded as an elementary school or basic education step. This education step is set as a mandatory education process. Private education is regarded as university or higher education.

[6]This function is assumed by Yasuoka and Miyake (2010) and Day (2012).

are assumed to be $0 < \hat{q} < \hat{z}$ and $0 < x < 1$. Given $\hat{q}$ and x exogenously, γ is determined endogenously to maintain a balanced budget ($0 < \gamma$).

Additionally, we consider the pension budget. Defining N_t as the population of the young generation in period t, $n_{t-1} \equiv \frac{N_t}{N_{t-1}}$ shows the intergenerational population ratio or population growth rate. Based on a balanced budget, pension benefits for older people are shown by (9). With a balanced budget, public education investment v_t is provided by (10).

3 Effects of Child Care Policies

This section presents analysis of the effects of two child-care policies on the human-capital growth rate and fertility: a child allowance ($x = 0$) and a subsidy for education investment for children ($\hat{q} = 0$).

3.1 Child Allowance

First, we examine how a child allowance affects fertility and human capital accumulation. If the government provides a child allowance, then the government budget constraint (8) becomes

$$\gamma w h_t = \hat{q} w h_t n_t. \tag{11}$$

Moreover, the human-capital growth rate $1 + g = \frac{h_{t+1}}{h_t}$ and fertility n are shown as presented below[7]

$$1 + g = \frac{\beta w}{1 - \epsilon} \left[\epsilon(\hat{z} - \hat{q})\right]^{\epsilon} \left[\theta(1 + \alpha) \frac{\hat{z} - \hat{q} - \frac{\tau(1-\epsilon)(1+g)}{(1+\alpha)(1+r)}}{1 - \tau - \theta - \gamma}\right]^{1-\epsilon}, \tag{12}$$

$$n = \frac{(1 - \epsilon)(1 - \gamma - \tau - \theta)}{(1 + \alpha)\left[\hat{z} - \hat{q} - \frac{1-\epsilon}{1+\alpha} \frac{\tau(1+g)}{1+r}\right]}. \tag{13}$$

Total differentiation of (11) and (12) with respect to $\hat{q},\ \gamma,\ g$ at the approximation of $\hat{q} = 0$ provides the effects of a child allowance on the growth rate as

$$\frac{dg}{d\hat{q}} = -\frac{(1 + g)\left[\frac{\epsilon}{\hat{z}} + \left(1 + \frac{1-\epsilon}{1+\alpha}\right) \frac{1-\epsilon}{\hat{z} - \frac{\tau(1-\epsilon)(1+g)}{(1+\alpha)(1+r)}}\right]}{1 + \frac{\tau(1-\epsilon)^2(1+g)}{(1+\alpha)(1+r)\left[\hat{z} - \frac{\tau(1-\epsilon)(1+g)}{(1+\alpha)(1+r)}\right]}} < 0. \tag{14}$$

A child allowance decreases the human capital growth rate. In addition, this paper presents an examination of whether child allowances can raise fertility or not. From total differentiation of (11)–(13) with respect to $\hat{q},\ \gamma,\ g$, and n at the approximation of $\hat{q} = 0$, the effects of a child allowance on fertility are given as

$$\frac{dn}{d\hat{q}} = \frac{n\left[1 - \frac{1-\epsilon}{1+\alpha}\left(1 + \frac{\tau\epsilon}{\hat{z}} \frac{1+g}{1+r}\right)\right]}{\hat{z} - \frac{\epsilon(1-\epsilon)}{1+\alpha} \frac{\tau(1+g)}{1+r}}. \tag{15}$$

[7] The right hand side of (12) is zero at $\theta = 0$ and infinity at $\theta = 1 - \tau - \gamma$. Therefore, we have $0 < \theta < 1$ to hold (12) as the intersect of the left hand side and the right hand side of (12).

Without a pension system ($\tau = 0$), child allowances can always raise fertility. However, in a pay-as-you-go pension system, the child allowance cannot always raise fertility. Now, $1+g$ depends on the level of θ and $1+g$ increases with θ as shown by (12). Defining θ^* as θ to hold $1 = \frac{1-\epsilon}{1+\alpha}\left(1 + \frac{\tau\epsilon}{\hat{z}}\frac{1+g}{1+r}\right)$, $\theta < \theta^*$ brings about $\frac{dn}{d\hat{q}} > 0$.[8] Then, the following proposition is established.

Proposition 1 Child allowances always decrease the human capital growth rate. In contrast, child allowances increase fertility if public education investment is low.

As shown by (15), without a pay-as-you-go pension, child allowances invariably increase fertility. In an economy with a pay-as-you-go pension system, however, fertility cannot always increase. A decrease in the income growth rate reduces the pension benefit. A decline in the pension benefit reduces household income and decreases fertility. If public education investment is maintained at a high level, then the income growth rate decreases greatly. Consequently, the negative effects of the decrease in a household's income on fertility are large. As shown by (14), child allowance reduces the income growth rate. Child allowance increases the relative cost of private education. If public education level is high, that is v_t is large, marginal decrease in human capital accumulation brought about by a decrease in private education is large. Therefore, large level of public education reduces the fertility.

Zhang (1997) derives the only result $\frac{dn}{d\hat{q}} > 0$. On the other hand, Fanti and Gori (2009) derives $\frac{dn}{d\hat{q}} > 0$. However, Yasuoka and Miyake (2014) derives the both positive and negative sign of $\frac{dn}{d\hat{q}}$, depending on the pension benefit level. This paper derives the both positive and negative sign of $\frac{dn}{d\hat{q}}$. However, if τ increases, θ^* decreases. That is, the range of θ to hold $\frac{dn}{d\hat{q}} > 0$ becomes narrow. The result show that both the contribution rate τ and public education level are high, it is hardly to hold $\frac{dn}{d\hat{q}}$.

3.2 Subsidy for Education

Next we discuss how a subsidy for education affects fertility and human capital accumulation. If the government subsidises education investment, then the government budget constraint (8) becomes $\gamma w h_t = x e_t n$, i.e.,

$$\gamma = \frac{\epsilon}{1-\epsilon}\frac{x\hat{z}}{1-x}n. \tag{16}$$

Then, the human-capital growth rate and fertility are expressed as

$$1+g = \frac{\beta w}{1-\epsilon}\left(\frac{\epsilon\hat{z}}{1-x}\right)^{\epsilon}\left[\theta(1+\alpha)\frac{\hat{z} - \frac{1-\epsilon}{1+\alpha}\frac{\tau(1+g)}{1+r}}{1-\tau-\theta-\gamma}\right]^{1-\epsilon}, \tag{17}$$

[8] Because $1+g$ is monotonic increasing function about θ, there exists θ^* to hold $1 = \frac{1-\epsilon}{1+\alpha}\left(1 + \frac{\tau\epsilon}{\hat{z}}\frac{1+g}{1+r}\right) = 0$.

$$n = \frac{1-\epsilon}{1+\alpha}\frac{1-\tau-\theta-\gamma}{\hat{z}-\frac{1-\epsilon}{1+\alpha}\frac{\tau(1+g)}{1+r}}. \tag{18}$$

We examine the effects of a subsidy for education on the human-capital growth rate and fertility. Substituting (16) into (17) and (18), and total differentiation of (16) and (17) with respect to x and g at the approximation of $x = 0$, it is apparent that this subsidy raises the human capital growth rate because of

$$\frac{dg}{dx} = \frac{\epsilon(1+g)\left(1+\frac{\hat{z}n}{1-\tau-\theta}\right)}{1-\frac{\frac{\tau(1-\epsilon)(1+g)}{1+r}}{\hat{z}+\frac{\tau(1+g)}{1+r}}} > 0. \tag{19}$$

Total differentiation of (16)–(18) with respect to x, n, and g at the approximation of $x = 0$ gives the effects of a subsidy for education as

$$\frac{dn}{dx} = \frac{\frac{\epsilon n}{1+\alpha}\left[\frac{\tau(1-\epsilon)(1+g)}{1+r}-\hat{z}\right]}{\hat{z}-\frac{\epsilon(1-\epsilon)}{1+\alpha}\frac{\tau(1+g)}{1+r}}. \tag{20}$$

Without a pension system, the subsidy for education investment reduces fertility. However, the pension system brings about $\frac{dn}{dx} > 0$. Defining θ^{**} as θ to hold $\frac{\tau(1-\epsilon)(1+g)}{1+r} = \hat{z}$ and given $\theta^{**} < \theta$, it is obtained that $\frac{dn}{dx} > 0$. Therefore, the following proposition is established.

Proposition 2 A subsidy for education investment can always raise the human-capital growth rate. In addition, this subsidy can raise fertility if public education investment is high.

A subsidy for education investment entails a tax burden, which reduces the household disposable income. Then, fertility decreases. However, if public education investment is high, then an increase in education investment raises the pension benefit to a great degree. Therefore, the household income increases. Fertility also increases.

Using a brief calculation, one obtains $\theta^{**} < \theta^{*}$. If $\theta < \theta^{**}$, then only child allowances can raise fertility. In the case where $\theta^{**} < \theta < \theta^{*}$, both child allowances and the subsidy for education investment can raise fertility. Moreover if $\theta^{*} < \theta$, then only the subsidy for education investment can raise fertility. In fact, the effects of child allowances and the subsidy for private education investment in fertility are changed by the extent of public education investment.

Zhang (1997) derives the result of $\frac{dn}{dx} < 0$. However, Yasuoka and Miyake (2014) and this paper derives both positive and negative sign of $\frac{dn}{dx}$. Being different from Yasuoka and Miyake (2014), we consider public education investment and therefore we derives the something new results. If τ increases, θ^{**} decreases. That is, the effect of the policy on the fertility depends on not only θ but also τ. If the pension benefit level is small, the public investment level must be high to hold $\theta^{**} < \theta$.

Now, to discuss about the policy implications, we consider the public education as compulsory education (elementary school, high school and others) and the private education as higher education (university and others.)

[Insert Table 1 around here.]

Sweden and France are considered as the countries that can increase the fertility thanks to the policy. In these countries, pension benefit and public education are high level. In addition, the education cost for university is low, as shown by Table 1. This shows high τ, v_t and x. Then, in real the fertility is pulled up by the education subsidy policy in Sweden and France.

On the other hand, Japan is considered as the country that can not increase the fertility because of the child care policy. However, the pension benefit and public education investment are low in Japan. In addition, the education subsidy for university is low compared with the other countries. This shows low τ, v_t amd x. Therefore, θ^{**} in Japan is larger than the one in Sweden and France. Then, the fertility does not increases.

4 Social Welfare and Policies

This subsection presents consideration of the manner in which the policies affect social welfare assumed by the following equation:[9]

$$\begin{aligned} W &= \sum_{s=0}^{\infty} \rho^s u_{t+s}, \\ &= \frac{1}{1-\rho}\ln n + \frac{\alpha}{1-\rho}\ln c_{t+1} + \frac{1+\alpha\rho}{(1-\rho)^2}\ln(1+g) + \frac{1}{1-\rho}\ln h_t. \end{aligned} \tag{21}$$

Here, ρ denotes the discount factor of each generation's utility ($0 < \rho < 1$).

4.1 Child Allowance

First, we examines the effect of child allowance on the welfare. We obtain the condition of $\frac{dW}{d\hat{q}} > 0$ as

$$\frac{dW}{d\hat{q}} = \left[1 - \frac{(1+\alpha\rho)(1-\epsilon)}{(1-\rho)(1+\alpha)}\right]\frac{dn}{d\hat{q}} - \frac{\alpha n}{(1+\alpha)\hat{z}} + \frac{\epsilon n(1+\alpha\rho)}{(1-\rho)(1+\alpha)\hat{z}} > 0. \tag{22}$$

With $\rho < \frac{\alpha+\epsilon}{1+2\alpha-\alpha\epsilon}$, the condition to raise social welfare by child allowances is given by,

$$\frac{dn}{d\hat{q}} > -\frac{\alpha+\epsilon-(1-\epsilon)\alpha\rho}{(1+\alpha)(1-\epsilon)}\frac{n}{\hat{z}}. \tag{23}$$

[9] Social welfare function is derived to sum each generation's utility. However, (23) does not include the utility of the older generation at t period. If one considers the utility of the older generation at t, then the social welfare function changes to $W = \sum_{s=0}^{\infty}\rho^s u_{t+s} + \alpha\ln c_t$. Noting that no child care policy exists at the $t-1$ period, we obtain $c_t = \frac{\alpha\hat{z}(1+r)wh_{t-1}}{1-\epsilon}n_{t-1}$ because of (4) and (5). n_{t-1} and h_{t-1} are unaffected by child care policy. Therefore, we can omit this term in considering the effect of child care policy on social welfare.

If child allowance can raise the fertility, the social welfare can be pulled up, too, because the fight hand side of (24) is negative. On the other hand, If $\rho > \frac{\alpha+\epsilon}{1+2\alpha-\alpha\epsilon}$, that is, the discount for the future generation's utility is small, the condition to raise social welfare is given by,

$$\frac{dn}{d\hat{q}} < -\frac{\alpha+\epsilon-(1-\epsilon)\alpha\rho}{(1+\alpha)(1-\epsilon)}\frac{n}{\hat{z}}. \tag{24}$$

Then, as long as $\frac{dn}{d\hat{q}} > 0$, the social welfare can not be pulled up by the child allowance.

4.2 Subsidy for Education

Second, we examine the effect of the subsidy for private education on the social welfare. We obtain the condition for $\frac{dW}{dx} > 0$ as

$$\frac{dW}{dx} = \frac{\alpha+\epsilon-(1+2\alpha-\alpha\epsilon)\rho}{n}\frac{dn}{dx} + \epsilon(1+\alpha\rho) > 0. \tag{25}$$

With $\rho < \frac{\alpha+\epsilon}{1+2\alpha-\alpha\epsilon}$, social welfare increases if

$$\frac{dn}{dx} > -\frac{\epsilon n(1+\alpha\rho)}{\alpha+\epsilon-(1+2\alpha-\alpha\epsilon)\rho}, \tag{26}$$

The right hand side of (26) is negative. Then, if $\frac{dn}{dx} > 0$, the social welfare can be pulled up. If $\rho > \frac{\alpha+\epsilon}{1+2\alpha-\alpha\epsilon}$, the condition to raise the social welfare is given by

$$\frac{dn}{dx} < -\frac{\epsilon(1+\alpha\rho)n}{\alpha+\epsilon-(1+2\alpha-\alpha\epsilon)\rho}. \tag{27}$$

The right hand side of (27) is positive. Even if the subsidy for education investment can be pulled up the fertility, the social welfare can not be always pulled up. If $\frac{dn}{dx}$ is positive large value, this inequality (27) is not held.

Then, the following proposition is established.

Proposition 3 With $\rho < \frac{\alpha+\epsilon}{1+2\alpha-\alpha\epsilon}$, that is, the discount rate for future generation's utility is large, the social welfare can be increased by both child allowance and subsidy for education when child allowance and subsidy for education can increase the fertility.

If the discount rate for future generation's utility is large, the inter-generational redistribution through the pay-as-you-go pension can increases the social welfare because the inter-generational redistribution can raise the present generation's utility instead of a decrease in the future generation's utility. Because of discount rate is large, the positive effect on the present generation's utility dominates the negative effect on the future generation's utility. An increase in fertility brought about by the policy raises the pay-as-you-go pension benefit. An increase in the fertility brought about by the policy financed by the taxation is substantial same with inter-generational distribution.

5 Concluding Remarks

This paper presents consideration of two child-care policies: one is for child allowances, which are provided for the quantity of children; the other is for subsidies for private education investment, which are provided for the quality of children. This discussion presents an examination of how child-care policies affect fertility and education investment. Although child allowances decrease private education investment and the income growth rate, they can increase fertility if the public education investment is small. In contrast, the subsidies for private education investment can always increase education investment and the income growth rate. This subsidy can increase fertility when public education investment is large. These results demonstrate that the policy to increase fertility should be chosen based on the extent of the public education investment. Using this model setting, it was possible to explain the conflicting results obtained by related studies.

Moreover, this paper presents an examination of whether child allowances and the subsidy for education investment can increase the pension benefit and social welfare or not. Child allowances can increase the pension benefit as long as fertility is low. However, the subsidy for education investment can increase the pension benefit by virtue of an increase in income growth even if the subsidy decreases fertility.

In addition, this manuscript presents an examination of whether these policies increase social welfare or not. If the future generation's utility is not greatly discounted and fertility is pulled up by child allowances, then child allowances cannot increase social welfare. However, the subsidy for education investment can increase social welfare even if the future generation's utility is not greatly discounted.

References

[1] Apps Patricia, and Ray Rees (2004). "Fertility, Taxation and Family Policy," *Scandinavian Journal of Economics* 106(4): 745-763.

[2] Becker, Gary (1960). "An Economic Analysis of Fertility," NBER Chapters, in: *Demographic and Economic Change in Developed Countries*: 209-240.

[3] Becker, Gary, Kevin Murphy, and Robert Tamura (1990). "Human Capital, Fertility, and Economic Growth," *Journal of Political Economy* 98(5)-2: S12-S37.

[4] Chen H.-J. (2014). "Child Allowances, Educational Subsidies and Occupational Choice," *Journal of Macroeconomics* 44: 327-342

[5] Day, Creina (2012). "Economic Growth, Gender Wage Gap and Fertility Rebound," *Economic Record* 88(s1): 88-99.

[6] de la Croix, David and Matthias Doepke (2003). "Inequality and Growth: Why Differential Fertility Matters," *American Economic Review* 93(4): 1091-1113.

[7] de la Croix, David and Matthias Doepke (2004). "Public versus Private Education when Differential Fertility Matters," *Journal of Development Economics* 73(2): 607-629.

[8] Fanti Luciano and Luca Gori (2009). "Population and Neoclassical Economic Growth: a New Child Policy Perspective," *Economics Letters* 104(1): 27-30.

[9] Fanti Luciano and Luca Gori (2011). "Child Policy Ineffectiveness in an Overlapping Generations Small Open Economy with Human Capital Accumulation and Public Education," *Economic Modelling* 28(1): 404-409.

[10] Ferrero, Dolores and Amaia Iza (2004). "Skill Premium Effects on Fertility and Female Labor Force Supply," *Journal of Population Economics* 17(1): 1-16.

[11] Glomm Gerhard and Michael Kaganovich (2003). "Distributional Effects of Public Education in an Economy with Public Pensions," *International Economic Review* 44(3): 917-937.

[12] Glomm, Gerhard and B. Ravikumar (1992). "Public versus Private Investment in Human Capital: Endogenous Growth and Income Inequality," *Journal of Political Economy* 100(4): 818-834.

[13] Omori, Tatsuya (2009). "Effects of Public Education and Social Security on Fertility," *Journal of Population Economics*, 22(3): 585-601.

[14] Sleebos, Joelle. (2003). "Low Fertility Rates in OECD Countries Facts and Policy Responses," *OECD Social, Employment and Migration Working Papers* No. 15.

[15] Tamura, Robert (1994). "Fertility, Human Capital and the Wealth of Families," *Economic Theory*, 4(4): 593-603.

[16] van Groezen, Bas, Theo Leers, and Lex Meijdam (2003). "Social Security and Endogenous Fertility: Pensions and Child Allowances as Siamese Twins," *Journal of Public Economics*, 87(2): 233-251.

[17] Yasuoka, Masaya and Naohisa Goto (2011). "Pension and Child Care Policies with Endogenous Fertility," *Economic Modelling*, 28(5): 2478-2482.

[18] Yasuoka, Masaya and Atsushi Miyake (2010). "Change in the Transition of the Fertility Rate," *Economics Letters*, 106(2): 78-80.

[19] Yasuoka, Masaya and Atsushi Miyake (2014). "Fertility Rate and Child Care Policies in a Pension System," *Economic Analysis and Policy*, 44(1): 122-127.

[20] Zhang, Jie (1997). "Fertility, Growth and Public Investments in Children," *Canadian Journal of Economics*, 30(4a): 835-843.

[21] Zhang, Jie, and Richard Casagrande (1998). "Fertility, Growth, and Flat-Rate Taxation for Education Subsidies," *Economics Letters*, 60(2): 209-216.

Appendix

Optimal Allocations

We set the following Lagrange equation as follows,

$$L = \ln n_t + \epsilon \ln e_t + \ln \beta + (1-\epsilon)\ln v_t + \alpha \ln c_{t+1} + \lambda \left((1-\gamma-\tau-\theta) w h_t + \frac{P_{t+1}}{1+r} - (z_t - q_t) n_t - (1-x) e_t n_t - \frac{c_{t+1}}{1+r} \right), \tag{28}$$

where λ denotes Lagrange multiplier. We obtain the following equations,

$$\frac{\partial L}{\partial n_t} = 0, \quad \frac{1}{\lambda} = (z_t - q_t + (1-x)e_t) n_t, \tag{29}$$

$$\frac{\partial L}{\partial e_t} = 0, \quad \frac{\epsilon}{e_t} = (1-x)\lambda n_t, \tag{30}$$

$$\frac{\partial L}{\partial c_{t+1}} = 0, \quad \frac{\alpha}{c_{t+1}} = \frac{\lambda}{1+r}, \tag{31}$$

$$\frac{\partial L}{\partial \lambda} = 0, \quad (1-\gamma-\tau-\theta) w h_t + \frac{P_{t+1}}{1+r} - (z_t - q_t) n_t - (1-x) e_t n_t - \frac{c_{t+1}}{1+r} = 0. \tag{32}$$

Then, we obtain $\frac{1}{\lambda} = \frac{1}{1+\alpha}\left((1-\gamma-\tau-\theta) w h_t + \frac{P_{t+1}}{1+r} \right)$ and the optimal allocations (4)-(6).

Social Welfare Function

The social welfare function (21) are derived as follows,

$$\begin{aligned} W &= \ln n + \ln h_{t+1} + \alpha \ln c_{t+1} + \rho \left(\ln n + \ln h_{t+2} + \alpha \ln c_{t+2} \right) + \cdots \\ &= (1 + \rho + \rho^2 + \cdots) \ln n + (1 + \rho + \rho^2 + \cdots) \ln h_t + \alpha (1 + \rho + \rho^2 + \cdots) \ln c_{t+1} \\ &+ (1 + 2\rho + 3\rho^2 + \cdots) \ln(1+g) + \alpha(\rho + 2\rho^2 + 3\rho^3 + \cdots) \ln(1+g) \\ &= \frac{1}{1-\rho} \ln n + \frac{\alpha}{1-\rho} \ln c_{t+1} + \frac{1+\alpha\rho}{(1-\rho)^2} \ln(1+g) + \frac{1}{1-\rho} \ln h_t. \end{aligned}$$

Considering (4) and (5), we obtain $c_{t+1} = \frac{\alpha(1+r)(\hat{z}-\hat{q}) w h_t n}{1-\epsilon}$, and then we can examine the effect of policies on the social welfare.

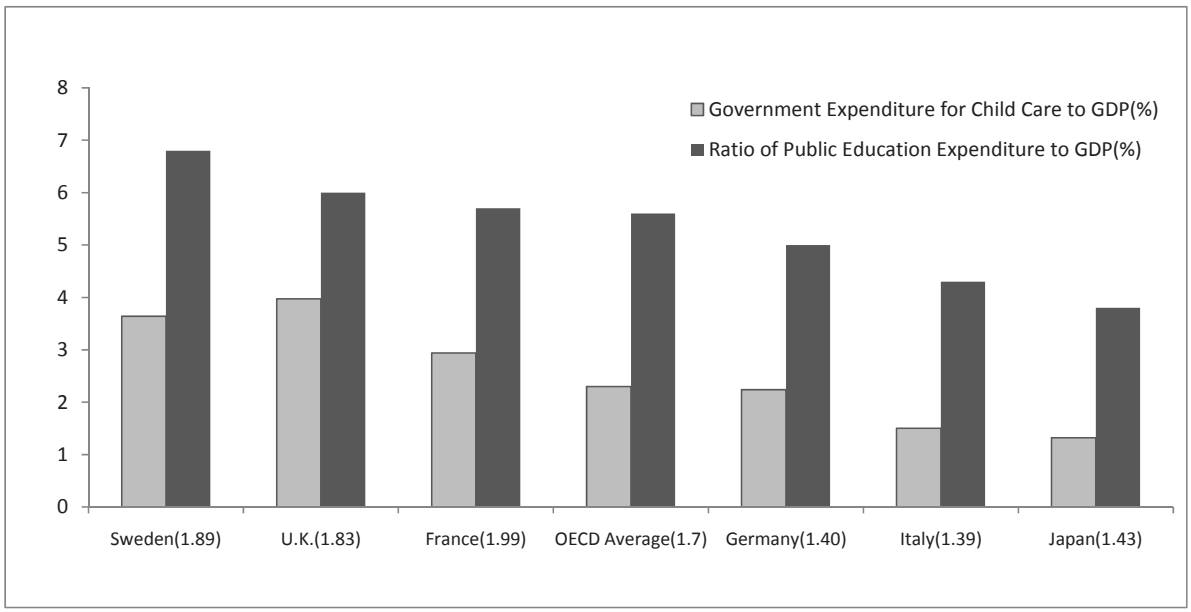

Fig. 1: Total Fertility Rate, Fiscal Support for a Family and Public Education Investment.

(Data: Cabinet Office, Government of Japan (2015) White Paper on Birthrate-Declining Society (*in Japanese*), OECD Social Expenditure Database (September 2014), OECD Education at a Glance 2014. Data years are the following: Data years of the total fertility rate are for 2013. Data of fiscal support for families are for 2011. Fiscal Support for Families includes in-kind benefits (day care, home help and other in-kind benefits) and cash benefits (family allowance, maternity and parental leave, and other cash benefits). Data of public education are those for 2011. The bracketed value represents fertility in each country in 2013.)

	(i)	(ii)	(iii)
Sweden	3.9	1.9	56%
France	3.6	1.2	68%
Japan	2.7	0.8	40%

(i) All Primary, secondary and post-secondary non-tertiary

(ii) All tertiary

(iii) Replacement of pension benefit (the ratio to pre-retirement earnings at 2014)

Table 1 Total public expenditure on education (2012)

(Data: OECD (2015) Education at a Glance, OECD Data Net Pensin Relpacement Rate.)