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Abstract

Activity based travel demand models are based on the idea that travel is derived from the demand to participate in different activities. Predicting travel demand should therefore include the prediction of demand for activity participation. Time-space constraints, such as working hours, restricts when and where different activities can be conducted, and plays an important role in determining how people choose to travel. Travelling is seen as a possibly costly link between different activities, that also implicitly leads to missed opportunities for activity participation.

With a microeconomic foundation, activity based models can further be used for appraisal and for accessibility measures. However, most models up to date lack some dynamic consistency that, e.g., might make it hard to capture the trade-off between activity decisions at different times of the day. In this paper, we show how dynamic discrete choice theory can be used to formulate a travel demand model which includes choice of departure time for all trips, as well as number of trips, location, purpose and mode of transport. We estimate the model on travel diaries and show that the it is able to reproduce the distribution of, e.g., number of trips per day, departure times and travel time distributions.

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1 Introduction

Travel demand models have during the last decades evolved from highly aggregated trip based models, through tour based models into activity based models that considers the choice of all activities and all transportation for a full day (or longer) on an individual (or household) level. The motivation behind this gradual increase in complexity is the realization that the demand for travel is derived from the demand for activity participation. Accurate predictions of how individuals will react to infrastructure investments and policy changes should therefore take into account to what extent individuals are flexible in their activity participation. There is only a limited amount of time each day, and some activities are more or less fixed in both location and time, such as working and picking up or dropping of children at school/daycare. These constraints severely restrict individuals' ability to adapt, and not considering them is therefore likely to result in unrealistic forecasts. This is especially the case when considering policy changes, such as congestion charge, that are becoming increasingly important in today's traffic planning.

The problem with considering the choice of full day activity schedules subject to time-space constraints is that the number of possible ways to plan a day is immense. One could definitely argue that people do not actually consider all these alternatives, but there are definitely complex aspects of the activity schedule that do influence how people plan their days. For example, when considering what time to leave for work, they are likely to take into consideration when they will get home and so the preferred departure time to work should be derived from a trade-off between time spent at home in the morning versus time spent at home (or on some activity) in the evening.

How to spend the limited time budget when the preferences for time is dependent on the time of day should be the key determinant for when, where and if people choose to conduct different activities in activity based models. Although many activity based models up to date result in full day activity schedules, most fall short in their treatment of time. For a comprehensive overview of activity based models, see e.g., Pinjari and Bhat (2011) or Rasouli and Timmermans (2014). As an extension to the tour-based approach, Bowman and Ben-Akiva (2001) developed a nested-logit structure that treats tours and activities sequentially based on their importance for the individual. The model consists of five nests: 1) the choice of activity pattern, including the number of tours carried out during the day; 2) the choice of time of day for the primary tour and all its trips; 3) the mode and destination for the primary tour; 4) the time of day for the secondary tours; and 5) the mode and destination of the secondary tours. The nested-logit structure ensures that higher decisions, such as the choice of activity pattern, includes the individual specific information about all available tour-combinations that the pattern includes. However, since secondary tours are not conditioned on each other, it is possible to end up with daily patterns that take more than a full day to complete. The model has been combined with a duration and departure time model, which is not integrated into the nested-logit structure (Bradley et al., 2010; Vovsha and Bradley, 2004) and that does not integrate upwards. Time-constraints does therefore not consistently influence how people choose to spend their time, nor can variation in preferences towards activity times with the time of day.

Some other models are worth mentioning. Habib (2011) presents a discrete-continuous random utility model for weekend traveling. Agents choose mode, destination and activity based on the utility of the combination. Future time is contained in a time-of-day dependent composite good, which is parameterized and estimated. In the Albatross model system, choices are also made sequentially in time (Arentze et al., 2000), and at every time step a heuristic decision rule determines determine the next action so that time-space constraints are fulfilled. A problem with these approaches is that they do not treat the value of future time consistently. A dynamically consistent model should either directly model the choice of full day-schedules or the value of future time that individuals consider should be the same as the expectation of the utility they can obtain during the remaining day according to the model.

Some models of travel demand do include the full time spent on different activities over a full day. A common feature of these approaches is that they are utility based and that individuals (or households) are searching for a day-path that maximize the sum of the utility gathered during the day. One idea is to simulate day-paths and through a search algorithm approach paths that maximize the sum of the utility gathered over a day Balmer et al. (2005). The utility of a day-path is once again the sum of the utility for respectively activity or travel episode, and individuals are choosing the alternative with the highest utility. The original model did not include any random or unknown term in the utility function but some kind of randomness was introduced through the search process. This was not enough when including location choice and (Horni et al., 2011) found that formulating the choice of day-paths as an MNL gave a realistic distribution of travel times.

The shortest path problem can equivalently be expressed as a mixed-integer programming problem. Time-space constraints that define when certain links in the network are available can be included as constraints in the programming problem, which makes this approach tractable. Recker (2001) show how the choice of an activity schedule including mode, activity duration and participation can be solved in this way. The model has been estimated using a genetic algorithm (Recker et al., 2008) and extended to include destination choice (Kang and Recker, 2013).

The frameworks mentioned above are promising in their attempts to model and simulate the choice of day-paths, but it is so far unclear how they can be used together with models of long-term decisions such as car ownership, household- and work-location. Further, although they are based on utility maximization, it is unclear how they should be used for other purposes than prediction. In a discrete choice framework, it is logical to use the expected (maximum) utility from a day-path as input to other models and for cost appraisal (Geurs et al., 2010). The expected utility could also be used to get detailed disaggregated measures of accessibility (Dong et al., 2006; Jonsson et al., 2013). With a dynamically consistent model, such measures of accessibility could be used to see how, e.g., the fact that some activities, such as picking up children and going shopping, are mandatory, influences the accessibility of different work locations.

The number of different daily activity patterns is immense, but the number of possible actions available to an individual at a specific time of the day is relatively easy to define. Dynamic discrete choice theory could therefore present a way of simplifying the activity scheduling problem without making any restrictions on the choice set (Karlström, 2005), and give detailed time-dependent individual accessibility measures (Jonsson et al., 2013). The choice of a day-path is modeled as a sequence of simultaneous choices of activity type, duration, mode of transport and location conditional on the expected future utility given respectively choice. The sequencing of actions and the expected future utility components ensures dynamic consistency and makes it easy to include time-space constraints. However, up until now, the curse of dimensionality have restricted the ability to estimate the model. Here, we propose an estimation method based on sampling of alternatives. Section 2 will present and discuss the modelling framework and specification; section 3 will discuss the estimation method proposed; section 4 the data; section 5 the utility specification; and section 6 estimation result and simulation validations.

2 Model

When individuals decide what time to leave for work, they take into consideration how it will influence their afternoon. If they leave ten minutes later it means that they get ten minutes less for activities in the afternoon. The difference in how they value afternoon-time versus morning-time will therefore be one factor determining when they leave for work. Even people with flexible working hours seem to prefer to go to work during rush hours, when roads and public transport are heavily congested. Probably they have other time-constraints – they might have to pick up their children from school, have a gym-class or are meeting friends – or time preferences of some other sort – they might value having dinner with their family at 6 p.m. Whatever the reasons, it is clear that the timing of trips in the morning is influenced by their plans for the afternoon, and that the benefits of traveling at less congested times would not outweigh the costs of changing these plans. The decision on when, where and how to travel should therefore be explained by trade-offs on how a limited amount of time should be spent and a correct representation of time is crucial in activity-based travel demand models. It therefore seems reasonable to assume that individuals consider full daily activity schedules when determining what activities to engage in as well as timing, location and mode of transport.

A daily schedule of trips and activities could be represented by a path between states, where a state s_t defines, among other things, the location and time of day t. An action a_t, defining activity, duration and mode of transport, gives a new state s_{t+1}, and a sequence of such actions starting in the morning and ending in the evening is what we will call a day-path. A rational agent in an uncertain environment that starts in a state s would behave according to a decision rule $a_t = d(s_t)$ that maximizes the expected future utility:

$$V(s_0) = \max_{a_0} E\left\{ \sum_{t=0}^{T} \beta^t u(s_t, a_t) \middle| s_0 = s \right\} \tag{1}$$

for some utility function $u(s_t, a_t)$ and discount factor β, assuming that the utility is additively separable. The expectation here is with respect to the stochastic process (s_t, a_t).

Finding the utility maximizing decision rule is a daunting task. Consider, for example, an individual with 10 h of free time during a day. If there are 8 different activities that can be conducted at 100 alternative locations and 4 available modes of transport, and each activity-travel episode is at least 1 h long, there are $(8 \cdot 100 \cdot 4)^{10} \approx 10^{35}$ alternative sequences of actions. The problem is thus immense, and we do not propose that people actually consider all these options. For one thing, a specific individual probably only considers a small set of locations for each activity. However, constructing consistent models for how individuals are constructing their choice sets is a very complex problem, and considering the universal choice set is thus a tractable option. Fortunately, even this immense problem can be solved with dynamic programming.

The value function $V(s)$ in (1) can be defined recursively through Bellman's equation as (Rust, 1987):

$$V(s_t) = \max_{a_t} \left\{ u(s_t, a_t) + \beta \int V(s_{t+1}) p(ds_{t+1}|s_t, a_t) \right\} \tag{2}$$

where $p(s_{t+1}|s_t, a_t)$ is the probability to reach state s_{t+1} when taking action a_t in state s_t. To make (2) computationally tractable, Rust (1987) introduce a number of assumptions. Firstly, the state s_t are divided into (x_t, ϵ_t), where x_t are known to both the econometricians and the individual whereas ϵ_t is unknown for the econometricians but known by the individual for the current period

t. The unknown part ϵ_t is further assumed to be Gumbel distributed and i.i.d. over alternatives and time. We will assume that the error term is translated to $\epsilon \sim G(-\gamma, 1)$, where the location is $-\gamma$ to ensure that the mean of ϵ is zero rather than γ. The utility function $u(s_t, a_t)$ is assumed to be additively separable into a known and unknown part: $u(s_t, a_t) = u(x_t, a_t) + \epsilon_t$. For a full set of assumptions and regularity conditions guaranteeing the existence of a solution to (2) given any time horizon (with $\beta < 1$), see Rust (1988).

We will make two additional restrictions that give a more tractable specification. Firstly, we will not include a discount variable ($\beta = 1$), which is possible as we have a finite time horizon. As the time frame of decisions is short, we do not see this as a big restriction and to our knowledge no current models of activity scheduling assumes that people discount within days. We will, secondly, assume that all uncertainty in the state transitions is captured by ϵ_t. This means that we, e.g., cannot include travel time uncertainty. Being able to explicitly model travel time uncertainty would definitely be of great value, and we will attempt to overcome this restriction in future work as individuals certainly take travel time uncertainty into account when planning their day. Currently, this will be have to be captured by ϵ.

With these two additional restrictions (and $\epsilon_t \sim G(-\gamma, 1)$), the expected value of the value function (the *expected value function*) becomes:

$$EV(x_t) = \int V(s_{t+1}) p(ds_{t+1}|s_t, a_t) = \log \left(\sum_{a \in A(x_t)} e^{u(x_t, a) + EV(x_{t+1})} \right) \tag{3}$$

and the probability that an individual n will choose alternative a when in state x is given by the well-known MNL-formula:

$$P_n(a|x) = \frac{e^{u_n(a,x) + EV_n(x'(a))}}{\sum\limits_{k \in A(x)} e^{u_n(k,x) + EV_n(x'(k))}} \tag{4}$$

$$= e^{u_n(a,x) + EV_n(x'(a)) - EV_n(x)} \tag{5}$$

where (3) together with (4) gives (5). It is worth noting that in this special case, a dynamic discrete choice model is equivalent to a nested-logit model where all nest-parameters are equal.

When modelling daily planning, there is logical terminal time T in the end of the day. We will restrict ourselves to a single feasible state x_T in the end of day with $EV_n(x_T) = 0$. This is not a restriction per se; multiple states in the end of the day could be included by adding a link from all of these states to a common fictive terminal state x_T. With $EV_n(x_T)$ defined, it is possible to use backward induction to calculate EV_n in all states using (3).

In the context of route choice modeling, Fosgerau et al. (2013) use that an MNL over routes in a directed network can be expressed by (5) and the same insight is useful here. Let $\mathbf{a} = (a_0, ..., a_{T-1})$ denote a sequence of actions and a let $\mathbf{x} = (x_1, ..., x_T)$ be the corresponding sequence of states that are traversed when the action sequence $\mathbf{a}$ is applied to the state x_0. The probability of this sequence being chosen conditional on the initial state x_0 is then:

$$\begin{aligned} P_n(\mathbf{a}|x_0) &= \prod_{i=0}^{T} P_n(a_i|x_i) = \prod_{i=0}^{T} e^{u_n(a_i, x_i) + EV_n(x_{i+1}) - EV_n(x_i)} \\ &= e^{u_n(\mathbf{a}, x_0) + EV_n(x_{T+1}) - EV_n(x_0)} \end{aligned} \tag{6}$$

where $u_n(\mathbf{a}, x_0) = \sum_{i=0}^{T-1} u_n(a_i, x_i)$. Observe that since $EV_n(x_T)$ and $EV_n(x_0)$ are the same for all alternative action sequences starting in x_0, the probability for each alternative is proportional to $e^{u_n(\mathbf{a},x_0)}$. If $\mathscr{A}(x_0)$ is the set of action sequences that, starting from x_0, satisfies all space-time constraints, then:

$$P_n(\mathbf{a}|x_0) = \frac{e^{u_n(\mathbf{a},x_0)}}{\sum\limits_{\mathbf{a}' \in \mathscr{A}(x_0)} e^{u_n(\mathbf{a}',x_0)}} \tag{7}$$

2.1 Specification

We have so far restricted the model to working days and people that arrive at work between 6 a.m. and 11 a.m. and return home before 11 p.m. We do currently not model lunch activities nor business trips. Work location, working time and whether people have fixed or flexible working schedules is taken as exogenous in the current implementation. Escort trips of children to or from school/daycare are mandatory for individuals that do the trip on the survey day, and drop off/pick up location is exogenous.

Below we will first describe the state space and choice set, and then how time-space constraints can be expressed in terms of restrictions on either the state space or on a state specific choice set.

2.1.1 States and actions

A state should include the information needed to determine available actions and utility of these actions. Here a state x consists of:

Time $t \in [5\,\text{am}, 11\,\text{pm}]$: Continuous variable for time of day. A day starts at 5 am and ends at 11 pm

Location $L \in [1, 1240]$: Current location. One of 1240 zones in the region of Stockholm.

Activity A: New activity, end activity, social, recreational, shop small, shop medium, shop large, home, work and escorting children are the alternative activity states.

The activity must be included in the state since the individual can choose to continue with the same activity for yet another time-period, but have to travel (possibly within the zone) to change activity. The purpose of the new activity and end activity states will be discussed below.

Errand indicator $E \in [0, 3]$: A state keeping track of the number of finished mandatory activities. The number of mandatory activities varies from 1 to 3 depending on the individual, as will be explained later.

Car dummy $\delta_{car} \in \{true, false\}$: Dummy for car availability. An individual have to travel with car if $\delta_{car} = true$ and if out of home and cannot travel with car if $\delta_{car} = false$.

The set of actions a that are available in a state x for individual n is denoted $A_n(x)$. The universal choice set consists of any combination of activity A, mode M and location L:

Activity A: Activity for new action.

Location L: New location.

Mode: Car, public transport, bike and walk are the modeled modes. When continuing with the same activity, the mode of the action is “no-mode”.

When starting a new activity with flexible duration it is initially conducted for one time-step, which we have chosen to 10 minutes. Depending on the activity and on time-space constraints it can be possible to continue with the same activity for another time step. The action-space is thus discrete and finite. This means that every 10th minute individuals can decide whether to continue with the current activity for another 10 minutes. Travel times are not divisible by these time step lengths, and it therefore makes sense to have a continuous state variable for time. Since there are a finite number of actions in each state there will still only be a finite number of reachable states. It is sometimes argued that activity length should be a continuous variable, as it is included in, e.g., (Habib, 2011), (Pinjari and Bhat, 2010) and (Kang and Recker, 2013), but from a behavioral perspective we think it makes at least as much sense to assume that people are considering whether to, e.g., spend 10, 20 or 30 minutes shopping as to assume that they decide to spend exactly 17.3123 minutes.

Working, escorting children and grocery shopping (in respectively category) has fixed duration (10 minutes for dropping of children and 10, 20 and 40 minutes for shopping in respectively category). The remaining alternatives can be continued for any number of time steps. Most computations come from calculating the log-sums in (3) for all states, and as activities and locations are both states and alternatives in each state, the computational time will increase quadratic with both the number of locations and the number of activities. To reduce computational time, the “start-activity” and “end-activity” states are added. The reason for this can be illustrated with an example. In each state, an individual can choose to either continue with the same activity or start a new activity at any location. This gives $1 + N_{act} \cdot N_m \cdot N_{loc}$ alternatives, and calculating EV in a state therefore requires summing up $1 + N_{act} \cdot N_m \cdot N_{loc}$ factors. In each time step, there are approximately $N_{act} \cdot N_{loc}$ states for which this operation is performed. In a “start-activity”-state, the only available alternatives are to start one of the available activities, so there are approximately N_{act} factors that needs to be summed together. Once we have EV in this state, we can divide the choice of a new activity into two steps, firstly the choice of a new location and mode, and secondly the choice of activity. This will not change the choice probability with choice probabilities according to (4). With this new state, the number of terms reduces to $1+N_m \cdot N_{loc}$ (where 1 is the alternative to continue with the same activity), and approximately decreases with a factor N_{act}. This comes at the expense of calculating the log-sum of N_{act} in N_{loc} states. When considering a new action, the future utility is independent of the current activity, and it is therefore possible to create an “end-activity” where the sums of all possible “start-activity” states is calculated. Instead of having to sum up $1 + N_{act} \cdot N_m \cdot N_{loc}$ factors in $N_{act} \cdot N_{loc}$ states we sum up the $N_m \cdot N_{loc}$ “start-activity” factors in N_{loc} “end-activity” states, so the computational savings can be significant. This also means that increasing the number of activities only have a minor effect on the computation time.

We currently do not consider any mixed modes, or mode chains, such as taking the bike to the train station. This would definitely be possible conceptually, but would increase the computation time. Car is only a possible choice if the individual has a car available at home. Further, if a car is used for a trip away from home, all consecutive trips on the same tour must be done using car. This is controlled through the car-dummy δ_{car}. Certainly it happens that individuals use a car for only some trips in a tour. They might leave their car at work and pick it up the next day or leave it for another family member. To make it possible to leave a car at any location, another state variable would be needed that remembered the location where the car was parked and the state space would explode. Since that kind of behavior is quite uncommon, we think this restriction on car usage is reasonable.

Each individual is considering all possible locations for each new action. As mentioned before, locations are both state variables and alternative actions so the computation time increases quadratically with the number of locations. Restricting the choice set of locations for individuals is therefore extremely tempting, but combining an activity scheduling model with a location choice set model in a consistent way seems extremely complex. This curse of dimensionality connected to the number of zones is sometimes solved by sampling a number of zones through some auxiliary model (see e.g., (Liao et al., 2013)), or by approximating the log-sums through importance sampling (similar to how Bradley et al. 2010 does in a nested framework). We want to avoid such approximations if possible. However, if the zones would be refined or increase for other reasons, we would likely have to resolve to some sort of sampling. Rust (1997) shows how randomization can be used to approximate EV in dynamic discrete choice models, and it would be one possible way to decrease computation time.

2.1.2 Time as a continuous variable.

Time is modeled as a continuous variable, but the number of states in which EV can be calculated is limited by computation time. It is therefore not possible to exactly calculate the expected value functions in all reachable states. Instead, EV is calculated on a discretized time-grid containing every 10:th minute and linear interpolation is used to approximate the value between these points. The linear interpolation approximation in a state x is denoted $\overline{EV}$ (x). The calculation of EV will therefore be based on approximations of EV in future states. We will therefore never know the exact expected value function in any state, but rather the (1:st order) approximate expected value function $\widetilde{EV}$, given by:

$$\widetilde{EV}(x_t) = \log \left(\sum_{a \in A(x_t)} e^{u(x_t,a)+\overline{EV}(x_{t'})} \right) \tag{8}$$

where $t_{k-1} \leq t' \leq t_k$ gives:

$$\overline{EV}(x_{t'}) = \alpha_1 \widetilde{EV}(x_{t_k}) + \alpha_2 \widetilde{EV}(x_{t_{k+1}}) \tag{9}$$

where $\alpha_1 = \frac{t_{k+1}-t'}{t_{k+1}-t_k}$ and $\alpha_2 = \frac{t'-t_k}{t_{k+1}-t_k}$

This approximation makes the order in which EV is calculated important. As mentioned before, backward induction is used to calculate EV, so one time step is calculated at a time. This works whenever all actions steps strictly forward in time, but if an action is less than 10 minutes long, the approximation in (9) will be based on $\widetilde{EV}$ in the current time step. The only way this can occur with the current discretization is if the new action involves a trip to a "new activity" state. As the only available alternative in a "new activity" state is to start an activity, $\widetilde{EV}$ in these states can be calculated first without risking self dependence. After $\widetilde{EV}$ has been calculated in the "new activity" states, it is safe to calculate it in the remaining states for that time step.

2.1.3 Time-space constraints.

Time space constraints define when and where an individual can participate in different activities and thereby impose a structure on the day. Time-space constraints can be of the type "I have to be at work by 7 a.m.", and both explicitly determine where an individual will be at 7 a.m. (at work) and implicitly influence where they can be at 6 : 50 a.m. (not more than 10 minutes away from work with available modes of transport). To check that a specific trip is possible, one must look multiple future trips into the future to ensure that all time-space constraints can be satisfied if that specific trip is carried out. Finding feasible activity schedules in a dynamic discrete choice model is trivial since expected value function $EV = -\infty$ in any explicitly or implicitly infeasible state, as by definition there are no actions leading from such a state to another state with $EV \neq -\infty$. Actions that are implicitly infeasible due to time-space constraints will therefore have zero probability.

Some activities are time constrained. Time constraints on when activities can be started or when they must be completed can easily be included by restricting the choice set at times that do not meet these constraints. Location constraints, i.e., constraints specifying where different activities can take place, are treated in the same way.

People can have fixed or flexible working hours. People with fixed working hours must arrive at work when the workday start and leave when the workday ends. People with flexible working hours can choose to arrive between 6 a.m. and 10 a.m., but the length of a working day is still fixed. The individual specifications on working hour type, working length, start and end hours must be provided from elsewhere. Children can be dropped off between 6 a.m. and 9 a.m.. Pick up trips must be completed between 12 a.m. and 6 p.m. All individuals must start and end their days at home. There is no need to restrict the state space in the start of the day. Such restrictions are ensured by the choice of the initial state used when, e.g., simulating day paths.

Picking up and dropping of children at school as well as going to work are considered mandatory activities with fixed location and time constraints. These three activities further have an internal order: dropping of children is done before going to work which must be done before picking up the children again. To model this order of activities, we introduce the errand indicator E. When $E = 0$, only dropping of children is possible. After having finished a drop-off activity, E increases by one and the only available activity in the group is work. Enforcing that all activities are finished during the day is done by restricting E in the end of the day, and time-constraints are treated as above.

More generally, a constraint could impose that some activity or a group of activities must be conducted a number of times N during a day. This can be modeled by introducing an errand indicator state variable, say Q, for each such group of mandatory activities and setting the expected value function to $-\infty$ whenever $Q \neq N$ in the end of the day. Whenever an activity in the group is started, Q is increased by one. If the day is started in a state with $Q = 0$, all feasible activity schedules will do activities in the group exactly N times. Introducing an extra state variable is not without costs. The number of states will increase linearly with N and the number of actions in each state will not decrease substantially, so the computation time will increase almost linearly with N in each basic activity constraint. If there are multiple groups of mandatory activities where the activities in a group i must be conducted N_i times, the number of states will increase with a factor $\prod_i (N_i + 1)$ times.

Correlation between alternatives It is common practice in route choice modelling to add a size attribute to each link to take correlation among paths that overlap into account, e.g., using Path-Size Logit (Ben-Akiva and Bierlaire, 1999). For their link-based route choice model, Fosgerau et al. (2013) obtains a size coefficient by calculating EV in each link using some pre-specified parameters and adding that to the link-utility. In the activity-scheduling model presented here, it is not as easy to define the overlapping of paths, as the network is dynamic. If two paths are identical besides that the start time for all activities in one path is 10 min after the start time in the other path, there can be practically no overlapping as defined by the Path-Size Logit although the two paths would be very similar. How to address this issue in a dynamic network and in the activity-scheduling framework is therefore an open question.

In trip-generation models, it is common to have nests for mode choice, location choice and activity choice, as in, e.g., Bowman and Ben-Akiva (2001). It would be possible to introduce different scales for the error term when solving (3) and obtaining choice probabilities in (4) where the scale (which is one here) would be state dependent. However, the probability of a path would then not reduce to (6) and sampling of alternative sequences would not be possible to use for estimation. Another issue is the correlation in preferences over time. Individuals' variances in preferences for, e.g., mode or activities are likely to be consistent over time and therefore to some extent be the same throughout the day. Including nests on a trip level would not capture this correlation. Bowman and Ben-Akiva (2001) solves this issue by introducing a dummy for previously chosen modes. Such a dummy could also be included in the day-path model specification, but would require that the state where dependent on the previously chosen modes, and thus increase the already high computation time. A possible solution to both of these problems would be to introduce mixed parameters for, e.g., activities and modes, that would be the same for each individual for the full day. Our estimation approach is based on sampling of alternatives and recent research by Guevara and Ben-Akiva (2013) shows that the same method gives consistent estimates for mixed logit models.

Finally, it is worth noting that the expected value function in (4) might pick up some of the correlation in the unobservable ϵ that is usually captured by introducing nests in a trip based model. Since a trip with walk, public transport and bike all share the same state, except for the arrival time, EV will be correlated for the three alternatives.

3 Estimation

As discussed in the introduction, the number of feasible day-paths for each individual is immense and this makes estimation on the full choice set problematic. It is a general property of MNL models that estimating over a subset of alternatives gives consistent estimates if a correction term is added to the utility function (McFadden, 1978). Since we have an MNL over day-paths, it is possible to sample a subset of the universal choice set and still obtain consistent estimates. The estimates are, however, not efficient and how the choice sets are constructed will determine the efficeny loss. Since the number of alternatives is immense, we need a smart way of sampling alternatives that somewhat resembles the model in order to obtain good estimates. One way of doing this would be to sample from the true model with some smart guesses of the parameter values. However, simulating alternatives is not easy when the number of alternatives is this large and one would probably have use some sort of Metropolis Hastings algorithm such as the one Flötteröd and Bierlaire (2013) developed in a route-choice context. In the previous section, we showed that a dynamic discrete choice model without discounting and uncertain state-transitions, giving the choice probability in (4), is equivalent to an MNL over action sequences, as in (7). This is good, because although it is too time consuming to calculate the value functions required for (4) (takes $\sim 9\,\mathrm{s}$/observation) every time one updates the parameter values during optimization, it is feasible to calculate them once for some set of parameters and then use (4) in order to sample a choice set.

Estimation using sampling of alternatives involves sampling a choice set $\tilde{\mathbb{C}}_n \subset \mathbb{C}_n$ and estimating using the conditional choice probability $P_n(\mathbf{a}_n|\tilde{\mathbb{C}}_n)$ instead of the $P_n(\mathbf{a}_n|\mathbb{C}_n)$. A maximum likelihood estimation on a choice set $\tilde{\mathbb{C}}$ gives consistent estimates if the correction term $\log(\bar{q}_n(\tilde{\mathbb{C}}_n|j))$ is added to each alternative and $\bar{q}_n(\tilde{\mathbb{C}}_n|j)$ satisfies the positive conditioning property, i.e., that if $j \in \tilde{\mathbb{C}}_n$ and

$\bar{q}_n(\tilde{\mathbb{C}}_n|i) > 0$ for some i, then $\bar{q}_n(\mathbb{C}_n|j) > 0$. This holds if $\tilde{\mathbb{C}}_n$ is sampled from the universal choice set $\mathbb{C}_n$ and all alternatives in $\mathbb{C}_n$ have a non-zero probability of being sampled.

Suppose we have N observations forming the set of observations $\mathscr{O}_N$. The log-likelihood function for $\mathscr{O}_N$ based on the conditional likelihoods becomes:

$$\bar{\mathscr{L}\mathscr{L}}(\mathscr{O}_N;\theta) = \sum_{n=1}^{N} \log\Big(\frac{e^{u(\mathbf{a}_n)+\log(q_n(\tilde{\mathbb{C}}_n|j))}}{\sum\limits_{\mathbf{a}^*\in\tilde{\mathbb{C}}_n} e^{u(\mathbf{a}^*)+q_n(\tilde{\mathbb{C}}_n|\mathbf{a}^*)}}\Big) \tag{10}$$

If all alternatives in $\mathbb{C}_n$ have equal probability of being sampled to the choice set $\tilde{\mathbb{C}}$, the correction term $q_n(\tilde{\mathbb{C}}_n|j)$ will also be the same for all alternatives and therefore cancel out from the likelihood function. However, if we sample alternatives according to some other sampling protocol, we need to calculate this probability. We use the same sampling protocol used to estimate an MNL model over the choice of routes in a traffic network in Frejinger et al. (2009). The sampling protocol consists of drawing R alternatives with replacement from the choice set $\mathbb{C}_n$ consisting of J_n alternatives, and then adding the observed choice to the choice set. The outcome of such a protocol is $(k_{n1}, k_{n2}, \ldots, k_{nJ})$ where k_{nj} is the number of times alternative j appears in the choice set, so that $\sum_{j=1}^{J} k_{nj} = R + 1$, since the observed alternative j is added once extra to the choice set. Let $q_n(i)$ denote the probability that alternative $j \in \mathbb{C}_n$ is sampled. The correction term can then be derived to: $q_n(\tilde{\mathbb{C}}_n|j) = K_{\tilde{\mathbb{C}}_n} \frac{k_{nj}}{q_n(j)}$. The constant $K_{\tilde{\mathbb{C}}_n}$ will cancel out from the likelihood function to give:

$$\bar{\mathscr{L}\mathscr{L}}(\mathscr{O}_N;\theta) = \sum_{n=1}^{N} \log\Big(\frac{e^{u(\mathbf{a}_n)+\log(\frac{k_{n\mathbf{a}_n}}{q_n(\mathbf{a}_n)})}}{\sum\limits_{\mathbf{a}^*\in\mathbb{C}_n} e^{u(\mathbf{a}^*)+\log(\frac{k_{n\mathbf{a}^*_n}}{q_n(\mathbf{a}^*_n)})}}\Big) \tag{11}$$

As previously mentioned, we will sample a choice set by using (4) with a single set of parameters. Selecting these parameters were not entirely trivial. We started with a simple specification of the model, only involving time and cost parameters, and changed them manually until travel times, mode choices and activity episodes were in line with the real observations. This resembles a method-of-moments estimation on part of the parameters.

4 Data

We have estimated the model using the Stockholm travel survey from 2004, where individuals report a full day travel diary. Estimating using full day-paths puts a high demand on the reported diaries, since the information for all trips in an observation must be correct in order for it to be usable. Further, travel times as reported in the diaries are rarely the same as the data we have on travel times or the travel times we calculate for the same origin-destination, and sometimes the discrepancy is huge. This is always a problem, since the observed travel times only are available for the observed origin-destination pair with the observed mode. One common way of dealing with this is to use the calculated travel times rather than reported travel times, and this is how we choose to proceed. A static traffic assignment model with a nested-logit based travel demand model gives the travel times and costs used for peak and off-peak periods. It is worth noting that when considering day-paths changing the travel time of an observed trip will influence the starting time and/or duration of all remaining actions in the same day. Travel cost with car is calculated as

1.4 kr/km, travel times with bike is calculated assuming a speed of 15 km/h and walk travel times assuming a speed of 4 km/h.

4.1 Dataset

We have so far restricted the model to individuals that go to work on a weekday. This leaves us with 5200 observations with sufficient information. Out of these, 3300 behave in a way that fits the model. The $\sim$ 2000 observations that are removed at this stage behave in ways that the model cannot handle, for example by ending the work day with a business trip, and therefore not ending the work day at their work location; having longer than $2h$ breaks in the middle of their work day; working late (later than 8pm) or starting early (earlier than 6am); leaving the car somewhere or not returning home in the end of the day.

We have demanded that car should be used for either all or no trips on a tour. As passengeres and drivers has been treated in the same way, this means that observations with passengers are likely to be removed. Besides individuals that report that they were passengeres, 3% of the observations included a trip of this kind. It is hard to say what is happening here. It is possible that the individual is beeing dropped off or is beeing picked up but is driving the car to/from the activity. It is also possible that individuals leave the car for a later day. Parameterizing this behaviour without knowing if the car will be picked up on a later occacion or if someone else is driving it back, and thereby not knowing the attributes associated with such a choice, is likely make the model responde incorrectly to changes in traffic conditions. We have therefore choosen to exclude these observations.

5 Utility specification

For an individual n, the instantaneous utility $u_n(a|s)$ is the sum of the (dis)utility of traveling $u_{n,m}$ and the utility of participating in an activity $u_{n,p}$. The utility of traveling is dependent on the travel cost, travel time and mode, which in turn will be dependent on time of day, origin and destination. The current state is given by $s = (t, l, p, e, c)$, where t is the time, l the location, p the activity (or purpose), e the errand indicator and c the car dummy; and the action is $a = (p', l', m)$ where p' is the new activity, l' the new location and m the mode of transport. The utility of travelling with mode m for individual n can then be written as $u_{n,m}(a|s) = u_{n,m}(l, l', t)$, as it is dependent on the individual, the origin, destination, time of day and mode. For respectively mode it is specified as:

$$
\begin{aligned}
u_{n,\mathrm{car}}(l,l',t) &= c_{\mathrm{car}} + \theta_{\mathrm{car},t} T_{\mathrm{car}}(l,l',t) + (\theta_c + \delta_{n,\mathrm{h.i}} \cdot \theta_{\mathrm{h.i}})\, C_{\mathrm{car}}(l,l',t) \\
u_{n,\mathrm{PT}}(l,l',t) &= c_{\mathrm{PT}} + \theta_{\mathrm{PT},t} T_{\mathrm{PT}}(l,l',t) + \theta_{\mathrm{wait,PT}} T_{\mathrm{wait,PT}}(l,l',t) + (\theta_c + \delta_{n,\mathrm{h.i}}\theta_{\mathrm{h.i}})\, C_{\mathrm{PT}}(l,l',t) \\
u_{n,\mathrm{bike}}(l,l',t) &= c_{\mathrm{bike}} + \theta_{\mathrm{bike},t} T_{\mathrm{bike}}(l,l',t) \\
u_{n,\mathrm{walk}}(l,l',t) &= c_{\mathrm{walk}} + \theta_{\mathrm{walk},t} T_{\mathrm{walk}}(l,l',t) + \theta_{\mathrm{d}\geq\,5\,\mathrm{km}} \delta_{\mathrm{d}\geq\,5\,\mathrm{km}}
\end{aligned}
$$

where $T_m(l_1, l_2, t)$ and $C_m(l_1, l_2, t)$ denote the travel time and cost with mode m at time t for a trip from origin l_1 to destination l_2, $T_{\mathrm{wait,PT}}$ is the waiting time when using public transport, $\delta_{n,\mathrm{h.i}}$ is a dummy indicating whether individual n has a high income (defined as greater than 40 000 SEK/month), $\delta_{\mathrm{d}\geq\,5\,\mathrm{km}}$ is a dummy for distances greater than 5 km and c_m are mode specific constants.

When arriving at the destination l' at time $t' = t + T_m(l, l', t)$, the new activity p is started and performed for an activity dependent duration Δt_p. Starting the activity gives a time-of-day dependent constant utility $c_p(t)$ and a duration and time-of-day dependent utility $U_{n,p}(t, \Delta t_p)$. Choosing to continue with the same activity for another time step only gives the duration utility $U_{n,p}(t, \Delta t_p)$. Not all activities have time-of-day specific parameters. In order to keep down the number of explanatory variables, the constant utility $c_p(t)$ is only time dependent for the work activity and the duration utility $U_{n,p}(t, \Delta t_p)$ is only time dependent for the home activity. The time-of-day dependent parameters are specified on discrete time steps T_k with values θ_{p,T_k} and c_{p,T_k}. The activity specific constant is given by linear interpolation between the closest defined parameters:

$$c_p(t) = \frac{c_{p,T_k}(T_{k+1} - t) + c_{p,T_{k+1}}(t - T_k)}{T_{k+1} - T_k}$$

where $t \in (T_k, T_{k+1})$. For the durational utility we specify the *marginal* utility of activity participation at time t as given by linearly interpolation between the closest parameters, so:

$$u_t(t, p) = \frac{\theta_{p,T_k}(T_{k+1} - T_k) + \theta_{p,T_{k+1}}(t - T_k)}{T_{k+1} - T_k}.$$

The utility of an activity episode of duration Δt_p, when $T_k \leq t$ and $t + \Delta t_p \leq T_{k+1}$, then becomes:

$$U_p(t, \Delta t_p) = \int_t^{t+\Delta t_p} u_t(\tau, p)\, \mathrm{d}\tau = \alpha_{T_k}\theta_{p,T_k} + \alpha_{T_{k+1}}\theta_{p,T_{k+1}} \tag{12}$$

where:

$$\alpha_{T_k} = \Delta t_p \frac{T_{k+1} - t - 0.5\Delta t_p}{T_{k+1} - T_k}$$
$$\alpha_{T_{k+1}} = \Delta t_p \frac{t + 0.5\Delta t_p - T_k}{T_{k+1} - T_k}.$$

Observe that $\alpha_{T_k} + \alpha_{T_{k+1}} = \Delta t_p$, so if $\theta_{p,T_k} = \theta_{p,T_{k+1}}$ the duration utility becomes $\Delta t_p \theta_{p,T_{k+1}}$. If $t + \Delta t_p > T_{k+1}$, $u_t(\tau, p)$ in (12) becomes a stepwise linear function but is otherwise treated in the same way.

Besides activity specific constants, each location l has size parameters representing the number of available opportunities for each activity at that location. This utility is given by:

$$u_{p,\text{size}}(l) = \theta_{p,\text{size}} \log\left(\sum_{s=1}^{s=S_p} x_{p,l,s} e^{\theta_{p,s}}\right)$$

where S_p is the number of size variables for activity p, and the size variables $x_{p,l,s}$ can be, e.g., the number of employees in a specific sector at location l. Since we have an activity specific constant, one of the parameters $\theta_{p,s}$ should be fixed for all activities. This also provides an alternative interpretation of the activity specific constants as scales for the size variables $x_{p,l,s}$. A complete list of size variables included for respectively activity is given in table 2.

6 Result

6.1 Estimation

Table 1 gives estimation result for all parameters except the size-parameters, which are given in 2. Most parameters are significant and have the expected sign. Cost is negative and spending time on activities is preferred to spending time on travelling. Home time is valued higher early in the morning and late in the evening. It also increases slightly around 6 p.m., possibly because people like having dinner together at home at this time, or because they return home after work before heading out for another activity. Since not all time parameters can be identified, $\theta_{\text{6PM Time}}$ is fixed, and the linear-in-time parameters can only be compared against each other. Although the choice of parameter to fix does not affect the theoretical properties of the estimates, it can have a huge impact on their variance. When $\theta_{\text{PT Time}}$ was fixed rather than $\theta_{\text{6PM Time}}$, the standard deviation of all time parameters was close to 0.006, rather than varying between 0.001 and 0.005. Travel time parameters are significantly smaller than activity duration parameters, so participating in an activity is preferred to travelling. Since time parameters can only be interpreted in relation to each other, it is not possible to directly calculate the value of time. A travel time saving with car that gives one minute extra home at 6 p.m. would be valued $(\theta_{\text{6PM Time}} - \theta_{\text{Car Time}})/\theta_{\text{Cost}} = 476\,\text{kr}$, which seems too large with a magnitude of approximately four. Observe that this is not the correct way of calculating the value of travel time savings from the model, which should rather, in some way, be obtained from EV.

The time-specific constants for work hours seem quite large in comparison to the time parameters, but this is mainly due to the scaling of the parameters. When comparing two alternative sequence, one that arrives at work at 6 a.m. and arrive back home at 4 p.m. and one that arrives at work at 7 a.m. and back home at 5 p.m., the difference in utility per minute at work from arriving at the different times will be $(\theta_{\text{Work ASC 6AM}} - \theta_{\text{Work ASC 7AM}})/60 = -0.0433$. This is of the same size as the difference between $\theta_{\text{Home 6AM}}$ and $\theta_{\text{Home 7AM}}$. The difference in the valuation of time spent at home at different times of the day and the difference in the valuation of time spent at work at different times of the day will therefore be of similar importance when determining departure time to work.

Interpreting the constants for mode and activities is not entirely straightforward. Firstly, they are all normalized by fixing $\theta_{\text{Home PM ASC}}$. Further, the scale of the size parameters is arbitrary and obtained through the fixing of one of the size parameters, and the activity specific constants will have to pick this error up. The parameter $\theta_{\text{Home PM ASC}}$ is further a "tour" constant rather than a home constant, as an extra tour give one extra arrival at home. That the home constant is negative therefore says more about the unwillingness to leave home than the unwillingness to go home.

6.2 Simulation result

To test how well the model manages to produce realistic behavior we simulated daily activity schedules for the observed individuals and compared some characteristics of the simulated data with the real data. Since we have parameters for travel time, travel cost, activity time, number of trips, etc., many quantities should be the same as in data (if we did not use sampling of alternatives). We are therefore focusing on quantities that we do not directly estimate but that are outcomes of the estimated parameters. Here we report result on activity timing, distribution of trips and tours, and trip length distribution.

Table 1: Estimates

Parameter		Estimate	Rob. t-test	Rob. Std.
Car	Time	-0.099	-22	0.0045
	ASC	-1.5	-18	0.084
PT	Time	-0.054	-10	0.0053
	ASC	-2.5	-28	0.087
	Total Wait	-0.0037	-0.57	0.0066
Walk	Time	-0.094	-21	0.0044
	same zone	-0.74	-6.6	0.11
	ASC	-0.2	-1.8	0.11
Bike	Time	-0.097	-24	0.004
	ASC	-2.5	-23	0.11
Cost		-0.012	-4.4	0.0028
	High Income	-0.0051	-1.7	0.003
Home	6AM Time	-0.019	-4.1	0.0046
	7AM Time	-0.054	-10	0.0054
	8AM Time	-0.072	-15	0.0048
	9-10AM Time	-0.055	-10	0.0053
	1-4PM Time	-0.014	-9.1	0.0016
	5-6PM Time	0	Fixed	
	7-8PM Time	-0.013	-8	0.0016
	9-10PM Time	0.0033	2.1	0.0016
	PM ASC	-2	Fixed	
	AM ASC	-4.8	-19	0.25
Rec.	Time	-0.0094	-9.5	0.00099
	ASC	-5.3	-14	0.37
	LSM Size	0.33	7.6	0.044
Social	Time	-0.012	-8.6	0.0014
	ASC	-8.6	-34	0.25
	LSM Size	0.039	1.7	0.023
Shop	Small ASC	0.91	1.4	0.64
	Medium ASC	1.1	1.8	0.64
	Large ASC	0.76	1.2	0.62
	LSM Size	1.1	13	0.083
Other	Time	-0.023	-14	0.0016
	ASC	-1.9	-2.8	0.68
	LSM Size	0.79	9.7	0.081
Work	ASC 6pm	-6.9	-13	0.54
	ASC 7pm	-4.1	-14	0.3
	ASC 8pm	0	Fixed	
	ASC 9pm	3	11	0.28
	ASC 10pm	2.8	5.3	0.53

Table 2: Estimates

	Parameter	Estimate	t-test	LL-ratio
Rec.	Population	1	0.7	0.074
	No Employed Rest.	0.027	9e-05	7.7
	No Emplyoyed OE	0.17	0.85	0.018
Social	Population	0.9	0.94	0.0033
	No Employed OE	-1.3	0.39	0.37
Shop	Population	4.5	1.3e-12	25
Other	Population	2.1	0.12	1.2
	No Employed OE	3.6	1.7e-06	11
	No Employed Rec.	2.6	0.37	0.4
	No Employed Rest.	5.8	0.00037	6.3

Three factors determine the timing of activities: firstly, time constraints on working hours and pickup times; secondly, preferences for when to arrive at work in the morning; and thirdly, preferences on when to be home. From figure 1 it seems as if these determinants are enough to predict when people go to work, when they get home after work and when they drop-off/pick-up children. However, there seems to be some missing determinants for when people prefer to be doing free time activities. The total number of free time activities seems to be correct, but they are overrepresented early in the morning and late in the evening. The lack of time restrictions on when free time activities can be conducted is likely one reason for this, as most activities cannot be conducted before and after certain hours due to closing hours of shops, etc. Further, it could be because the utility is linear in time for all spare time activities. If some activities need a minimum duration to be undertaken, it would both reduce their occurrences in the morning and make them start earlier in the evening.

The distribution of the number of trips and tours in a day is determined by a vast number of factors, but most directly by the tour and trip constants. The tour constant is equal to the home constant, as each additional tour will include an additional trip home. A specific trip constant would not be possible to identify given the constants we already have for modes and activities, but these constants will ensure that the number of trips is correct. The total number of tours and trips is the same for the simulated and real data should therefore be the same. However, we do not have any constants governing the distribution of the number of trips per day, number of tours per day or number of trips per tour. The overall structure of the model will therefore determine these distributions, and seems to be enough to give good predictions, as figure 2 show. This is interesting when comparing to, e.g., Bowman and Ben-Akiva (2001) that introduce multiple constants for alternative patterns of tours and trips.

The length of trips with respectively mode will mainly be determined by the utility of time and money for respectively mode and by network characteristics. This gives a good distribution of travel times, as can be seen in figure 3. Since a large share of the trips are made to and from work, where the location is fixed and the trip is mandatory, the model is guaranteed to reproduce a large share of the trips well. However, the observed mode for the trip to work will only be the chosen mode in some of the simulated observations, and these restrictions will not give the distribution directly.

As we are using sampling of alternatives and the real number of alternatives is so vast, it would be possible that the obtained estimates were very inefficient. When validating the estimation on simulated data with known parameters and using parameters far away from the true parameters, it was not possible to estimate the model. Since the simulations reproduce the characteristics of the observed data well, it seems likely that the estimation succeeded. We have also compared accumulated travel time, cost, activity duration, activity episodes, and all other attributes that was used for estimation, with simulation result, and there is at most a few percentages difference. At the true estimate, this difference should be zero, but as the estimates are inefficient (since we use sampling of alternatives), some deviations are to be expected. It is also possible that the approximations used to calculate $\widetilde{EV}$ are causing the observed deviation between observed and simulated data. The small difference between the data and the simulations does also indicate that any bias produced by this approximation is small.

The "same zone" dummy parameter $\theta_{\text{same zone}}$ for walk is negative, which seems contra-intuitive as one would expect walk to be the preferred mode of transport for shorter distances. From figure 3 it is clear that for same-zone trips (trips with zero travel time) walk is still the preferred mode of transport. The reason for this is that the mode specific constant C_{walk} is the largest of the constants, even after adding $\delta_{\text{same zone}}$, and when the travel time is small it will therefore have the highest utility.

The parameters for travel time are almost the same for car, walk and bike but significantly smaller for PT. Since the travel time is longer for bike and walk, they will be less common for longer trips. PT has the smallest time coefficient and trips with longer travel time are therefore more common with public transport. The lower speed of bike and walk in comparison with the motorized modes will make them less common for longer trips, as can be seen in figure 3. The lower alternative specific constants, and the fact that more locations will become available with the same travel time with PT and car also make bike and walk less common.

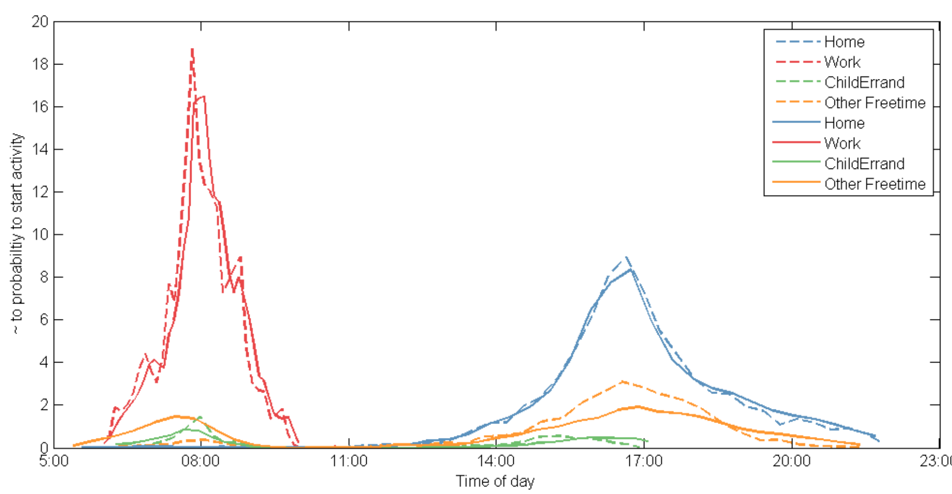

Figure 1: Time of day when respectively group of activities are started for simulated (solid) and real data (dashed).

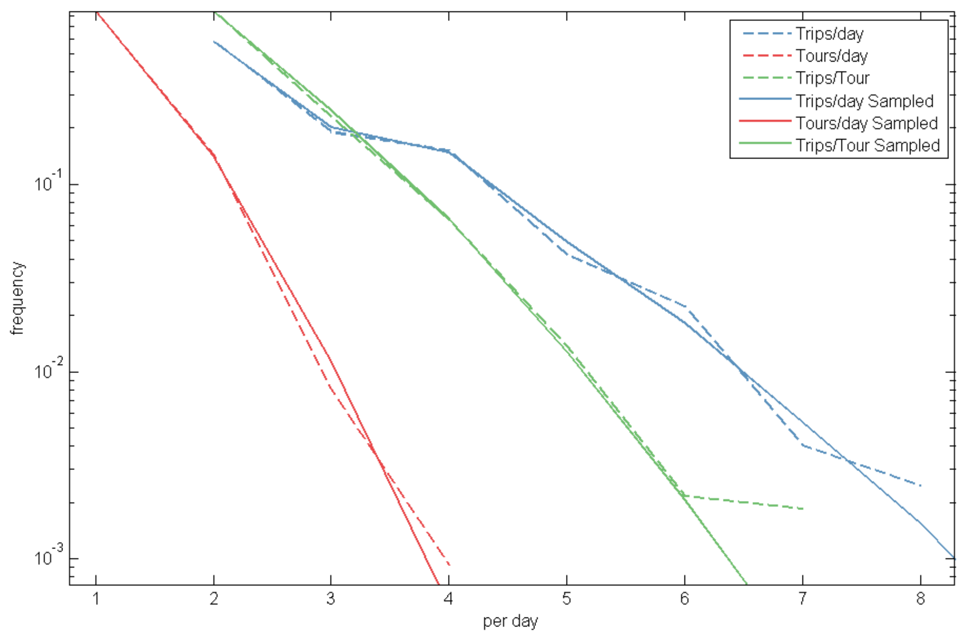

Figure 2: Distribution of number of trips, tours and trips per tour for simulated (solid) and real data (dashed).

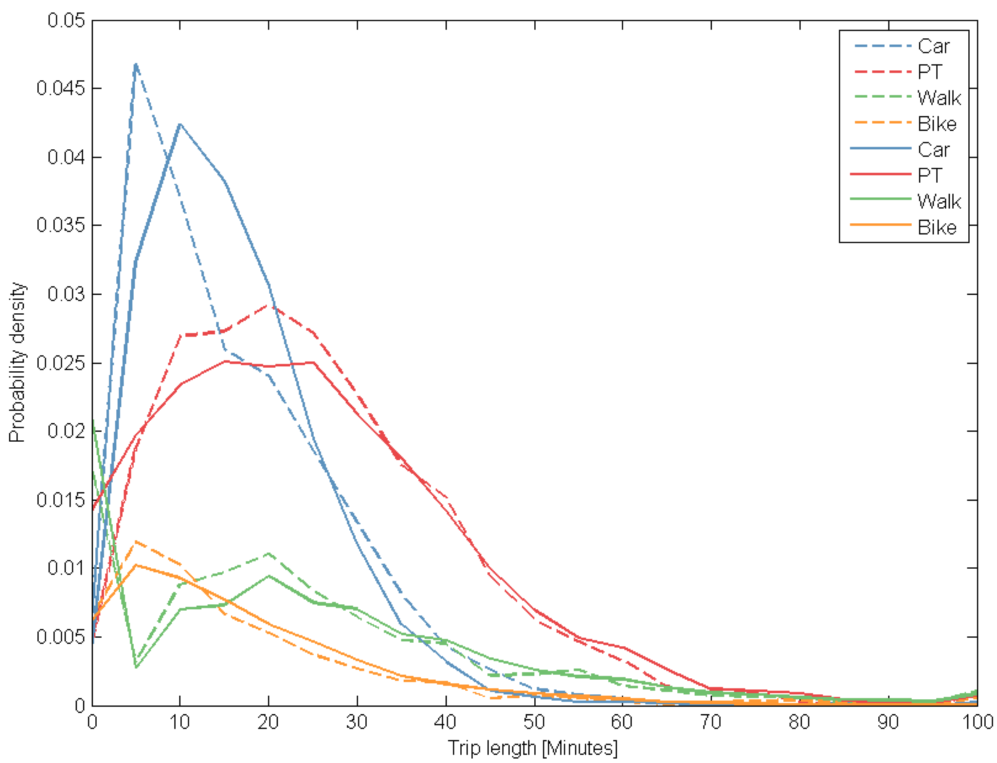

Figure 3: Distribution of trip lengths of respectively mode for simulated (solid) and real data (dashed).

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