A note on CES Preferences in Age-Structured Models

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A note on CES Preferences in Age-Structured Models[†]

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Abstract

In a biomass model a CES function generates an exploitation rate that is directly proportional to the scarcity of the resource: resources with less biomass are subjected to lower exploitation rates. In this paper we investigate the implications of introducing invariant intertemporal preferences as to yield stability in age-structured fishery problem. Our results show that a CES function in an age-structured bioeconomic model produces links between the scarcity of the resource (measured as the weighted sum of the size of the cohorts, which is similar to the Shannon index) and the exploitation of the resource over a complete cycle, the duration of which is equivalent to the number of age groups of the resource. Given that multiple paths can be constructed that regenerate the population of the resources (the age pyramid) over the course of the cycle, optimum harvest allocation means selecting the one that permits the biggest catch at the beginning of the cycle. Smoother exploitation path towards the stationary values are achieved by catching more in periods when there is less biomass in exchange for catching less when the biomass recovers, which results in exploitation rates that are not directly proportional to the scarcity of the resource. Moreover, we show that introducing non-constant discount rates into age-structured models enables exploitation rates proportional to the scarcity of the resource to be recouped.

Keywords: Optimisation in age-structure models, Stability preferences, Natural resource management, Constant-elasticity-of-substitution utility function.

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1 Introduction

Natural resource management assesses the policy implications of inter-temporal choices: the choice of balancing the benefits from harvesting now with the potential gains from harvesting in the future. Management is therefore affected by users' willingness to substitute harvesting over time and preferences as to yield/income stability. In fisheries, for example, these preferences as to economic stability may vary across stakeholders, with the fishing industry giving greater weight to stable and therefore more predictable catch opportunities (TACs) in long-term management plans (e.g. Pascoe et al., 2009; Dichmont et al., 2010; Aanesen et al., 2014; Sampedro et al., 2016).

This paper explores the implications of incorporating preferences for smooth harvesting into natural renewable resource problems, which include information about age classes to better address optimal harvesting decisions.[1] Managers' preferences for stability on the optimal allocation of harvests over time are modeled in this paper focusing on a fisheries management problem and assuming that utility is isoelastic with constant relative risk aversion (CRRA), i.e. using a Constant Elasticity of Substitution (CES) utility function.[2] In the recent literature, this utility function has been used for example in (Quaas and Requate, 2013) represent consumers preferences over the consumption of fish speci; and (McGough et al., 2009) to capture uncertain environmental fluctuations and risk aversion in a biomass-fishery analysis. In this paper, we focus on an age-structure fishery managers decision problem to incorporate the desire for smooth rather than excessively changeable exploitation paths, and question the implicit CES assumption of constant elasticity of intertemporal substitution (EIS). When this property holds, over time a manager is equally averse to proportional fluctuations in

[1]The rationale behind age-structured bio-economic models is the need to regulate the size and volume of harvests at ecosystem level in order to account for increasing harvesting pressures in the internal age structure of biological populations that may impact on reproduction and/or non-monetary benefits from ecosystem services dependent on age composition (e.g. Kronbak and Vestergaard, 2013), and to avoid fishing down the marine food webs (Ravn-Jonsen, 2011). Moreover such models form the centerpiece of fisheries management in real practice; most stock assessment methods used by fisheries agencies rely on age-structured populations (e.g. Lassen and Medley, 2000).

[2]CRRA was used, for example, by McGough et al. (2009).

yields. The closer to (farther from) zero the EIS is the greater (lesser) the desire for intertemporal smoothing is. When the EIS parameter tends to infinity the utility is linear and coincides with the present value of the natural resource exploitation profits. In this case current and future catches are perfect substitutes and the desire of stakeholders for a stable harvesting path is minimal. By contrast, when EIS tends to zero current and future harvests are complementary and the desire for a stable catch from one period to another is maximal. In this paper we theoretically proof that in contrast to what occurs in biomass models, in age-structured models the use of a CES utility function does not guarantee time-invariant proportionality between harvest and escapement throughout the transitional paths toward the steady state. In age-structured models a CES utility function links changes in discounted yield over a cycle (more than one period) to changes in relative abundance of age-class population sizes (measured by an Shannon index-like indicator, capturing the uniformity in the relative size of age-classes with respect to their stationary targeted levels).

Most importantly, we find that increasing captures can be the optimal harvest response to biomass drops if constant discount is used. This result is related to the nature of the stationary solution of age structured problems: A discounted yield for the cycle that completes the whole age structure of the resource population, i.e. a sequence of yields, rather than a constant yield for all years (Tahvonen, 2009; Da Rocha et al., 2013). Consequently, the implicit CES assumption of constant intertemporal elasticity of substitution does not hold in the case of constant discounting. The way to recover the proportionality between yield and biomass along the transition path and thus maintain the assumption of invariant preferences over intertemporal substitution is to assume a non constant discount rate. Given that multiple paths can be constructed that regenerate the population of the resources (the age pyramid) over the course of the cycle, optimum exploitation means selecting the one that permits the biggest catch at the beginning of the cycle. This means that the smoothest exploitation path is achieved by catching more in periods when there is less biomass in exchange for catching less when the biomass recovers, which results in exploitation rates that are not directly proportional to the scarcity of the resource. We analytically show that introducing non-constant discount rates into age-structured models enables exploitation rates proportional to the scarcity of the resource to be recouped.

These findings contain crucial insights for fishery management. They show the potential severity of exploring harvesting control rules under a modeling approach with constant discount factors, in particular in those situations where a precautionary principle needs to be adopted to avoid the threat of fishery overexploitation.

The paper proceeds as follows. In Section 2 optimal harvesting is analyzed in a simple biomass fisheries model incorporating a CES utility function. Section 3 extends the analysis to age-structured models. Section 4 concludes.

2 CES in the biomass model

We consider a simple fishery biomass model. Escapement, s_t, (the biomass, x_t, that remains in the ecosystem after the harvest, y_t) is considered as the state variable. Given a level of escapement available for reproduction at time t, $s_t = x_t - y_t$, the growth of biomass from t to $t+1$ is described by the equation $x_{t+1} - x_t = F(x_t - y_t) - y_t$, such that $x_{t+1} = F(s_t) + s_t = G(s_t)$, where $G(s_t)$ satisfies $\frac{\partial^2 G(.)}{\partial s^2} < 0$. Given these constraints, fishermen maximize the discounted utility derived from harvesting by choosing the optimal escapement level sequence

$$\max_{s_t} \sum_{t=1}^{\infty} \beta^t U(G(s_{t-1}) - s_t),$$

where $0 < \beta < 1$ is the discount factor, and harvest is expressed in escapement terms as, $y_t = G(s_{t-1}) - s_t$. This management problem can be formulated as a recursive problem with Bellman equation given by:

$$V(s_{t-1}) = \max_{s_t} U(G(s_{t-1}) - s_t) + \beta V(s_t).$$

The first order condition, $U'(G(s_{t-1}) - s_t) = \beta V'(s_t)$, and the envelope condition, $\frac{dV(s_t)}{ds_t} = U'(G(s_t) - s_{t+1})G'(s_t)$, can be combined to obtain the following familiar optimality condition for the choice of escapement time-path policy,

$$U'(G(s_{t-1}) - s_t) = \beta U'(G(s_t) - s_{t+1})G'(s_t),$$

which indicates that the marginal utility of an additional unit of resource harvested in period t (left-hand-side term) equals its opportunity cost in terms of the discounted marginal utility that an unharvested unit would convey in the period $(t+1)$ (right-hand-side term). Note that steady-state condition is given by $G'(s_t) = 1/\beta$.

Fishermen's preferences as to the intertemporal allocation of harvesting are captured in this model by using a constant elasticity of substitution (CES) utility function

$$U(y) = \frac{y^{1-\sigma}}{1-\sigma},$$

where $\rho = \frac{1}{\sigma} = -\frac{U'(y_t)}{U''(y_t)y_t}$ is the constant elasticity intertemporal of substitution (EIS), and its inverse, σ is the relative risk parameter. When $\sigma \to \infty$ ($\rho \to 0$), individuals are very risk averse, and the intertemporal smoothing harvesting motive, i.e. a preference for avoiding inequality in captures over periods, is strong for fishermen. Howevwe if $\sigma \to 0$ ($\rho \to \infty$), the concern over large fluctuations in harvesting is weak, as fisherman perceive captures over periods as substitutes.

We log-linearize the first order condition around the steady-state, which we denote as y_{ss} in order to assess the effect of intertemporal preferences on the optimal trajectory in the proximity of a targeted steady-state future level. This yields the following: [3]

$$\frac{y_{t+1} - y_t}{y_t} = -\frac{1}{\sigma}\frac{s_{t+1} - s_t}{s_t} \Rightarrow \rho = -\frac{\frac{y_{t+1}-y_t}{y_t}}{\frac{s_{t+1}-s_t}{s_t}}. \tag{1}$$

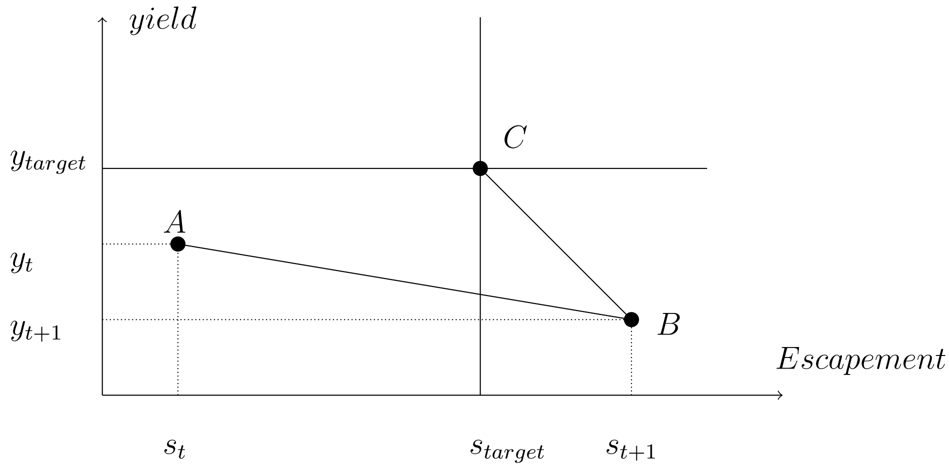

Figure 1: Transition path with smoothing intertemporal harvesting preferences (fluctuation risk averse fishermen), $\rho \to 0$ ($\sigma > 0$).

This shows that preferences for stability in yields can be represented by σ, and therefore assumed to be constant over the planning period, with the EIS in yield reflecting the response of harvest growth to fluctuations in escapement. EIS represents the ratio of the percentage variation in yield to the percentage variation in escapement. Note that along the transitional path to the steady state the escapement and harvest growth rates are negatively correlated,

[3]Log-linearising the first order condition around steady-state

$$\underbrace{U''(y_{ss})y_{ss}\ln\frac{y_t}{y_{ss}}}_{\simeq U'(y_t)-U'(y_{ss})} = \beta\underbrace{V''(s_{ss})s_{ss}\ln\frac{s_t}{s_{ss}}}_{\simeq V'(s_t)-V'(s_{ss})},$$

and assuming that $V'(s_{ss}) = \lambda_{ss}s_{ss}$, and using the first order condition, we have

$$U''(y_{ss})y_{ss}\ln\frac{y_t}{y_{ss}} = \underbrace{\beta\lambda_{ss}s_{ss}}_{U'(y_{ss})}\ln\frac{s_t}{s_{ss}} = U'(y_{ss})\ln\frac{s_t}{s_{ss}},$$

which enables us to express the elasticity of intertemporal substitution as a percentage deviation of harvest between consecutive periods,

$$\ln\frac{y_t}{y_{ss}} = \frac{U'(y_{ss})}{U''(y_{ss})y_{ss}}\ln\frac{s_t}{s_{ss}} \Rightarrow \ln\frac{y_t}{y^{ss}} = -\frac{1}{\sigma}\ln(s_t/s^{ss}) \Rightarrow \Delta\ln\ y_t = -\frac{1}{\sigma}\Delta\ln\ s_t.$$

Therefore, along the transitional path we have equation (1).

and the response of escapement to a change in harvest is affected by the spawning stock biomass.

These results imply that the EIS determines the co-movement between harvest and escapement, and hence the power of fishery policy to smooth fluctuations in the fishery stock. For example, regulators often face situations in which the fishery stock is below a stationary target level, where they can either adopt a precautionary policy and reduce harvesting with the option of an increase in future harvests, or by contrast they can increase harvesting for short-term gain. Here we argue that if the answer to this management problem depends on social preferences as to yield fluctuations over time, the precautionary approach is consistent with a strong preference for stabilization of harvest levels as illustrated in Figure 1. This figure shows, consistently with Equation (1) , that a reduction in captures which allows the relative change in yield to be smaller than the consequent relative change in escapement is a transitional path to a steady state target with smoothing harvesting preferences.[4] Summarizing, we show the implications of a CES utility function in representing intertemporal preferences. We now investigate the use of CES preferences in an age-structured bio-economic fishery model.

3 CES in an age-structured model

This section uses an age-structured modeling approach for a fish population with two age classes (juveniles and adults) where, as before, the management problem involves optimizing the discounted utility derived from harvesting. We denote the number of juveniles and adults at period t by N_1 and N_2, respectively.

The stock-recruitment function is represented by φ, and μ is the maturity fraction of the juvenile population. The dynamics of the population are summarized in Table 1 following Da Rocha et al. (2013), where cycles of 2 periods are assumed. This means that each year a

[4]This outcome is consistent with the literature on uncertainty in fisheries management as described in (Holland and Herrera, 2012) and (McGough et al., 2009).

proportion of the juveniles become adults in the next period, depending on the fraction h_t of individuals (juveniles and adults) harvested in period t, with p being the fishing selectivity parameter for those of age 1. Therefore, harvesting yield in period t is $y_t = h_t(pN_1 + N_2)$. This simple dynamic enables the population of juveniles and adults in the next two periods $t+2$, denoted as N_1' and N_2', respectively, to be written as a function of N_1 and N_2, i.e. the population distribution at t.

	period t	period $t+1$	period $t+2$
number of juveniles	N_1	$\varphi(\mu N_1 + N_2)$	$N_1' = \varphi(\mu\varphi(\mu N_1 + N_2) + (1 - h_t p)N_1)$
number of adults	N_2	$(1 - h_t p)N_1$	$N_2' = (1 - h_{t+1}p)\varphi(\mu N_1 + N_2)$

Table 1: Fish population dynamics in a two-age-class model

3.1 CES Preferences from one cycle to another

We begin by considering a scenario in which preferences represented by the utility function $U(.)$, are assumed to be defined over stationary cycles given by the fish life/age cycle, where we denote $\mathbf{Y}$ as the discounted yield obtained in a two-period cycle

$$\mathbf{Y}(N_1, N_2, N_1', N_2') = y_t + \beta y_{t+1} =$$
$$h_t(N_1, N_2, N_1')(pN_1 + N_2) +$$
$$\beta h_{t+1}(N_1, N_2, N_2')[p\varphi(\mu N_1 + N_2) + (1 - h_t(N_1, N_2, N_1')p)N_1]$$

The management problem for the optimal exploitation of this fishery can be formulated as a recursive problem with a Bellman equation given by

$$V(N_1, N_2) = \max_{N_1', N_2'} U(\mathbf{Y}(N_1, N_2, N_1', N_2')) + \beta^2 V(N_1', N_2')$$

The first order conditions and envelope theorem are as follows

$$U'(\mathbf{Y}(N_1, N_2, N_1', N_2'))\frac{\partial \mathbf{Y}(N_1, N_2, N_1', N_2')}{\partial N_i'} + \beta^2 \frac{\partial V(N_1', N_2')}{\partial N_i'} = 0, \quad i = 1, 2 \tag{2}$$

$$\frac{\partial V(N_1', N_2')}{\partial N_i'} = U'(\mathbf{Y}(N_1', N_2', N_1'', N_2''))\frac{\partial \mathbf{Y}(N_1', N_2', N_1'', N_2'')}{\partial N_i'}$$

thus characterizing the solution

$$\frac{U'(\mathbf{Y}(N_1, N_2, N_1', N_2'))}{U'(\mathbf{Y}(N_1', N_2', N_1'', N_2''))} = -\beta^2 \frac{\partial \mathbf{Y}(N_1', N_2', N_1'', N_2'')/\partial N_i'}{\partial \mathbf{Y}(N_1, N_2, N_1', N_2')/\partial N_i'}, \quad i = 1, 2$$

This condition gives the harvesting quota that maximizes utility derived from the discounted fishery yield in a given cycle. It illustrates that each potential harvesting decision in the current cycle should take into account the effect of h_t and h_{t+1}, on the resulting age-structure of the population in future cycles (the state of juveniles and adults at $t+2$ and at $t+4$); i.e. it shows that current harvest decisions change population conditions and their contribution to future yields. The left-hand side represents the rate of change in the marginal utility derived from the yield in the current cycle (starting at t) with respect to the marginal utility in the next cycle (the cycle starting at $t + 2$). The right-hand side captures the opportunity costs of the current cycle's harvesting, which are given by the discounted ratio of the marginal productivity of juveniles and adults in the next cycle with respect to their productivity in the current cycle. This condition therefore acknowledges that the impact of current harvesting decisions is reflected in the marginal utility from fishing and resource productivity of future cycles.

In order to explore the trajectories in the proximity of the stationary solution, we log-linearize following a procedure equivalent to that described in the previous section (see Appendix A.1 for further details),

$$\frac{U''(\mathbf{Y_{ss}})\mathbf{Y_{ss}}}{U'(\mathbf{Y_{ss}})} \ln(\frac{\mathbf{Y_t}}{\mathbf{Y_{ss}}}) = \sum_{j=1}^{2}(\varepsilon_{i,j}^{p} - \varepsilon_{N_j'}) \ln(\frac{n_j'}{N_j}) - \sum_{j=1}^{2} \varepsilon_{N_j} \ln(\frac{n_j}{N_j}) \tag{3}$$

where n_1, n_2, n_1', n_2' represent states of the juvenile and adult population in periods t and $t+2$ in proximity to the stationary solution, and the terms $\varepsilon^p_{i,j}$, ε_{N_j}, $\varepsilon_{N_j'}$, which we denote as "elasticities", are given by

$$\varepsilon^p_{i,j} = \frac{\partial^2 V(N_1', N_2')/\partial N_j' \partial N_i'}{\partial V(N_1', N_2')/\partial N_i'} N_j$$

$$\varepsilon_{N_j} = \frac{\frac{\partial^2 \mathbf{Y}(N_1, N_2, N_1', N_2')}{\partial N_j \partial N_i'}}{\frac{\partial \mathbf{Y}(N_1, N_2, N_1', N_2')}{\partial N_i'}} N_j$$

$$\varepsilon_{N_j'} = \frac{\frac{\partial^2 \mathbf{Y}(N_1, N_2, N_1', N_2')}{\partial N_j' \partial N_i'}}{\frac{\partial \mathbf{Y}(N_1, N_2, N_1', N_2')}{\partial N_i'}} N_j$$

Elasticity $\varepsilon^p_{i,j}$ measures the responsiveness of the fishery's value function over the coming cycles with respect to future capital (i.e. those fish that survive to the next cycle). Elasticity ε_{N_j} measures the responsiveness of fishing yield ("income") to an additional increase in the number of individuals of class-age j at the beginning of the current cycle. Similarly, elasticity $\varepsilon_{N_j'}$ shows the effect on the flow of fishing yield ("income") of a change in the survival to the next cycle of an individual of class j.

Now assume CES preferences, so that the manager maximizes $U(\mathbf{Y}) = \frac{1}{1-\sigma}\mathbf{Y}^{1-\sigma}$, which implies that equation (3) can be written as

$$\sigma \ln(\frac{\mathbf{Y_t}}{\mathbf{Y_{ss}}}) = \sum_{j=1}^{2} (\varepsilon^p_{i,j} - \varepsilon_{N_j'}) \ln(\frac{n_j'}{N_j}) - \sum_{j=1}^{2} \varepsilon_{N_j} \ln(\frac{n_j}{N_j}) \tag{4}$$

Note that n and n' are not equal as they represent a state in the optimal trajectory in the proximity of the the stationary solution. In order to simplify this expression further and relate the rate of growth of the resource to its current state, we assume that a certain proportion of each age group is maintained throughout the optimal trajectory, and denote $\phi(N)_j$ as the proportion of the population of individuals of age j. The RHS of equation (4) can be approximated by

$$\sum_{j=1}^{2}(\varepsilon_{i,j}^{p}-\varepsilon_{N'_j})\ln(\frac{\phi(n')_j}{\phi(N)_j})-\sum_{j=1}^{2}\varepsilon_{N_j}\ln(\frac{\phi(n)_j}{\phi(N)_j})$$

denoting $b_j^{ss}=\ln(\phi(N)_j)$ and $b_j^{n}=\ln(\phi(n)_j)$, it results that

$$\ln(\frac{\mathbf{Y_t}}{\mathbf{Y_{ss}}})=\left[\sum_{j=1}^{2}(\varepsilon_{i,j}^{p}-\varepsilon_{N'_j})(b_j^{n'}-b_j^{ss})-\sum_{j=1}^{2}\varepsilon_{N_j}(b_j^{n}-b_j^{ss})\right]\frac{1}{\sigma}$$

again with an RHS term that can be written as

$$-\frac{1}{\sigma}\left[\sum_{j=1}^{2}(\varepsilon_{i,j}^{p}-\varepsilon_{N_j}-\varepsilon_{N'_j})b_j^{ss}-\sum_{j=1}^{2}(\varepsilon_{i,j}^{p}-\varepsilon_{N'_j})b_j^{n'}+\sum_{j=1}^{2}\varepsilon_{N_j}b_j^{n}\right]$$

Furthermore, if we assume the particular case in which n_j and n'_j represent a steady-state cycle, $n'_j=n_j$, the equation above can be expressed as

$$\sum_{j=1}^{2}\left(\varepsilon_{i,j}^{p}-\varepsilon_{N_j}-\varepsilon_{N'_j}\right)(b_j^{n}-b_j^{ss})=\sum_{j=1}^{2}\left(\varepsilon_{i,j}^{p}-\varepsilon_{N_j}-\varepsilon_{N'_j}\right)[\ln(\phi(n)_j)-\ln(\phi(N)_j)]$$

This gives the expression for the EIS as follows, using equation (4)

$$\Delta\ln\mathbf{Y}=-\frac{1}{\sigma}\sum_{j=1}^{2}\left[\varepsilon_{i,j}^{p}+(\varepsilon_{N'_j}-\varepsilon_{N_j})\right]\Delta b_j$$

where $\Delta b_j=(b_j^{n}-b_j^{ss})=\ln(\frac{\phi(n)_j}{\phi(N)_j})$ is a measure of the relative abundance of each size-class in any state n during a cycle with respect to the relative abundance of each size class in the stationary solution. For a given number of size classes, this measure is maximized when the relative frequency of each size classes in a cycle is identical to its relative abundance in the stationary level. That is, Δb_j increases with the degree of uniformity in the relative size of each age-class towards the stationary path.

This result shows that in an age-structured fishery model the EIS $(1/\sigma)$ measures the changes

in discounted yield over a cycle, $\mathbf{Y}$, with the change in the abundance of individuals in different age-classes in the fishery population. Those changes in the relative abundance of the different age-classes are assessed in terms of how the abundance of each age classes differs from its stationary targeted levels. This is an intuitive result as the relative abundance of each age-class affects recruitment and consequently population levels and fishery yields. Note also that Δb_j is moderated by the elasticities measuring how the survival to the next cycle of an individual of class j would change the fishery value function, and the productivity in fishing yield between two consequent cycles. For age class j, where Δb_j is negative (i.e. the relative abundance of age class j is smaller than its target), the relationship along the transitional path between yield and age class abundance is positive, leading to biggest catches being optimal to reach targeted stationary abundance levels. The results show that risk averse preferences (harvesting fluctuations), i.e. resistance to intertemporal substitution in catches ($\sigma > 0$), would require relatively small temporal fluctuations in discounted yields with respect to the magnitude of the increase in the abundance of age class j over its stationary level.
Most importantly, this result means that EIS depends on discounted yield over a cycle, so the assumption implicit in the specification of a CES utility function, i.e. constant intertemporal elasticity of substitution, does not hold in this case as a constant discounting process is used. This means that the value of EIS is expected to change in this model, and the EIS cannot be modeled via a single parameter. In order to maintain the assumption of constant EIS, there must be fluctuating yields. However, this contradicts the social preference for yield stability that the use of the CES utility function is intented to capture. An immediate question that arise from this remark is whether allowing the discount factor to vary would generate a constant EIS, and what implications this would have for the optimal transition of the fishery population from given initial conditions to the stationary reference target level. We explore this issue in the next section, based on year catches for the sake of simplicity and consistency with the biomass model shown above.

3.2 CES preferences as to yearly yield

In this section we use additive separate utility functions and denote $\mathbf{U}$ as the discounted utility obtained in a 2 periods cycle,

$$
\begin{aligned}
&\mathbf{U}(N_1, N_2, N_1', N_2') = u(y_t) + \mathbf{Q}u(y_{t+1}) = \\
&u(h_t(N_1, N_2, N_1')(pN_1 + N_2)) + \\
&\mathbf{Q}u(h_{t+1}(N_1, N_2, N_2')[p\varphi(\mu N_1 + N_2) + (1 - h_t(N_1, N_2, N_1')p)N_1])
\end{aligned}
$$

where $\mathbf{Q}$ is a given discount factor, and where no assumption is made at this stage as to the invariability of the discounting process.

Given a management problem for the optimal exploitation for this fishery as formulated above, and following similar steps, the f.o.c. is the following:

$$
\frac{\partial}{\partial N_i'}\mathbf{U}(N_1, N_2, N_1', N_2') = -\beta^2 \, \frac{\partial}{\partial N_i'}\mathbf{U}(N_1', N_2', N_1'', N_2'') \quad i = 1, 2
$$

This condition gives the fraction of the fish population to be harvested, h_t and h_{t+1}, that maximizes the discounted utility derived from yield over a cycle. For ease of interpretation it can be re-written as

$$
\begin{aligned}
&u'(y_t)\frac{\partial \mathbf{y_t}(N_1, N_2, N_1', N_2')}{\partial N_i'} + \mathbf{Q}u'(y_{t+1})\frac{\partial \mathbf{y_{t+1}}(N_1, N_2, N_1', N_2')}{\partial N_i'} = \\
&-\beta^2 \, \frac{\partial}{\partial N_i'}\mathbf{U}(N_1', N_2', N_1'', N_2'') \quad i = 1, 2
\end{aligned}
$$

Similarly to the previous section, the left-hand side is the rate of change in the discounted marginal utility derived from the current cycle's yield with respect to the number of juveniles and adults in the next cycle. It considers (i) the effect of choosing the level of yield on the marginal utility; and (ii) the effect of altering the age structure of the population through the harvesting decision as to the yield. The right-hand side represents the opportunity costs of undertaking additional harvesting in the current cycle. This is given by discounted marginal utility in the following cycle with respect to the fish population age structure. This condition captures the effects that current harvesting decisions may therefore cause in the distribution of ages in the fish population, altering the contribution of each age class to the yield (and reproduction), and affecting utility in the future.

We log-linearize following a procedure equivalent to that described in the previous section. If within each cycle a steady state solution is assumed, i.e. $u'(y_t) = u'(y_{t+1})$:

$$u'(y_t) = -\beta^2 \mathbf{H_i}(N_1, N_2, N_1', N_2')$$

where

$$\mathbf{H_i}(N_1, N_2, N_1', N_2') = \frac{\frac{\partial V(N_1', N_2')}{\partial N_i'}}{\frac{\partial \mathbf{y_t}(N_1, N_2, N_1', N_2')}{\partial N_i'} + \mathbf{Q} \frac{\partial \mathbf{y_{t+1}}(N_1, N_2, N_1', N_2')}{\partial N_i'}} \quad i = 1, 2$$

In the proximity of the stationary solution, where n and n' represent (as above) a state in the optimal trajectory in the proximity of that stationary solution, note that now the optimal condition depends on the yield harvested during the cycle,

$$u'(y_t) = -\beta^2 \mathbf{H_i}(n_1, n_2, n_1', n_2', y_t, y_{t+1})$$

where

$$\mathbf{H_i}(n_1, n_2, n_1', n_2', y_t, y_{t+1}) = \frac{\frac{\partial V(n_1', n_2')}{\partial N_i'}}{\frac{\partial \mathbf{y_t}(n_1, n_2, n_1', n_2')}{\partial N_i'} + \mathbf{Q} \frac{u'(y_{t+1})}{u'(y_t)} \frac{\partial \mathbf{y_{t+1}}(n_1, n_2, n_1', n_2')}{\partial N_i'}}$$

Proposition 1. *Assuming a constant elasticity of substitution over yearly catches,* $u(y) = \frac{1}{1-\sigma} y^{1-\sigma}$*, if we consider the following*

1. *Constant discount factor,* $\mathbf{Q} = \beta$*, then*

$$\sigma \ln(\frac{y_t}{y_{ss}}) - C_{i,1} \ln(\frac{y_{t+1}}{y_t}) = -\sum_{j=1}^{2} \left[\varepsilon_{i,j}^p + (\varepsilon_{N_j'} - \varepsilon_{N_j}) \right] \Delta b_j \tag{5}$$

2. Stochastic discount factor, $\mathbf{Q} = \beta \frac{u'(y_t)}{u'(y_{t+1})}$*, then*

$$\sigma \ln(\frac{y_t}{y_{ss}}) = -\sum_{j=1}^{2} \left[\varepsilon^p_{i,j} + (\varepsilon_{N'_j} - \varepsilon_{N_j}) \right] \Delta b_j \tag{6}$$

Proof See Appendix A.2.

Consider that $\sum_{j=1}^{2} \left[\varepsilon^p_{i,j} + (\varepsilon_{N'_j} - \varepsilon_{N_j}) \right] \Delta b_j > 0$. Given that $C_{i,1} < 0$ if we start a transition path from a "biomass level" lower than the steady state then

1. The discount factor is constant, $\mathbf{Q} = \beta$, then

$$\sigma \ln(\frac{y_{t+1}}{y_t}) - C_{i,1} \ln(\frac{y_{t+2}}{y_{t+1}}) < 0$$

2. The stochastic discount factor, $\mathbf{Q} = \beta \frac{u'(y_{t+1})}{u'(y_t)}$, then

$$\sigma \ln(\frac{y_{t+1}}{y_t}) < 0$$

Under a stochastic discount rate scenario, the implicit property of the CES utility function, i.e., the stationarity requirement of EIS is met (6). Therefore implementing stability preferences with the introduction of a CES function is a suitable option for capturing smoothing over catches in fishery management problems. This result means that it is possible to maintain a given proportionality along the transitional path between the rate of variation in yield and the rate of variation in age structure of the population over periods. Averse preferences as to fluctuations in yearly catches are associated with an optimal allocation of harvesting activities on the transition path with relatively small decreases in yields if the biomass falls below target levels. These changes in yields would result in an increase in the relative abundance of the age classes to more than the stationary level for which $\left[\varepsilon^p_{i,j} + (\varepsilon_{N'_j} - \varepsilon_{N_j}) \right] > 0$, i.e. more individuals means higher productivity and a greater value of the fishery. This is illustrated in Figure (3), which shows that this strategy is in accordance with the Precau-

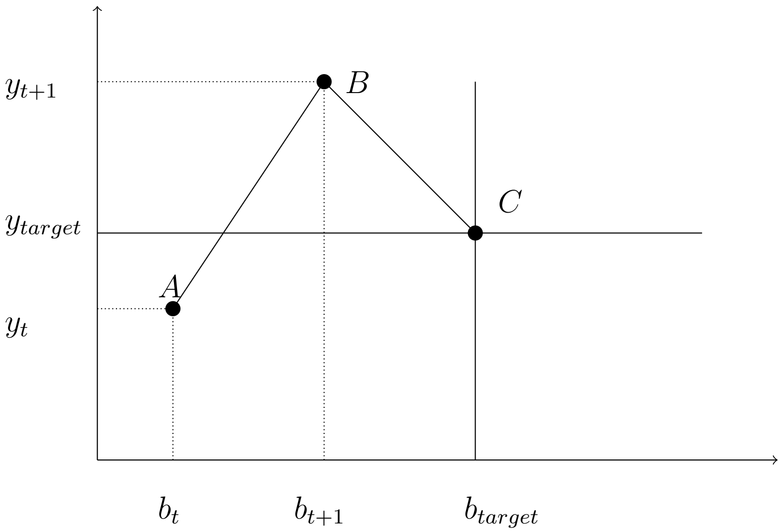

Figure 2: Transition path with $\mathbf{Q} = \beta$

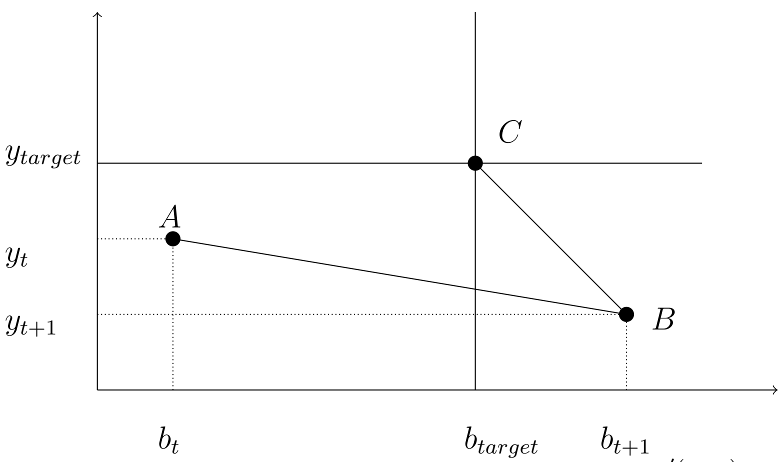

Figure 3: Transition path with $\mathbf{Q} = \beta \frac{u'(y_{t+1})}{u'(y_t)}$

tionary Principle. However, equation (7) implies that the use of a time constant discount rate would lead to a situation where an increase in harvesting is the optimal response to a fall in biomass levels, as illustrated in Figure (2).

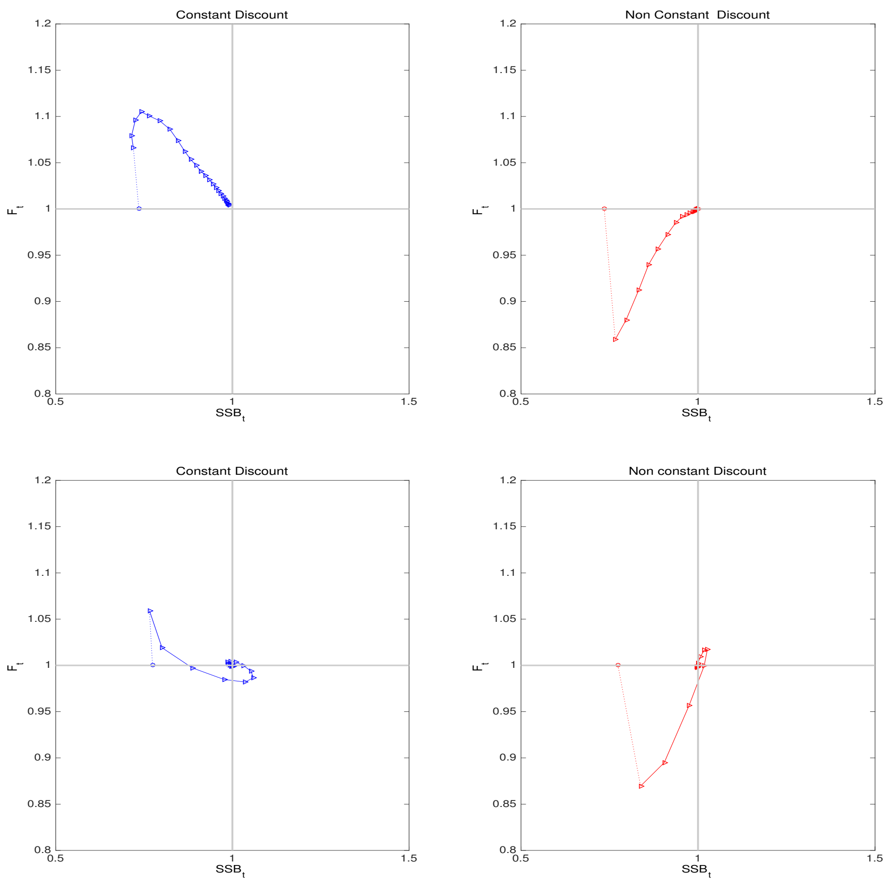

Figure 4: Combinations of yield and SSB along optimal exploitation paths. The parameterization for the age-structured population is the same as that shown in Da Rocha et al. (2012). The left-hand panels (blue) represent preferences for smoothness (with $\sigma = 1$ at the top, and $\sigma = 2.5$at the bottom) and constant discount rates. The right-hand panels (red) represent the same preferences for smoothness and non constant discount rates. The usual positive relationship between fishing mortality and SSB is observed in the right-hand panels: when the SSB is below its stationary value the optimal fishing mortality is reduced to below the stationary value and vice versa. However this relationship is reversed in the left-hand panels.

4 Conclusions

Fisheries management increasingly acknowledges that the variability in returns of a fishery should be considered when seeking management strategies aimed at the MEY.[5] In this context, we investigate the implications of introducing invariant intertemporal preferences as to yield stability in natural resource management problems where age-structured is crucial.

Intertemporal preferences as to yield stability are compromise approaches that allow management to adapt to natural fluctuations in stock abundance and reduce the uncertainty of future fishing opportunities and the financial risk for fishermen, who depend on a stable economic return over time. Moreover, when there is low yield variability in fisheries there is also a more stable food supply, which prevents market saturation or price spikes, and contributes to food security. These are common features shared with agriculture food production, also characterised by high financial risk as a result of high year-to-year variation particularly for farmers in developing countries, and by the economic importance of having reliable food production (see e.g. Kasperski and Holland, 2013; Deepak et al., 2015). In forestry, continuous cover forestry projects are attracting increasing interest as alternatives to rotation systems of forest management (see e.g. Pukkala et al., 2010; Price and Price, 2006; Tahvonen, 2015) in order to best combine timber production with other multiple benefits, where it is not only the timber stock in a long term objective that is important but also the impact that tree harvesting has on the maintenance of other benefits (recreational, biodiversity, non-timber products, carbon storage, etc) (see e.g. Touza et al., 2008; Goetz et al., 2010; Kuuluvainen et al., 2012).

Our results analytically show that in an age-structured model with a CES utility function, if some proportionality is to be maintained between growth rates of annual catches and the age-structure equivalent to biomass (i.e. the changes in the age-diversity of the population

[5] For example, Current fishery regulations often include a cap on interannual TAC variations (e.g. Kell et al., 2006; Penas, 2007; Baudron et al., 2010), and catch-quota balancing schemes such as retroactive catch balancing are applied for example in Iceland where fishermen have the ability to carry forward unused quota or borrow from the next year's allocation (e.g. Woods et al., 2015).

with respect to the steady-state target), then the discount rate should be non constant. We can therefore conclude that preferences over time expressed as to the discounting process are key in managing age-structured resources. Constant discounting would not guarantee proportional growth rates, and would also have implications for the transition of the resource towards the steady state. The use of a time-constant discount rate would lead to a situation where an increase in harvesting was the optimal response to a fall in biomass levels. Such a policy goes against the precautionary principle followed by most fisheries agencies, where the desirability of implementing alternative harvesting control rules is explored in line with their ability to steer toward an optimal exploitation level, stabilizing catches and driving stock to safe levels (Da Rocha et al., 2016).

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A Appendix

A.1 Log-linearizing yield over cycles

Log-linearization explores the trajectories in the proximity of the stationary solution in section 3.1, with equation (2) being written as follows:

$$U'(\mathbf{Y}(N_1,N_2,N_1',N_2')) = -\beta^2 \mathbf{H_i}(N_1,N_2,N_1',N_2'),$$

where

$$\mathbf{H_i}(N_1,N_2,N_1',N_2') = \frac{\frac{\partial V(N_1',N_2')}{\partial N_i'}}{\frac{\partial \mathbf{Y}(N_1,N_2,N_1',N_2')}{\partial N_i'}} \quad i=1,2.$$

By log-linearizing $\mathbf{H_i}(n_1,n_2,n_1',n_2')$ around $\mathbf{H_i}(N_1,N_2,N_1',N_2')$, where n_1, n_2, n_1', n_2' represent states of the juvenile and adult population in periods t y $t+2$ in the proximity of the stationary solution, we obtain,

$$\begin{aligned}
\mathbf{H_i}(n_1,n_2,n_1',n_2') \;-\; & \mathbf{H_i}(N_1,N_2,N_1',N_2') \\
= \; & \sum_{j=1}^{2} \underbrace{\left(\frac{\partial \mathbf{H_i}(N_1,N_2,N_1',N_2')}{\partial N_j} N_j\right)}_{B_{i,j}} (\ln n_j - \ln N_j) \\
+ \; & \sum_{j=1}^{2} \underbrace{\left(\frac{\partial \mathbf{H_i}(N_1,N_2,N_1',N_2')}{\partial N_j'} N_j'\right)}_{A_{i,j}} (\ln n_j' - \ln N_j),
\end{aligned}$$

where

$$B_{i,j} = -\mathbf{H_i}(N_1,N_2,N_1',N_2') \left(\frac{\frac{\partial^2 \mathbf{Y}(N_1,N_2,N_1',N_2')}{\partial N_j \partial N_i'}}{\frac{\partial \mathbf{Y}(N_1,N_2,N_1',N_2')}{\partial N_i'}} \right) N_j$$

$$A_{i,j} = \mathbf{H_i}(N_1,N_2,N_1',N_2') \left(\frac{\partial^2 V(N_1',N_2')/\partial N_j' \partial N_i'}{\partial V(N_1',N_2')/\partial N_i'} - \frac{\frac{\partial^2 \mathbf{Y}(N_1,N_2,N_1',N_2')}{\partial N_j' \partial N_i'}}{\frac{\partial \mathbf{Y}(N_1,N_2,N_1',N_2')}{\partial N_i'}} \right) N_j'.$$

Given that in a stationary solution $N_j' = N_j$, $\Delta\mathbf{H_i} = \mathbf{H_i}(n_1, n_2, n_1', n_2') - \mathbf{H_i}(N_1, N_2, N_1', N_2')$ can be expressed as

$$\Delta\mathbf{H_i} = A_{i,1}\ln(\frac{n_1'}{N_1}) + A_{i,2}\ln(\frac{n_2'}{N_2}) + B_{i,1}\ln(\frac{n_1}{N_1}) + B_{i,2}\ln(\frac{n_2}{N_2}).$$

If we denote

$$\varepsilon_{i,j}^p = \frac{\partial^2 V(N_1', N_2')/\partial N_j' \partial N_i'}{\partial V(N_1', N_2')/\partial N_i'} N_j,$$

$$\varepsilon_{N_j} = \frac{\frac{\partial^2 \mathbf{Y}(N_1,N_2,N_1',N_2')}{\partial N_j \partial N_i'}}{\frac{\partial \mathbf{Y}(N_1,N_2,N_1',N_2')}{\partial N_i'}} N_j,$$

$$\varepsilon_{N_j'} = \frac{\frac{\partial^2 \mathbf{Y}(N_1,N_2,N_1',N_2')}{\partial N_j' \partial N_i'}}{\frac{\partial \mathbf{Y}(N_1,N_2,N_1',N_2')}{\partial N_i'}} N_j,$$

and log-linearize the left-hand term of the f.o.c. as given in equation (A.1) yields equation (3)

$$\frac{U''(\mathbf{Y_{ss}})\mathbf{Y_{ss}}}{U'(\mathbf{Y_{ss}})}\ln(\frac{\mathbf{Y_t}}{\mathbf{Y_{ss}}}) = \sum_{j=1}^{2}(\varepsilon_{i,j}^p - \varepsilon_{N_j'})\ln(\frac{n_j'}{N_j}) - \sum_{j=1}^{2}\varepsilon_{N_j}\ln(\frac{n_j}{N_j}).$$

A.2 Proof of Proposition 1

Log-linearizing $\mathbf{H_i}(n_1, n_2, n_1', n_2', y_t, y_{t+1})$ around $\mathbf{H_i}(N_1, N_2, N_1', N_2')$, where n_1, n_2, n_1', n_2', as above, represent states of the juvenile and adult population in periods t y $t+2$ in the proximity of the steady state, yields,

$$\begin{aligned}
\mathbf{H_i}(n_1, n_2, n_1', n_2', y_t, y_{t+1}) \quad - \quad & \mathbf{H_i}(N_1, N_2, N_1', N_2') \\
= \quad & \sum_{j=1}^{2}\underbrace{(\frac{\partial \mathbf{H_i}(n_1, n_2, n_1', n_2', y_t, y_{t+1})}{\partial N_j}N_j)}_{B_{i,j}}(\ln n_j - \ln N_j) \\
+ \quad & \sum_{j=1}^{2}\underbrace{(\frac{\partial \mathbf{H_i}(n_1, n_2, n_1', n_2', y_t, y_{t+1})}{\partial N_j'}N_j')}_{A_{i,j}}(\ln n_j' - \ln N_j) \\
+ \quad & \sum_{j=0}^{1}\underbrace{(\frac{\partial \mathbf{H_i}(n_1, n_2, n_1', n_2', y_t, y_{t+1})}{\partial y_{t+j}}y_{ss})}_{C_{i,j}}(\ln y_{t+j} - \ln y_{ss}),
\end{aligned}$$

and

$$\frac{u''(y_{ss})y_{ss}}{u'(y_{ss})}\ln(\frac{y_t}{y_{ss}}) - (C_{i,1}\ln y_{t+1} + C_{i,0}\ln y_t) + (C_{i,1}+C_{i,0}))\ln y_{ss}$$
$$= \sum_{j=1}^{2}(\varepsilon_{i,j}^{p} - \varepsilon_{N_j'})\ln(\frac{n_j'}{N_j}) - \sum_{j=1}^{2}\varepsilon_{N_j}\ln(\frac{n_j}{N_j}).$$

When $u(y) = \frac{1}{1-\sigma}y^{1-\sigma}$, and given that (in steady state $C_{i,0} = -C_{i,1}$), we have

$$\sigma\ln(\frac{y_t}{y_{ss}}) - C_{i,1}\ln(\frac{y_{t+1}}{y_t}) = \sum_{j=1}^{2}(\varepsilon_{i,j}^{p} - \varepsilon_{N_j'})\ln(\frac{n_j'}{N_j}) - \sum_{j=1}^{2}\varepsilon_{N_j}\ln(\frac{n_j}{N_j}).$$

Therefore, using a procedure similar to that in section 3.1, we have equations (5) and (6).