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Abstract

When individual stay/exit decisions depend on the opportunity cost of exiting, capital malleability is endogenously determined by the instruments used for stock rehabilitation. In a General Equilibrium framework, we characterize the transitional dynamics caused by stock rehabilitation policies. We show that a management policy based on input controls generates less exit, a less productive fleet, and overcapitalization, as input controls require a higher number of firms to achieve the same biological targets. Using data from the Multiannual Plan for the Western Mediterranean, we show that the use of input controls generates a Spanish fleet around 14 percent higher than the one that would result from a non distortionary instrument.

Keywords: Firm dynamics, Investment, General Equilibrium, Fisheries.
JEL codes: Q22

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1 Introduccion

We extend Da-Rocha and Sempere (2016) to study the transitional dynamics of firms when individual stay/exit decisions depend on expected future opportunity benefits that are afected by public policy. As a particular case, we use the model to study the dynamics of firms when a a policy of stock rehabilitation is implemented in a marine fishery.

One of the most important challenges for the management of marine fisheries, as is the case in other industries, is over-capitalization. In their seminal paper, Clark et al. (1979) showed that different assumptions about capital malleability[1] have a significant influence on the form of physical capital dynamics. In particular, overcapitalization is associated to non malleability of capital.

Even when disinvestment in physical capital is not feasible for an individual vessel, the depreciation rate is equal to zero and capital has a negligible scrap value, malleability of physical capital at the fishery level is closely related to the decisions of entry and exit taken by firms. In this paper, we show that when, as in Ikiara and Odink (1999), individual stay/exit decisions depend on the opportunity cost of exiting, capital malleability is endogenously determined by the instruments used for stock rehabilitation. In particular, we characterize the transitional dynamics caused by stock rehabilitation policies in a fishery and show that, along the stock rehabilitation path, capital malleability depends on the type of management control, as some of the policies induce capital reductions (through enough exit of firms) and some others not.

As Weninger and Just (2002), we assume that individual firm's abilities follow a stochastic process and that there is a fixed operating cost that firms must incur if they want to remain in the fishery. Those two assumptions generate firm dynamics over time. Therefore, in our environment, individual rational decisions will not be based only on current profits

[1] The term "malleability" is commonly used to refer to the existence or not of constraints on the disinvestment of capital assets. If these constraints do not exist, then capital is fully malleable.

and the whole transitional dynamic –induced by the instrument used to achieve the stock rehabilitation objective– must be computed to capture firm behavior and its consequences on the economic variables. We also assume that the fishery is operated by heterogeneous agents as in (Clark, 1980; Terrebonne, 1995; Heaps, 2003) to relate (expected) future opportunity benefits for firms with the policy instruments.

Following Homans and Wilen (1997), we assume that the instruments chosen by managers to achieve the biological targets are exogenously determined.[2]. We compute fleet dynamics based on individual stay/exit decisions when managers use a non distortionary instrument (i.e. taxes, ITQs), and also when managers use input controls – the basic instruments in the command and control management approach used currently in many fisheries. We show that a management policy based on input controls generates less exit, a less productive fleet, and more overcapitalization. In particular, we show that this policy leads to smaller vessels with lower yield and individual profits, and lower wages. The less productive vessels stay in the fishery and pay the iddling cost waiting for better times and this reduces average factor productivity of the fleet. The result of input controls is that a higher number of vessels is required to achieve the same biological targets, and this implies an over-capitalized fleet.

Our results would be supported by the empirical evidence provided by the spanish fleet. As the management system in the Mediterranean is mainly based on effort restrictions (limitations on average days at sea and other measure of time per vessel), our results would imply that we shoud expect more overcapitalization in fleets operating in the Mediterranean than in fleets operating in the Atlantic. Figure 1 shows the status of the Spanish fleets. The long-term economic profitability of vessels as measured by the Return on Fixed Tangible Assets (ROFTA) is plotted on the y-axis, and the Sustainable Harvest Indicator (SHI) on the x-axis. Values of SHI greater than 1.2 represent that fleets are operating under biological imbalance. The figure shows indeed that Mediterranean fleets operate under lower ROFTA

[2]For an analysis of the optimal combination of instruments under stock uncertainty see Da-Rocha and Gutiérrez (2012).

and greater biological imbalance than Alantic fleets. This suggests (more) overcapitalization in the Mediterranean (than in Atlantic) fleets.

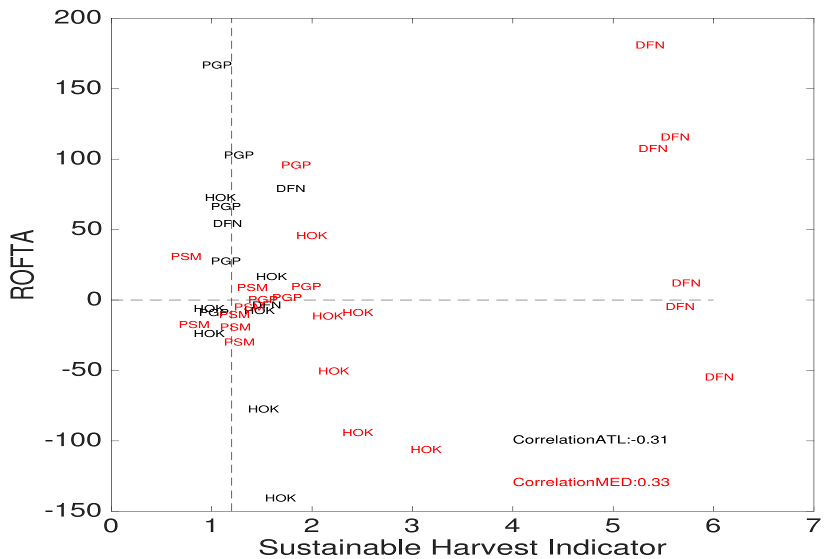


Figure 1: ROFTA (Rofta (%)= Net profit/Capital Value, This measures the sector's long-term economic profitability) and Sustainable Harvest Indicator (Measures how much a fleet segment depends on overexploited stocks at levels above MSY for its revenues; Greater than than 1.2 = biological imbalance) for different Spanish fleet segments. Red Med Black Atlantic. Source: MAAMMA (2014)

The analysis of firms' dynamics based on individual stay/exit decisions has received much less attention in the economics literature than the analysis of optimal capacity investment paths under the assumption of a sole fleet owner. Indeed, in the spirit of (Smith, 1968, 1969) the literature has mostly focused on models in which capital is assumed to be equal to the number of vessels in fleets composed by homogeneous vessels. Stay/exit decisions are modeled as an investment/disinvestment decision, and (usually) a sole fleet owner choses the optimal fleet size, or the capacity utilization under different assumptions on investment cost (Boyce, 1995; Nøstbakken, 2008; Sandal et al., 2007), stock dynamics (Botsford and Wainwright, 1985), stock uncertainty (Hannesson, 1987; Singh et al., 2006; Da-Rocha et al., 2014b) or the strategic effect of irreversible investment decisions under an strategic environment (Sumaila, 1995).[3]

We depart significantly from this literature. Our paper follows closely Weninger and Just (2002) and Da-Rocha and Sempere (2016) where individual (exit/stay) decisions depend on the (expected) future opportunity cost of exiting. In fact, in our model, in each moment of time individual firms assess the expected value of remaining in the industry, and compare it to the present discounted value of profits associated with exiting the industry. Based on this comparison, individual firms decide to stay in or exit the industry. The aggregate behavior of individual firms, and not the decision of a monopolistic fleet owner, determines the dynamics of capital in the industry.

The rest of the paper is organized as follows: Section 2 describes the model and characterizes the equilibrium of the model. Section 3 discusses de case study. Section 4 presents our results distinguishing those refering to the steady state from those regarding the transitional dynamics of the model. Finally Section 5 presents some conclusions.

2 The Model

We consider a natural resource industry with heterogeneous firms. This industry is output constrained by a regulatory agency in order to achieve the rehabilitation of a given stock.

There are two markets in the economy: a final goods and a labor (which is used to produce the final good) market. Taking output price as the numeraire, we denote wages by $w(t)$. We asume that a continuum of identical households, which own the firms, consume the final good and supply labour by solving a consumption-leisure maximization problem.

We assume that firms, which have a finite lifespan, are heterogeneous. Let $g(z, t)$ be the measure of firms over time (i.e. the number of firms with productivity z at time t). Incumbent

[3]For an excellent summary of the literature see Nøstbakken et al. (2011).

firms' decision rules at period t depend on z. We denote as $y(z,t)$ and $l(z,t)$ the optimal choice of output and labor.

As Weninger and Just (2002), we assume that individual firms' abilities follow a stochastic process and that there is a fixed operating cost of c_f. That is, if a firm wants to remain active in the industry then it must pay the fixed cost. These two assumptions make that individual firms change over time. In each particular moment, some of them expand production, hiring staff; others contract production, firing staff; and others exit the industry.

The incumbent firms' decision problem produces two types of decision rule. There are continuous decision rules for the optimal choice of output $y(z,t)$ and labor $l(z,t)$, and there is a discrete decision rule for the optimal stay/exit decision.

Therefore, on one hand, we have *endogenous exit*. This decision depends on each period's employment $l(z,t)$ and output $y(z,t)$. Conditional on each period's choices, $l(z,t)$ and $y(z,t)$, the firm must assess the expected value of remaining in the industry, and must compare it to the present discounted value of profits associated with exiting the industry $S(t)$ –a scrap value. On the other hand, a finite vessel lifespan implies *depreciation*. Finally, managers of the fisheries allow *entry* when quota exceeds fleet capacity. Note that in contrast to the standard framework, the distribution of firms' productivity is not exogenous. In our model it is endogenously determined by the firms decisions on exit. Therefore, $g(z,t)$ evolves over time.

We analyze the model in three steps. First we solve the individual problems of firms and households. This establishes the relationship between input controls and exit decisions. Later, we specify the dynamics of the distribution of firms and the feasibility conditions. Finally, we define the equilibrium.

The problem of incumbent firms Let τ_l be a constraint on effort (in particular, τ_l will be the maximum number of hours of labor per vessel). Conditional on this constraint, firms maximize profits subject to their available technology, $y = \sqrt{z\, l}$.[4] Thus, at time t, the intra-temporal profit maximization problem is

$$\max_{l(t),y(t)} \quad y(t) - w(t)l(t) - c_f,$$
$$s.t. \quad y(t) = \sqrt{z \;\; l(t)},$$
$$l(t) \leq \tau_l,$$

where profits are defined as revenues $y(t)$ less labor costs $w(t)l(t)$ less the fixed operation cost c_f. Note that we assume that fishermen behavior is non affected by stock variability, –which is consistent with the findings of Ward and Sutinen (1994)– and that physical capital at vessel level is non-malleable (and therefore we can normalize capital per vessel to one). Solving for the first order conditions of this problem, we have that labor demand, given by

$$l(t,z) = \begin{cases} \frac{z}{4w(t)^2} & \text{if} \quad z \leq z^c(t), \\ \\ \tau_l & \text{if} \quad z > z^c(t), \end{cases}$$

and profits, given by

$$\pi(t,z) = \begin{cases} \pi(t)z - c_f & \text{if} \quad z \leq z^c(t), \\ \\ \sqrt{z\tau_l} - w\tau_l - cf & \text{if} \quad z > z^c(t). \end{cases}$$

depend on the input constraint.

We assume that the productivity shock z follows a stochastic process with a negative expected growth rate, μ, i.e.

$$dz = -\mu dt + \sigma_z dW,$$

[4] Our technology is in accordance with the fifty-fifty rule, i.e. 50% of net revenues are accounted for by payments to crew members.

where σ_z is the per-unit time volatility, and dW is the random increment to a Weiner process. The dynamic incumbents' problem is an *stopping time problem* defined by:

$$\begin{aligned} v(z,t) &= \max_{\tau} E_0 \int_0^{\tau} \pi(z,t) e^{(\rho+\lambda)t} dt + S(t) e^{\rho t}, \\ & s.t. \quad dz = -\mu z dt + \sigma_z dw_z. \end{aligned}$$

where λ is the exogenous death rate of firms.[5] Let, z be such that the establishment does not exit. Then the following Hamilton-Jacobi-Bellman (HJB) equation holds

$$(\rho+\lambda) v(z,t) \;\; = \pi(t) z - c_f + \mu z \partial_z v(z,t) + \frac{\sigma_z^2}{2} \partial_{zz} v(z,t) + \partial_t v(z,t).$$

The value matching and the smooth pasting conditions at the switching point $\underline{z}$ are $v(\underline{z},t) = S(t)$ and $v'(\underline{z},t) = 0$, respectively. For z lower than the exit threshold, $z \leq \underline{z}$, we have $v(z,t) = S(t)$. The incumbent's problem can also be witten as a HJB variational inequality, i.e.

$$\min_{I_{\text{exit}}(z,t)} \left\{ (\rho+\lambda) v(z,t) - \pi(t) z + c_f - \mu z \partial_z v(z,t) - \frac{\sigma_z^2}{2} \partial_{zz} v(z,t) - \partial_t v(z,t), v(z,t) - S(t) \right\} \quad (1)$$

where $I_{\text{exit}}(z,t)$ is an indicator function that summarises the endogeneous decision of exit.

Household's problem Each representative household solves a static consumption-leisure maximization problem:

$$\max_{C,L} \log C - eL,$$

subject to the budget constraint $C = w(t)L + \Pi(t)$, where the right-hand side of the budget constraint is given by the wage income wL and the total profits of operating firms, Π.[6]

[5] The death rate of firms is equal to the inverse of the vessel lifetime.

[6] Controls on inputs/outputs per vessel generate unemployment and (potentially) introduce heterogeneity in househodls. We apply a convenient technical devise developed by Hansen (1985) and Rogerson (1988) to simplify the problem and use the representative household framework to solve the problem. That is, we

Notice that wages are determined by

$$w(t) = e[w(t)L(t) + \Pi(t)].$$

Firm dynamics For prices to be calculated, the dynamics of firms must be computed. In our economy, the evolution of the measure of firms is determined endogenously by entry/exit decisions made by firms themselves. Formally, $g(z,t)$ follows a Kolmogorov-Fokker-Planck (KFP) equation

$$\partial_t g(z,t) = -\partial_z[\mu z g(z,t)] + \frac{\sigma_z^2}{2}\partial_{zz} g(z,t) - (I_{\text{exit}}(z,t) + \lambda)g(z,t) + g^e(z,t). \tag{2}$$

where, entry, when it is allowed, is given by the distribution $g^e(z,t)$.

Notice that the mass of firms, $N(t) = \int_{\underline{z}(t)}^{\infty} g(z)dz$ represents the number of firms. Therefore, $N(t)$ is equal to capital in period t in Clark et al. (1979). Therefore, investment in "capital", satisfies $N(t+dt) = N(t) + I(t)$. Then, investment is equal to

$$I(t) = \int_{\underline{z}(t)}^{\infty} \left[g^e(z,t) - (I_{\text{exit}}(z,t) + \lambda)g(z,t)\right] dz.$$

The term "non-malleability" is commonly used to refer to the existence of constraints on the disinvestment of capital assets utilized in exploiting the resource stock. Therefore, capital is non-malleable if $0 \leq I(t) \leq \infty$.

Feasibility conditions To close the model we need to define feasibility conditions. The

assume the existence of a lottery such that each household has the same probability p_n of being selected to work. Therefore, in expected terms, each household will work $p_n L$ hours. Note that the rules of this lottery imply that there is perfect insurance in the sense that every household gets paid whether she works or not. Hence, they will have identical consumption, i.e. $C = wL + \Pi$. Under these conditions, the utility function associated with the lottery is quasilinear in labour.

$(I_{\text{exit}}(z,t)+\lambda)\ \ g(z,t)$ exit and death

Incumbents at time t $\nearrow$

$g(z,t)$ $\searrow$

$1-(I_{\text{exit}}(z,t)+\lambda)\ \ g(z,t)$ survivors

$\downarrow$

Incumbents at time t+dt

$\uparrow$

entrants $g^e(z,t)=g^{\text{ss}}(z)$

Figure 2: Firm dynamics

household budget constraint implies that the final output market is in equilibrium. That is,

$$C \ = \ wL+\Pi \Rightarrow C=\int_{\underline{z}(t)}^{\infty} y(z,t)g(z,t)dz - c_f N(t),$$

where $c_f N(t)$ is the value of ouput allocated to produce the fixed operating cost.[7] The manager of the fishery sets the input control such that the individual decisions given by

$$y(t,z)=\begin{cases} y(t,z)^{*}=\frac{z}{2w(t)}- & \text{if} \quad z\leq z^{c}(t) \\ \\ y(t,z)^{c}=\sqrt{z\tau_l} & \text{if} \quad z>z^{c}(t) \end{cases}$$

satisfy the quota path, $Q(t)$. Therefore, feasibility conditions in the labour and output

[7]Note that C is equal to

$$wL+\Pi=\int_{\underline{z}(t)}^{\infty} w(t)l(t)g(z,t)dz+\int_{\underline{z}(t)}^{\infty}\left(y(t)-w(t)l(t)-c_f\right)g(z,t)dz=\int_{\underline{z}(t)}^{\infty} y(z,t)g(z,t)dz-c_fN(t).$$

markets are given respectively by

$$\int_{\underline{z}(t)}^{\infty} l(z,t)g(z,t)dz = L(t), \tag{3}$$

$$\int_{\underline{z}(t)}^{z^c(t)} y^*(t)g(z,t)dz + \int_{z^c(t)}^{\infty} y(t,z)^c g(z,t)dz = Q(t). \tag{4}$$

Note that, given $Q(t)$, equations (3 -4) jointly determine $w(t)$ and $\underline{z}(t)$. Moreover, after some manipulation, we can write the wage as a function of e, Q and the mass of firms, $N(t)$, i.e

$$w(t) = e\left[Q(t) - c_f N(t)\right].$$

2.1 Definition of equilibrium

Given an output restriction, $Q(t)$, and an input control τ_l, an equilibrium is a measure of firms $g(z,t)$, wages $w(t)$, incumbents' value functions $v(z,t)$, individual decision rules $l(z,t)$, $y(z,t)$ and a threshold $\underline{z}(t)$, such that:

i) (Firm optimization) Given prices $w(t)$, the exit rule, $I_{\text{exit}}(z,t)$ and $v(z,t)$ solve incumbent problem, equation (1), and $l(z,t)$, $y(z,t)$, are optimal policy functions.

ii) (Firm measure) $g(z,t)$, satisfies the Kolmogorov-Fokker-Planck equation (2).

ii) (Market clearing-feasibility) Given individual decision rules, and the firms' measure function, $w(t)$ and $\underline{z}(t)$, solve equations (3 -4).

Steady State The economy can be represented by the following system of equations

$$\min_{I_{\text{exit}}(z)} \left\{ \rho v(z) - \pi(z) + c_f - \mu z \partial_z v(z) - \frac{\sigma_z^2}{2} \partial_{zz} v(z), \quad v(z) - S \right\},$$

$$-\partial_z[\mu z g(z)] + \frac{\sigma_z^2}{2} \partial_{zz} g(z) - (I_{\text{exit}}(z) + \lambda) g(z,t) + g^e(z,t) = 0,$$

$$\int_{\underline{z}}^{\infty} g(z) dz = N,$$

$$\int_{\underline{z}}^{z^c} y^* g(z) dz + \int_{z^c}^{\infty} y(z)^c g(z) dz = Q,$$

$$e \left[Q - c_f M \right] = w.$$

Finally, note that in an stationary equilibrium $g^e(z) = g(z)$ and $I = 0$.

3 Case study

We apply the model to assess the impact of inputs controls on the Spanish demersal fleet in the Mediterranean Sea. Data comes from the Expert Working Group on Multiannual plan for demersal fisheries in the Western Mediterranean elaborated by the Scientific, Technical and Economic Committee for Fisheries (STECF-16-21).

The EU demersal fisheries in the Western Mediterranean include the EU fleets from Spain, France and Italy. According to the Annual Economic Report for 2016 (STECF), 2016a), which presents data corresponding to 2014, the fleet potentially targeting demersal fisheries covered by the Multiannual Plan included around 9,000 vessels, with a combined gross tonnage of 56,331 GT and engine power of 473,615 kW. There were accounted 932,798 days at sea, and the estimated employment in these fisheries was equal to 14,119 jobs corresponding to 10,717 full time equivalent jobs.

The main species caught by demersal fisheries in the Western Mediterranean are: hake, red mullet, blue whiting, monkfishes, deep-water rose shrimp, giant red shrimp, blue and red

Table 1: **Species and References points catched by the Spanish demersal fisheries in the Mediterranean Sea**

GSA	3A code	Scientific name	Ref year	FMSY	Fcurr/FMSY
1 7	HKE	Merluccius merluccius	2014	0.39	3.59
1	ARA	Aristeus antennatus	2014	0.41	3.41
1	ANK	Lophius budegassa	2013	0.16	1.56
1	MUT	Mullus barbatus	2013	0.27	4.85
1	DPS	Parapenaeus longirostris	2012	0.26	1.65
5	ARA	Aristeus antennatus	2013	0.24	1.75
5	ANK	Lophius budegassa	2013	0.08	10.50
5	MUT	Mullus barbatus	2012	0.14	6.64
5	DPS	Parapenaeus longirostris	2012	0.62	1.24
6	ANK	Lophius budegassa	2013	0.14	6.50
6	MUT	Mullus barbatus	2013	0.45	3.27
6	DPS	Parapenaeus longirostris	2012	0.27	5.19
7	ANK	Lophius budegassa	2011	0.29	3.34
7	MUT	Mullus barbatus	2013	0.14	3.21
Source: (STECF-16-21)					

shrimp and Norway lobster. In 2014, the volume of landings of European hake, red mullet and deep water rose shrimp amounted to 10,000 tonnes that accounted about 69 million euros (which is around 25% of the overall demersal production). The first species, both in volume and value, is hake, followed by red mullet and deep water rose shrimp. Hake, at Geographical Sub Areas 1-7, is principally targeted by Spanish vessels (which land a 58 percent of total). The average price of the red mullet, deep water rose shrimp, and hake landed by Spanish vessels are (on average) 5.92 euros/kg, 16.15 euros/kg, and 6.68 euros/kg, respectively.

We consider a stock rehabilitation policy associated with a a reduction in the fishing mortality level from the status quo, to the maximum sustainable yield fishing mortality level. Table 1 provides the details of the reduction in fishing mortality for each of the 14 different stocks considered by the Expert Working Group.

In order to compute the output constrains faced by the spanish fleet associated with the stock rehabilitation policy, we use the value added path generated by the age structured

Table 2: **Species and Prices of Spanish demersal fisheries in the Mediterranean Sea**

		Species				
		hake	red mullet	DW Red Shrimp	Monk fish	blue and red shrimp
country	GSA	HKE	MUT	DPS	ANK	ARA
Spain	1		x	x	x	x
Spain	5		x	x	x	x
Spain	6		x	x	x	x
France /Spain	7	x	x			
		HKE	MUT	DPS	ANK	ARA
Share of each Species	1-7	0.58	1.00	1.00	1.00	1.00
		HKE	MUT	DPS	ANK	ARA
Prices of each Species	1-7	6.68	5.93	16.15	=HKE	=DPS

models of each species (see Appendix A.1). Table 2 provides prices.

3.1 Calibration

Table 3: Calibration

Parameter	Value	Statistic	
Q	1	TAC	Normalization
e	1.5339	utility parameter	L=1/3
ρ	0.04	discount rate	Da-Rocha et al. (2014a)
λ	0.04	vessel lifespan	25 years
μ	-0.04	Productivity Drift	Weninger and Just (2002)
σ^2	0.01	Productivity Drift	Da-Rocha and Sempere (2016)
S	0	Scrap value	No decommissioning scheme
c_f	0.2403	fixed cost	(STECF-16-21)

We select the values of μ from Weninger and Just (2002) and σ^2 from Da-Rocha et al. (2014a). Given this stochastic process, it is necessary to calibrate six parameters Q, λ, S, c_f, e and ρ. We start by selecting a value of the annual interest rate $\rho = 0.04$ which is standard for the US economy in the macroeconomics literature.[8] We set $Q = 1$. We consider a vessel life span of 25 years ($\lambda = 0.04$). We assume the non-existence of decommissioning schemes. $S = 0$. We use data from Structure and economic performance estimates by MS fleets operating in the Mediterranean & Black Sea region, 2014[9] to compute the fixed cost. Finally, we calibrate utility parameter e by solving the model when the economy is non-distorted ir order to match a labor supply of 1/3. This is a standard normalization in the macroeconomics literature.

4 Results

This section is divided in two sub-sections. The first one presents the main results regarding the steady state solution of the model. The second presents the results obtained from the analysis of the transitional dynamics implied by stock rehabilitation policies leading to a situation in which all stocks are on their maximum sustainable yield fishing mortality level.

4.1 Steady State

The mass of vessels, $N(t) = \int_{\underline{z}(t)}^{\infty} g(z)dz$ represents the number of "standardized" firms (fishing vessels). Firms operate capital (the vessel) and stay *active* if they find it optimal to pay the idling cost, c_f. Note that the marginal firm (the less efficient active vessel) is indifferent between paying the idling cost or exiting the market. This marginal firm makes negative instantaneous profits, i.e. $\pi(\underline{z}, t) - c_f = -\frac{\sigma^2}{2}\partial_{zz}v(\underline{z}, t) < 0$, and the total expected value of operating the vessel is zero.[10]

[8]See, for instance Restuccia and Rogerson (2008).

[9] See Table 4.3 of the (STECF-16-21).

[10]If the marginal active firm decides to leave the market, it obtains the value, $v(\underline{z}) = S = 0$. From the smooth pasting condition and stationarity, $(\partial_z v(\underline{z}, t) = \partial_t v(\underline{z}, t) = 0)$ we have equation (1) $-\pi(\underline{z}, t) + c_f + \frac{\sigma^2}{2}\partial_{zz}v(\underline{z}, t) = 0$.

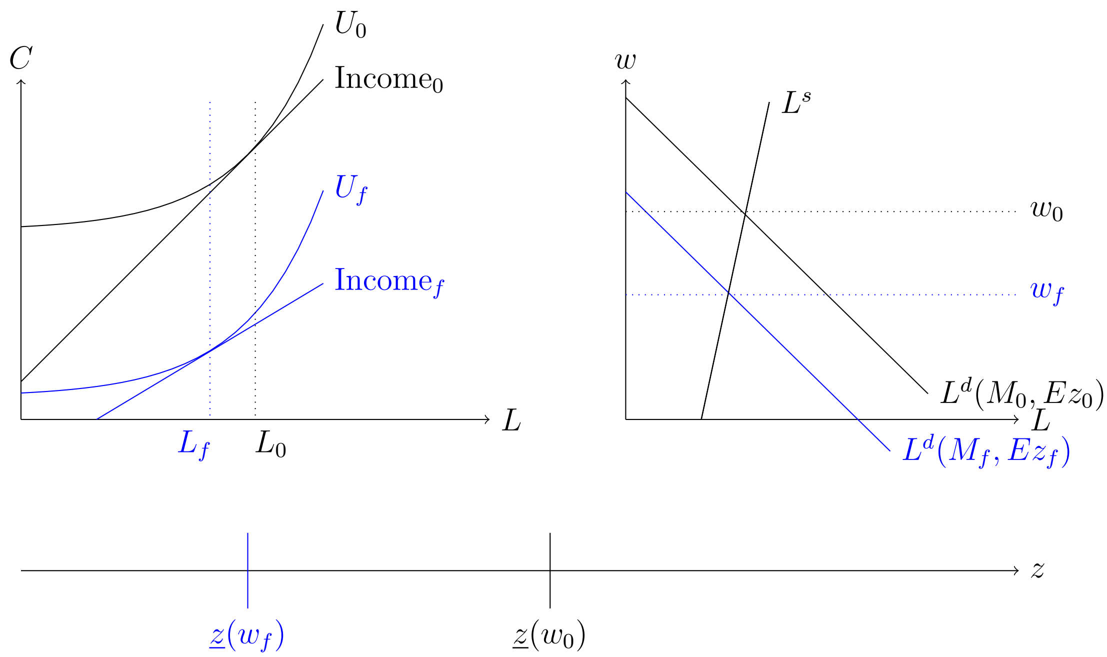


Figure 3: General Equilibrium effect of a higher Quota in the Steady State

To evaluate the macroeconomic and welfare implications of effort controls (changes in τ_l) the model generates the optimal response in three (management) variables: (1) average catch per unit effort (C.P.U.E.) per day at sea per vessel, Total Factor Productivity $TFP = E[y(z)/l(z)]$; (2) average days at sea per vessel, $E[l(z)]$, and (3) the number of vessels, $N(t)$.[11]

Effort controls –i.e., days-at-sea scheme– change the three management variables at the same time. First notice that effort controls imply a lower wage. The intuition of this result is as follows. If effort controls are active, more vessels are active for the same quota. Notice too that more vessels imply higher operating cost, $c_f N$, and remember from the household problem in section 2 that higher operating costs imply a lower consumption level $C = [Q - c_f M]$. This lower consumption level increases the marginal utility of labour.

[11]Given that

$$Y(t) = N(t) \int_{\underline{z}(t)}^{\infty} \left(\frac{y(z,t)}{l(z,t)} \right) l(z,t) f(z,t) dz.$$

Therefore, in equilibrium, wages have to decrease so that the next equation holds

$$\partial_C U(C)w(t) = -\partial_L U(L) \Rightarrow \frac{w(t)}{C(t)} = e(t),$$

Note that for the new wage rate (induced by the effort control), the labour supply is lower. (The graph at the top left in figure (3) illustrates this). Lower wages induce changes in nominal effort composition. On one hand, the demand of labour for each vessel is reduced, i.e. effort control is active and $E[l(z)]$ is lower for each vessel (each vessel spends less days at the sea). On the other hand, lower wages induce some vessels (that otherwise would exit) to stay, as $\underline{z}$ decreases, and the average productivity of the fleet, $f(z, t)$ decreases. Therefore an increment in the fleet size is compatible with less total days at the sea $L(t) = N(t)E[l(z)]$ and lower effort per vessel $E[l(z)]$ generated by effort controls. The graph at the top right in figure (3) illustrates this last effect.

Summarising, effort controls generate fleets with higher number of vessels. Productivity of vessels (TFP= $E[y(z)/l(z)]$) is reduced, vessels stay less days at the sea (lower $E[l(z)]$), and total catches per vessel, $E[y(z)]$ are lower. As a result, both profits per vessel and the vessel's value ($E[\pi(z)]$ and $E[v(z)]$, respectively) are lower.

Table 4 shows the steady state associated with different levels of effort control τ_l (measured as the % of z constrained). This table illustrates what was argued in the previous paragraphs with some more precise details. For instance, the line showing fleet size shows clearly how it increases monotonicaly with more restrictive output controls, and the next line show how this effect is accompanied by a monotonic reduction in the wage rate. Next lines show a decrease in total factor productivity, employment per vessel, profits per vessel and the value per vessel. The next lines show the values of several economic variables of interest for policymakers and their sensitivity to different degrees of input controls.

Table 5 shows the steady state associated with different level of output constraints with and

Table 4: **Effects of different levels control on days**

		Q =1				
Control on Inputs	τ_l (% of z constrained)	0.000	0.187	0.375	0.562	0.750
Fleet Size	M	0.132	0.133	0.138	0.149	0.175
wage	w	1.485	1.485	1.483	1.479	1.469
		data per vessel				
TFP	$E[y(z)/l(z)]$	2.971	2.970	2.966	2.958	2.939
Employment per vessel	$E[l(z)]$	2.553	2.505	2.348	2.047	1.536
Yield per vessel	$E[y(z)]$	7.583	7.509	7.254	6.724	5.710
Profits per vessel	$E[\pi(z)]$	3.551	3.550	3.532	3.456	3.213
Wealth per vessel	$E[v(z)]$	28.778	28.785	28.764	28.502	27.202
		Inequality: Gini Coeff.				
Revenues	$E[y(z)]$	0.573	0.573	0.573	0.573	0.573
Wealth	$E[v(z)]$	0.624	0.624	0.623	0.620	0.606
		Aggregate Accounts				
Operating Cost	$c_f M$	0.032	0.032	0.033	0.036	0.042
Consumption	$Q - c_f M$	0.968	0.968	0.967	0.964	0.958
Compensation of employees	wL	0.500	0.495	0.480	0.450	0.395
Gross operating surplus	Π	0.468	0.473	0.487	0.514	0.563
		Days at the see				
Total days at the see	L	0.337	0.334	0.324	0.304	0.269
Impact of effort control	L/L^*	1.000	0.991	0.961	0.904	0.799

without effort control. The first part of the table shows values for the variables of interest for different levels of Q and a 25% of restriction in effort. The second part shows values for the same variables and levels for the Q but for unrestricted effort. The table allows two types of comparisons. One type is that for the same Q, different levels of restriction in effort imply different values of the variables. The other is that for the same level of effort constraint, different Qs imply different values of the variables. Some regularities can be observed. For instance, a larger Q implies larger fleet size, higher wage rate, higher TFP, lower profits and lower employment per vessel, for any level of restriction in effort. On the other hand, for the same Q, more restriction in effort implies larger fleet size, lower wage rate, lower TFP, lower profits, and lower employment per vessel.

4.2 Transitions

This section focuses on the caracterization of the transition dynamics caused by stock rehabilitation policies leading the fishery, from a given status quo, to a stationary situation where all stocks are on their maximum sustainable yield fishing mortality level ($Fmsy$). Our strategy follows two steps. First, we set a drastic reduction to $Fmsy$ for all species in the fishery and compute the Value of landings (VA) using the age estructured model (discussed in Appendix 1). Second, given the VA for the Spanish fleet associated to a reduction to $Fmsy$ for all species, we compute the transition dynamics associated with the non distortionary instrument $\tau(t)$[12] that drive the fishery from the status quo conditions (the VA associated with the fishing mortality in the status quo) to the stationary solution where fishing mortality is equal to $Fmsy$ for all species. Formally, we assume that the non distortionary instrument is such that the VA target in each period is implemented. That is

$$\pi(t) = (1 - \tau(t))y(t) \Rightarrow Q(t) = \int_{\underline{z}(t)}^{z^c(t)} y^*(t)g(z,t)dz + \int_{z^c(t)}^{\infty} y(t,z)^c g(z,t)dz$$

[12] $\tau(t)$ can be interpreted as a tax rate or as the price of an ITQ in a system of fully tradable individual quotas.

Table 5: **Effects of different output constraints**

		τ_l (% of z constrained) =0.25				
Statistic	Q	1.000	1.352	1.704	2.056	2.407
Fleet Size	M	0.175	0.268	0.365	0.485	0.606
wage	w	1.469	1.844	2.217	2.582	2.946
		data per vessel				
TFP	$E[y(z)/l(z)]$	3.717	4.670	5.625	6.556	7.495
Employment per vessel	$E[l(z)]$	1.536	1.014	0.747	0.566	0.455
Yield per vessel	$E[y(z)]$	5.710	4.733	4.202	3.710	3.412
Profits per vessel	$E[\pi(z)]$	3.213	2.624	2.306	2.009	1.831
Wealth per vessel	$E[v(z)]$	27.202	22.097	19.310	16.744	15.183
		Inequality: Gini Coeff.				
Revenues	$E[y(z)]$	0.573	0.558	0.533	0.522	0.503
Wealth	$E[v(z)]$	0.606	0.600	0.581	0.577	0.563
		Aggregate Accounts				
Operating Cost	$c_f M$	0.042	0.064	0.088	0.117	0.145
Consumption	$Q - c_f M$	0.958	1.202	1.445	1.683	1.921
Compensation of employees	wL	0.395	0.500	0.604	0.709	0.812
Gross operating surplus	Π	0.563	0.702	0.841	0.974	1.109
		Welfare				
Utility (society welfare)	$u(C) - eL$	−0.456	−0.232	−0.050	0.100	0.230
Total employees	L	0.269	0.271	0.273	0.275	0.276
Employment constraint	L	1.000	1.008	1.013	1.021	1.025
		τ_l (% of z constrained) =0.00				
Fleet Size	M	0.132	0.202	0.275	0.366	0.457
wage	w	1.485	1.868	2.250	2.626	3.001
		data per vessel				
TFP	$E[y(z)/l(z)]$	2.971	3.737	4.501	5.251	6.002
Employment per vessel	$E[l(z)]$	2.553	1.680	1.238	0.936	0.753
Yield per vessel	$E[y(z)]$	7.583	6.277	5.573	4.915	4.521
Profits per vessel	$E[\pi(z)]$	3.551	2.898	2.546	2.217	2.020
Wealth per vessel	$E[v(z)]$	28.778	23.350	20.393	17.663	16.008
		Inequality: Gini Coeff.				
Revenues	$E[y(z)]$	0.573	0.558	0.533	0.522	0.503
Wealth	$E[v(z)]$	0.624	0.619	0.601	0.598	0.584
		Aggregate Accounts				
Operating Cost	$c_f M$	0.032	0.048	0.066	0.088	0.110
Consumption	$Q - c_f M$	0.968	1.218	1.467	1.712	1.956
Compensation of employees	wL	0.500	0.633	0.767	0.900	1.033
Gross operating surplus	Π	0.468	0.585	0.700	0.812	0.923
		Welfare				
Utility (society welfare)	$u(C) - eL$	−0.549	−0.323	−0.139	0.012	0.143
Total employees	L	0.337	0.339	0.341	0.343	0.344
Employment constraint	L	1.000	1.007	1.012	1.018	1.023

We assume that the tax revenue is returned to household in the form of a non-distortionary lump sum transfer. Tax revenue is equal to $T(t) = \int_{\underline{z}(t)}^{\infty} \tau(t) y(t) g(z,t) dz$.[13]

The transitional dynamics are described by the following system of equations

$$\min_{I_{\text{exit}}(z,t)} \left\{ \rho v(z,t) - \pi(t) z + c_f - \mu z \partial_z v(z,t) - \frac{\sigma_z^2}{2} \partial_{zz} v(z,t) - \partial_t v(z,t), \quad v(z,t) - S(t) \right\},$$

$$-\partial_z[\mu z g(z,t)] + \frac{\sigma_z^2}{2} \partial_{zz} g(z,t) - (I_{\text{exit}}(z,t) + \lambda) g(z,t) + g^e(z,t) = \partial_t g(z,t),$$

$$\int_{\underline{z}(t)}^{\infty} g(z,t) dz = N(t)$$

$$\int_{\underline{z}(t)}^{z^c(t)} y^*(t) g(z,t) dz + \int_{z^c(t)}^{\infty} y(t,z)^c g(z,t) dz = Q(t),$$

$$e\left[Q(t) - c_f N(t)\right] = w(t)$$

$$\text{Exit}(t) = \int_{\underline{z}(t)}^{\infty} I_{\text{exit}}(z,t) g(z,t) dz$$

The equilibrium depends on the instrument $\tau(t)$. We solve this system using the following algorithm. First, we compute the stationary value functions $v(z|Q)$ and fleet distributions, $g(z|Q)$, associated with the status quo, $Q_0 = 1$ and the stock rehabilitation $Q_T = 2.407$. Second, guess a function $\tau(t)$. Next, follow the next iterative procedure:

1. Given $w(t)$, compute $v(z,t)$ by solving the HJB equation (5) with terminal condition $v(z|Q_T)$ and compute also $I_{\text{exit}}(z,t)$

2. Given $I_{\text{exit}}(z,t)$, compute $g(z,t)$ by solving the KFP equation (5) using $g(z|Q_0)$ as the initial conditions,

3. Given $g(z,t)$, calculate $w^1(t)$ using equation (5) and update $w(t)$. Stop when $w^1(t)$ is sufficiently close to $w(t)$.

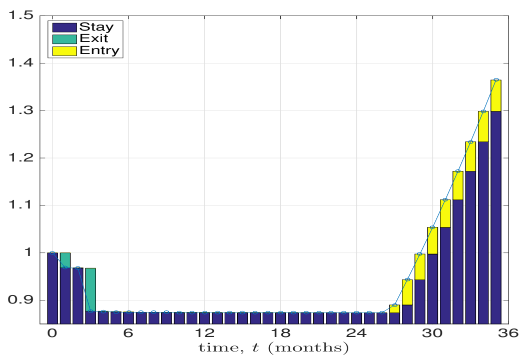
4. Given $w(t)$, compute $Q(t)$. Allow entry if $Q(t)$ is lower than the VA path associated with the stock rehabilitation policy. Stop when $Q(t)$ is sufficiently close to the VA

[13] Then $C = w(t)L(t) + \Pi(t) + T(t) = Q(t) - c_f N(t)$.

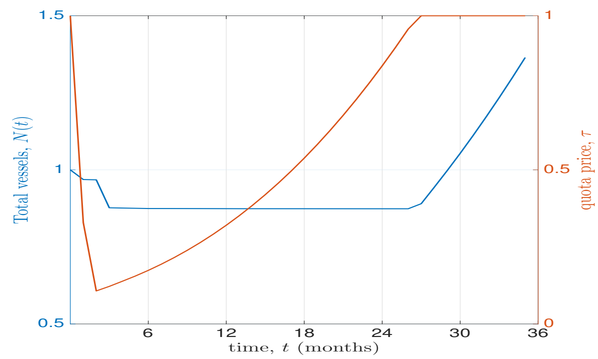
path. Otherwise update $\tau(t)$

We compute two transitions. The first one is computed when input controls τ_l are used. This is related to a stationary constraint (% of z constrained) equal to 25 percent. The second one is computed for the case of no input controls. Note that, along the transition, the fraction of z constrained is endogenous (it is a function of $w(t)$). That is, C.P.U.E. is given by

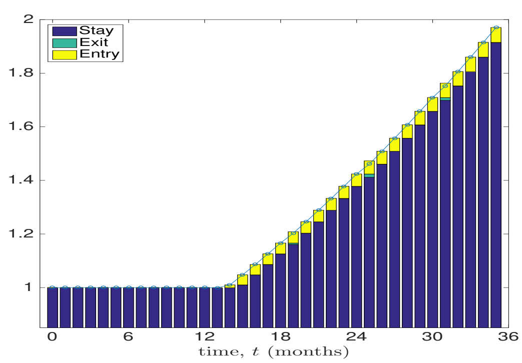
$$\frac{y(z,t)}{l(z,t)} = \begin{cases} 2w(t) & \text{if} \quad z \leq z^c(t) \\ \\ \sqrt{\frac{z}{\tau_l(t)}} & \text{if} \quad z > z^c(t) \end{cases} \tag{5}$$



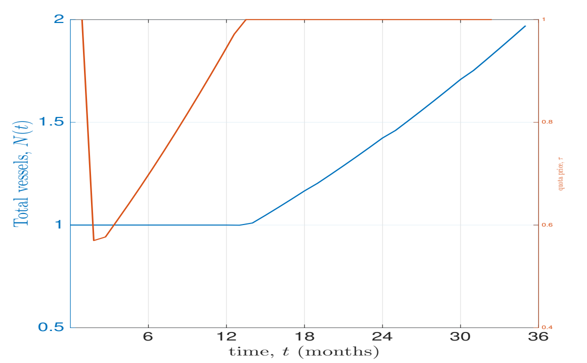
(a) entry / exit without effort controls



(b) K dynamics without effort controls



(c) entry / exit with effort controls



(d) K dynamics with effort controls

Figure 4: Capital dynamics

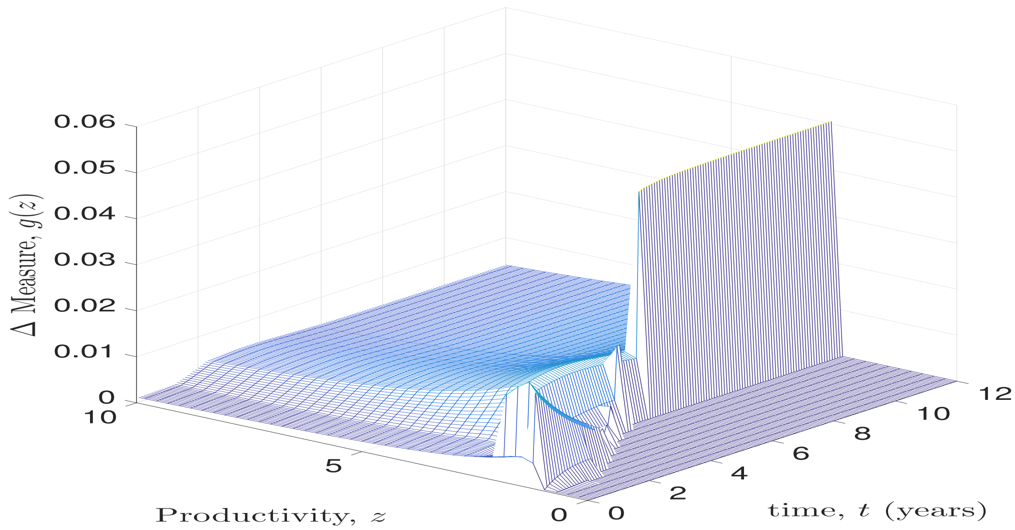
In our model, malleability of capital is associated with the fleet size dynamics caused by the existence of heterogeneous agents and endogenous entry/exit. In an stationary solution, we observe that capital is non-malleable as $N(t) = N(t + dt) = N$ and $I(t) = 0$.

Along the stock rehabilitation path, capital malleability depends on the use (or not) of input controls. Figure 4(a) shows that without effort controls, some of the firms exit during the first months and there is entry of firms at the end of the period considered. Therefore, without input controls capital is malleable as exit of firms produces a reduction in capital in the fishery. The figure shows how, starting from the status quo number of vessels (normalized to 1), some vessels exit during the first four months, then the size of the fleet is stable for several months until the stock is recovered enough and entry is allowed. Entry occurs at a constant rate during the last months. Figure 4(b) shows the capital dynamics. First there is a drop in capital (i.e. malleability), then it remains constant, and, finally when the stock rises enough, the capital rises.

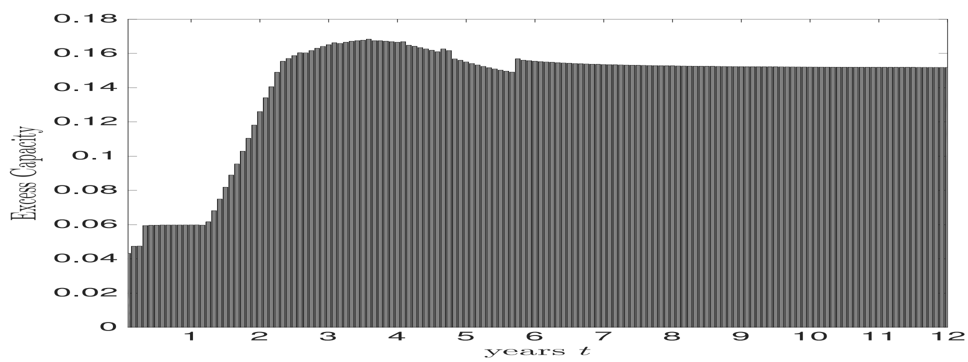
However, Figure 4(c) shows that when input controls are used, capital is non-malleable as no exit occurs and no reduction in capital is produced. This figure shows how for this type of policy the number of vessels remains constant for more than a year. Then, once the stock of fish is recovered enough, entry is allowed. Then entry is produced at a constant rate until the final period. Figure 4(d) show how the capital remains constant (i.e. non-malleability) until it starts to rise at a constant rate.

We can summarize those findings by computing the excess of capacity associated with the use of input controls. We compute excess of capacity asociated to the distortionary policy as the difference, in each period, between capital in the fishery regulated with input controls and capital in the fishery regulated with a nos distortionary instrument. In some more precise terms, we first compute the measure of firms along the transitional path under the two policy regimes, and once we have these measures we compute the adequate differences that are represented in figures 5(b) and 5(a).

Figure 5(b) shows that excess of capacity, measured as the difference between measures



(a) $\Delta g(z,t)$



(b) ΔK

Figure 5: Impact of distortions

$g(z,t)$ associated to the different policies for each z and t, is positive for each z and t. This difference is larger for low productivity levels (i.e. for z closer to zero). This implies that the excess of capacity is also associated with lower average levels of productivity as it is relatively more concentrated in vessels with low productivity.

Figure 5(a) represents, for each moment in time, the difference between the number of vessels (in percentages) associated to a regulatory policy based in input controls and the number of vessels associated to policy based on non distortionary instruments. We name this number as “the excess of fleet”. The figure shows that the excess of fleet is always positive. It is increasing during the first periods and it can be close to 16 percent for some period. Later in time, it remais positive and stabilizes about 14 percent.

The conclusion of the section would be that if the fleet in a given fishery is already overcapitalizated, a policy of input controls makes the problem even worse as the excess of capital is always positive with respect that resulting from other less distortionary policies.

5 Conclusions

We show that a management policy based on input controls generates less exit, a less productive fleet, and more overcapitalization. In particular, we show that in the steady state equilibrium, this policy leads to smaller vessels with lower yield and individual profits and lower wages. The lower wages allow less productive vessels (that otherwise would exit) to stay in the fishery, reducing the average productivity of the fleet. The result of input controls is that a higher number of vessels is required to achieve the same biological targets, and this implies an over-capitalized fleet.

On the other hand, we also caracterize the transition dynamics caused by stock rehabilitation policies leading the fishery, from a given status quo, to a stationary situation where all stocks are on their maximum sustainable yield fishing mortality level. We show that along the stock rehabilitation path, capital malleability depends on the use (or not) of input controls. We show that without input controls capital is malleable as exit of firms is produced in the transition path and the stock of capital in the fishery is reduced. However, we also show that when input controls are used, capital is non-malleable as no exit of firms is produced along the transition path and the capital is not reduced.

We also show that the excess of capacity associated to input controls will also produce lower average levels of productivity as it is relatively more concentrated in vessels with low productivity. Furthermore, the excess of fleet associated to this type of policies is always positive. Therefore, if the fleet in a given fishery is already overcapitalizated, a policy of input controls makes the problem even worse as the excess of capital is always positive with respect that resulting from other less distortionary policies.

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A Appendix

A.1 Age structured Stock dynamics

For each of the species, we use an age structured model (see figure 6) to evaluate the impact of each *fishing mortality*, $F(t)$, trajectory to Fmsy (see figure 7) on landings generated by the transitional dynamics of the stocks, $n(a,t)$ (see figure 8). Let $n(a,t)$ be the number of fish of age a at time t. As in Botsford and Wainwright (1985), the conservation law is described by the following McKendrick-von Foerster partial differential equation.[14]

$$\frac{\partial n(a,t)}{\partial t} = -\frac{\partial n(a,t)}{\partial a} - [m(a) + p(a)F(t)]n(a,t). \tag{6}$$

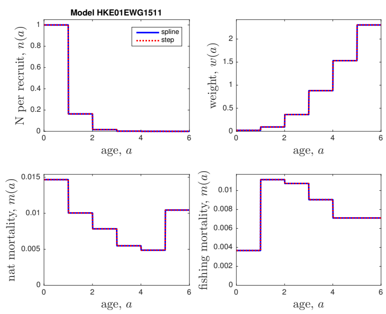
Equation (6) shows that the rate of change on the number of fish in a given age interval, $\frac{\partial n(a,t)}{\partial t}$, is equal to the net rate of departure less the rate of deaths. Given all fish age, the net rate of departure is equal to $\frac{\partial n(a,t)}{\partial a}$. The rate of deaths at age a is proportional to the number of fish of age a, i.e. $[m(a) + p(a)F(t)]n(a,t)$. Recruitment and maximum age occurs as boundary conditions. We assume that fish die at age A, and constant recruitment i.e $n(0,t) = 1$ and $n(A,t) = 0$.[15] For a given $F(t)$ trajectory, catches at age a are equal to $p(a)F(t)n(a,t)$, therefore $Q(t)$, is equal to

$$Q(t) = \left(\int_0^A \omega(a)p(a)n(a,t)da \right) F(t).$$

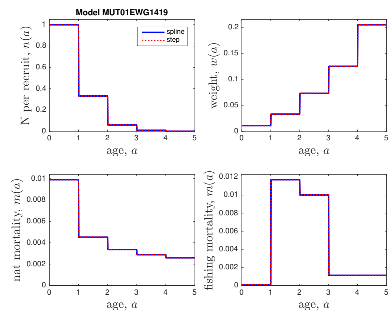
[14] See Von Foerster (1959) and McKendrick (1926).

[15] It can be assumed a Stock Recruitment relationship. In that case, each period, the number of fish at age zero are given by $n(0,t) = \Psi(\int_0^A \omega(a)\mu(a)n(a,t)da)$, where, $\int_0^A \omega(a)\mu(a)n(a,t)da$ is the SSB. See Da-Rocha et al. (2012).

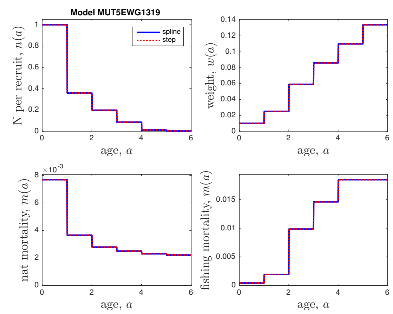
Figure 6: Age Structured Models



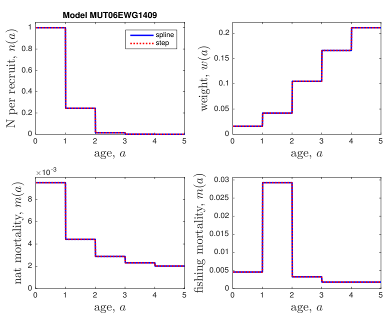
(a) HKE



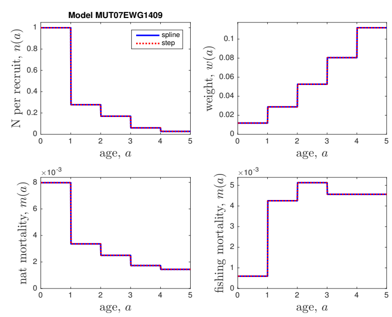
(b) MUT 1



(c) MUT 2



(d) MUT 3



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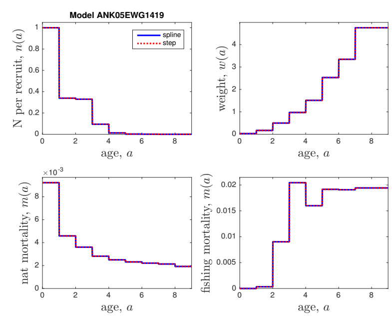
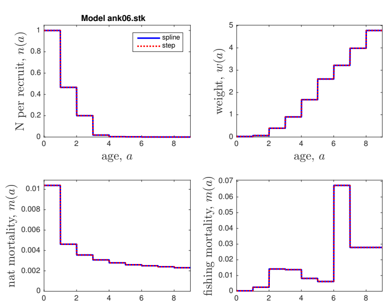
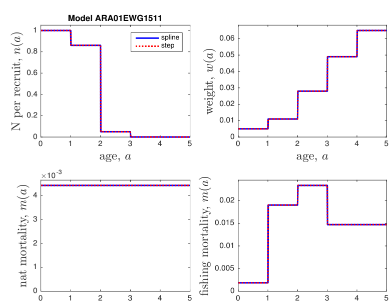


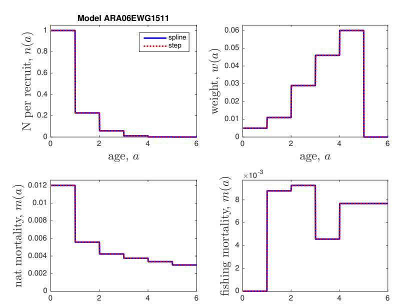
(f) ANK 1



(g) ANK 2



(h) ARA 1



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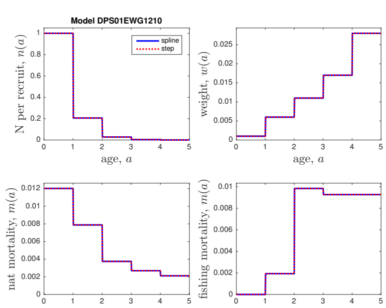
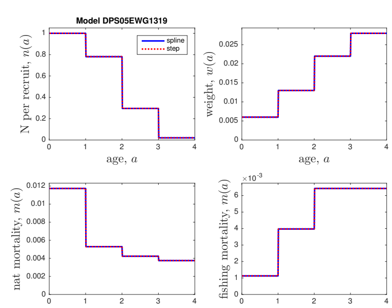
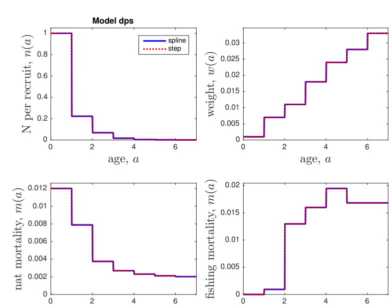


(j) DPS 1

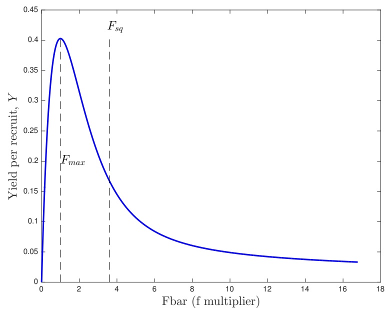


(k) DPS 2

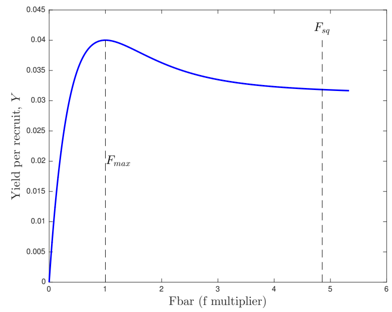


(l) DPS 3

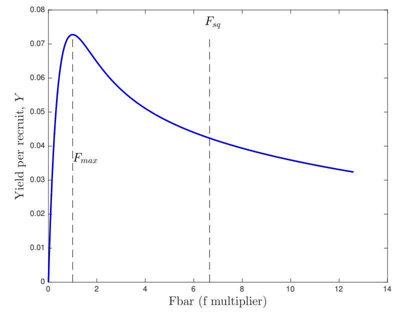
Figure 7: Targets



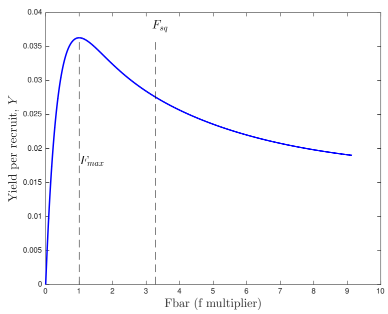
(a) HKE



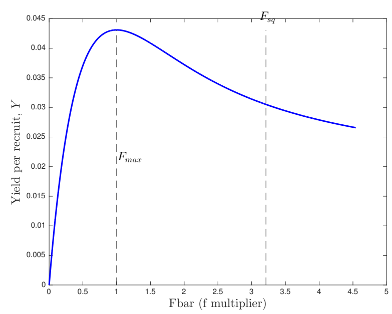
(b) MUT 1



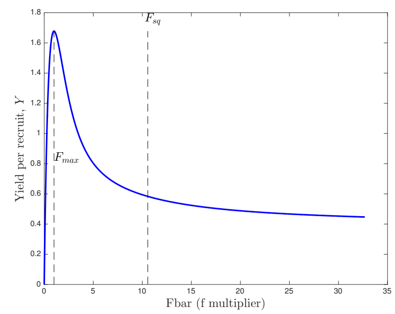
(c) MUT 2



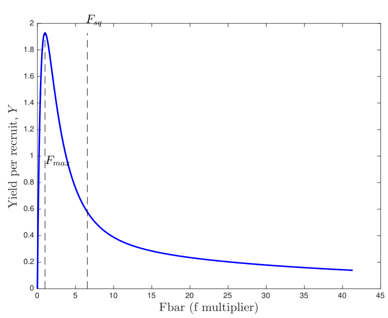
(d) MUT 3



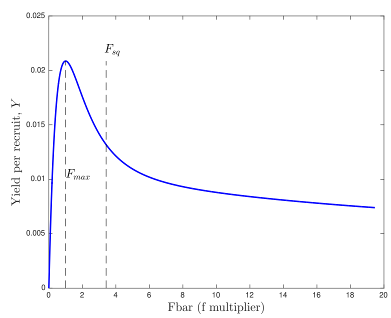
(e) MUT 4



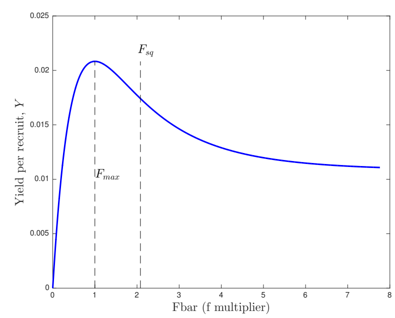
(f) ANK 1



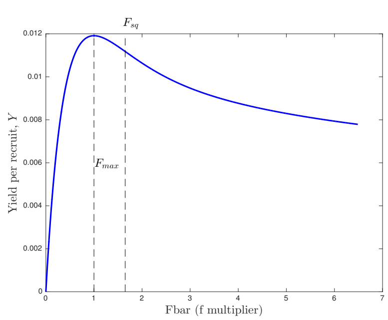
(g) ANK 2



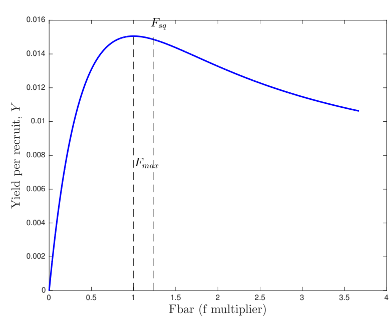
(h) ARA 1



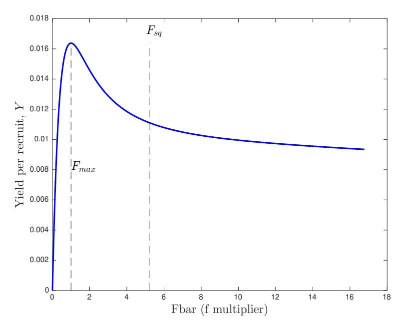
(i) ARA 2



(j) DPS 1

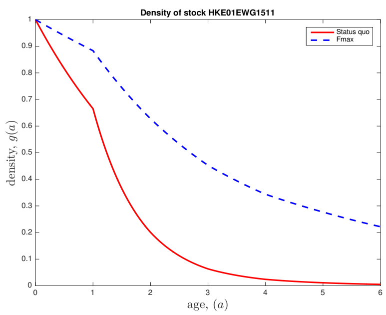


(k) DPS 2

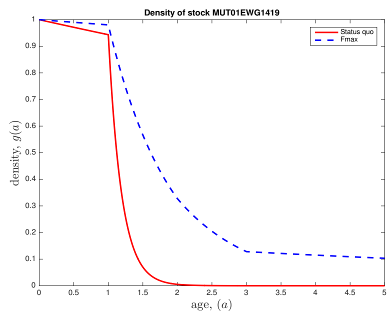


(l) DPS 3

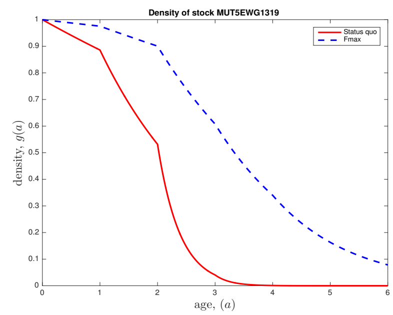
Figure 8: Equilibrium Distributions by age



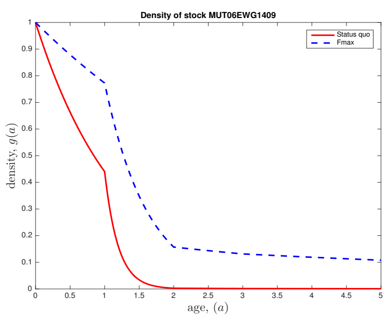
(a) HKE



(b) MUT 1



(c) MUT 2



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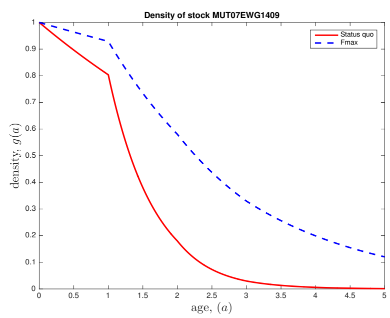
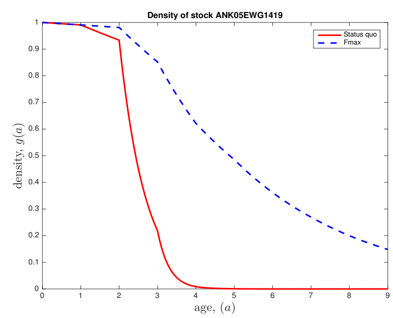
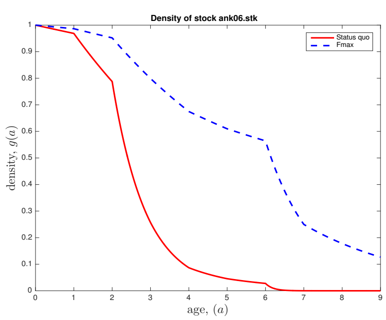


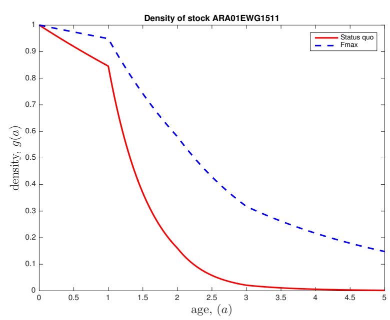
(e) MUT 4



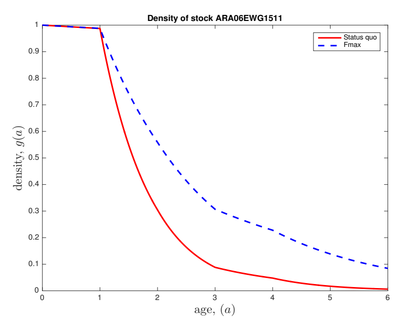
(f) ANK 1



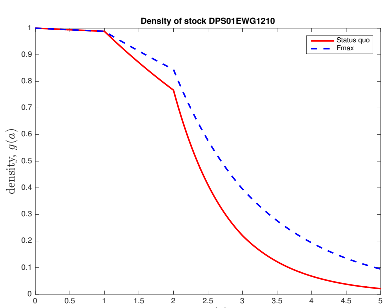
(g) ANK 2



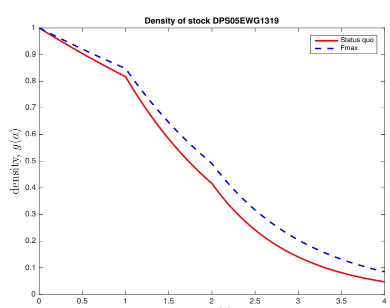
(h) ARA 1



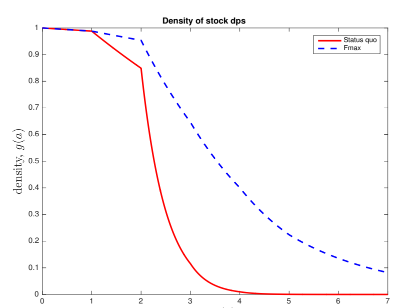
(i) ARA 2



(j) DPS 1



(k) DPS 2



(l) DPS 3

A.2 Finite difference method

Following Achdou et al. (2014) Achdou et al. (2015) we use a finite difference method and approximate the fuctions $v(z,t)$ and $g(z,t)$ (equations 1 and 2). We use the shorthand notation $v_i^n = v(z_i, t_n)$ and $g_i^n = g(z_i, t_n)$.

Linear Complementarity Problems (LCP). We approximate (1)

$$\rho v_i^n \;= \pi_i^n + [\mu_i]^+ \left(\tfrac{v_{i+1}^n - v_i^n}{\Delta z}\right) + [\mu_i]^- \left(\tfrac{v_i^n - v_{i-1}^n}{\Delta z}\right) + \tfrac{\sigma_z^2}{2}\left(\tfrac{v_{i+1}^n - 2v_i^n + v_{i-1}^n}{\Delta z^2}\right) + \left(\tfrac{v_i^{n+1} - v_i^n}{\Delta t}\right),$$

where $[\mu_i]^+ = \max\{\mu_i, 0\}$ and $[\mu_i]^- = \min\{\mu_i, 0\}$. Therefore, collecting terms, we have

$$\begin{aligned}
\rho v_i^n &= \pi_i^n + a_i v_{i-1}^n + b_i v_i^n + c_i v_{i+1}^n + \left(\frac{v_i^{n+1} - v_i^n}{\Delta t}\right), \quad \text{where} \\
a_i &= -\frac{\min\{\mu_i, 0\}}{\Delta z} + \frac{\sigma_z^2}{2\Delta z^2}, \\
b_i &= -\frac{\max\{\mu_i, 0\}}{\Delta z} + \frac{\min\{\mu_i, 0\}}{\Delta z} - \frac{\sigma_z^2}{\Delta z^2}, \\
c_i &= \frac{\max\{\mu_i, 0\}}{\Delta z} + \frac{\sigma_z^2}{2\Delta z^2}.
\end{aligned}$$

Note that $a_i + b_i + c_i = 0$. Then, equation (7) in matrix form

$$\rho v^n = \pi^n + \mathbf{A} v^n + \frac{1}{\Delta t}\left(v^{n+1} - v^n\right),$$

where (for i=1,2,3,4)

$$\mathbf{A} = \begin{bmatrix} b_1 & c_1 & 0 & 0 \\ a_2 & b_2 & c_2 & 0 \\ 0 & a_3 & b_3 & c_3 \\ 0 & 0 & a_4 & \hat{b}_4 \end{bmatrix}.$$

Boundary conditions: from $\partial_z v(\infty, t) = 0$ we have, $v_I^n = v_{I+1}^n$, then $\hat{b}_I = b_I + \frac{\sigma_z^2}{2\Delta z^2}$ such that $a_I + b_I = 0$.

To solve equation (1) we follow Huang and Pang (1988). They show that the variational inequality problem (the discretized version of equation (1))

$$\min_{I_{\text{exit}}(z,t)} \left\{ \rho v^n - \pi^n - \mathbf{A} v^n - \frac{1}{\Delta t}\left(v^{n+1} - v^n\right), v^n - S^n \right\},$$

can be formulated as Linear Complementarity Problems (LCP), i.e

$$\begin{aligned} (v^n - S^n) \perp \left(\mathbf{B}(v^n - S^n) + \mathbf{q}^{m+1}\right) &= 0, \\ (v^n - S^n) &\geq 0, \\ \mathbf{B}(v^n - S^n) + \mathbf{q}^{m+1} &\geq 0, \end{aligned}$$

where $\mathbf{B} = \left(\rho + \frac{1}{\Delta t}\right)\mathbf{I} - \mathbf{A}$ and $\mathbf{q}^{m+1} = \mathbf{B}S^n - \pi^n - \frac{1}{\Delta t} v^{n+1}$.[16]

Kolmogorov Forward equation. We approximate the KFP equation (2) using the following approximation for $\partial_z[\mu z v(z,t)]$

$$\partial_z[\mu z v(z,t)] \simeq \left[\left(\frac{[\mu_i]^+ g_i^n - [\mu_{i-1}]^+ g_{i-1}^n}{\Delta z}\right) + [\left(\frac{[\mu_{i+1}]^- g_{i+1}^n - [\mu_{i-}]^- g_i^n}{\Delta z}\right)\right].$$

Therefore, we have

$$\begin{aligned} \left(\frac{g_i^{n+1} - g_i^n}{\Delta t}\right) &= c_{i-1} g_{i-1}^n + b_i g_i^n + a_{i+1} g_{i+1}^n - I_i^n g_i^n + \delta^n, \quad \text{where} \\ a_{i+1} &= -\frac{\min\{\mu_{i+1}, 0\}}{\Delta z} + \frac{\sigma_z^2}{2\Delta z^2}, \\ b_i &= -\frac{\max\{\mu_i, 0\}}{\Delta z} + \frac{\min\{\mu_i, 0\}}{\Delta z} - \frac{\sigma_z^2}{\Delta z^2}, \\ c_{i-1} &= \frac{\max\{\mu_{i-1}, 0\}}{\Delta z} + \frac{\sigma_z^2}{2\Delta z^2}. \end{aligned}$$

[16] Matlab provides Yuval Tassa's Newton-based LCP solver, download from http://www.mathworks.com/matlabcentral/fileexchange/20952.

Note that (7) in matrix form

$$\frac{1}{\Delta t}\left(g^{n+1}-g^{n}\right)=\mathbf{A}^{T}g^{n}$$

where (for i=1,2,3,4)

$$\mathbf{A}^{T}=\begin{bmatrix} b_1 & a_2 & 0 & 0 \\ c_1 & b_2 & a_3 & 0 \\ 0 & c_2 & b_3 & a_4 \\ 0 & 0 & c_3 & b_4 \end{bmatrix}.$$

and $g^n = I^n_{\text{exit}} g^n$. Finally density is computed as $f_i = \frac{g_i}{\sum_{i=1}^{I} g_i \Delta z}$, where $\sum_{i=1}^{I} g_i \Delta z$ is the mass of firms.