The Rise of the Working Rich, Market Imperfections, and Income Inequality

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2 December 2016

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Abstract

This paper attempts to explain the recent substantial increase in income inequality—which is largely due to the explosion of the very top labor incomes (i.e., the rise of the working rich)—in the rich countries especially the United States. This paper points to technological progress and (less importantly to) capital accumulation as the main cause of the (universal) increase in income inequality starting from the late 1970s; also, this paper points to the differences in the nature of demand as the main cause of the large cross-country differences. This paper will build a simple general equilibrium model to formalize this idea and to provide some new insights into the analysis of income inequality.

Keywords: Income inequality, trickle-down, entrepreneurs, supermanagers.
JEL Classification: D31, D63.

1. INTRODUCTION

In the "Capital in the Twenty-First Century," Piketty (2014) reports that income inequality in the rich countries, especially the United States, has increased significantly in the last several decades. Piketty also reports that the recent increase in income inequality is largely due to the explosion of the very top labor incomes (including wages, salaries, and
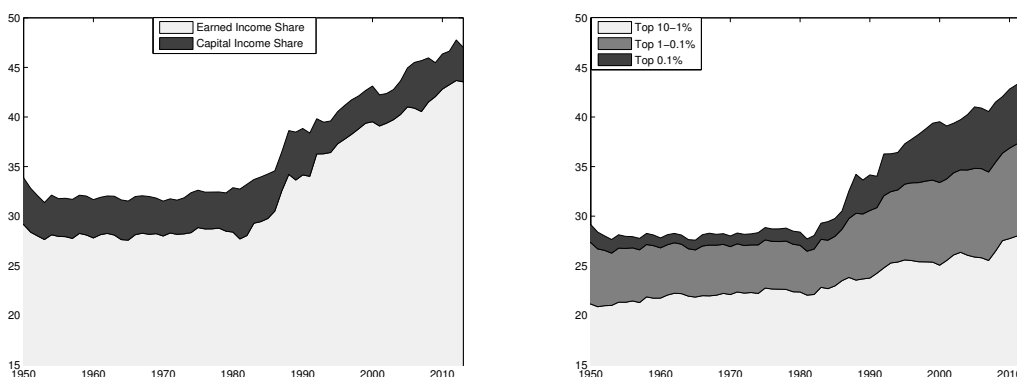
Figure 1: Top Decile Income Share and Composition in U.S., 1950-2013.

Note: The left panel displays the top decile earned income share and capital income share, while the right panel decomposes the top decile earned income share into the top 10-1%, top 1-0.1%, and top 0.1%.

Source: The World Wealth and Income Database.

entrepreneurial income). As shown in the left panel of Figure 1, for example, the top decile earned income share in the United States has increased significantly since the late 1970s, while the top decile capital income share (including dividends, interest income, and rents) has remained roughly unchanged. Also, as shown in the right panel, the increase in the top decile earned income share is largely due to the increase at the very top. Concretely, while the top decile earned income share increases by 15.1% (from 28.4% in 1980 to 43.5% in 2013), the top 1% alone increases by 9.3% and the top 0.1% increases by 4.7%. Piketty refers to this explosion of the very top earnings as "the rise of the supermanagers." The term "supermanagers" might obscure the importance of the very top entrepreneurs who have comparable contribution to the increase in income inequality. In the United States, for example, the top decile entrepreneurial income (resp. wages and salaries) increases by 5.4% from 2.7% in 1980 to 8.1% in 2013 (resp. 9.7% from 25.7% to 35.4%), and the top percentile increases by 4.4% from 1.1% to 5.5% (resp. 4.9% from 4.9% to 9.8%). Accordingly, without trouble with terminology, we instead use the term "the working rich," rather than the supermanagers.[1]

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[1]The term "the working rich" was used in Piketty and Saez (2003) who conclude that the working class have replaced the coupon-clipping rentiers at the top, but they later admit in Atkinson et al. (2011)
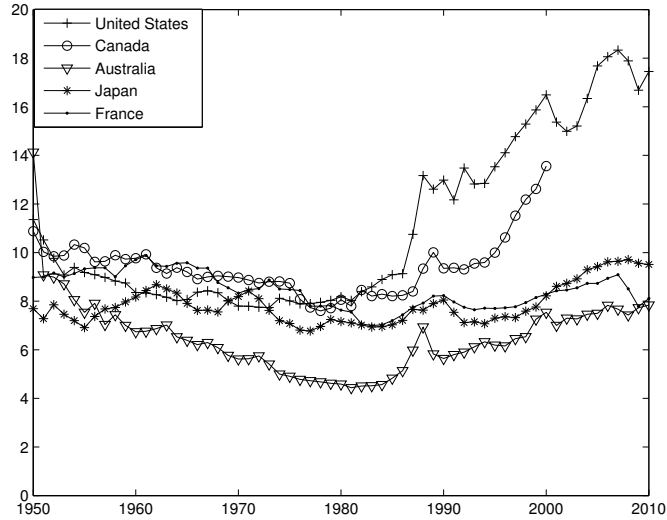
Figure 2: Top Percentile Income Shares in the Rich Countries, 1950-2010.

Source: The World Wealth and Income Database.

The previous discussion is just a matter of terminology. Regardless of whether we use the working rich or the supermanagers, as shown in Figure 2, the recent increase in income inequality has two important properties, namely the (universal) takeoff in the late 1970s and early 1980s,[2] and the large differences across the rich countries (see Figure 8-9 in Atkinson et al. (2011) for more countries). The former suggests that we should focus on the factors, occurring especially during and after 1970s, which are not unique to any country and have widespread effects. This paper points to technological progress—especially, advances in ICT and automated production, which can be considered as the third industrial revolution starting in the early 1970s—and (less importantly to) capital accumulation in the post-war periods. At the same time, because the rich countries have arguably experienced similar economic development, to explain the large cross-country differences, we should also focus on country-specific factors (e.g., social norms, the nature of demand, and institutions). This paper argues that because of market imperfections,[3]

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that the conclusion is strong and needs to be qualified. Regardless of whether Piketty and Saez (2003) are right or wrong, as discussed above, this term in our view is more appropriate than the supermanagers.

[2]Note that even in Japan and France, income inequality was on the rise (although relatively slowly) prior to the great recession.

[3]Income inequality exists even under perfect competition, in which case individuals are paid at their

the differences in the nature of demand play significant role. Before we can say anything more, let us first consider a selective literature review.

A Selective Literature Review

The rise of the working rich is a huge and controversial literature. Here, we only provide a selective literature review (see Frydman and Jenter (2010); Gabaix and Landier (2008); Kaplan and Rauh (2010) for more discussions). First and foremost, Rosen (1981) argues that goods are imperfect substitutes and the less desirable goods are poor substitutes for the highly desirable ones. Thus, technological progress benefits the highly talented individuals (i.e., superstars) more. For instance, the invention of CD enables the superstars to enlarge market size with disproportional costs. Technological progress can also be skilled-biased (see Autor et al. (2006); Katz and Murphy (1992)); that is, technological progress shifts demand toward skilled labors and hence opens the gap between the wages of skilled and unskilled labors.

In addition to market size, technological progress also provides the means to enlarge firm size. Gabaix and Landier (2008) and Terviö (2008) argue that the bigger the firm size, the higher the executive compensation. Gabaix and Landier (2008), in particular, argue that when firm size increases by 500%, CEO productivity and hence compensation also increases by 500%. However, Gabaix and Landier (2008) and Terviö (2008) tell us nothing about the weak correlation between executive compensation and firm size prior to the mid 1970s, reported by Frydman and Saks (2010), who argue that the strong correlation in the recent decades might be the result of the upward trend in both variables. Thus, we should focus on the changes occurring during and after 1970s, and should go beyond technological progress to explain the rise in the executive compensation. One possible explanation is the increase in the importance of general managerial skills relative to firm-specific managerial skills as firms get bigger and more complex (see Murphy and Zábojník (2004, 2007)). In such case, demand in the market for managers increases—because firms in various industries have to compete for managers in the same market—and

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marginal social contributions and hence there is little interest in the analysis of income inequality. With market imperfections, however, the rise in income inequality—when those at the bottom and in the middle are worse off—might be the signal that trickle-down is not working, and that the top monopolists have incentives in the wrong directions (see subsection 3.1).

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so do executive compensations. Similarly, Cuñat and Guadalupe (2009) argues that globalization and foreign trade raise competition among firms, leading to higher demand for top managers. Another possible explanation is the theory of managerial power (see Bebchuk and Fried (2005); Bertrand and Mullainathan (2001); Yermack (1997)), which states that managerial influence over pay-setting process (say due to manager's power to benefit directors, friendship and loyalty, and directors' incentive to be re-elected) enables managers to claim more credits when firms are doing well and to obtain pays without performances.

A major drawback of the above works is the inability to explain the large cross-country differences. With this regard, Piketty (2014) points to the differences in social norms. Particularly, Piketty (2014) argues that because it is impossible to estimate marginal productivity, it is inevitable that managers who set their own salaries can treat themselves generously. This is exacerbated by social norms which have evolved (in some countries especially the United States) in a way that tolerates extremely generous pays.

An Overview

The idea we have in mind is that technological progress and capital accumulation, in a similar fashion to Rosen (1981), allow the very top entrepreneurs and firms to reap relatively high benefits. To capture this fact, this paper puts some restrictions on demand functions. To put it differently, this paper focuses on the nature of demand, which is also important for the discussion of cross-country differences. To give an example, according to the Big Mac index of The Economist, the price of a Big Mac in July 2016 is $3.5 in Japan, $5 in the United States, and $6.6 in Switzerland. Although price is determined by both supply and demand, this example suggests that the nature of demand plays a role in the explanation of the large cross-country differences. In this paper, we show that the impacts of technological progress and capital accumulation on income inequality depend on the nature of demand, specifically, on a distribution of a variable, called "desirability," which we will introduce into demand function in section 2.

The widespread effects of technological progress and capital accumulation give rise to the top firms in various industries, which raise demand for the very top managers and hence executive compensation (and bargaining power). This also depends on the

nature of demand, i.e., how desirable the managers (relative to workers) are to firms. For instance, Khurana (2002) argues that investors in 1980s suddenly started to look for leaders with charisma and good public image (i.e., superstar CEOs), rather than just with (firm-specific) talents. Such change causes firms to overly value CEOs and to overstate the impact of CEOs. As a result, the board of directors is more likely to go after those with prior experience as CEO as reported in Murphy and Zábojník (2007) (and less likely to promote their own employees), leading to higher demand in the market for the very top CEOs. In section 2, we will discuss how this can lead to a substantial increase in executive compensation relative to wage.

In section 2, we provide a preliminary discussion to convey and to formalize the above idea. In practice, it is difficult to formalize the above idea in a unified model. Thus, in section 3 and thereafter, we only consider a simple general equilibrium model, which is particularly appropriate for the discussion of entrepreneurs and firms, rather than for the discussion of the managers (which is left to future works). The model is closely related to Melitz and Ottaviano (2008)—although we only allow for endogenous exit and entry as an extension in section 5—in the sense that we use linear demand function and there is a sufficient statistic, which can be used to discuss, say, trickle-down.[4] The model is also closely related to Behrens et al. (2014) and Behrens and Robert-Nicoud (2014), who use the Melitz-like models to analyze the links between market size, self-selection into entrepreneurship, and inequality. While these works focus on the effects of market size (i.e., population) on inequality, this paper focuses on the effects of economic development, i.e., technological progress and the increase in capital stock. Concretely, we show in section 4 that (depending on the interaction between the top capital income and earned income) income inequality can increase even in the later stage of development when productivity and capital stock are sufficiently high. However, there is an upper limit to income inequality, which depends only on the distribution of desirability (i.e., on the nature of demand). Also, in section 5, we show that government can use progressive taxation to keep income inequality in check. Finally, we give a final remark in section 6.

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[4]There are numerous works using the random growth model to generate the Pareto tails of the income distribution (see for example Gabaix et al. (2016); Jones and Kim (2014)). Here, in this paper, we use a different approach, which is relatively closely related to the Melitz-like model of heterogenous firms.

2. A PRELIMINARY DISCUSSION

To emphasize the importance of the nature of demand, we assume imperfect competition, which is plausible given relatively limited numbers of those at the very top and given imperfect substitution. Also, we introduce a new dimension of heterogeneity, i.e., "desirability" (defined below)—which is the main source of imperfect substitution—into demand function.

Because of imperfect information, in practice, how firms value different workers and how consumers value different goods inevitably involve subjective valuation. Desirability is a measure of this subjective valuation based on (for workers) talent, skill, connection and on (for goods) taste, quality, advertisement, brand, and so on. It is conceivable, for instance, that CEOs are desirable when they have good friendship with the board of directors; and, for goods, when consumers receive new information from advertisement, they will update their valuation and hence desirability. Using desirability as an exogenous variable allows us not only to ignore the problem of how firms and consumers value different workers and goods (which is obviously beyond the realm of economics), but also to capture the importance of the differences in the nature of demand across countries. For instance, we can say that the desirability of manager is relatively high in the countries where the board of directors overly value managers (say because of high faith). We can also say that desirability of luxurious goods is relatively high in materialistic countries.

2.1. *Demand Function*

Let us begin with the properties of demand functions, which are conceivably important for the rise of the working rich. The two most apparent properties are: the higher the desirability, (i) the higher and (ii) the less elastic the demand (holding everything else constant). The first property is obvious. The second property can be justified by the fact that it is hard or costly to find replacement for highly desirable individuals and goods. For instance, compared to normal workers, potential CEOs are rare because they are extremely talented individuals. For goods, in the spirit of Rosen (1981)—who states that "hearing a succession of mediocre singers does not add up to a single outstanding performance" (p. 846)—less desirable goods are poor substitutes for the highly desirable ones. Thus,

holding everything else constant, demand for the latter should be less elastic.

Although the first and second properties might lead to (or overlap with) a third property, we state it as a separate property: (iii) elasticity of demand function is decreasing in demand. Zhelobodko et al. (2012) refer to this property as the "increasing relative love for variety," meaning that when consumption rises, elasticity of substitution among varieties falls. In other words, consumers perceive varieties as being more differentiated and thus are willing to pay more for each variety. For managers, we can say that when the board of directors hires more managers, they have more faith and can rest assured, and therefore are willing to pay higher compensation to each manager. This can be justified by the fact that more faith means higher desirability.

The second and third properties imply that those at the top have high market powers. Because higher price means lower demand, there can be another conceivably important property; i.e., (iv) demand function exhibits increasing returns to desirability. This property is in the spirit of Murphy et al. (1991); that is, individuals choose occupation exhibiting increasing returns to their desirability to obtain extraordinary returns. Indeed, this property along with the second and third properties implies that a small differences in desirability leads to a large differences in earning.

The four properties above might overlap one another, and some properties (say, the fourth property) can be relaxed. Still, as a starting point, we allow for all the four properties and for simplicity use the following linear demand function:

$$y\left(p(i)\right) = \bar{y}(i) - m(i)p(i), \qquad \forall i, \tag{1}$$

where $\bar{y}(i) > 0$ is desirability, $m(i) > 0$ is the slope, and $p(i) > 0$ is price or wage. In addition to the four properties discussed above and obviously simplicity, the virtue of linear demand function (1) is its compatibility with empirical analysis. Particularly, given the data of prices and demands, we can use equation (1) to estimate the slope $m(i)$ and desirability $\bar{y}(i)$ with a simple OLS regression.

It is worth mentioning that, in a fully fledged model, we have to derive demand function (1) from utility/profit maximization problem. It is relatively easy to introduce desirability (a subjective valuation) as a parameter into utility function. We cannot just do this with production function. Thus, to derive demand function for managers, it is more appropriate to consider a model with say endogenous bargaining which depends on desirability and

the competition among firms for the managers. This is beyond the scope of this paper and the fully fledged setup is left to future work. Here, as a preliminary discussion to convey the idea we have in mind, we proceed with linear demand function (1) and, in the case of managers, use markup as a measure of market power.

Before proceeding, it is also worth mentioning that, in the case of managers, equation (1) is the demand function of each firm; and, managers choose the firms with high $\bar{y}(i)$ and low $m(i)$, i.e., firms which are willing to pay the highest wages. If there is a new firm entering the competition for the very top managers, we say the slope $m(i)$ falls (while desirability $\bar{y}(i)$ is unchanged); that is, firms have to pay higher wages to attract or to hold on to their highly-rated managers. In contrast, in the case of good, equation (1) is the market demand function. Thus, if there is a new buyer, we have to add his or her demand function to market demand function and hence desirability rises while the slope needs not fall.

2.2. *The Rise of the Working Rich*

Assume that each individual i faces a constant marginal cost (or marginal disutility of labor) $rc(i) > 0$. Then, maximizing profit subject to market demand function (1), we can write profit maximizing price/wage and profit function of each individual i as follow:

$$p(i) = \frac{\bar{y}(i) + z(i)}{2m(i)}, \tag{2}$$

$$\pi^i = \frac{(\bar{y}(i) - z(i))^2}{4m(i)}, \tag{3}$$

where $z(i) \equiv m(i)rc(i)$ is called "market toughness" for individual i in the sense that when $z(i)$ increases, demand falls and it is more likely that individual i cannot survive the market. From equation (2), we can write the markup $\mu(i) \equiv p(i)/rc(i)$, a measure of the market power of individual i, as follow

$$\mu(i) = \frac{\bar{y}(i) + z(i)}{2z(i)}. \tag{4}$$

It is clear from equation (4) that (a) the higher the desirability, the higher the markup, (b) the lower the market toughness, the higher the markup, and (c) when market toughness falls, the higher the desirability, the higher the increase in markup: $|\partial\mu(i)/\partial z(i)| = \bar{y}(i)/2z(i)^2$.

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The fourth property of demand function and (a)—which arises from the second property—imply in the spirit of Rosen (1981) that the earning is a strictly convex function of desirability and hence a small differences in desirability lead to a substantial differences in earning. Specifically, if desirability increases α-fold ($\alpha > 1$), earning given by equation (3) will increase more than α^2-fold. Then, it is appropriate to point to the (relative) increases in desirability as the explanation for the explosion of the very top earnings in both absolute and relative terms. This is particularly likely to be the case for the top entrepreneurial income because the very top entrepreneurs are innovators and inventors, i.e., those who know how to raise/promote their (relative) desirability. As a starting point, however, this paper does not provide a fully fledge model with endogenous desirability. We take desirability exogenously and, when applicable (say regarding to trickle-down), we discuss the effect of an exogenous increase in desirability. In this paper, we focus more on the effect of economic development (i.e., the rises in technology/productivity and capital stock) through market toughness $z(i)$,[5] and show that this effect depends on the distribution of desirability $\bar{y}(i)$, which can explain the large cross-country differences. This is the task of the following sections. In this section, we use the remaining space for the discussion of the rise of the supermanagers.

(b), which arises from the third property of demand function, implies that when the competition among firms for the managers is tough, managers can exert higher market powers on firms and thus can claim higher shares in revenues. Then, it immediately follows that anything occurring during and after 1970s—which allows firms in various industries to realize remarkable developments, and leads to higher demands (and hence lower market toughnesses) for the top managers—can explain the strong correlation between executive compensation and firm size in the last several decades which was weak prior to 1970s as reported by Frydman and Saks (2010). Although we cannot show explicitly in this paper, we point to technological progress—e.g., advances in ICT and automated production which started particularly in the early 1970s—and (less importantly) the increase in capital stock, which respectively provide the means and resources for firms in various industries

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[5]Note from profit function (3) that even when $z(i) = z$ for all i, a fall in z (due to a fall in either the slope m or marginal cost rc) benefits the more desirable individuals more as stated in (c). Still, this does not guarantee that inequality in (earned) income must increase when z falls. As we shall see in section 4, it depends on the interaction between capital income and earned income.

to enlarge firm size and market size. The resulting increase in scale (i.e., the rise of firm size) allows the top firms to afford extremely general pays; and, the resulting increase in the number of top firms in various industries—along with skill-biased technological changes and the change in general managerial skills—raises demand and hence the relative bargaining power of the top managers as stated in (c) above.

To give an illustrative example, suppose that the ratio of average executive compensation to average wage—denoted by c/w—rises from 30 to 300 in forty years. One way to explain this is to discuss why average compensation grows faster than average wage. For instance, if average compensation annually grows 6% faster than average wage (i.e., if the growth rate of c/w is 6%), c/w will increase from 30 to around 96, 172, and 309 respectively in 20, 30, and 40 years. At first glance, a 6% difference might not seem much, but it represents a significant disparity in absolute terms. To see this, suppose that initially wage is 10,000 dollars and compensation is 300,000 dollars (30 times 10,000 dollars). If the growth rate of wage is 1% and that of compensation is 7%, in absolute terms, workers receive a 100-dollar raise, while CEOs receive a 21,000-dollar raise. The difference is bigger, the bigger the initial amount. It is difficult to justify such great disparity especially within firms. Thus, in addition to time horizon, we should also look into the firm horizon. Suppose for illustrative purpose that there are new 500 (resp. 1000) top firms in 40 years entering the competition for the top managers. If an additional firm raises the ratio of average compensation to average wage c/w by 0.1%, 0.2%, and 0.3%, holding everything else constant, c/w will increase from 30 to respectively 48, 81, and 134 (resp. 81, 221, and 600).

The effect of an additional firm on compensation conceivably depends on the nature of demand, i.e., how much firms value managers relative to workers. To get a clearer picture, imagine an auction for the very top managers. The more the firms value the manager, the more the firms involve in the auction and the higher the bargaining powers of the managers; meanwhile, technological progress and capital accumulation raise the capacity of firms in the bidding. As a result, given that a small change can lead to a substantially large amount (as shown in the example above) and that the nature of demand can amplify this change (as shown in (c)), it is appropriate to expect that the change along firm horizon can not only explain the rise of the supermanagers, but along with the differences in the nature

of demand also explain the large differences (regarding to the supermanagers) across the rich countries, which have experienced similar economic development.

3. A BASELINE MODEL

This section considers a baseline general equilibrium model to discuss inequality arising from profit given by equation (3). Concretely, the economy is populated by a continuum of individual producers with mass $\mathcal{J}$. Each individual $j \in \mathcal{J}$ is endowed with (or inherits) k^j units of capital and can produce one good.

To derive linear demand function (1), by convention, we can use quadratic utility function. To enrich the microeconomic foundations of desirability, we find it crucial to discuss the general form and microeconomic foundations of quadratic utility function. Still, to prevent this discussion drawing attention away from the main objectives, we leave it to Appendix A in our companion paper cited in the references. Here, let us just say we use a variant of the quadratic utility function in Melitz and Ottaviano (2008):

$$U_t^j = \sum_{s=t}^{\infty} \beta^s \left[\Phi^j \left(k_{s+1}^j \right) - V^j \left(\left\| \bar{x}^j - x_s^j \right\| \right) + \Gamma^j \left(\bar{x}^j \right) \right], \qquad \forall j \in \mathcal{J},$$

where $\beta \in (0,1)$ is discount factor, k_{s+1}^j is capital holding, x^j is consumption vector of differentiated goods, and $\bar{x}^j \gg 0$ is the vector of "desired demand," highlighting desirability. $\|\cdot\|$ is Euclidean norm, and Φ^j and V^j are twice continuously differentiable and strictly increasing functions where the former is concave while the latter is convex. The third term $\Gamma^j \left(\bar{x}^j \right)$ does not affect utility maximization problem and hence can be ignored.

To see how the above (instantaneous) utility function is a variant of that in Melitz and Ottaviano (2008), ignore the first term for a moment and note that

$$\alpha \int_i x_i di - \frac{\gamma}{2} \int_i x_i^2 di - \frac{\eta}{2} \left(\int_i x_i di \right)^2 = -\frac{1}{2} d \left(\frac{\alpha}{\gamma + \eta}, x \right)^2 + \frac{1}{2} \frac{\alpha^2}{\gamma + \eta},$$

where α, γ, and η are strictly positive and $d(\bar{x}, x)^2 = \gamma \int_i \left(\bar{x}_i - x_i \right)^2 di + \eta \left[\int_i \left(\bar{x}_i - x_i \right) di \right]^2$, and where we have implicitly assumed for simplicity that $\int_i di = 1$. Obviously, Melitz and Ottaviano (2008) uses a different notion of distance, while we use Euclidean norm to simplify the analysis and to keep generality regarding to Φ^j, the utility attained from capital holding. The introduction of Φ^j into utility function does not affect linear demand function (1) and thus the discussion in section 2. We include Φ^j because capital is a

numeraire, which differs from differentiated goods. Also, without Φ^j, it is not possible to determine each individual's capital holding. Moreover, the introduction of Φ^j along with the assumption that $d\Phi^j/dk_{s+1}^j > 0$ ensures that in general consumer does not consume at desired demand.

Without loss of generality, let us normalize the price of capital to unity. For simplicity, assume that capital does not depreciate and aggregate capital stock $\kappa = \int_{j \in \mathcal{J}} k^j dj > 0$ is constant over time. The timing is as follow. At the beginning of period s, individual j lends capital holding k_s^j out for production with interest rate r_s. At the end of period s, he or she will receive $\pi_s^j + (1 + r_s) k_s^j$, from the production of good j and the rent of capital. Then, he or she will determine the amount of consumption x_s^j and capital holding k_{s+1}^j. Therefore, we can write flow budget constraint of each individual $j \in \mathcal{J}$ at time $s \geq t$ as follow

$$p_s \cdot x_s^j + k_{s+1}^j = \pi_s^j + (1 + r_s) k_s^j,$$

where $p \gg 0$ is price vector and the dot sign denotes inner product. Because each consumer is a producer, we abstract this paper from labor and consider π^j as aggregate earned income and rk^j as capital income. To avoid confusion, it is worth mentioning that profit paid to individuals in the form of dividend is considered as capital income. However, in our setting, individuals own firm not because of shareholdings but because of the uniqueness of their desirability.

From utility maximization problem, we can write demand function of each consumer $j \in \mathcal{J}$ as

$$x^j(i) = \begin{cases} \bar{x}^j(i) - m^j p(i) & \text{if } \bar{x}^j(i) > m^j p(i) \\ 0 & \text{if } \bar{x}^j(i) \leq m^j p(i) \end{cases}, \qquad \forall i \in \mathcal{J} \qquad (5)$$

where $m^j > 0$ is an endogenous variable and where time subscript is omitted to save notation. Adding up individual demand function (5) across individual $j \in \mathcal{J}$, we obtain market demand function (1), where $(\bar{y}(i), m(i)) = \int_{j \in \mathcal{J}_i} (\bar{x}^j(i), m^j) \, dj$ and $\mathcal{J}_i \subseteq \mathcal{J}$ is the set of consumers who consume good i. We can infer from individual demand function (5) that when there is technological progress, producers can lower prices or raise desirability and hence can increase the market size (i.e., the number of buyers). For instance, advance in automated production in the spirit of Rosen (1981) allows firms to supply to a large market with low cost. Also, advance in ICT, e.g., the creation of social network provides

an easy access to a large number of audiences so firms can promote their brands (i.e., to raise their desirability) say through advertisement. This allows firms not only to raise demands and market powers, but also to bring in new buyers.

3.1. *Trickle-Down and Market Toughness*

To complete the model, assume that utility maximization problem has interior solution and, hence, we know from individual demand function (5) that market demand functions of all goods have the same slope, i.e., $m(i) = m, \forall i \in \mathcal{J}$. Then, from equations (1)-(2) and market clearing condition for capital—given by $\int_i c(i) y_s(i) di = \kappa$, where $1/c_i$ is (capital) productivity of producer i, assumed to be constant over time—we have

$$mr = \left(\int_{i \in \mathcal{J}} c(i) \bar{y}(i) di - 2\kappa \right) / \|c\|^2 < \min_i \{ \bar{y}(i)/c(i) \}, \tag{6}$$

where c is a vector, and the inequality (which will be relaxed in section 5) ensures that all producers survive the market. Because mr is identical across producers and is proportional to the market toughness of each producer, we can analyze the effect of each producer on the whole economy through mr. Here, we are particularly interested in the so-called "trickle-down," which states that the good performance of those at the top benefits the rest (through a smaller mr). Before we do this, it is worth mentioning beforehand that producers in our economy face a direct competition with one another; that is, they compete to get a piece of the limited input κ, and their competition determines the size of the economic pie.

From equation (6), it is straightforward to show that, for all $j \in \mathcal{J}$, we have

$$\|c\|^4 \frac{\partial mr}{\partial c(j)} = \int_i c(i) \left[\bar{y}(j)c(i) - \bar{y}(i)c(j) \right] di + c(j) \left(4\kappa - \int_i \bar{y}(i)c(i) di \right).$$

It is obvious that the effect of an increase in productivity of any firm on other firms depends on the joint distribution of productivity and desirability. Suppose, for example, that the most desirable producers are also the most productive. Then, from preceding equation, we know that when the market for those at the top is not tough (i.e., when κ and $1/c(j)$ are sufficient high), the increase in the productivity of those at the top benefits the whole economy (since $\partial mr/\partial c(j) > 0$); that is, there is indeed trickle-down. The reason is that the increase in productivity enlarges the size of the pie (for a given input) and loosens the

competition for input,[6] benefiting all producers.

The above discussion suggests that it is good for the whole economy if the top monopolists have incentives to invest in say innovation, aiming at raising productivity. However, if they instead have incentives to invest in say advertisement or in the improvement of attractiveness of the products to raise desirability and hence prices and market powers (say because of the lack of competition), their investment will hurt other producers since $\partial mr/\partial \bar{y}(j) > 0$; that is, there is no trickle-down. Loosely speaking, this is because the purpose of the investment is not to enlarge the size of the pie but to establish themselves in the consumption behaviors of consumers and to obtain a larger share of the pie. If this is the case, it is tougher (i.e., $mr \uparrow$) for the rest to get a piece. Also, not only will inequality rise, but economic efficiency and growth will also fall because, with tougher market, some firms might go out of business and entry is difficult. This discussion makes it clear that trickle-down needs not always work. Also, provided that incomes are increasing at the top but stagnated at the bottom and in the middle, it is important to re-examine the idea of trickle-down, which can always be used as an excuse for those at the top.

To be sure, here, we do not say advertisement or the endeavor to improve attractiveness is bad. Indeed, advertisement is a good way to convey information. Also, improvement in attractiveness (say quality enhancement) will surely improve the quality of the pie. The important point to be taken here is that raising desirability—either through advertisement or other means—is a perfect strategy because not only will the top monopolists benefit, but the resulting tougher market also impedes entry. Thus, from the perspectives on equity and prosperity, it is good to improve productivity, i.e., to enlarge the size of the pie. As commonly believed, to do this, it is important to promote competition among producers in the output markets and, as one can infer from our model, to loosen the competition in the input market, i.e., to ensure that those at the bottom—who have both low productivity and desirability—and potential entrant have access to input and have high production capacity.

So far, we have kept the heterogeneity of productivity. For the rest of this paper,

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[6]Be cautious that, to expand production, those at the top also raise demand for input and hence toughen the competition. Then, if the input market is very tough (say because of limited supply), i.e., if mr is high, those at the top have incentives to raises production rather than to raise markup (see equations (1) and (4)). In such case, the increase in productivity of those at the top can be harmful to other producers.

we give up this generality and assume that $c(i) = c, \forall i \in \mathcal{J}$ to analyze the effect of productivity on income inequality. In such case, market toughness $z = mrc$ is identical across producers and, from equation (6), can be written as

$$z = \left(c \int_{i \in \mathcal{J}} \bar{y}(i) di - 2\kappa \right) / \mathcal{J}c > 0. \tag{7}$$

It is obvious that market toughness is strictly decreasing in both capital stock and productivity. In other words, along economic development as productivity and capital stock rise, market toughness falls, benefiting every individual; but those at the top can reap relatively higher benefit (recall from footnote 5). In section 4, we show that a fall in the market toughness along economic development gives rise to the working rich,[7] and (depending on the interaction between the top capital and earned incomes) raises income inequality in the later stage of development; and, the magnitude of the increase in income inequality depends on the distribution of desirability. Before we can do these, we complete the model in the following subsection with the discussion of steady state.

3.2. *Steady State*

From utility maximization problem, we can write Euler equation and transversality condition (TVC), respectively, as

$$\frac{\beta \left(1 + r_{s+1} \right) v_{s+1}^j}{\| p_{s+1} \|} = \frac{v_s^j}{\| p_s \|} - \phi_s^j, \tag{8}$$

$$\lim_{s \to \infty} \frac{\beta^s v_s^j}{\| p_s \|} k_{s+1}^j = 0, \tag{9}$$

where v^j and ϕ^j are the first-order derivatives of V^j and Φ^j respectively. The dynamic of the economy is determined by Euler equation (8), TVC (9), and flow budget constraint. To keep heterogeneity among individuals, it is important to put restriction on utility function. By convention, we can assume that utility function is quasilinear in capital holding, i.e., $\phi^j = 1$, and use the quadratic utility function, i.e., $v(d) = d$. However, Appendix A shows that, in such case, some individuals can accumulate infinite wealth while the others have infinite debt. Intuitively, because utility is quasilinear in capital, there is no wealth effect on consumption. Thus, individuals will accumulate infinite wealth if they earn sufficiently

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[7]Consistently, we can say that the tough market in the early stage of development prevents the rise of the working rich and ensures that the coupon-clipping rentiers dominate at the top of income hierarchy.
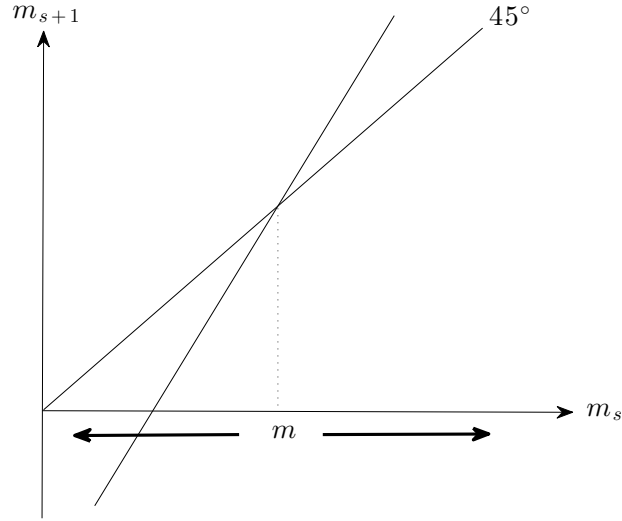
Figure 3: The Dynamics of the Economy.

high income and can raise wealth in any period. This case provides little qualitative implications. Accordingly, here, we assume instead that $\phi^j = \phi$ with $\phi' < 0$ and $v^j = 1$ for all $j \in \mathcal{J}$. Then, from Euler equation (8), we have $k^j = \kappa/\mathcal{J}$ for all $j \in \mathcal{J}$ and thus we can rewrite Euler equation (8) as

$$m_{s+1} = \frac{1}{\beta}m_s - rm - \frac{m\|p\|}{\beta}\phi\left(\frac{\kappa}{\mathcal{J}}\right), \qquad \forall s \geq t, \tag{10}$$

where rm and $m\|p\|$ are constant over time. Equation (10) is displayed in Figure 3. As we can see, the economy can take three paths, but it is possible to show that the economy jumps immediately to the steady state, at which m and r are constant and are given by

$$m = \frac{\beta}{1-\beta}mr + \frac{m\|p\|}{1-\beta}\phi\left(\frac{\kappa}{\mathcal{J}}\right), \qquad r = z/mc.$$

To show this, we can use TVC to rule out the other paths, on which m either rises indefinitely or falls indefinitely. Particularly, we can rewrite equation (10) as

$$m_s - m = \lim_{T\to\infty} \beta^{T-s}(m_T - m).$$

Then, it immediately follows that TVC is violated if the economy is on the indefinitely-rising (resp. indefinitely-falling) path, which requires $m_s > m$ (resp. $m_s < m$).

4. INCOME INEQUALITY

This section focuses on pre-tax income inequality and leaves after-tax income inequality to section 5.

4.1. *Income Inequality along Economic Development*

As the measure of inequality, we use Gini coefficient, which will be convenient for decomposition. To show the robustness of the results, nevertheless, we also consider coefficient of variation in Appendix B-C.

Without loss of generality, assume that desirability follows a distribution F with support on $[a, b]$, where $\int_a^b dF = \mathcal{J}$ and $a > z$. Because the economy jumps immediately to the steady state, inequality in income in each period is identical to inequality in lifetime income. Then, letting $\Pi = \int_a^b \pi(u) dF(u)$ be aggregate earned income, we can write Gini coefficient of income GC_I as follow

$$GC_I\left(z(\omega), \omega\right) = 1 - \frac{2}{\mathcal{J}} \int_a^b \int_a^s \frac{\pi(u) + r\kappa/\mathcal{J}}{\Pi + r\kappa} dF(u) dF(s) = \frac{\Pi}{\Pi + r\kappa} GC_\pi,$$

where $\omega \equiv \kappa/\mathcal{J}c$ and GC_π is Gini coefficient of earned income and is given by

$$GC_\pi(z) = 1 - \frac{2}{\mathcal{J}} \int_a^b \int_a^s \frac{\pi(u)}{\Pi} dF(u) dF(s).$$

From equation (3), we can tell that inequality in earned income GC_π depends only on market toughness z, while GC_I also depends on ω. Appendix B shows that inequality in earned income is strictly increasing in market toughness, i.e., $GC_\pi'(z) > 0$. To see why, recall that when market toughness falls, those at the top can reap relatively higher benefits. Thus, the range of earned income distribution $\pi(b) - \pi(a)$ is bigger. However, because all producers benefit, the increase in aggregate earned income is more than proportional to the increase in the range. As a result, the gap between top and bottom earned income shares $[\pi(b) - \pi(a)]/\Pi$ falls and so does Gini coefficient. This is hardly surprising because the model underestimates the top earnings, and overestimates the bottom earnings by assuming that all producers survive the market (see subsection 5.2). Interestingly, even in such setting, income inequality is actually rising as market toughness falls, i.e., $\partial GC_I/\partial z < 0$ (see Appendix B) because aggregate earned income share $\Pi/(\Pi + r\kappa)$ rises (for a given ω). Intuitively, this occurs because capital income lowers the impact of market toughness

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through aggregate income, and one can verify that the gap between the top and bottom income shares $[\pi(b) - \pi(a)]/(\Pi + r\kappa)$ is strictly decreasing in z for a given ω. To be more specific, let Y and R be aggregate income and the range of income distribution, respectively. Then, we must have

$$\frac{\partial \log(R/Y)}{\partial z} = \frac{\partial R/\partial z}{R} - \frac{\partial Y/\partial z}{Y}. \tag{11}$$

Suppose for illustrative purpose that initially earned income π is the only source of income. Then, if we add another constant source of income $\omega > 0$, R, $\partial R/\partial z$, and $\partial Y/\partial z$ are unaffected, but $Y = \Pi + \mathcal{J}\omega$ is higher. In such case, the higher the additional source of income ω, the more likely the income inequality rises as market toughness falls.

Before proceeding, it is first noteworthy that the first term on the right hand side of equation (11) highlights the growth rate of the spread of income distribution, and the second term highlights the growth rate of GDP (along economic development). Equation (11) is an important identity linking growth rate and the variation in income inequality. In practice, R can be replaced by the gap between the earnings of say the top 10% and bottom 10%. If capital income is unequally distributed, R is higher and the variation in income inequality will also depend on whether the top capital income and top labor income move in the same direction. For instance, if the top capital owners (who need not be the working rich) invest in the top firms, a fall in market toughness—which disproportionally raises top earnings and thus top capital income—is likely to raise income inequality.

Now, let us turn to the effect of capital stock and productivity on income inequality. Appendix C shows that a rise in ω (due to a rise in either capital stock or productivity) lowers income inequality, i.e., $\partial GC_I/\partial\omega < 0$. This is not surprising because the increase is perfectly equally distributed among individuals. Even in such case, Appendix C also shows that a rise in either productivity or capital stock still raises income inequality through a fall in market toughness when ω is sufficiently high and thus market toughness is sufficiently low. We can easily see this by simply differentiating $[\pi(b) - \pi(a)]/(\Pi + r\kappa) \equiv R/Y$ with respect to ω:

$$\text{sign}\left\{\frac{d(R/Y)}{d\omega}\right\} = \text{sign}\left\{\int_a^b u^2 dF(u) - \mathcal{J}z(a + b - z)\right\}.$$

Then, when market toughness is sufficiently small, $d(R/Y)/d\omega$ must be strictly positive. As already discussed with equation (11), this is because the higher the ω, the stronger

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the indirect effect of ω on income inequality through market toughness. As a result, in the later stage of development when productivity and capital stock are sufficiently high and thus market toughness is sufficiently small, income inequality rises along economic development as productivity and capital stock rise. Moreover, in such case (i.e., when z is sufficiently small), we can also show that an increase in ω raises earned income share $\pi^j/(\Pi + r\kappa)$ and the ratio of earned income to capital income $\pi^j/(r\kappa/\mathcal{J})$ of each individual $j \in \mathcal{J}$. The increases in both $\pi^j/(\Pi + r\kappa)$ and $\pi^j/(r\kappa/\mathcal{J})$ are more pronounced at the top. These results are consistent with the evidences reported in Figure 1; and, it is appropriate to state that, in the latter stage of development when capital stock and productivity are sufficiently high, the low value of market toughness allows the working rich to excel, raising the importance of earned income in the top income share. To put it differently, high capital stock and advance in technology in the later stage of development make it relatively easy for an individual to make substantial earnings.

Because capital income is perfectly equally distributed, we avoid stressing the rise of top earned income share relative to top capital income share. In fact, even in the United States, Atkinson et al. (2011) point out that top capital income including capital gain (which is ignored in this paper) also rises significantly in recent years. One possible explanation is that the working rich and coupon-clipping rentiers co-habitate, as suggested by Wolff and Zacharias (2009), because in the long run the former can accumulate substantial wealth. Another possible explanation is that the top capital owners invest heavily in the top firms.

4.2. *The Convergence of Income Inequality*

In our model, there is no obvious market force against the rising in income inequality, in spite of the fact that all individuals benefit from economic development. Although this is obviously in contrast to the optimistic inverse-U curve hypothesis of Kuznets (1955), income inequality in our model does not rise unboundedly; and, the path of income inequality along economic development in the later stage looks like the graph displayed in Figure 4, where $\overline{GC}_I$ is given by

$$\overline{GC}_I \equiv \lim_{z \to 0} GC_I = 1 - \frac{2}{\mathcal{J}} \int_a^b \int_a^s \frac{u^2}{\int_a^b v^2 dF(v)} dF(u) dF(s) < 1.$$
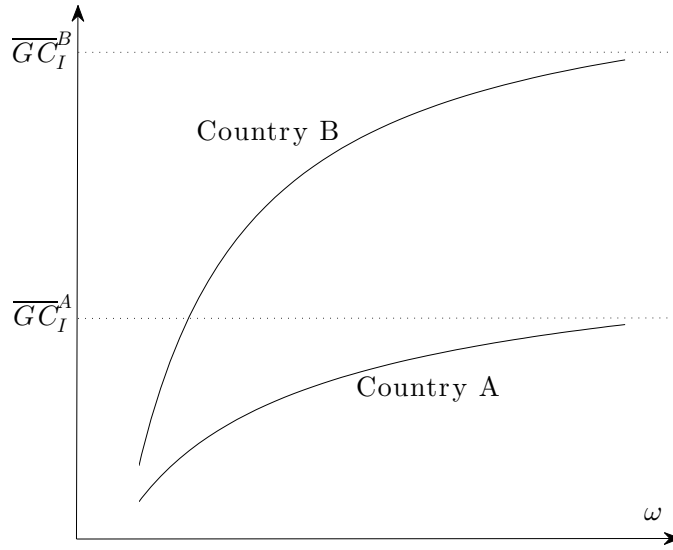
Figure 4: Income Inequality along Economic Development in the Later Stage

To be more specific, note from equation (7) that as productivity and capital stock rise, market toughness will converge to zero. Thus, we can say that, along economic development income inequality will converge to $\overline{GC}_I$, which is obviously independent of productivity and capital stock, and depends solely on the distribution of desirability. As shown in Figure 4, countries with different distributions of desirability (highlighting the nature of demand) will converge to different limits. This explains why there can be large differences among the rich countries, which have experienced similar economic development. Given the relatively stable income inequality in say France and Japan (see Figure 2), it is appropriate to expect that $\overline{GC}_I$ in these countries is low, whereas that in say the United States high.

Once we establish that $\overline{GC}_I$ can explain the cross-country differences, the naturally ensuing question should be whether low value of $\overline{GC}_I$ is better. The answer is yet to be known. More research is obviously needed. What we can say in this paper is the following. Recall from subsection 3.1 that the endeavor of those at the top to raise desirability can improve the quality of the economic pie. Thus, the high value of $\overline{GC}_I$ per se need not be bad. What is likely to be bad is the high sensitivity of $\overline{GC}_I$ to the top monopolists' endeavors. That is, if desirability can be easily swung by say advertisement, the top monopolists have strong incentives to invest in advertisement which, as we already discussed

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in subsection 3.1, will hurt the economy and increase the number of the left out.[8] In contrast, if desirability is adequately stable, the top monopolists will instead have incentive to invest in say innovation to lower productivity (or to invent a new variety in which case we have little to say about). If this is the case, even when the desirability of those at the top is relatively very high, their investment—which (intentionally or unintentionally) raises the size of the economic pie—will bring the left out into the equation (through trickle-down).

The above discussion is easier said than done. In practice, it is difficult to design a policy to ensure the stability of $\overline{GC}_I$, which is obviously an elusive goal and requires a great corporation among the whole population. (Note that $\overline{GC}_I$ depends on aggregate level of desirability $\bar{y} = \int_{j \in \mathcal{J}} \bar{x}^j dj$, not on individual level $\bar{x}^j$). Also, the realization of such goal is undoubtedly beyond the scope of economics, and requires other fields such as psychology and politics. This brings us to the next question which can be answered in the realm of economics; that is, given that income inequality converges to $\overline{GC}_I$, if $\overline{GC}_I$ is very high and is socially unacceptable, does this mean capitalism is doomed to fail as predicted by some classical economists (e.g., Karl Marx)?[9] In the next section, we will provide an answer against this question. Particularly, we will show that government can use progressive taxation to reduce the limit to after-tax income inequality.[10]

Before proceeding, it is worth mentioning that $\overline{GC}_I$ can be an important index tracking income inequality. Although $\overline{GC}_I$ is a theoretical construct, in practice, it is not impossible to estimate $\overline{GC}_I$ say by using linear demand function (1).

5. EXTENSIONS

This section provides two extensions. In subsection 5.1, we take government income redistribution into account and consider the effect of taxes on income inequality. In subsection

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[8]Note that $\overline{GC}_I$ is derived under the assumption that all producers face the same market toughness z. If this is not the case, it is possible that those at the bottom are left out of the equation.

[9]The high level of income inequality in the rich countries already causes discontent, as reflected by the Occupy Wall Street movement in the United States and other similar movements around the world.

[10]As Piketty (2014) suggests, inherited wealth should play non-trivial role in the discussion of income inequality. Appendix A shows that wealth inequality diverges when steady state depends on the distribution of initial wealth. Thus, we should be cautious not to be too optimistic given that inherited wealth (which is ignored in this paper) plays no role in the derivation of $\overline{GC}_I$; it is better to consider $\overline{GC}_I$ as the limit to inequality in earned income, rather than in total income.

5.2, we will relax the assumption that each individual produces one good in equilibrium, and allow for endogenous entry.

5.1. *Income Redistribution*

Suppose that government redistributes income by imposing income taxes and making transfers to all individuals to balance its budget. Let τ_π, τ_k, and t^j (where t^j satisfies $\int_j t^j dj = 1$) be the rate of earned income tax, the rate of capital income tax, and a fraction of tax revenue transferred to individual j. Then, we can write flow budget constraint of each individual j as follow

$$p_s \cdot x_s^j + k_{s+1}^j = (1 - \tau_\pi) \pi_s^j + (1 - \tau_k) r_s k_s^j + k_s^j + t^j (\tau_\pi \Pi_s + \tau_k r_s \kappa).$$

In such case, market toughness given by equation (7) is unchanged while Euler equation (8) becomes

$$\frac{\beta [1 + (1 - \tau_k) r_{s+1}]}{\|p_{s+1}\|} = \frac{1}{\|p_s\|} - \phi_s.$$

From the above Euler equation, we know that each individual still earns the same amount of capital income, and the economy jumps immediately to the steady state. Then, we can write the after-tax Gini coefficient of income GC_I^A as follow

$$GC_I^A = (1 - \tau_\pi) GC_I + \frac{\tau_\pi \Pi + \tau_k r \kappa}{\Pi + r \kappa} GC_T, \tag{12}$$

where GC_T is the Gini coefficient of government transfer and is given by

$$GC_T = 1 - \frac{2}{\mathcal{J}} \int_a^b \int_a^s t(u) dF(u) dF(s).$$

To understand equation (12), one should note that total income consists of earned income, capital income, and government transfer. Income inequality arises from the unequally distributed sources of income, namely earned income and government transfer, respectively, highlighted by the first and second terms on the right hand side of equation (12). Suppose first that tax revenue is equally distributed among individuals, i.e., $t^j = 1/\mathcal{J}$ for all $j \in \mathcal{J}$. Then, GC_T is zero and after-tax Gini coefficient equals $(1 - \tau_\pi) GC_T$, which depends only upon the rate of earned income tax and is strictly smaller than before-tax Gini coefficient (as long as $\tau_\pi > 0$). This is because earned income tax affects the distribution of income and induces progressive income tax scheme in the sense that the higher

the income, the higher the fraction of income the individual has to pay, i.e., the higher the $\tau_\pi (\pi - \Pi/\mathcal{J})/(\pi + r\kappa/\mathcal{J})$. Suppose instead that tax revenue is unequally distributed. In such case, the rates of earned income tax and capital income tax appear in the second term on the right hand side of equation (12) because they affect the distribution of government transfer, and their effects depend on whether GC_T is negative. For instance, if government transfer is regressive—in the sense that $t(u)$ is decreasing in desirability u and thus in income—the Lorenz curve of t must be strictly above the $45°$ line and the Gini coefficient of t must be strictly negative, i.e., $GC_T < 0$.[11] As a result, the regressive government transfer negatively affects income inequality and the effect is higher, the higher the aggregate income share of tax revenue.

We know from equation (12) that, as market toughness converges to zero, after-tax Gini coefficient will converge to $\overline{GC}_I^A$ given by

$$\overline{GC}_I^A \equiv \lim_{z \to 0} GC_I^A = (1 - \tau_\pi)\overline{GC}_I + \tau_\pi GC_T.$$

Then, as long as $GC_T < \overline{GC}_I$ (say because $GC_T < 0$), $\overline{GC}_I^A$ is strictly smaller than $\overline{GC}_I$ and is strictly decreasing in the rate of earned income tax τ_π.

5.2. *Endogenous Entry*

In this subsection, we allow for the possibility that some producers cannot survive the market; i.e., we relax the assumption that $z < a$. Then, there are only $\int_z^b dF < \mathcal{J}$ surviving producers. In such case, assuming that those, who cannot survive the market (i.e., those whose desirability belongs to [a,z]), live off capital holdings, we can rewrite equation (7) as follow

$$\int_z^b (u - z)dF(u) = 2\kappa/c.$$

Obviously, market toughness is still strictly decreasing in capital stock and productivity and, thus, eventually (as capital stock and productivity rise) market toughness will fall below a as in previous sections.

It is straightforward to show that capital stock is still equally distributed. Then, there is no income inequality within the bottom $\int_a^z dF(u)/\mathcal{J}$ percent. In such case, Gini

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[11]To be more specific, suppose that $t(\cdot)$ is continuously differentiable and $t'(u) < 0$. Then, we have $GC_T < 0$ because $B(s) = \int_a^s (t(u) - 1/\mathcal{J})\, dF(u)$ is strictly concave and satisfies $B(a) = B(b) = 0$.

coefficient of income can be simplified to

$$GC_I = 1 - \frac{2}{\mathcal{J}} \int_z^b \int_z^s \frac{\pi(u) + \nu^{-2} r\kappa/\mathcal{J}}{\Pi + r\kappa} dF(u)dF(s),$$

where $\nu = \int_z^b dF/\mathcal{J}$ is the the fraction of the surviving producers and $\Pi = \int_z^b \pi(u)dF(u)$ is aggregate earned income. Given that $\pi(z) = 0$, the effect of market toughness on income inequality through a change in the number of surviving producers is only captured by a change in the fraction ν. Then, we can tell from the above equation that a fall in z raises the number of surviving producers (i.e., raises ν) and thus, ceteris paribus, puts upward pressure on income inequality. To see why, suppose that market toughness falls from z to z'. Then, income inequality among producers within the range $[z', z]$ rises from zero to strictly positive. Furthermore, it is possible to show that Gini coefficient is still strictly decreasing in market toughness for a given ν. Thus, as already noted, the assumption that all individuals are producers underestimates the effect of market toughness on income inequality.

6. FINAL REMARK

Piketty (2014) proposes that if interest rate is higher than growth rate, wealth inequality is self-reinforcing. Whether his argument is right, the evidence is clear that wealth in many of the rich countries is highly concentrated. This makes the analysis of income inequality particularly significant because (given highly concentrated wealth) the high level of income inequality—which leads to low level of economic and social mobility (see for example Corak (2013))—is likely to create social unrest. In fact, the high level of inequality—especially in the rich countries such as the United States where the standard of living is high—has started gathering attention and already caused discontent (as reflected by the Occupy Wall Street movement). Accordingly, inequalities deserve more attention.

Some people still have optimistic views that inequalities are merely the result of economic development, and that the social unrest (e.g., the Occupy Wall Street movement) is the politics of envy. Needless to mention that those at the top have incentives to change the rules in their favours (see Stiglitz (2012)), at this juncture, little do we know about inequalities. Even when those believing in the politics of envy turn out to be true, they do not provide the answer to the problem because envy created by inequalities affects the

economic, social and political orders, while it is impossible to restrain everyone from being envious. Despite these obvious facts, it is surprising that inequalities receive (relatively) little attention especially in the mainstream economics. Perhaps, one of the main reasons is technical difficulty. Concretely, allowing for heterogenous agents in general equilibrium models generates insoluble problem. Restrictive assumptions (e.g., indirect utility function with Gorman form) are required to ensure tractability and the resulting models often have similar characteristics to those with representative agent (see, for example, Caselli and Ventura (2000)).

This paper presents a simple general equilibrium model with monopolistic competition. With imperfect competition, this paper can use the nature of demand (in addition to initial endowment of physical and human capitals) as the main source of inequality (not to mention that this paper does not assume Gorman form). This paper shows that the nature of demand affects income inequality, and can explain not only the rise of the working rich but also the large cross-country differences. Although depending on some restrictive assumptions (say regarding to the specific form of utility function) which need to be qualified, this paper presents a simple model, which can be used as a starting point.

To the best of my knowledge, the analysis of the rise of the very top entrepreneurs is very limited relative to that of the rise of the supermanagers which is one of the focal points in corporate finance. While mainly making contributions to the former, we hope that the preliminary discussion in section 2 also ignites the interest and evokes more studies regarding to the latter.

A. QUASILINEAR UTILITY FUNCTION

Suppose that $v^j(d) = d$ and $\phi^j = 1$ for all $j \in \mathcal{J}$. Then, Euler equation (8) in the text becomes

$$\beta \left(1 + r_{s+1}\right) m_{s+1}^j = m_s^j - 1. \tag{A1}$$

Adding both sides of (A1) over $j \in \mathcal{J}$, we obtain

$$\beta \left(1 + r_{s+1}\right) m_{s+1} = m_s - \mathcal{J}. \tag{A2}$$

Similar to the discussion in the text, TVC ensures that the economy jumps immediately to the steady state and, thus, m and r are constant over time. From (A1), it follows that

m^j/m is also constant over time. Then, flow budget constraint of each individual j can be rewritten as

$$\Delta k_{s+1}^j = \left(\pi^j - p \cdot x^j\right) + r k_s^j. \tag{A3}$$

Because $\left(\pi^j - p \cdot x^j\right)$ is constant over time, it immediately follows from (A3) that capital k^j of each individual j rises (falls) indefinitely if $\left(\pi^j - p \cdot x^j\right) + r k_0^j$ is strictly positive (negative). In such case, it is possible to show that TVC is not violated. To see this, suppose that $\lim_{s\to\infty} k_{s+1}^j = \infty$. Then, from (A3), we know that the growth rate of k_{s+1}^j converges to r. As a result, we must have

$$\lim_{s\to\infty} \beta^s k_{s+1}^j = 0,$$

because we know from (A2) that $\beta(1+r) < 1$.

B.   THE IMPACTS OF MARKET TOUGHNESS

This appendix gives mathematical calculations of comparative statics. To save notation, we can write Gini coefficients and coefficient of variations of earned income and aggregate income in general forms as follows

$$
\begin{aligned}
GC &= 1 - \frac{2}{\mathcal{J}} \int_a^b \int_a^s \frac{\pi_z(u) + \omega}{\Pi_z + \mathcal{J}\omega} dF(u)dF(s), \\
CV &= \left[\frac{1}{\mathcal{J}} \int_a^b \left(\frac{\pi_z(u) - \bar{\pi}_z}{\bar{\pi}_z + \omega}\right)^2 dF(u)\right]^{1/2},
\end{aligned}
$$

where $\pi_z(u) = (u-z)^2/4z$, $\Pi_z = \int_a^b \pi_z(u)dF(u)$, $\bar{\pi}_z = \Pi_z/\mathcal{J}$, and ω equals 0 for earned income and equals $\kappa/\mathcal{J}c$ for aggregate income. Then, with straightforward calculation, we have

$$\frac{\partial GC}{\partial z} = \int_a^b \int_a^s \int_a^b \frac{u_z v_z(v_z - u_z) + 2\omega\left(u^2 - v^2\right)}{4z^2 \mathcal{J}\left(\Pi_z + \mathcal{J}\omega\right)^2} dF(v)dF(u)dF(s), \tag{B1}$$

$$
\begin{aligned}
\frac{\partial CV}{\partial z} &= \int_a^b \frac{\pi_z(u)}{\mathcal{J}\left(\bar{\pi}_z + \omega\right)^3 CV} \left\{\left[\bar{\pi}_z \frac{d\pi_z(u)}{dz} - \pi_z(u)\frac{d\bar{\pi}_z}{dz}\right] + \left[\frac{d\pi_z(u)}{dz} - \frac{d\bar{\pi}_z}{dz}\right]\right\} dF(u), \\
&= \int_a^b \int_a^b \frac{u_z^3 v_z(u_z - v_z) + 2\omega u_z^2\left(v^2 - u^2\right)}{32z^3 \mathcal{J}^2\left(\bar{\pi}_z + \omega\right)^3 CV} dF(v)dF(u), \tag{B2}
\end{aligned}
$$

where we wrote $u - z = u_z$ and $v - z = v_z$ to save notation.

Earned Income: Let $E\left(g(v)\right) = \int_a^b g(v)dF(v)$. Then, because $\omega = 0$, (B1)-(B2) can be

simplified to

$$\frac{\partial GC_\pi}{\partial z} = \frac{1}{4z^2 \mathcal{J}\Pi_z^2} \int_a^b \int_a^s u_z \left[E\left(v_z^2\right) - u_z E\left(v_z\right)\right] dF(u) dF(s) > 0,$$

$$\frac{\partial CV_\pi}{\partial z} = \frac{1}{32z^3 \mathcal{J}^2 \bar{\pi}_z^3 CV_\pi} \left[E\left(v_z^4\right) E\left(v_z\right) - E\left(v_z^3\right) E\left(v_z^2\right)\right] > 0,$$

where $\partial GC_\pi/\partial z > 0$ because $B(s) \equiv \int_a^s u_z \left[E\left(v_z^2\right) - u_z E\left(v_z\right)\right] dF(u)$ is strictly positive on $[a,b]$ (because $B(s)$ is increasing and then decreasing on $[a,b]$ and satisfies $B(a) = B(b) = 0$). It remains to show that $\partial CV_\pi/\partial z > 0$. Using Cauchy-Schwarz inequality—i.e., $E\left(u^2\right) E\left(v^2\right) > E(uv)^2$ when $E(u) > 0$, $E(v) > 0$—we have

$$E\left(v_z^4\right) E\left(v_z^2\right) E\left(v_z\right) > E\left(v_z^3\right)^2 E\left(v_z\right) > E\left(v_z^2\right)^2 E\left(v_z^3\right).$$

Then, because $E\left(v_z^2\right) > 0$, we obtain $\partial CV_\pi/\partial z > 0$ as desired.

Aggregate Income: using equation (7), i.e., $z = -2\omega + E(v)/\mathcal{J}$, we can rewrite (B1)-(B2) as follow

$$\frac{\partial GC_I}{\partial z} = \int_a^b \int_a^s \frac{\left[E\left(v^2\right) - \mathcal{J}z^2\right] \left[u - E(v)/\mathcal{J}\right]}{4z^2 \mathcal{J}\left(\Pi_z + \mathcal{J}\omega\right)^2} dF(u) dF(s) < 0, \quad \text{(B3)}$$

$$\frac{\partial CV_I}{\partial z} = \frac{\left[E\left(v^2\right) - \mathcal{J}z^2\right] \left[2\omega E\left(v_z^2\right) - E\left(v_z^3\right)\right]}{32z^3 \mathcal{J}^2 \left(\bar{\pi}_z + \omega\right)^3 CV_I} < 0. \quad \text{(B4)}$$

To show $\partial GC_I/\partial z < 0$ and $\partial CV_I/\partial z < 0$, note first that $2\omega = E\left(v_z\right)/\mathcal{J}$ and

$$E\left(v^2\right) - \mathcal{J}z^2 = \int_a^b \left(u - \frac{E(v)}{\mathcal{J}} \right)^2 dF(u) + 4\omega(E(v) - \mathcal{J}\omega) > 0.$$

Then, with similar methods used above, we will immediately obtain the desired results.

C.   THE IMPACTS OF PRODUCTIVITY AND CAPITAL

Differentiating Gini coefficient and coefficient of variation with respect to ω yields

$$\frac{\partial GC_I}{\partial \omega} = \frac{1}{2z\mathcal{J}\left(\Pi_z + \mathcal{J}\omega\right)^2} \int_a^b \int_a^s \left[\mathcal{J}u_z^2 - E\left(v_z^2\right)\right] dF(u) dF(s) < 0,$$

$$\frac{\partial CV_I}{\partial \omega} = \frac{E\left(v_z^2\right)^2 - \mathcal{J}E\left(v_z^4\right)}{16z^2 \mathcal{J}^2 \left(\bar{\pi}_z + \omega\right)^3 CV_I} < 0.$$

Then, using equation (7), (B3)-(B4), and the above equation, we can write the overall effects of ω on Gini coefficient $dGC_I/d\omega$ and coefficient of variation $dCV_I/d\omega$ as follows

$$\frac{dGC_I}{d\omega} = \int_a^b \int_a^s \frac{\left[E\left(v^2\right) - \mathcal{J}z^2\right] (E(v) - \mathcal{J}u) + \mathcal{J}z \left[\mathcal{J}u_z^2 - E\left(v_z^2\right)\right]}{2z^2 \mathcal{J}^2 \left(\Pi_z + \mathcal{J}\omega\right)^2} dF(u) dF(s),$$

$$\frac{dCV_I}{d\omega} = \frac{\left[E\left(v^2\right) - \mathcal{J}z^2\right] \left[E\left(v_z^3\right) - 2\omega E\left(v_z^2\right)\right] + z \left[E\left(v_z^2\right)^2 - \mathcal{J}E\left(v_z^4\right)\right]}{16z^3 \mathcal{J}^2 \left(\bar{\pi}_z + \omega\right)^3 CV_I}.$$

It is straightforward to show that the direct effect of ω is negative, i.e., $\partial GC_I / \partial \omega < 0$ and $\partial CV_I / \partial \omega < 0$. However, because the indirect effect through market toughness is positive, the overall effect is ambiguous and can be positive. To see this, note from the above equation that if $z = 0$, the numerators of $dGC_I / d\omega$ and $dCV_I / d\omega$ are strictly positive. Then, given continuities of the numerators of $dGC_I / d\omega$ and $dCV_I / d\omega$, overall effect of ω on income inequality must be positive when ω is sufficient high and, thus, z is sufficiently low.

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