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Bayesian Expectations and Strategic Complementarity: Implications for Macroeconomic Stability

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Abstract

This paper examines the heterogeneous market in which economic agents of different information-processing abilities interact. In the theoretical framework, the market is composed of three different types of agents, "sophisticated" agents with rational expectations, "naive" agents with adaptive expectations, and Bayesian agents endowed with learning abilities. The behavior of these agents in the context of an important economic problem of nominal price adjustment after a fully anticipated one-time negative monetary shock is examined. If sophisticated agents with their perfect foresight find it profitable to imitate the biased behavior of naive agents, then the interaction of agents exhibits strategic complementarity. Thus the naive agents will have a disproportionately large effect on sluggish price adjustment towards equilibrium. However, the introduction of Bayesian agents with learning abilities into the market will have a compensatory effect by mitigating the price rigidity. Since Bayesian learning is allowed in heterogeneous market, Bayesian agents that first start as naive will undergo a learning process to become sophisticated after a certain period. In conclusion, the proportion of naive agents decreases in favor of sophisticated agents as depicted in the simulation model. As a result, the price adaptation towards equilibrium is accellerated.

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Keywords: nominal inertia; strategic complements; behavioral dynamics; heterogeneous expectations; Bayesian learning

1. Introduction

In macroeconomic models of heterogeneous market structure, agents' information-processing abilities to form expectations differ from each other. The implications of heterogeneity are examined in macroeconomic models that exhibit an environment of strategic complementarity. The concept of "strategic complements" plays a key role in

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investigating the behaviors of agents that are divided into two categories. The first category comprises the sophisticated agents that are assumed to form rational expectations. The second category comprises naive agents that are considered to have adaptive expectations. If macroeconomic interaction between these two types of agents follows a course of action that has a disproportionately large effect on macroeconomic equilibrium then the environment exhibits strategic complementarity. In heterogeneous markets, sophisticated and naive agents are mixed and interact in a strategic environment. If strategic substitution is valid between different types of agents, then the expectational errors of naive agents will be corrected by sophisticated agents. The adjustment process to equilibrium after fully anticipated one-time shock will follow a path similar to that of a homogeneous market in which only sophisticated agents exist. However, if sophisticated agents find it profitable to imitate naive agents, then strategic complementarity will take place between market participants. Strategic complementarity offers an explanation of limited rationality as a source of both nominal and real inertia.

The macroeconomic implications of heterogeneity in terms of agents' abilities to process information and form expectations in an environment of strategic complementarity are considered by papers dealing with Keynesian coordination problems. Since heterogeneous expectations cause serious theoretical problems under the rational expectations hypothesis, the different views of modern Keynesians and Lucasians are still central to contemporary macroeconomic debate (Blinder, 1987). Recent economic literature on this debate develops agent-based behavioral modeling of micro foundations of macroeconomics. Therefore, in this paper, we compare two economic systems: Keynesian system of non-neutrality and Lucasian system of neutrality of money. Keynesian coordination problems cause backward price rigidity and heterogeneous expectations are one of the reasons behind coordination problems. Our model explains price-setting phenomenon in a strategic oligopolistic market structure in which different types of agents with heterogeneous expectations interact. As a solution to Keynesian coordination problems, we show that the introduction of rational learning agents into the market lowers the level and shortens the duration of nominal inertia. We use the Hidden Markov Model (HMM) of learning and this is a new approach in the relevant economic literature.

2. Literature Review

One of the most important economic problems to date is the macroeconomic implications of the effect of nominal price adjustment after a fully anticipated and exogenous monetary shock. The phenomenon of nominal price stickiness is examined by macroeconomists to explain nominal inertia as a source of monetary non-neutrality. In relevant literature, considerable attention is paid to informational frictions (Lucas, 1972), menu costs (Mankiw, 1985), and the theory of wage contracts (Fischer, 1977). But the sources and effects of nominal inertia are still controversial among economists and the phenomenon is not yet clearly explained.

The theoretical works on the Keynesian coordination problems include Diamond (1982), Bryant (1983), Heller (1984), Akerlof and Yellen (1985a,b), Haltiwanger and Waldman (1985, 1989), Russell and Thaler (1985), Cooper and John (1988), Ball and Romer (1991). These papers provide a theoretical analysis for the role of strategic interaction between market participants which is a source of coordination failure and nominal rigidity in various contexts. For example, Diamond (1982) shows the relevance of strategic complementarity when search frictions with positive trading externalities are taken into account. Bryant (1983) deals with informational frictions.

Under the heterogeneous expectations assumption, the explanation to limited rationality given by the Haltiwanger-Waldman approach is due to the disproportionate effect of naive agents on the persistence of disequilibrium after a fully anticipated one-time shock. Since naive agents have adaptive expectations and follow the rule-of-thumb, establishment of new equilibrium position will be slow. On the other hand, sophisticated agents that forecast the actions of naives will imitate their behavioral bias because of strategic complementarity. Bulow, Geanakoplos, and Klemperer (1985) originally developed the concept of strategic complements and substitutes.

The heterogeneity regarding the ability of agents to forecast is fundamental to the analysis of the Haltiwanger-Waldman approach formalized by Evans and Ramey (1992) and Sethi and Franke (1995). Naive agents that follow the rule-of-thumb have less ability to forecast and are thereby subject to expectational errors. Although sophisticated agents have "traditional" rational expectations, they find it profitable to imitate the behavioral bias of naives and the interaction of agents in the market exhibits strategic complementarity. The introduction of even a small number of naive agents into the market will have a disproportionately large effect on the equilibrium position of the economy. Bomfim (2001) used a dynamic general equilibrium model to extend the Haltiwanger-Waldman approach. According to Bomfim "the work of Haltiwanger and Waldman (1985, 1989) brought a new perspective to the analysis of forecast heterogeneity" (Bomfim, 2001, p. 146).

However, in the macroeconomic models of Haltiwanger and Waldman (1989) and Bomfim (2001), the element of time is taken as exogenous. The element of time can be endogenized in the model only by taking into consideration the learning behavior of different types of agents. According to Haltiwanger and Waldman (1989) and Bomfim (2001), agents differ in terms of their abilities to form expectations and they don't undergo a learning process. Hence their model is not actually 'dynamic'. Agents with adaptive expectations obey their adaptive nature for all periods before and after the shock. Beside their limited ability to forecast, they have no ability to learn, and this is unrealistic. As Boland wrote "Unless the model builder is willing to recognize simultaneously many different ways to process information—that is, many different learning strategies little progress can be expected in a realistic way. Recognizing learning can help solve the problem of time's arrow since one cannot unlearn" (Boland, 2005, p. 129).

The more realistic model with the element of time taken endogenously and thereby really dynamic is only possible by the introduction of learning agents into the market. This aspect allows us to dynamize the model. Bayesian learning is examined by Arrow and Green (1973) and Jacobs and Jones (1977). However, these models are not set in a strategic environment. Cyert and DeGroot (1971, 1974) develop an oligopoly model in which the concept of Bayesian learning is introduced. Dosi et al. (2001) is a comprehensive study of learning in evolutionary environments.

There is an increasing interest in the learning models of economic agents in relation to the theory of expectations. For example, Branch (2004) develops a model of rationally heterogeneous expectations with which agents form their forecasts of inflation. Honkapohja and Mitra (2006) analyses the conditions for convergence of adaptive learning towards rational expectations equilibrium. Kalai and Lehrer (1993) show that rational learning leads to Nash Equilibrium. On the other hand, there is also a cognitive cost to form rational expectations as examined by Evans and Ramey (1992, 1998) and Brock and Hommes (1997). The principle of expectation formation depends on the cost-benefit analysis of the sophisticated agents acting upon the adaptive agents. Therefore, the windfall profit rate due to price changes caused by monetary shocks is an important motive.

The importance of heterogeneous expectations is shown by the recent macroeconomic literature. For example, Anufriev et al. (2009) uses the stylized macro model of Howitt (1992) to investigate inflation dynamics and examines the agents with heterogeneous expectations that update their beliefs based on past performance of prices.

The experimental model of Fehr and Tyran (2008) is based on the theoretical work by Haltiwanger and Waldman (1989). But the psychological content of their paper is money illusion and anchoring. Fehr and Tyran (2008) examine the effect of a strategic environment on agents' decision making problem. Zandt and Vives (2007) analyze interim beliefs in a framework of comparative statics. Potters and Suetens (2009), motivated by Haltivanger and Waldman (1985, 1989) and Fehr and Tyran (2008), assert that there is significantly more collusion (cooperation) under strategic complementarity than under strategic substitutability in a laboratory experiment. However, experimental analyses designed in a laboratory with computerized subjects are criticized for being far from real life practices of economic agents.

3. The Dynamic Bayesian Model of Expectations and Strategic Complementarity

In this section we analyze the effect of strategic complementarity on price adjustment after fully anticipated one-time monetary shock in a dynamic macroeconomic model with Bayesian expectations. There are three types of agents in the population and they are assumed to be risk neutral. Naive agents have adaptive expectations with limited abilities whereas sophisticated agents have rational expectations in the "traditional" sense; i.e., they have unlimited abilities to forecast future prices and they know the structure of the economy with all its relevant parameters. The third type of agents has Bayesian expectations with additional abilities to learn. In the case of oligopolistic price competition, if a firm sets its price above the Nash equilibrium level, then the other competitive firms will also set their price above the Nash equilibrium level (Potters and Suetens, 2009). Strategic complementarity within the context of oligopolistic competition results in Pareto-inefficient Nash equilibrium. And the degree of strategic complementarity will vary according to subjective preferences for conditional cooperation. For example, a negative monetary shock induces sophisticated agents to imitate the naives' biased behavior in favor of profit opportunities, and thereby nominal prices are kept high over certain periods of time after the shock is implemented. But the introduction of learning agents with Bayesian expectations mitigates the price persistence since the proportion of naive agents decreases over time as they learn by doing how to forecast the forecasts of other agents to restore Bayesian Nash equilibrium in a strategic interaction and finally become sophisticated with perfect foresight. Learning agents with Bayesian expectations will start the course as naive agents and then undergo a 'hidden' (learning) Markov process to end up with sophisticated agents. Since the only possible way to endogenize the time element into a real dynamic model is to include a learning process, we use a state-space Hidden Markov Model (HMM) of learning with dynamic Bayesian approach which serves best for the purpose and this is a new approach in the relevant economic literature. Learning agents with Bayesian expectations starting as naive will undergo a mutation through certain states and finally be sophisticated. The effect of this learning process is to mitigate the disproportionate impact of naive agents on Bayesian Nash equilibrium.

3.1. The model specification and methodology

Consider a market environment consisting of n agents. The proportion of sophisticated agents in the population is denoted by q while the proportion of naive agents is $1 - q$. Learning agents are those that begin as naive and then become sophisticated. For this reason, they have an effect on changing the proportion of naive to sophisticated agents. The profit function of agent i at time t is defined as $\pi_{i,t}$ and it's formulated as follows

$$\pi_{i,t} = \pi_{i,t}(P_{i,t}, \overline{P}_{-i,t}, M) \tag{3.1.1}$$

where $P_{i,t}$ denotes agent i's nominal price at time t, whereas $\overline{P}_{-i,t}$ represents nominal average price for the rest of the population $n - 1$ and M is the nominal money supply. To avoid money illusion, it's assumed that the profit function is homogeneous of degree zero with respect to individual and average prices as well as money supply. The price expectation of a particular agent P_i^e depends on the average price of other agents $\overline{P}_{-i}$ so that they have to form expectations on the basis of forecasting the average price forecast of population. This implies that the sophisticated agents imitate the naive agents' biased behavior in an attempt to gain unduly profit from conditional cooperation. The introduction of learning agents with Bayesian expectations allows naives to learn this process and share the profit, and thereby quicken the price adjustment towards Bayesian Nash equilibrium. Naive agents with limited abilities to forecast have adaptive expectations about future prices and this assumption is formulated as a general extrapolative expectations model:

$$P^e{}_{i,t} = \alpha_1 P_{t-1} + \alpha_2 P_{t-2} + \ldots + \alpha_l P_{t-l} \tag{3.1.2}$$

where $\alpha_1 > \alpha_2 > \ldots > \alpha_l$ such that $\sum_{j=1}^{l} \alpha_j = 1$. In a dynamic model, there are l past periods. Naive agents do not take into account the current information. They only form their expectations by backward-looking adaptive behavior and because of this their choices are biased. The factor of price revision is denoted by α parameter in (3.1.2).

The information set for the sophisticated agents with rational expectations includes the current information denoted by Ω_t . If ω denotes possible information available for an agent *i*, then $I_i(\omega)$ represents agent *i*'s information and $\omega \in \Omega$ contains the knowledge of actions in the past and present. For sophisticated agents to imitate naives, all past and present information are required and contained in the equation below (Bergin and Bernhardt, 2004).

$$(\Omega, \Pi) = [(\omega_{t-1}, \pi_{t-1}), ..., (\omega_{t-l}, \pi_{t-l})] \tag{3.1.3}$$

On the other hand, Bayesian learning behavior is based on naive agents' expectations and their own experience $I_i(\omega)$, so their information set must include their own recent history of actions and profits:

$$I_i(\omega) = [(\omega_{i,t-1}, \pi_{i,t-1}), ..., (\omega_{i,t-l}, \pi_{i,t-l})] \tag{3.1.4}$$

A Bayesian agent updates its beliefs as it learns from new information set included in its own experience in each period. A naive agent becomes Bayesian as it begins to learn from new flows of information and updates its price estimates towards new equilibrium level in every period and behaves like a sophisticated agent with rational expectation when making forecasts (Cogley & Sargent, 2005). This updating behavior of Bayesian agents is represented in the price forecasts by the parameter α in equation (3.1.2). Every new period, updated price estimates by the Bayesian agents make the forecasts rational and thereby price deviation decreases.

3.2. The Bayesian vectorautoregressive model in the form of price expectations

The Bayesian Vector Autoregressive (BVAR) function can be used in the form of an expectation model to analyze the state-space Hidden Markov Model of learning and the effect of strategic interaction between agents in a heterogeneous market. A variety of expectation formation models can be derived from the general form of the extrapolative expectations model (3.1.2) such as adaptive expectations, static expectations, and Bayesian expectations by utilizing some specific computational codes for the parameter α on an *ad hoc* basis.

We can rewrite (3.1.2) in a typical VAR model in the form of a price vector to derive the function of Bayesian expectations formation:

$$P_{i,t} = \alpha_1 P_{t-1} + \alpha_2 P_{t-2} + ... + \alpha_l P_{t-l} + \delta z_t + \varepsilon_t \tag{3.2.1}$$

where $P_{i,t}$ is a $n \times 1$ vector of endogenous variables (past prices) fort the *i*'th agent and *n* is the number of agents in the market. The sequence of parameter α is set in descending order $\alpha_1 > \alpha_2 > ... > \alpha_l$ such that $\sum_{j=1}^{l} \alpha_j = 1$. ε_t is $n \times 1$ vector of error terms independently, identically and normally distributed with zero mean and variance-covariance matrix σ , $\varepsilon_t \sim i.i.d(0, \Sigma)$; δ s are $n \times n$ and $n \times d$ matrices of parameters respectively and z_t is a $d \times 1$ vector of exogenous variables, i.e. price expectation of particular agent *i* at time *t*. To state Bayes rule for the learning process of the agents, equation (3.3.1) can be transformed into a Bayesian VAR model:

$$P_{i,t} = X_t \mathrm{A} + \varepsilon_t \tag{3.2.2}$$

where *t* is a discrete time unit *t* = 1, . . ., *T*. X_t is the Kronecker product of $(I_n \otimes W_{t-1})$ that is $n \times nk$ where $W_{t-1} = (P'_{t-1}, ..., P'_{t-l}, z'_t)$ is $k \times 1$. $\mathrm{A} = vec(\alpha_1, \alpha_2, ..., \alpha_l, \delta)$ is $nk \times 1$ (Ciccarelli & Rebucci, 2003). Unknown parameters of the model; i.e., $\mathrm{A} = vec(\alpha_1, \alpha_2, ..., \alpha_l, \delta)$ and σ are contained in the information set of (3.1.4) and they are updated in every period by the direct observation of the individual agent whose expectations are Bayesian.

3.3. Modeling dynamic Bayesian learning

We use the state-space Hidden Markov Model (HMM) for analyzing the Bayesian learning process. The HMM allows us to study the heterogeneous market with different types of agents endowed with different learning abilities. The existing research on learning is based on static model of agent behavior with the standard learning curve framework. The HMM captures the dynamic learning behavior of individual agents, their own experience and also their interaction with other agents in the market by explicitly setting the dynamics of heterogeneous market structure. Therefore the HMM has important advantages over the standard learning curve framework (Singh, T. & Youn, 2011). HMM comprises of finite set of hidden (unobserved) *learning states* and observed outcome and it processes past and present actions of individual agents in a strategic environment to analyze learning dynamics. In our model, since the only strategic variable is price, the outcome for each of the learning states will be the observed price. Therefore the HMM provides some information about the sequence of states. States are hidden in the HMM in contrast to the simple Markov model where states are directly observed. However, the only difference between the HMM and a simple Markov model is the introduction of learning agents and the transition from one state to another follows the usual Markov process.

In a particular period, a hidden state transforms into an observed outcome (actual price) through the learning process. Thereby in a dynamic Bayesian market network of strategically interacting agents, a particular agent transits from one state to another by its own learning behavior. The first-order Markovian process follows a transition from one learning state to another where subsequent learning state depends on the present learning state. Let the set of learning states is given by $S=\{S_t, S_{t-1}, ..., S_{t-l}\}$ where S_t is the highest state and S_{t-l} is the lowest state of learning. The observed (actual) Bayesian Nash equilibrium price at time *t* is generated by the learning process that is *hidden* from the agent at the relevant state, S_t . A particular state sequence for agent *i* is $S_i=\{S_{i,t}, S_{i,t-1}, ..., S_{i,t-l}\}$ and the corresponding sequence of actual prices (observed outcomes) is arrived at by agent *i* after the transition from a prior state to a present one through learning. Thereby, the locus of interim Bayesian Nash equilibria that corresponds to the learning curve are the path of price inertia between the pre-shock and after-shock Nash equilibrium level.

3.3.1. The specification of state-space hidden Markov model

In our model, the probability distribution of the Hidden Markov Model (HMM) of a sequence of states and observed outcomes (actual prices) is defined by *l*th order Markov process. Extending a first-order Markov process to *l*th order allows us to model higher order interactions between expected and actual prices. In an *l*th order Markov model, we define the set of prices as $P=\{P_t, P_{t-1}, ..., P_{t-l}\}$. Price observations are dependent on a set of learning state variables S_t in the HMM. Given the sequence of states of *l*th order of Markov process, S_t is determined by the set of states $\{S_{t-1}, ..., S_{t-l}\}$ which is independent of S_τ for $\tau < t$ – n. The factorization of the joint probability distribution function (Φ) of sequence of states (*S*) and price (*P*) observations for the state-space *l*th order HMM is as follows:

$$\Phi(S_{1:T}, P_{1:T}) = \Phi(S_1)\Phi(P_1 \left| S_1 \right.)\prod_{t=2}^{T}\Phi(S_t \left| S_{t-1}, ..., S_{t-l} \right.)\,\Phi(P_t \left| S_t \right.) \tag{3.3.1.1}$$

where $S_{1:T}$ means $S_1, ..., T$. The state transition probability of *l*th order of Markov process with a deterministic state variable and a stochastic variable can be written as;

$$S_t = f_t(S_{t-1},...,S_{t-l}) + w_t \tag{3.3.1.2}$$

The probability of observed price conditional upon the state variable can also be written with a stochastic variable as follows (Ghahramani, 2001).

$$P_t = g_t(S_t) + v_t \tag{3.3.1.3}$$

Bayesian learning starts with some prior knowledge about the parameters and price observations in the form of prior probability distribution over sequence of price observations and the relevant parameters. This prior knowledge is updated by using price observations in the past and present to obtain a posterior probability distribution function over future prices and parameters. In order to compute the expected price of agent *i*, $P^e_{i,t+1}$, a sequence of price observations in the past and present is used. The expected price as a function of current and past prices with their relevant parameters is formulated as follows:

$$f(P_{i,t+1}) = \alpha_0 P_{i,t} + \alpha_1 P_{i,t-1} + ... + \alpha_l P_{i,t-l} \tag{3.3.1.4}$$

And the Bayesian prediction function representing the posterior probability distribution of expected price is

$$\langle f(P) \rangle = \int f(P)\Phi(P|S,\alpha)dP \tag{3.3.1.5}$$

The expected price depends on learning state variables and parameters of the price variable. The Bayesian prediction function (3.3.1.5) averages over the uncertainty in price expectations and in the parameters (Ghahramani, 2001). Bayesian learning agents in our model use the Bayesian prediction function (3.3.1.5), which is normally distributed. Each learning agent uses its own price observations in the market and its own learning states in the strategic interaction with other agents to calculate parameters that are weighted average of distributed lag model (3.3.1.4).

3.3.2. Parametrization and simulation of the model

The parameters of the Bayesian price expectation formation equation (3.3.1.4) are generated according to our model structure on an *ad hoc* basis by using the forward-backward algorithm that we have created by writing codes in Mathematica 8.0. The forward-backward algorithm is applied to the Bayesian learning from strategic interaction of agents with heterogeneous expectations in a market network corresponding to a Hidden Markov Model (Ghahramani, 2001). The forward pass recursively computes the parameter of strategic price variable α_t defined as the joint probability of S_t and the sequence of future price observations from P_{t+1} to P_{t+l}. The backward pass computes the conditional probability of past and present price observations from P_t to P_{t-l} given S_t (Ghahramani, 2001). The time is symmetric between the forward pass and backward pass. In our Hidden Markov Model, there are 9 learning states (l = 9) within 10 time periods (t = 10) and 10 representative agents (n = 10) each of which is indexed as i = (1, 2,. . ., n). The choice of the integer 10 for the number of representative agents will allow us to interpret the simulation results in percentage terms as we change the market composition of different types of agents to compare the effects of strategic complementarity in the next section. Every agent transits from 9 learning states with 9th degree of Markov process:

$$S_t = f_t(S_{t-1},...,S_{t-9}) \tag{3.3.2.1}$$

In the 10th period, learning agents become sophisticated. Every agent assigns higher learning states to the near past and lower learning states to the distant past in the time horizon. So the parameters are weighted accordingly. The price observation in a particular period of an agent is based on learning state of the agent, and the price expectation of the next period is based on the parameter of the strategic price variable. We generate 9 random real numbers on an *ad hoc* basis that range between 0 and 0.3 as learning parameters for the revision of price expectations in the simulation model. The range of the numbers is selected according to the magnitude of nominal shocks implemented in the simulation. In compliance with the distributed lag model, the sum of the weighted averages of parameters equals 1 which is also a threshold for learning. If the sum of these 9 real numbers is less than 1, then the 10th parameter is obtained by subtracting the sum of 9 parameters from 1. When the total sum of price parameters for a learning agent reaches the threshold of the highest learning state and be 1, the Bayesian learning agent becomes a sophisticated agent with rational expectations.

The expected price functions of a learning agent *i* are formulated for 9 learning states as follows:

$$P^e_{i,t+1} = (\alpha_0 + \alpha_{10})P_{i,t} + \alpha_1 P_{i,t-1} + ... + \alpha_9 P_{i,t-9} \tag{3.3.2.2}$$

$$P^e_{i,t+2} = (\alpha_0 + \alpha_{10} + \alpha_9)P_{i,t+1} + \alpha_1 P_{i,t} + ... + \alpha_8 P_{i,t-8} \tag{3.3.2.3}$$

$$P^e_{i,t+3} = (\alpha_0 + \alpha_{10} + \alpha_9 + \alpha_8)P_{i,t+2} + \alpha_1 P_{i,t+1} + ... + \alpha_7 P_{i,t-7} \tag{3.3.2.4}$$

$$\begin{matrix} . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \end{matrix}$$

$$P^e_{i,t+9} = (\alpha_0 + \alpha_{10} + \alpha_9 + \alpha_8 + \alpha_7 + \alpha_6 + \alpha_5 + \alpha_4 + \alpha_3 + \alpha_2)P_{i,t+8} + \alpha_1 P_{i,t+7} + ... + \alpha_1 P_{i,t-1} \tag{3.3.2.5}$$

The equations (3.3.2.2), (3.3.2.3), (3.3.2.4), . . ., (3.3.2.5) with 9 learning states construct a Learning State Transition Matrix for each agent. The expected price for a learning agent at time $t+1$ is the weighted average of observed prices for 9 learning states. The calculated expected price at time $t+1$ enters into the price expectation equation at time $t+2$ and the price observation of distant past $t-9$ is omitted from the equation at time $t+2$. This conforms with the behavioral aspect of forgetting. The 9th degree of Markov process goes cumulatively throughout 9 learning states in the same sequence. When an agent transits from the highest 9th level of learning state to 10th period forward in the future ($t+10$), the agent's price expectation equation becomes equivalent to the rational expectations equation:

$$P^e_{i,t+10} = \left(\alpha_0 + \alpha_{10} + \alpha_9 + \alpha_8 + \alpha_7 + \alpha_6 + \alpha_5 + \alpha_4 + \alpha_3 + \alpha_2 + \alpha_1\right)P_{i,t+10} \tag{3.3.2.6}$$

where the price expectation of an agent *i* instantaneously adapts to the actual rational equilibrium price in the market. In other words, at the period 10, the learning agent becomes sophisticated and sets its expected price at rational equilibrium level.

4. The Effect of a Fully Anticipated One-Time Monetary Shock

In this section, we examine the effect of a fully anticipated one-time monetary shock on the price adjustment process in a strategic complementarity of mixed agents. There are three kinds of agents that interact and differ from each other in terms of their abilities to form expectations. According to this assumption, sophisticated agents with rational expectations dominate the market whereas naive agents with adaptive expectations have biased behavior. In this case, existence of even a small number of naive agents is enough to have large effect on price adjustment and to slow it down towards equilibrium. However, the introduction of Bayesian agents into a heterogeneous market with strategic complements dynamizes the model and mitigates the sluggishness of price adjustment.

In order to compare how the degree of strategic complementarity changes when the market composition changes with different sizes of fully anticipated one-time monetary shocks, we designed two different market structures with $q_1 = 0.90$ and $q_2 = 0.60$; i.e., the proportion of sophisticated agents in the population is 90 percent in the first market and 60 percent in the second market. In the simulation of the learning process, the naive agents with learning abilities use the Bayesian prediction function (3.3.1.5) to become sophisticated after a certain period of time. In a price-setting model, all agents are informed about the shock at the beginning of the post-shock period $t + 1$. In the pre-shock period $t - 1$, money supply M_0 is given, and then at period t a fully anticipated exogenous negative monetary shock is implemented. The degree of strategic complementarity is associated with the profit size as a percentage of price and the form of profit function with respect to price is assumed to be linear such as

$$\pi(P_t) = \pi_0 + \pi_1 P_t \tag{4.1}$$

In respect to this assumption, increasing profit size will increase the degree of strategic complementarity. The stronger the strategic complementarity, the longer the price adjustment process will be. However, as the profit size increases, Bayesian actors' incentive to learn more quickly will increase as well. So they will rapidly transform from being naive to become sophisticated and thereby alleviate the effect of nominal inertia on price adjustment.

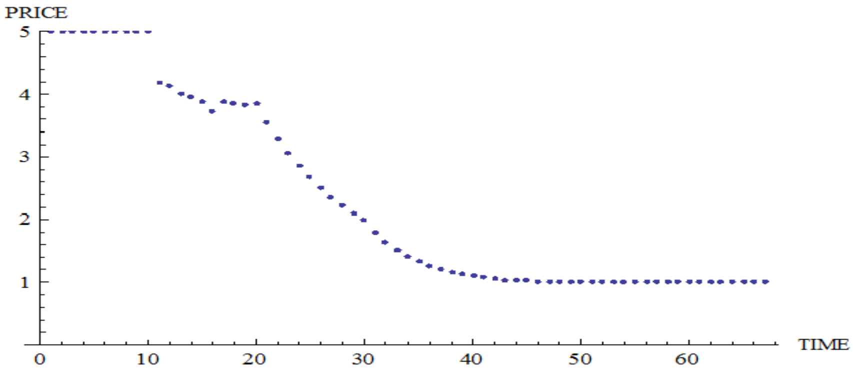
5. Simulation Results of the Model

In this section, we investigate the simulation results of a fully anticipated one-time monetary shock to the economy. Although Haltiwanger and Waldman (1989) deals with the real side of the economy with a hypothetical production project of economic agents, they indicate in their paper that fully and even relatively anticipated nominal monetary shocks have real effect due to the biased behavior of naive agents. The implications of nominal shocks and their effects on the real output corresponds to the price persistence caused by the interaction of sophisticated and naive agents in a heterogeneous market.

5.1. The adjustment of prices

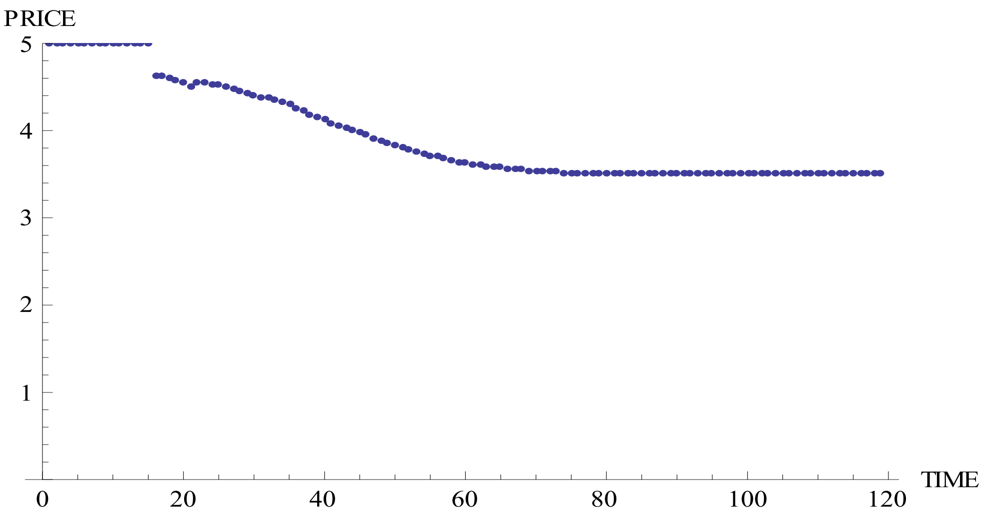
In before-shock periods, the economy is in a steady state. After a fully anticipated one-time shock, the behavior of the naive agents with adaptive expectations is to move slovely to the new equilibrium level. Since the price persistence is associated with a windfall profit, sophisticated agents will use the biased behavior of navies in their favor to reap the reward. Therefore, the price persistence will linger for longer than that within a homogeneous market in which only sophisticated agents exist. However, the introduction of Bayesian agents will have a compensatory effect on the price deviation from equilibrium. Given the presence of Bayesian agents, the convergence of the prices to the new equilibrium level in post-shock periods will be faster. Agents that were naive just after the post-shock periods will learn the dynamic characterization of price adjustment by counteracting the imitation of the sophisticated agents.

In the simulation model, the economy is in a steady state with the pre-shock equilibrium average price level $\overline{P_0^*}$ = 5 and the nominal money supply M_0 = 200. In the first scenario depicted in Figure 1, the economy experiences a fully anticipated one-time negative monetary shock exogenously at the 10th period. The shock is implemented by decreasing the money supply 80 percent from the steady state level of 200. The post-shock money supply $M_1 = \frac{M_0}{5}$ is 40 and the post-shock equilibrium average price level is $\overline{P_1^*}$ = 1. This corresponds to a relatively high level of profit rate due to the price change (ΔP = 4) and it's expressed as a first derivative of the profit function $\pi'(P_t) = \pi_1$ in (4.1). The profit rate π_1 also indicates the degree of strategic complementarity in the market. Therefore as the profit rate increases, the proportion of sophisticated agents in the market increases too. Because profit rate is an incentive for sophisticated agents to participate in the market and set themselves in a strategic interaction with naive agents. Hence, in the first scenario, the proportion of the sophisticated agents is 90 percent due to the relatively high level of profit.



[ΔP = 4 and q = 0.90]
Fig. 1. The Effect of A Fully Anticipated One-Time Negative Monetary Shock

In the second scenario depicted in Figure 2, the magnitude of the shock is smaller than the first scenario. Again, the economy is in a steady state at the pre-shock equilibrium average price level $\overline{P_0^*}$ = 5 and the nominal money supply M_0 = 200. But this time the shock is implemented by decreasing the money supply by 30 percent from the equilibrium level of 200 at the 10th period. The post-shock money supply M_1 is 140 and the post-shock equilibrium average price level is $\overline{P_1^*}$ = 3.5. Since there is a relatively small size of profit (π_1) as a function of price change (ΔP = 1.5), the incentive for sophisticated agents to imitate the naive agents will be less. In this case, the degree of strategic complementarity is low.



[ΔP = 1.5 and q = 0.60]
Fig. 2. The Effect of A Fully Anticipated One-Time Negative Monetary Shock

In this dynamic simulation model, analytical characterization of price convergence to post-shock equilibrium depends on the windfall profit caused by the price disturbance and the introduction of Bayesian agents with learning abilities into the market. After the one-time shock, if the market is homogeneous in which only sophisticated agents exist with rational expectations, then their forecast will be perfect and the price convergence to post-shock equilibrium level will be instantaneous. However, the existence of even a small number of naive agents will have a disproportionately large effect on price persistence (Haltiwanger & Waldman, 1989). This effect is important especially for the first post-shock periods.

6. Conclusion

In this paper the heterogeneous market structure in a strategic environment is examined. The heterogeneous market is composed of three different types of agents in terms of their abilities to form expectations. Firstly, sophisticated agents with perfect foresight have rational expectations. Secondly, naive agents have adaptive expectations for they are prone to make systematic errors because of their backward-looking nature. And thirdly, Bayesian agents are endowed with learning abilities. They start as naive and end up with becoming sophisticated. In this manner, they compensate the disproportionate effect of naive agents on sluggish price adjustment in favor of sophisticated agents. The strategic interaction between these three types of agents was examined in the context of the adjustment of nominal prices after a fully anticipated one-time negative monetary shock. The bounded rationality attributed to the adaptive agents plays an important role in this model. Starting with a steady economy at a before-shock equilibrium level of prices, an exogenous negative monetary shock is implemented in a certain period. In search of the price adjustment process after the shock, we analyzed the strategic interaction of market participants.

Under strategic complementarity, sophisticated agents have an incentive to mimic the biased behavior of naive agents because they find it profitable. After the shock, the naive agents' biased behavior is predicted and imitated by sophisticated agents, and thereby the price adjustment process slows down considerably. The degree of strategic complementarity plays a key role in this framework and it is expressed in terms of profit rate, which is a function of price change. If the magnitude of profit rate increases, the incentive for the sophisticated agents to take strategic action in the market is also increased. As more sophisticated agents enter into the market, learning and price adjustment process will be accellerated. Since a little rate of windfall profit from price change due to monetary shock will attract only limited number of sophisticated agents, a small deviation from equilibrium level of prices is more prone to price persistence than a big deviation. The introduction of Bayesian agents with learning abilities has a counter effect on the nominal inertia. The learning agents observe the strategic environment and accordingly update their information in every period, and use this experience to form gradually more accurate price forecasts. Naive agents become Bayesian in this stepwise updating of beliefs, and they process information to form rational expectations. As a result, the percentage of naive agents decreases in favor of sophisticated agents and the prices converge more quickly to the post-shock equilibrium level.

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