Competitive Supply of Money in a New Monetarist Model

Waknis, Parag

11 September 2017

Competitive Supply of Money in a New Monetarist Model

Parag Waknis*

September 11, 2017

Abstract

Whether currency can be efficiently provided by private competitive money suppliers is arguably one of the fundamental questions in monetary theory. It is also one with practical relevance because of the emergence of multiple competing financial assets as well as competing cryptocurrencies as means of payments in certain class of transactions. In this paper, a dual currency version of Lagos and Wright (2005) money search model is used to explore the answer to this question. The centralized market sub-period is modeled as infinitely repeated game between two long lived players (money suppliers) and a short lived player (a continuum of agents), where longetivity of the players refers to the ability to influence aggregate outcomes. There are multiple equilibria, however we show that equilibrium featuring lowest inflation tax is weakly renegotiation proof, suggesting that better inflation outcome is possible in an environment with currency competition.

JEL Codes: E52, E61.

Keywords: currency competition, repeated games, long lived- short lived players, inflation tax, money search, weakly renegotiation proof.

*The paper is based on my PhD dissertation completed at the University of Connecticut (UConn). I thank Christian Zimmermann (Major Advisor, St.Louis Fed), Ricardo Lagos (Associate Advisor, NYU) and Vicki Knoblauch (Associate Advisor, UConn) for their guidance and support. I thank participants at various conferences and the anonymous referees at *Economic Inquiry* for their helpful comments and suggestions. All errors are mine. Email:parag.econ@gmail.com.

1 Introduction

Whether currency can be efficiently provided by competitive money suppliers is arguably one of the fundamental questions in monetary theory. It is also one with practical relevance because of the emergence of multiple competing financial assets as well as competing cryptocurrencies like Bitcoin and Ethereum among others as means of payments in certain class of transactions. Starting with Hayek in 1976, several authors have explored this question. In general, the answer is a straightforward yes if the money suppliers could operate under full commitment, but quite circumspect under lack of commitment and the associated time inconsistency.

In this paper, I explore this question in a money search model with two competing fiat monies and no commitment. The two money suppliers are modeled as utility-maximizing strategic players in a money search model that is a variant of Lagos & Wright (2005)(LW henceforth). The centralized market sub-period interaction between the money suppliers and the anonymous agents is modeled as an infinitely repeated game as in Waknis (2014). Under reputation, the equilibrium is characterized using the theory of repeated games for more than one long-lived player and a short-lived player.

1.1 Modeling Approach and Related Literature

With the inclusion of two utility-maximizing money suppliers, the differential nature of the players in the centralized market becomes important. The $(0,1)$ continuum of economic agents trade competitively, anonymously, and their individual actions are unobservable. Only their aggregate play is important and consequential. On the other hand, the two money suppliers' individual actions are observable and affect the aggregate centralized market outcome. Games with such a composition of players are called games between multiple large or long-lived players and a small or a short-lived player comprised of a continuum of long-lived players (Fudenberg *et al.* (1990) and Mailath & Samuelson (2006)). Following this literature, we model the money suppliers as a long-lived players and the $(0,1)$ continuum of agents as a short-lived player (alternatively called a short-run player). The reference to size or longevity of the player is based on his or her ability to influence aggregate outcomes, anonymity, and observability of individual actions.

As an important implication of unobservability and informational irrelevance of agent's individual actions, there cannot be any belief based dynamics in this model. This is especially important given the exchange rate indeterminacy in this model as argued in Kareken & Wallace (1981) [1]. It could be argued that this indeterminacy could make predicting exchange rate difficult increasing uncertainty for the agents. As a result of such uncertainty, agents' beliefs about the exchange rates could influence the behavior of the money suppliers altering the equilibrium dynamics. Such dynamics, however is not plausible in the set up of this paper. This is because modeling agents as a continuum on an unit interval ensures that any idiosyncratic uncertainty does not survive aggregation and therefore does not play any role in shaping aggregate outcomes. This result follows from the seminal paper by Robert Aumann (Aumann (1964)) and subsequent literature.

There are several strands of literature that are directly or indirectly related to this paper. One such strand is the dual currency literature that deals with emergence of an iternational currency and coexistence of it with other currencies, Examples include but are not limited to Curtis & Waller (2000), Trejos & Wright (2001), Head & Shi (2003), Camera *et al.* (2003), Craig & Waller (2004), Lotz (2004), Martin (2006), and Zhang (2014). Models with dual currencies based on the LW framework have also been used to model use of multiple assets as means of payments or collateral. Some of the examples from this literature are Lagos & Rocheteau (2008), Lester *et al.* (2012), Lagos (2010, 2011), and Geromichalos *et al.* (2007), and Geromichalos & Simonovska (2014)[2]. However, in many of these models, there is some ex-ante difference in the available assets- they are not perfect substitutes. Since, the two currencies in this paper are perfect substitutes to start with, the results here could throw some light on possible equilibrium outcomes when two or more assets with similar profile compete as means of payments.

Because of absence of government in this paper and that the monies are useless pieces of paper issued by two private competing money suppliers, it is closely related to the idea and literature on Free Banking[3]. According to Selgin (n.d.) free banking primarily refers to banking without

[1]Market with two currencies here represents dual currency payment systems. The exchange rate in such set up is indeterminate as there are not enough equations to determine the value of both the currencies. See (Nosal & Rocheteau, 2017, pp. 307-08)

[2]Lagos *et al.* (2015) provides a comprehensive summary of this literature.

[3] A detailed analysis of the Free Banking idea and a thematic presentation can be found in Selgin & White (1994) and a summary argument in Selgin (2008)

government deposit insurance or a lender of last resort and free of legal restrictions on interest rates, bank portfolios, branch banking, and, most interestingly, private and competitive note (currency) issuance. Based on Bertrand competition, efficiency of a competitive market in currency was argued by Hayek ((Hayek (1976), Hayek (1978)). Following this, the free banking literature elaborates more on the desirability of monopoly in currency issue given the strong case against it in other areas of economic analysis.

For example, Klein (1974) studies efficiency of competing private monies and is part of the Free banking literature according to Selgin & White (1994). She builds a monopolistic competution model of money where different private monies have different characteristics giving them a brand value. In such a setup intentionally depreciating the exchange value of money is not consistent with profit maximization for the money issuer and hence the equilibrium is not characterized by over issue. Unlike this paper, Kelin's model is micro-founded only from money suppliers point of view. The demand for money is assumed to be exogenously given or what is now termed as included in reduced form. [4].

Answering whether monopoly private money supply is feasible, Berentsen (2006) finds that the revenue-maximizing policy is time-consistent if the trading history of the issuer is public information and if money demanders respond to the revelation of defection by playing barter. Li (2009), Williamson (1999), and Sun (2007) are examples of models where private competitive money supply improves welfare. On the other hand Marimon *et al.* (2012) and Sanches (2016) show that private competitive money supply reduces welfare relative to a monopoly supply of fiat money. Another recent paper in this literature, Fernández-Villaverde & Sanches (2016), is the closest to this paper in terms of research question and the modeling framework. In the Lagos-Wright environment, they include multiple utility maximizing entrepreneurs who can issue their own fiat currencies. Their main result is similar to this paper in that there exists an equilibrium in which price stability is consistent with competing private monies, but also that there exists a continuum of equilibrium trajectories with the property that value of private currencies monoton-

[4]The competing notes issued by the New England colonies, the notes issued by various branches of the Second Bank of the United States, and the free banking era of the antebellum United States are some of the examples when monies issued by several independent or interdependent entities circulated at par or at discount with each other (Klein, 1974, p.439-440).

ically converges to zero.

In comparison to these papers, the novelty of this paper is its game theoretic approach and the use of an equilibrium refinement to narrow the set of equilibria in an economy with only competing fiat monies. It does not require some form of government intervention as in Sanches (2016) or an introduction of a competing asset like capital in Fernández-Villaverde & Sanches (2016) to narrow the set of equilibria. The rest of paper is organized as follows. Section two outlines the dual currency model. Section three analyzes monetary policy under no commitment but where the money suppliers are concerned about reputation. Section four provides concluding comments.

2 The Model

The model is a variant of Lagos & Wright (2005) with two fiat monies. Goods and money are perfectly divisible. There are two sub-periods, a day sub-period where special goods are traded in a decentralized market and a night sub-period where a general good is traded in a centralized Walrasian market. The decentralized market is characterized by trading frictions and hence money gets valued for the liquidity services it provides. The night trading, though centralized, is anonymous and is used by agents to trade in the general good and rebalance their portfolios. The economy is characterized by imperfect memory and record keeping to rule out credit transactions as stressed in Kocherlakota (1998) and Wallace (2001).

2.1 Behavior of Agents

To describe the equilibrium we begin by describing the value functions, taking as given the terms of trade and distribution of monies. The state variables for the individual include his real money balances and a vector of aggregate states s. Let $s = (\pi^R, \pi^B)$, where π^R and π^B are the growth rates of currency R and B respectively. Let $\mathbf{\Phi} = (\phi^R, \phi^B)$, where ϕ^R and ϕ^B is the value of money in currency R and B respectively, in the centralized market. It represents the units of general good that be bought by one unit of the respective currency in the centralized market. [5]

Consider an agent who holds R and B units of currency at the beginning of the first sub-period. Then, his real portfolio is given by $m = [r \;\; b] = [\phi^R R \;\; \phi^B B]$. Before entering the decentralized market in the first sub-period the agent decides how much of this money to carry along as a means of payment. He or she carries both the monies to the market and can pay with either of them or both.

Let the probability of a meeting be ω , that of single coincidence meeting be τ, and that of barter exchange be θ. In a single coincidence meeting a seller's production L must equal the buyer's consumption x. Let us denote the common value as $q(m, \tilde{m}, s)$ and money that changed hands as $d(m, \tilde{m}, s)$, where m and $\tilde{m}$ are buyer's and seller's real money balances. $E(m, \tilde{m}, s)$ is the payoff in a double coincidence meeting.

Let $V(\mathbf{m}, \mathbf{s})$ be the value function for an agent entering the decentralized market with m portfolio and $W(m, s)$ be the value function for the centralized market. $V(\mathbf{m}, \mathbf{s})$ is given by

$$\begin{aligned} V(\mathbf{m}, \mathbf{s}) \;\; = \;\; & \omega\tau \int \{u[q(\mathbf{m}, \tilde{\mathbf{m}}, \mathbf{s})] + W[\mathbf{m} - d(\mathbf{m}, \tilde{\mathbf{m}}, \mathbf{s})]\}\, dZ(\tilde{\mathbf{m}}) \\ & + \omega\tau \int \{-c[q(\tilde{\mathbf{m}}, \mathbf{m}, \mathbf{s})] + W[\mathbf{m} + d(\tilde{\mathbf{m}}, \mathbf{m}, s)]\}\, dZ(\tilde{m}) \\ & + \omega\theta \int E(\mathbf{m}, \tilde{\mathbf{m}}, \mathbf{s})\, dZ(\tilde{\mathbf{m}}) + (1 - 2\omega\tau - \omega\theta) W(\mathbf{m}, \mathbf{s}) \end{aligned} \tag{1}$$

where $Z(\cdot)$ is the distribution of currency portfolios. Let $\mathbf{w}$ be the vector of choice variables (r_i, b_i) in the centralized market and L be the number of labor hours worked. The value of entering the centralized market with portfolio $\mathbf{m}$ is

$$W(\mathbf{m}, \mathbf{s}) = \max\{U(X) - TL + \beta V(\mathbf{m_{+1}}, \mathbf{s_{+1}})\} \tag{2}$$

$$\text{s.t. } X = L + \mathbf{m} - \mathbf{w} \tag{3}$$

[5]The material in this section follows Lagos & Rocheteau (2008) who add capital to the model in Lagos & Wright (2005) to address the coexistence of capital and money in a monetary economy. In this paper, capital is substituted with a second currency to create a dual currency version of the model.

Substituting (3) in (2) and normalizing $T = 1$, we get:

$$W(\mathbf{m}, \mathbf{s}) = \mathbf{m} + \max_{X,w}\{U(X) - X - \mathbf{w} + \beta V(\mathbf{m_{+1}}, \mathbf{s_{+1}})\} \tag{4}$$

We can see clearly that $U'(X) = 1$ implying that $X = X^*$. Secondly, the choice of w is independent of m. All that agents care about is the real value of their portfolio and not how it is constituted. Thirdly, W is linear in a and $W(\mathbf{m} + \mathbf{d}, \mathbf{s}) - W(\mathbf{m}, \mathbf{s}) = \mathbf{d}$. The payoff in the double coincidence meeting is $E(\mathbf{m}, \tilde{\mathbf{m}}, \mathbf{s}) = u(q^*) - c(q^*) + W(\mathbf{m}, \mathbf{s})$, where no money changes hands.

2.2 Behavior of Money Suppliers

There are two monetary authorities, $Bank_R$ and $Bank_B$ issuing R and B currency respectively. New money is issued by the money suppliers in the centralized market to consume the general good- a privilege derived from access to record keeping technology (Fernández-Villaverde & Sanches, 2016, pp.12).

The money suppliers choose money growth rates to maximize the utility from consumption. For simplicity, the utility function is assumed to be linear in consumption. As the money suppliers' are long run players, utility maximization amounts to choosing the money growth rate to maximize the average discounted payoff:

$$(1 - \delta) \sum_{t=0}^{\infty} \delta^t \mu^R(R_{t-1}, \pi_t^R, \pi_t^B) \tag{5}$$

$$(1 - \delta) \sum_{t=0}^{\infty} \delta^t \mu^B(B_{t-1}, \pi_t^R, \pi_t^B) \tag{6}$$

Under no commitment, the money suppliers do not have access to any commitment device and therefore take their decisions period by period [6]. Neither of the currencies is redeemable for any good or service- both are pure fiat monies with no intrinsic value. Unlike Martin (2011, 2013), there is no debt in this environment and the money suppliers consume the general good

[6] It is however assumed that within period commitment is possible for the money suppliers. I thank one of the anonymous referees for pointing it out.

and do not return it to the economy as a public good. Hence, for a given increase in monetary growth rate, the interplay between the rise in current demand for money and a higher required rate of return on debt in future as a mechanism determining a time consistent debt level today is not available in this model. The behavior of money suppliers here could be understood as a simplified representation of use of seignorage to finance government consumption of goods and services akin to the nature of government spending in many developing countries.

Being a duopoly supply of money, a change in the growth rate of one currency affects the value of both currencies through the resulting change in the total stock of money as well as relative proportion of currencies. As there is no difference between these two currencies in terms of the payment services they provide, there is a total symmetry between them. We can also think of them as two currencies but one unit of account[7]. The quasi-linearity of utility function for the centralized market implies that all the agents will carry a portfolio with same real value and not necessarily with same currency composition out of the centralized market.

3 Centralized market game under no commitment

We model the choice of monetary growth rate under no commitment as an infinitely repeated game.Fudenberg *et al.* (1990) provide an analysis of games between a short lived player (a continuum of long lived players) and one long lived player. Because the short lived player optimizes myopically i.e., is concerned only with optimizing current period consumption and the money that it carries out of the centralized market—it always plays Nash response and hence the equilibrium outcomes lie on its best response function. Fudenberg & Levine (1994) extend this analysis to the case of a general game of imperfect public monitoring with more than one long lived players and a short lived player. We consider a game of perfect monitoring, which is a special case of imperfect monitoring as described below. Mailath & Samuelson (2006) serves as a one point reference for this and related literature on repeated games and reputation[8]. This section uses the modeling apparatus and terminology provided by them.

[7]I thank one of the anonymous referees for pointing this out.

[8]Sannikov (2007) and Faingold & Sannikov (2011) offer a continuous time perspective on such games and others involving reputation.

3.1 Centralized Market Game of Perfect Monitoring

After every decentralized trading period, the centralized market opens with two long-run players—the money suppliers, $Bank_B$ and $Bank_R$, and a $(0,1)$ continuum of economic agents (a short-run player). To maximize consumption of the generic good X in the stage game, the money suppliers and the agents choose the monetary growth rate and work hours respectively .

Following is the description of centralized market stage (one-shot) game:

Players: Two money suppliers (long-run players) and $(0,1)$ continuum of anonymous agents (short-run player).

Actions: money suppliers choose monetary growth rate, $\pi_i \in (A_i) = [\pi_i^{min}, \pi_i^{max})$ and the agents choose $L \in [L^{min}, L^{max})$ to maximize utility in the centralized market. The action set is either finite or a convex subset of the Euclidean subspace $\Re^k$ for some k. $Y = A$ and $\rho(y|a) = 1$ if $y = a$ and 0 otherwise.

Payoffs: Agents: $W(L, \pi^R, \pi^B) = U[(L^* - (1-\delta)\sum_{t=0}^{\infty} \delta^t \mu^R(R_{t-1}, \pi_t^R, \pi_t^B)) - (1-\delta)\sum_{t=0}^{\infty} \delta^t \mu^B(B_{t-1}, \pi_t^R, \pi_t^B))) - L^*] + \beta V_{t+1}$

Payoffs: money suppliers: $(1-\delta)\sum_{t=0}^{\infty} \delta^t \mu^R(R_{t-1}, \pi_t^R, \pi_t^B))$ and $(1-\delta)\sum_{t=0}^{\infty} \delta^t \mu^B(B_{t-1}, \pi_t^R, \pi_t^B))$

Preferences: Preferences are given by the payoff functions. They are decreasing in L and π for the agents and increasing in own monetary growth rate but decreasing in the other bank's monetary growth rate for the money suppliers.

As in the LW framework, the hold-up problem caused by any increase in rate of monetary growth rate causes a decrease in q. Any such decrease in the quantity traded in the decentralized market along with the increase in work hours for the agents in the centralized market can be interpreted as an *inflation tax* imposed by the money suppliers' consumption. To maintain the quasi-linearity of the environment however, we assume that the upper bound on the agents' labor supply does not bind. This constrains the maximum inflation tax to some $\epsilon \in (0,1)$ neighborhood of L^{max}. The choice sets for both money suppliers and the agents, therefore, are shown right open in the game description above. If this were not the case, then wealth effects become operational: individual real balances become dependent on past trading histories, value functions become strictly concave in money holdings, and the distribution of money holdings ends up non-degenerate (Rocheteau *et al.* (2015)) [9].

After the bilateral trading in the decentralized market, agents enter the centralized market to rebalance their portfolios before going to the next decentralized market. They discount the future at the rate of β. The payoff for the agents is the value function they face for participating in the centralized Walrasian market. It captures the incentives faced by the agents in this market. Their level of consumption and the money balances they carry out to help their trade in the next decentralized market period depends on their willingness to work, consumption choices of money suppliers, and the money they carried to the centralized market. As noted earlier, quasilinear preferences ensure that each agent carries a portfolio with same value and not necessarily of same currency composition when he or she exits the Walrasian market.

The money suppliers compete with each other as well as with the agents for consumption of the generic good X, putting an upward pressure on its price and a downward pressure on the value of both the currencies. This has consequences not only for the work effort and consumption of agents in the centralized market but also for the consumption in decentralized market. Because the agents discount future at the rate of β, they prefer having more money for use in the decentralized market to ensure the required quantity of the specialized good in a single coincidence meeting. Therefore, when the money suppliers decide to increase their consumption of X by printing more money, agents best response is to adjust the work hours to minimize the impact of increased money supply on the value of the two monies. The later happens because the rate of return on either monies is lower than the discount rate for a positive money growth rate implying a positive liquidity premium (Nosal & Rocheteau, 2011, pp. 68).

This centralized market stage or one shot game has a unique Nash equilibrium. The following proposition states it formally:

Proposition 3.1. Equilibrium-Stage game under no commitment: *Assuming that any single agent*

[9] I thank one of the anonymous referees for pointing this out and providing the reference.

believes that monies from the centralized market will be accepted by other agents in the decentralized market, the one-shot game between the two banks is a Prisoner's Dilemma and $(\epsilon L^{max}, \epsilon\pi^R_{max}, \epsilon\pi^B_{max})$ *for all* $0 < \epsilon < 1$ *is the only Nash equilibrium of the stage game.*

Proof. Because of being a short lived player, the agents myopically optimize by playing their best response for the one-shot game. Therefore, any equilibrium is constrained to be on the short lived players' best response set. Because the agents always play their best responses, we only have to see if the money suppliers have any incentive for a unilateral deviation. As each of the banks has a strong incentive to deviate from the minimum required monetary growth rate, assuming that the other bank will stay put, the game becomes a prisoner's dilemma. Accordingly, even though there is a better equilibrium, $(\epsilon L^{max}, \epsilon\pi^R_{max}, \epsilon\pi^B_{max})$ for all $0 < \epsilon < 1$, is the only Nash equilibrium of the one-shot game [10]. □

3.2 Infinite Repetition of the Centralized Market Game

What happens if this one-shot game is repeated infinitely? In the general case of infinitely repeated games, there are multiple equilibria associated with different degrees of patience. This follows from the folk theorem for infinitely repeated games, which, however cannot be applied to the game being considered here. This is because the presence of short-lived players means that some of the extreme points of the equilibrium payoff set can be produced only by mixtures and hence do not have corresponding pure strategy representation. Fudenberg *et al.* (1990) characterize the equilibrium pay off set and present a folk theorem for the case of a single long lived player and one or more short lived players and Fudenberg & Levine (1994) do so for more than multiple long lived players. We use the apparatus provided by them and Mailath & Samuelson (2006) to characterize the equilibrium payoff set for the centralized market game in this paper.

Another technical issue relates to the number of short lived players in a game. Equilibrium behavior with a large but finite number of short lived players might be very different under per-

[10] As mentioned before, maintaining quasi-linearity of the environment requires that the feasibility constraint on the labor supply does not bind. Hence, the Nash equilibrium is stated as one in the epsilon neighborhood of the upper bound. As under utility theory, a positive affine transformation of payoffs preserves the game structure, we can think of this equilibrium as that of an ϵ-scaled down version of centralized market game where $0 < \epsilon < 1$.

Variable	Description
π^{min}	Minimum Money Growth rate
π^{max}	Maximum Money Growth Rate
ϕ^{h}	Highest Value of a currency
ϕ^{l}	Lowest Value of a currency
ϵ	parameter $\in (0,1)$
L	Hours worked-Centralized Market

fect monitoring than with short lived anonymous players (a continuum) (Mailath & Samuelson, 2006, p.269). Under imperfect monitoring this discontinuity disappears and therefore we use the imperfect monitoring setup to characterize the equilibrium pay off set and then use the perfect monitoring characterization of the folk theorem below. This works because under perfect monitoring, all action profiles of all long lived players have full rank- a sufficient condition to enable Nash threats. Also, because intertemporal incentives need to be provided only for the long lived players, it is sufficient that their actions are perfectly monitored (Mailath & Samuelson, 2006, pp. 305).

We state the proposition giving the lower and upper bound of the equilibrium payoff set here and proof in the appendix along with perfect monitoring characterization of the game in this paper.

Proposition 3.2. Equilibrium Payoff Set: *Minmax and least favorable payoffs for money supplier B are given by* $\underline{g}_B = \pi_B^{min}\phi_B^h B_{t-1}$ *and* $\bar{g}_B = \epsilon\pi_B^{max}\phi_B^l B_{t-1}$, $\forall\epsilon \in (0,1)$. *Similarly, Minmax and least favorable payoffs for the central bank R are given by* $\underline{g}_R = \pi_R^{min}\phi_R^h R_{t-1}$ *and* $\bar{g}_R = \epsilon\pi_R^{max}\phi_R^l R_{t-1}$, $\forall\epsilon \in (0,1)$.

Figure 1 depicts the equilibrium payoff set and the relationship of labor supply, money growth rate and seignorage for a money supplier in this economy. This diagram is drawn for a given supply of the competing money supplier. As argued before, as an implication of increased money supply by either of the money supplier, agents need to work more to compensate for the impact of resulting decline in value of money on their consumption of the generic good. You can see this from the first panel on the left side in the figure and subsequent effect on value of money and seignorage revenue from the remaining two panels. The incentives for agents are clear from their payoff function above.

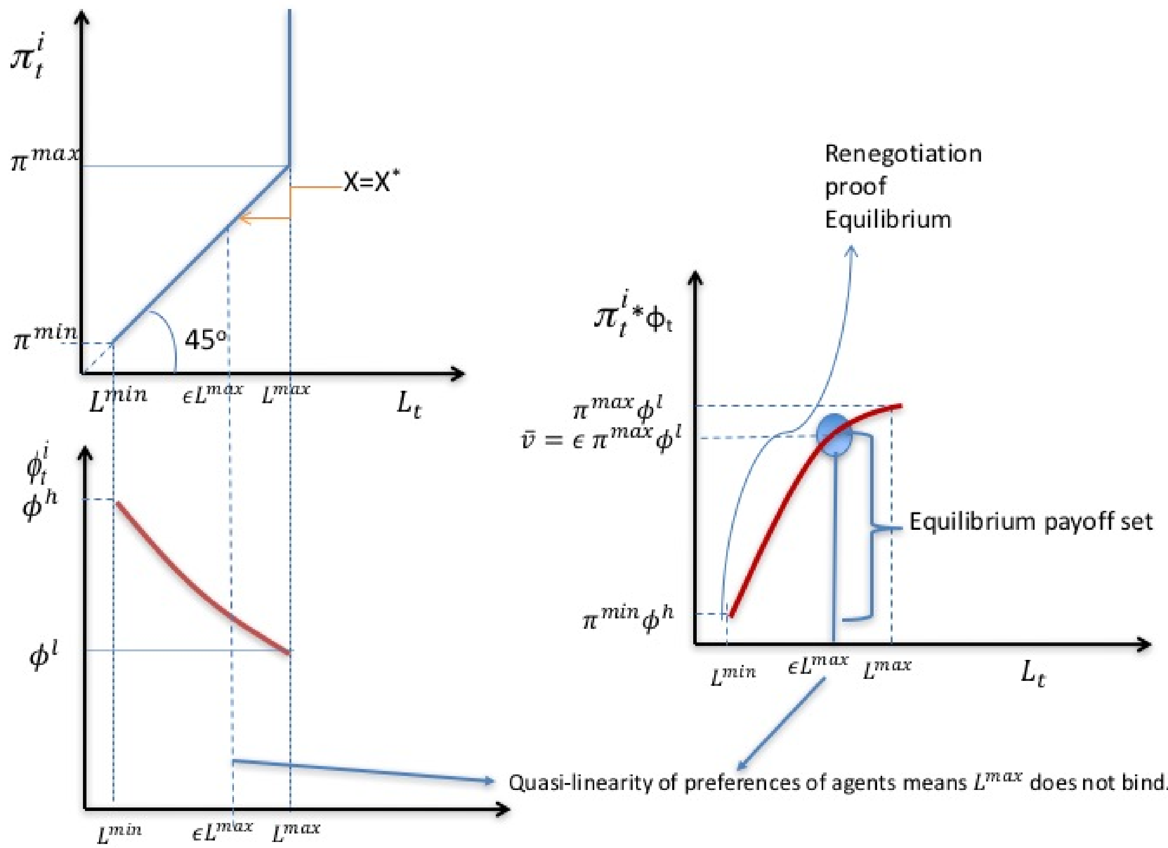

Figure 1: Relationship of labor supply, money growth rate, and seignorage under currency competition

Given the above equilibrium payoff set, the following proposition describes which of these payoffs can be supported as an equilibrium. In the spirit of folk theorems for repeated games, it basically highlights the fact that there are multiple equilibria in this infinitely repeated centralized market game.

Proposition 3.3. Long lived–Short-lived player theorem for the Two Money Suppliers LW Economy: *Suppose there are two money suppliers (long-lived players) along with a continuum of agents (a short-lived player). With the equilibrium payoff set as defined in Proposition 3.2, for every* $g \in int\mathfrak{F}^*$*, satisfying* $g_i < \bar{g}_i$ *for* $i = (Bank_B, Bank_R)$*, there exists a* $\underline{\delta}$ *such that for all* $\delta \in (\underline{\delta}, 1)$*, there exists a subgame perfect equilibrium of the repeated game with perfect monitoring with value g.*

Proof. It is a straight forward application of proposition 9.3.2 from (Mailath & Samuelson, 2006, pp.305) for multiple long-lived players and a short lived player as stated below:

Suppose A is finite and $\mathfrak{F}^\dagger$ *has nonempty interior. For every* $v \in int\mathfrak{F}^*$ *satisfying* $v_i < \bar{v}_i$ *for* $i = 1..., n$*, there exists a* $\underline{\delta}$ *such that for all* $\delta \in (\underline{\delta}, 1)$*, there exists a subgame perfect equilibrium of the repeated game with perfect monitoring with value v.* □

Thus, there are multiple equilibria in the two money suppliers model. Any of those, could be sustained depending on the money suppliers' level of patience. The payoffs depend on the history of money suppliers' actions and only the aggregate play of the agents. Given that agents always play Nash strategies, their incentive constraints are always satisfied. So to ensure that appropriate incentives exist for the money suppliers to stay on a given money growth rate, only specifying Nash reversion for the short lived players after observing a deviation is enough (Mailath & Samuelson, 2006, p. 93). Here Nash reversion would imply reverting to lower level of L and hence increase in price of X and decrease in value of money.

The threat of barter is credible because the agents are allowed to skip the centralized market if they anticipate a deviation from the range of money growth rates corresponding to the equilibrium payoff set in Proposition 3.2. At higher money growth rates, the value of money declines faster because of increase in total money supply as well as supply of one currency relative to another making more agents indifferent between having any kind of money and not having it. This increases the probability that a greater number agents skip the centralized market explaining the flattening of the seignorage curve. Thus, the money suppliers will have to choose their money growth rate from a finite interval determined by agents' willingness to work more. This gives the following proposition:

Proposition 3.4. Incentive Feasibility of payoffs: *The monetary growth rates corresponding to the equilibrium payoff set are the only ones that are incentive feasible in case of both the money suppliers.*

Proof. To be incentive feasible, the allocations should satisfy participation constraints and should be incentive compatible. Money being essential in this environment because of imperfect memory and lack of commitment, agents are better off using it than not. Similarly, the money suppliers would enjoy positive utility supplying money than not. This suggests that the participation constraints for both the players are satisfied. Considering the incentive constraints, because the equilibrium payoff set maps money suppliers' strategies to choose consumption to the Nash responses of the agents, it corresponds to the best response function **B**. Hence it satisfies the incentive constraints for both the players. □

3.2.1 Equilibrium Selection for the Infinitely Repeated Game

To explore the possibility of narrowing down of the equilibrium set, we need some refinement mechanism. In what follows, we use renegotiation-proofness of an equilibrium as an equilibrium refinement and show that the low inflation tax equilibrium $(L^{min}, \pi^R_{min}, \pi^B_{min})$ is weakly renegotiation proof.

A subgame perfect equilibrium σ is weakly renegotiation proof (WRP) if there do not exist continuation equilibria σ^1, σ^2 of σ such that σ^1 strictly pareto dominates σ^2. If an equilibrium σ is WRP, then the associated payoffs are also WRP (Farrell & Maskin (1989)).

In simple words, renegotiation-proofness implies that players at the beginning of the game or at any point in the game are able to coordinate on an efficient equilibrium. In all the range of equilibriums leading to the payoffs from the equilibrium payoff set, only the one with minimum inflation tax Pareto dominates all other equilibriums. The following proposition states it formally.

Proposition 3.5. *Equilibrium Selection for Infinitely Repeated Game of Two competitive money suppliers in an LW Economy: Even though there are multiple equilibria following Proposition 3.3, the only equilibrium that is weakly renegotiation proof,* $(L^{min}, \pi^R_{min}, \pi^B_{min})$*, features lowest inflation tax (cooperative outcome).*

Proof. Farrell & Maskin (1989) and van Damme (1989) show that cooperation in a infinitely repeated prisoners dilemma is weakly renogtiation-proof outcome if the discount factor is sufficiently close to 1. Given that the one-shot game here is a prisoner's dilemma (following Proposition 3.1), the cooperative equilibrium, $(L^{min}, \pi^R_{min}, \pi^B_{min})$, becomes WRP if the both the money suppliers are near perfectly patient. If this is the case, then $(L^{min}, \pi^R_{min}, \pi^B_{min})$ will be the equilibrium most likely to be played in the infinite repetition. □

From the point of view of efficient currency provision under competitive markets this is an important result. It suggests that the issue of time inconsistency arising because of lack of full commitment may be resolved provided both the money suppliers are perfectly patient and are able to renegotiate their behavior at any point in the game. However, note that because the money suppliers are utility-maximizing agents, even the cooperative equilibrium features a positive inflation tax.

4 Conclusion

In this paper, we develop a dual currency version of Lagos & Wright (2005) money search model to explore the nature of optimal monetary policy under competitive supply of money. One of the fundamental and often debated questions in monetary theory is whether competitive money supply is more efficient than a monopoly. It also seems to be of practical importance, given the emergence of multiple cryptocurrencies as well as multiple financial assets as a means of payment in certain class of transactions. To address the question, we analyze the nature of optimal monetary policy under currency competition and no commitment in a micro funded model of money. In the centralized market game modeled as a dynamic game between two utility maximizing private money suppliers and a continuum of agents, the Nash equilibrium of the stage game features highest inflation tax similar to the single money supplier case in Waknis (2014).

There are multiple equilibria in the infinite repetition of this game and any of those could be supported as an equilibrium in infinite repetition as the agents are free to skip the centralized market. This determines the monetary growth rates that could be supported as incentive feasible equilibrium payoff set. Any growth rate outside of this set triggers barter as agents tend towards indifference between having money and not having it and skip the centralized market.

It is possible to narrow down the possibilities further using an equilibrium refinement mechanism. The competition between money suppliers and the fact that agents play only Nash responses transforms the centralized market game to a prisoners dilemma between the two money suppliers, allowing us to use the idea of renegotiation proof equilibria as a refinement mechanism. Accordingly, the refinement suggests that if both the money suppliers are patient enough, then the equilibrium with lowest inflation tax (cooperative equilibrium) is weakly renegotiation proof, implying that currency competition is likely to generate a low inflationary outcome.

References

Aumann, Robert J. 1964. Markets with a Continuum of Traders. *Econometrica*, **32**(1/2), 39–50.

Berentsen, Aleksander. 2006. On the private provision of fiat currency. *European Economic Review*, **50**(7), 1683–1698.

Camera, Gabriele, Craig, Ben, & Waller, Christopher J. 2003. *Currency competition in a fundamental model of money*. Working Paper 0311. Federal Reserve Bank of Cleveland.

Craig, Ben, & Waller, Christopher J. 2004. Dollarization and currency exchange. *Journal of Monetary Economics*, **51**(4), 671 – 689.

Curtis, Elisabeth S., & Waller, Christopher J. 2000. A search-theoretic model of legal and illegal currency. *Journal of Monetary Economics*, **45**(1), 155–184.

Faingold, Eduardo, & Sannikov, Yuliy. 2011. Reputation in Continuous-Time Games. *Econometrica*, **79**(3), 773–876.

Farrell, Joseph, & Maskin, Eric. 1989. Renegotiation in repeated games. *Games and Economic Behavior*, **1**(4), 327 – 360.

Fernández-Villaverde, Jesús, & Sanches, Daniel. 2016 (Apr.). *Can Currency Competition Work?* NBER Working Papers 22157. National Bureau of Economic Research, Inc.

Fudenberg, Drew, & Levine, David K. 1994. Efficiency and Observability with Long-Run and Short-Run Players. *Journal of Economic Theory*, **62**(1), 103–135.

Fudenberg, Drew, Kreps, David M., & Maskin, Eric S. 1990. Repeated Games with Long-Run and Short-Run Players. *The Review of Economic Studies*, **57**(4), pp. 555–573.

Geromichalos, Athanasios, & Simonovska, Ina. 2014. Asset liquidity and international portfolio choice. *Journal of Economic Theory*, **151**, 342 – 380.

Geromichalos, Athanasios, Licari, Juan M, & Suarez-Lledo, Jose. 2007. Monetary Policy and Asset Prices. *Review of Economic Dynamics*, **10**(4), 761–779.

Hayek, Friedrich A. 1976. *Choice in Currency: A Way to Stop Inflation*. Institute of Economic Affairs, London.

Hayek, Friedrich A. 1978. *Denationalization of Money: The Argument Refined.* Institute of Economic Affairs, London.

Head, Allen, & Shi, Shouyong. 2003. A fundamental theory of exchange rates and direct currency trades. *Journal of Monetary Economics*, **50**(7), 1555–1591.

Kareken, John, & Wallace, Neil. 1981. On the Indeterminacy of Equilibrium Exchange Rates. *The Quarterly Journal of Economics*, **96**(2), 207–222.

Klein, Benjamin. 1974. The Competitive Supply of Money. *Journal of Money, Credit and Banking*, **6**(4), 423–53.

Kocherlakota, Narayana R. 1998. Money Is Memory. *Journal of Economic Theory*, **81**(2), 232–251.

Lagos, Ricardo. 2010. Asset prices and liquidity in an exchange economy. *Journal of Monetary Economics*, **57**(8), 913–930.

Lagos, Ricardo. 2011. Asset Prices, Liquidity, and Monetary Policy in an Exchange Economy. *Journal of Money, Credit and Banking*, **43**(October), 521–552.

Lagos, Ricardo, & Rocheteau, Guillaume. 2008. Money and capital as competing media of exchange. *Journal of Economic Theory*, **142**(1), 247–258.

Lagos, Ricardo, & Wright, Randall. 2005. A Unified Framework for Monetary Theory and Policy Analysis. *Journal of Political Economy*, **113**(3), 463–484.

Lagos, Ricardo, Rocheteau, Guillaume, & Wright, Randall. 2015. Liquidity: A new monetarist perspective. *Journal of Economic Literature, forthcoming*.

Lester, Benjamin, Postlewaite, Andrew, & Wright, Randall. 2012. Information, Liquidity, Asset Prices, and Monetary Policy. *Review of Economic Studies*, **79**(3), 1209–1238.

Li, Yan. 2009. The theory of fiat money and private money as alternative media of exchange. *International Review of Economics & Finance*, **18**(4), 568–582.

Lotz, Sebastien. 2004. Introducing a new currency: Government policy and prices. *European Economic Review*, **48**(5), 959–982.

Mailath, George J., & Samuelson, Larry. 2006. *Repeated Games and Reputations*. Oxford University Press.

Marimon, Ramon, Nicolini, Juan Pablo, & Teles, Pedro. 2012. Money is an experience good: Competition and trust in the private provision of money. *Journal of Monetary Economics*, **59**(8), 815–825.

Martin, Antoine. 2006. Endogenous Multiple Currencies. *Journal of Money, Credit and Banking*, **38**(1), 245–262.

Martin, Fernando M. 2011. On the joint determination of fiscal and monetary policy. *Journal of Monetary Economics*, **58**(2), 132–145.

Martin, Fernando M. 2013. Government Policy In Monetary Economies. *International Economic Review*, **54**(1), 185–217.

Nosal, Ed, & Rocheteau, G. 2011. *Money, Payments and Liquidity*. The MIT Press.

Nosal, Ed, & Rocheteau, G. 2017. *Money, Payments and Liquidity*. The MIT Press.

Rocheteau, Guillaume, Weill, Pierre-Olivier, & Wong, Tsz-Nga. 2015 (December). *Working through the Distribution: Money in the Short and Long Run*. Working Paper 21779. National Bureau of Economic Research.

Sanches, Daniel. 2016. On the Inherent Instability of Private Money. *Review of Economic Dynamics*, **20**(April), 198–214.

Sannikov, Yuliy. 2007. Games with Imperfectly Observable Actions in Continuous Time. *Econometrica*, **75**(5), 1285–1329.

Selgin, George A. *Free Banking*. http://people.terry.uga.edu/selgin/freebanking.html (Accessed on June 30, 2016).

Selgin, George A. 2008. Milton Friedman and the Case against Currency Monopoly. *Cato Journal*, **28**, 287–301.

Selgin, George A., & White, Lawrence H. 1994. How Would the Invisible Hand Handle Money? *Journal of Economic Literature*, **32**(4), 1718–1749.

Sun, Hongfei. 2007. Aggregate uncertainty, money and banking. *Journal of Monetary Economics*, **54**(7), 1929–1948.

Trejos, Alberto, & Wright, Randall. 2001. International Currency. *Advances in Macroeconomics*, **1**.

van Damme, Eric. 1989. Renegotiation-proof equilibria in repeated prisoners' dilemma. *Journal of Economic Theory*, **47**(1), 206 – 217.

Waknis, Parag. 2014. A Leviathan Central Bank: Modeling Seignorage in a Money Search Model. *Economics Letters*, **125**(3), 386–391.

Wallace, Neil. 2001. Whither Monetary Economics? *International Economic Review*, **42**(4), 847–69.

Williamson, Stephen D. 1999. Private Money. *Journal of Money, Credit and Banking*, **31**(3), 469–491.

Zhang, Cathy. 2014. An Information-Based Theory of International Currency. *Journal of International Economics*, **93**(2), 286–301.

A Perfect Monitoring Characterization of Games of Imperfect Public Monitoring

In a game with imperfect public monitoring, players $1, \ldots, n$ are long-lived players and players $n+1, \ldots, N$ are short-lived players with player i having a set of pure actions A_i, which is a compact subspace of the Euclidian $\Re^k$ for some k. Players choose actions simultaneously.

$\mathbf{B} : \prod_{i=1}^{n} \triangle(A_i) \rightrightarrows \prod_{i=n+1}^{N} \triangle(A_i)$ is the mapping of any mixed action profile for the long-lived players to the corresponding set of static Nash equilibria for the short-lived players.

At the end of the one-shot game players observe a public signal y, drawn from a signal space Y. $\rho(y|a)$ is the probability that the signal y is realized, given the action profile $a \in A = \prod_i A_i$. Player i's payoff after realization (y, a) is given by $u_i^*(y, a)$.

The set of public histories is $\mathbb{H} \equiv \bigcup_{i=0}^{\infty} Y^t$.

The set of histories for player i is $\mathbb{H}_i \equiv \bigcup_{i=0}^{\infty} (A_i \times Y)^t$.

We assume that each short-run player in period t observes only the public history.

A pure strategy for player i is a mapping from all possible histories into the set of pure actions, $\sigma_i : \mathbb{H}_i \rightarrow A_i$ and a mixed strategy is a mapping $\sigma_i : \mathbb{H}_i \rightarrow \triangle(A_i)$.

A behavior strategy σ_i is public if, in every period t, it depends only on the public history $h^t \in Y^t$ and not on i's private history. A perfect public equilibrium (PPE) is a profile of public strategies σ that for any public history h^t specifies a Nash equilibrium for the repeated game, i.e. for all t and all $h^t \in Y^t$, $\sigma|_{h^t}$ is a Nash equilibrium. A PPE is strict if each player strictly prefers his equilibrium strategy to every other public strategy. The set of PPE payoff vectors of the long-run players is denoted by $\mathcal{E}(\delta) \subset \Re^n$.

Given these definitions, we can describe a game of perfect monitoring as a special case where $Y = A$ and $\rho(y|a) = 1$ if $y = a$ and 0 otherwise. Accordingly, the game between the two money suppliers (long-lived players) and a continuum of economic agents (a short-lived player) in the second sub-period is a game of perfect monitoring. This is because the set of actions for the money suppliers is observable along with the aggregate play of the agents, $Y = A$ and there is no noise in the suppliers' actions being conveyed as $\rho(y|a) = 1$ if $y = a$ and 0.

B Proof of Proposition 3.2

Let the equilibrium set of strictly individually rational payoffs be $\mathcal{F}^{\dagger}$. This set is contained in the set of payoffs that is generated by all the pure actions $\mathcal{F}$. To guarantee enforceability of α even when it is not an equilibrium, we need that the signals generated by any action α_i be statistically distinguishable from those of any other mixture α_i'. This is ensured if the profile α has a full rank. In the game under consideration, there are two long-lived players and the infinitely lived agents are the short-lived players. In such games, if A is finite and there is perfect monitoring ($Y = A$ and $\rho(a'|a) = 1$ if $a' = a$ and 0 otherwise), then all action profiles have pairwise full rank for all pairs of long-lived players (money suppliers). This implies that $\bigcap_{\lambda \in \Lambda^n} H^*(\lambda) = \mathcal{F}^{\dagger}$.

Given a direction $\lambda \in \Re^n$, and constant $k \in \Re$, $H(\lambda, k)$ denotes the half space such that $g \in \Re^n : \lambda.g \leq k$. Let $\mathfrak{B}(\mathcal{W}; \delta, \alpha)$ be the set of payoffs that can be decomposed by α on $\mathcal{Q}$, when the discount factor is δ. Then, for a fixed λ and α, define:

$$k^*(\alpha, \lambda, \delta) = \max_{v} \lambda.g \tag{7}$$

subject to $g \in \mathfrak{B}(H(\lambda, \lambda.v); \delta, \alpha)$.

It can be shown that $k^*(\alpha, \lambda, \delta)$ is independent of δ and so can be written as $k^*(\alpha, \lambda)$. However, $k^*(\alpha, \lambda) \leq \lambda.u(\alpha)$ with the equality holding only if α is orthogonally enforced in the direction λ. Thus, the equilibrium payoff set can be approximated $H(\lambda, k^*(\alpha, \lambda))$, only if α is chosen appropriately. A way out is to choose an action profile that maximizes $k^*(\alpha, \lambda)$, that is set $k^*(\lambda) \equiv \sup_{\alpha \in \mathbf{B}} k^*(\alpha, \lambda)$ and $H^*(\lambda) \equiv H(\lambda, k^*(\lambda))$. This is referred to as the maximal half space in the direction λ. It is the largest $H(\lambda, k)$ half space with the property that a boundary point of the half space can be decomposed with respect to that half space (Mailath & Samuelson, 2006, p. 293).

The coordinate direction $\lambda = e_j$ and therefore, $k^*(e_j)$, defines the least favorable payoff ($\bar{v}_j$) for a long-lived player and as mentioned above $k^*(-e_j)$ gives the minmax payoff ($\underline{v}_j$). Thus, these two give the stretch of the maximal half space and given that the equilibrium payoff $\mathcal{F}^{\dagger}$ can be approximated by the intersection of the maximal half space, $k^*(-e_j)$ and $k^*(e_j)$ form the limit points or bounds of this equilibrium payoff set.

In the case of the model in this paper, the long-lived players are the money suppliers. Given the definitions of $\underline{v}_j$ and $\bar{v}_j$ in , to prove proposition (**??**), we have to prove that for $Bank_B$

$$k^*(-e_j) = -\min_{\alpha \in \mathfrak{B}} \max_{a_j} u_j(a_j, \alpha_{-j}) \quad = \quad \pi_B^{min} \phi_B^h B_{t-1} \tag{8}$$

$$k^*(e_j) = \max_{\alpha \in \mathfrak{B}} \min_{a_j \in Supp(\alpha_i)} u_j(a_j, \alpha_{-j}) \quad = \quad \pi_B^{max} \phi_B^l B_{t-1} \tag{9}$$

An analogous proof will hold for $Bank_R$.

Proof. The first part of equation 2 above states the coordinate direction $\lambda = -e_j$ corresponds to minimizing long-lived player j's payoff because $-e_j.g = -g_j$. Given that $\pi_B^{min} \phi_B^h B_{t-1}$ is the maximum payoff $Bank_B$ can hope to achieve if agents only put in minimal work, it becomes the *minmax* payoff.

The first part of the equation 3 suggests that the maximum payoff either long-lived player could attain cannot be greater than $k^*(e_j)$. Alternatively, the payoff from best response to a strategy, α_{-j} played by all other players cannot give a payoff better than the upper bound. The payoff $\pi_B^{max} \phi_B^l B_{t-1}$ is the maximum payoff that $Bank_B$ could hope to reach if the agents play any other strategy than "do minimal additional work". In fact to maintain quasilinearity of preferences in the centralized market, neither of the money suppliers would be able to exactly achieve it. They can only hope to get $\epsilon \pi_B^{max} \phi_B^l B_{t-1}$ for all $\epsilon \in (0, 1)$.

□