Hysteresis in a New Keynesian Model

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Abstract

This paper examines “hysteresis” in which persistent unemployment takes on structural characteristics over time. Hysteresis is modeled as deterioration in labor market matching efficiency as the average duration of unemployment increases. This is embedded in a simple New Keynesian macro model. A decline in labor market matching efficiency would be consistent with the observed rightward shift of the Beveridge curve since the 2007-09 recession. Hysteresis is shown to lead to larger and more persistent responses of the unemployment rate and unemployment duration to productivity, intertemporal preference and monetary shocks. Hysteresis also generates an increase in the natural rate of unemployment.

JEL: E24, J64, E32

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1 Introduction

Persistent elevated levels of long-term unemployment in the wake of the 2007-09 recession have generated renewed concern about “hysteresis” effects in which cyclical unemployment takes on structural characteristics over time. One mechanism by which this could take place would be a decline in the matching efficiency of the labor market. The apparent rightward shift of the Beveridge curve relationship between vacancies and unemployment in the US (Fig. 1) and other countries indicates such a decrease may have occurred.

This paper introduces a mechanism for modeling hysteresis in a simple New Keynesian model. Hysteresis takes the form of a decrease in labor market matching efficiency as the average duration of unemployment rises. This reflects the deterioration of labor market connections between workers and firms (e.g., the loss of information generated through informal networks) as workers are unemployed for long periods. Hysteresis is shown to generate larger and more persistent responses of the unemployment rate and unemployment duration to productivity, intertemporal preference and monetary shocks.

By endogenously increasing the persistence of responses of model variables to shocks, hysteresis provides a mechanism that can help address what Chari, Kehoe and McGrattan (2002) call the “persistence problem” - the well-known difficulty of macroeconomic models in matching the dynamics of aggregate variables like output following shocks.

The notion of hysteresis as a mechanism for understanding persistent high unemployment was raised by Blanchard and Summers (1986) in the context of Europe in the 1980s with a model that featured an “insider-outside” mechanism a la Lindbeck and Snower (1988) in which unemployed “outsiders” lose influence over the labor market\(^1\).

\(^1\)This was recently revisited by Galí (2015) which integrates an insider-outsider mechanism into a New Keynesian model to account for the apparent nonstationarity of European
More recently, concerns about whether persistent unemployment might take on structural characteristics or increase the natural rate have been widely expressed. For example, at his Nov. 2, 2011 press conference, then-Federal Reserve Board Chairman Ben Bernanke said:

Cyclical unemployment, left untreated, so to speak, for a long time can become structural unemployment as people lose skills, as they lose attachment to the labor force, as their work networks dry up and so on. So in that respect, it’s important for us to try to address the unemployment problem while it’s still amenable to monetary policy.

As shown in Fig. 2, the both the median and mean duration of unemployment spells in the US have reached unprecedented levels, and remain considerably elevated even relative to the rates prevailing following earlier recessions.

A number of studies have shown channels through which matching may be more difficult for long-term unemployed workers. In Ljungqvist and Sargent (1998), workers’ skills decline during periods of unemployment. Based on survey evidence, Krueger and Mueller (2011) find that search intensity declines with unemployment duration. Kroft et al. (2012), Ghayad (2013) and Eriksson and Rooth (2014) find that employers are less likely to respond to job applications from candidates in long spells of unemployment.

A decline in matching efficiency would be reflected in an outward shift of the Beveridge curve. That such a shift has occurred has been widely noted and examined - e.g., Diamond (2013), Hobijn and Sahin (2012) and Dickens and Triest (2012). Further evidence of a decline in matching efficiency in the US is provided by Lubik (2013) and Gregory et al. (2014). In an estimated New Keynesian model with shocks to matching efficiency, Furlanetto and Groshenny (2013), find that decreases in efficiency led to higher unemployment as well as a higher natural rate of unemployment in the Great Recession. Ghayad and Dickens (2012) show that the shift of the US Beveridge Curve is largely unemployment rates.
driven by the long-term unemployed and Arpaia and Turrini (2014) find that long-term unemployment plays a key role in explaining decreases in matching efficiency in EU countries\textsuperscript{2}. Barnichon and Figura (2015) examine the role of the composition of unemployment in terms of worker characteristics and dispersion across markets; their estimates show that the increased duration of unemployment spells has made a substantial contribution to the decline in matching efficiency in the US in recent years. This is consistent with the fact that most of the apparent shift in the Beveridge curve is attributable to the long-term unemployed; Fig. 3 shows separate Beveridge curves generated with persons unemployed more than 27 weeks and those unemployed 27 or fewer weeks as a percentages of the civilian labor force.

A number of papers have examined the relationship between monetary policy and hysteresis. Ball (1999) finds that countries that responded to recessions in the early 1980s with expansionary monetary policy were spared the persistent increases in unemployment that occurred in those which did not. Stockhammer and Stern (2012) also find less hysteresis (measured by changes in natural rate estimates) in recessions that are followed by more expansionary monetary policy. Ball (2009) finds that large changes in estimated NAIRUs are generally preceded by disinflation, and also that run-ups in inflation rates lead to NAIRU decreases. In a study of OECD countries, Llaudes (2005) shows that the fit of the Phillips curve relationship is improved by assigning a lower weight to long-term unemployment; this is particularly true for Europe. Using more recent US data, Rudebusch and Williams (2016) also find that long-term unemployed have less influence on the Phillips Curve and Ball and Mazumder (2014) find that using short run unemployment improves estimates of the inflation-unemployment

\textsuperscript{2}Ahn and Hamilton (2014) and Hall and Schulhofer-Wohl (2015) show compositional effects are important in the rightward shift of the Beveridge curve. Workers who involuntarily lost a job permanently are disproportionally represented among the long-term unemployed and these workers have lower exit rates from unemployment.
relationship. Krueger et al. (2014) find evidence that the long-term unemployed not only have less influence on inflation, but also that lower matching probabilities for long-term unemployed persons can help account for the rightward shift of the Beveridge curve.

The labor market issues considered in this paper contribute to a broader reduction in aggregate supply resulting from a period of low aggregate demand. Summers (2014) calls hysteresis “the shadow cast forward on economic activity by adverse cyclical developments.” Reifschneider et al. (2013) find evidence that this has occurred in the US post-2008. The US Congressional Budget Office (2014) cited elevated long-term unemployment as a contributing factor in revising downward its estimates of potential output. Ball (2014) reports substantial declines in potential output across OECD countries.

The model presented below follows the setup of Ravenna and Walsh (2011) but, unlike that model, also includes real wage rigidity. This is part of a growing literature integrating labor market search and matching into New Keynesian models, including Walsh (2005), Krause and Lubik (2007), Faia (2008), Thomas (2008), Trigari (2009), Blanchard and Gali (2010), Tang (2010) and Christiano, Trabandt and Walentin (2011). This line of research builds on earlier work by Merz (1995) and Andolfatto (1996) that introduced labor market search into Real Business Cycle models. An alternative approach to understanding persistent unemployment in a model with labor market search is given by Farmer (2012).
2 Model

2.1 Households

A representative household of unit measure maximizes expected lifetime utility from consumption,

\[ U = E \sum_{t=0}^{\infty} \beta^t D_t \frac{C_t^{1-\sigma}}{1-\sigma} \] (1)

where \( \sigma \) is the inverse of the intertemporal elasticity of substitution. Consumption, \( C \), is an aggregate of market produced goods, \( C^m \), and output from home production, where \( z^h \) is the productivity of unemployed household members and \( L \) is the fraction of the household that is employed. Hence, \( C = C^m + z^h (1 - L) \).

As is standard in the literature using labor market search in business cycle models, following Merz (1995), the household members are assumed pool consumption so consumption of employed and unemployed household members is the same. The discount factor, \( \beta \), is multiplied by an intertemporal preference shifter, \( D \), which represents variation in the household’s impatience - shocks to this variable can be thought of as representing shocks to desired saving which might, for example, result from a financial crisis. This discount shock evolves in an autoregressive fashion,

\[ \ln D_t = \rho D \ln D_{t-1} + \varepsilon_{D,t} \] (2)

and \( D \) has a mean of 1. The household can purchase one-period nominal bonds with a payoff of 1 for price \( q \), and market-produced consumption goods at price \( P \). It earns a wage, \( w \), and receives the profits, \( \Xi \), from the final goods firms (described below) which it owns. Its budget constraint is thus

\[ P_t C^m_t + q_t B_{t+1} \leq w_t L_t + \Xi_t + B_t. \] (3)

\(^3\text{For a discussion of intertemporal shocks, see Primiceri et al. (2006).}\)
Household optimization implies

\[ q_t = \beta \mathbb{E} \frac{D_{t+1}}{D_t} \left( \frac{C_t}{C_{t+1}} \right)^\sigma \frac{1}{\pi_{t+1} + 1} \]  

(4)

where \( \pi_{t+1} \) is the inflation rate between \( t \) and \( t+1 \). This implies that an increase in \( D_t \), ceteris paribus, lowers the price of bonds (or, alternatively, raises the nominal interest rate, \( i \), where \( q_t = \frac{1}{1 + i_t} \)).

### 2.2 Intermediate Goods Production

Perfectly competitive intermediate goods firms hire workers and produce output, which is sold to final goods firms that transform it into differentiated final products. This separation between intermediate and final goods production is for analytical convenience: separating the wage-setting problem, which involves the intermediate goods firm, and the price-setting problem which occurs at the final goods stage removes the complications of interactions between the two.\(^4\)

Firms hire labor and use it to produce a homogeneous intermediate good using a linear technology. To post a vacancy, intermediate goods firms pay a cost, \( \kappa \). Intermediate goods output is thus given by

\[ Y_t = z_t L_t - \kappa V_t \]  

(5)

where \( V \) is the number of vacancies. Productivity, \( z \), has a mean of one and follows the process

\[ \ln z_t = \rho_z \ln z_{t-1} + \varepsilon_{z,t} \]  

(6)

\(^4\)This is a commonly-made assumption; Thomas (2011) examines the implications of relaxing it.
2.3 The Labor Market

Separations are assumed to occur at an exogenous rate, $\delta$, and hires are given by $H$, so the labor employed at date $t$ is given by

$$L_t = (1 - \delta)L_{t-1} + H_t$$  \hspace{1cm} (7)$$

The number of unemployed at the beginning of period $t$ is given by

$$U_t = 1 - (1 - \delta)L_{t-1}$$  \hspace{1cm} (8)$$

Matches are formed according to the matching function,

$$H_t = A_tV_t^\alpha U_t^{1-\alpha}$$  \hspace{1cm} (9)$$

where $A$ represents the productivity of the matching technology, which depends on the duration of unemployment in a manner described below. Labor market tightness is the ratio of vacancies to unemployed, $x = \frac{V}{U}$. The probability that a vacancy is successfully filled is thus $Ax^{\alpha-1}$, which is decreasing in $x$, while the probability that an unemployed worker will find a job is given by $Ax^\alpha$.

The average duration of unemployment, denoted $M_t$, is

$$M_t = \frac{\delta}{U_t} \left\{ L_{t-1} + \sum_{i=2}^{\infty} \left[ \prod_{j=1}^{i-1} (1 - A_{t-j}x_{t-j}^\alpha) \right] L_{t-i} \cdot i \right\}$$  \hspace{1cm} (10)$$

which evolves according to

$$M_t = \frac{U_{t-1}}{U_t}(1 - A_{t-1}x_{t-1}^\alpha)M_{t-1} + 1$$  \hspace{1cm} (11)$$

Shimer (2012) finds that increases in unemployment are predominately accounted for by decreases in job-finding rather than increases in the rate of exit from employment.
Using bars to denote steady state values, the steady state beginning-of-period duration of unemployment is given by $\bar{M} = \frac{1}{Ax^\alpha}$. Hysteresis enters the model as a decrease in matching efficiency as the duration of unemployment increases according to

$$A_t = \bar{A} e^{-\gamma (\frac{M_t}{\bar{M}} - 1)}$$

(12)

Where $\bar{A}$ is a normalization constant. The degree of hysteresis can be governed by the parameter $\gamma$ where $\gamma = 0$ is the “no hysteresis” case. The functional form allows for “reverse hysteresis” (Ball, 1999) where a tight labor market can reduce the natural rate of unemployment over time.

Letting $W$ denote the real wage, $w_P$, and $V_E$ and $V_U$ the values of being employed and unemployed, respectively, $V_E$ is given by:

$$V_E^t = W_t + \beta E_t \frac{D_{t+1}}{D_t} \left\{ \left( \frac{C_t}{C_{t+1}} \right)^\sigma \left[ (1 - \delta + \delta A_{t+1} x_{t+1}^\alpha) V_E^{t+1}_{t+1} + \delta (1 - A_{t+1} x_{t+1}^\alpha) V_U^{t+1}_{t+1} \right] \right\}. \quad (13)$$

The second term is the discounted expected future value, where the probability of separating and being rehired is $\delta A_{t+1} x_{t+1}^\alpha$. The value of unemployment is:

$$V_U^t = z^h + \beta E_t \frac{D_{t+1}}{D_t} \left( \frac{C_t}{C_{t+1}} \right)^\sigma \left[ (1 - \delta + \delta A_{t+1} x_{t+1}^\alpha) V_U^{t+1}_{t+1} + A_{t+1} x_{t+1}^\alpha V_E^{t+1}_{t+1} \right]. \quad (14)$$

The household’s surplus, $S_H$, from a match is the difference between the value of employment and unemployment. That is, $S_H = V_E^t - V_U^t$. Substituting and rearranging yields the following expression,

$$S_H^t = W_t - z^h + \beta (1 - \delta) E_t \frac{D_{t+1}}{D_t} \left( \frac{C_t}{C_{t+1}} \right)^\sigma (1 - A_{t+1} x_{t+1}^\alpha) S_H^{t+1}. \quad (15)$$

Firms will post vacancies until the expected value of doing so equals the cost,
which implies

\[ A_t x_t^{\alpha-1} V_t' = \kappa \]  

where \( V_t' \) the value to a firm of a filled job (which is identical to the firm’s surplus, since the value to a firm of an unfilled job is zero).

A Nash bargain would divide match surplus according to the rule \( (1-\xi) S^H = \xi V' \) where \( \xi \) is the firm’s share resulting from is relative bargaining power. Applying this yields, after some algebra, an expression for the Nash-bargained real wage,

\[ W_t^N = z^h + \frac{\xi}{1-\xi} A_t x_t^{1-\alpha} \]

\[ - \beta (1-\delta) \frac{\xi}{1-\xi} E_t \left( \frac{C_t}{C_{t+1}} \right)^\sigma \left( \frac{1}{A_{t+1} x_{t+1}^{1-\alpha} - x_{t+1}} \right) \]  

Shimer (2005) and Hall (2005) showed that the search and matching model has difficulty matching cyclical properties of unemployment and vacancies with Nash-bargained real wages. This is because wage movements leave firms with little incentive to adjust vacancies; the inclusion of real wage rigidities can improve its performance in this regard. As in Kruse and Lubik (2007), Faia (2008) and Blanchard and Gali (2010) real wage rigidities are implemented by assuming that real wages are a weighted average of the wage that would be arrived at through Nash bargaining and their steady state value, \( \bar{W} \),

\[ W_t = \lambda W^N + (1-\lambda) \bar{W} \]  

where \( \lambda \) governs the degree of real wage flexibility.
2.4 Price-Setting

Letting $p^I$ denote the price of the intermediate good, a producing firm chooses its vacancy posting (and, thus, its labor) to maximize expected profits, discounted according to the utility of the households that own it. Its first-order condition yields the following expression for the intermediate goods price, $p^I$, expressed relative to the final goods price, $P$,

$$\frac{p^I_t}{P_t} = \frac{1}{z_t} \left[ W_t + \frac{\kappa}{A_t} x_t^{1-\alpha} - \beta (1 - \delta) \kappa E_{t+1} D_{t+1} \left( \frac{C_t}{C_{t+1}} \right)^{\sigma} x_{t+1}^{1-\alpha} \right]$$ (19)

The price of goods reflects not only the cost of labor, but also the cost of job posting, as well as the discounted value of future job posting costs saved by hiring in the current period.

The competitive intermediate goods firms sell their output to final goods firms that transform it, costlessly, into differentiated final products. From their point of view, the real price of intermediate goods represents their real marginal cost, i.e., $\frac{P^I}{P} = MC$.

Product varieties are indexed by $0 \leq i \leq 1$ and the household’s consumption bundle is a constant elasticity of substitution aggregate of all the varieties,

$$C = \left[ \int_0^1 c(i) \frac{1}{\nu} di \right]^\frac{\nu}{\nu-1}$$ (20)

where $\nu$ is the elasticity of substitution. If prices were flexible, the final goods firms would set their prices at a constant markup, $\mu = \frac{\nu}{\nu-1}$ over (nominal) marginal cost, i.e., $P = \mu p^I$.

Following Calvo (1983), a fraction, $1 - \theta$, of firms can reset their prices in each time period. Letting $\tilde{p}_t$ denote the price set by price-changing firms, the
overall price index evolves according to

\[ P_t = \left[ (1 - \theta)\tilde{p}_t^{1-\nu} + \theta P_{t-1}^{1-\nu} \right]^{\frac{1}{1-\nu}}. \]  

(21)

The price-changing firms maximize

\[ \sum_{s=0}^{\infty} \theta^s E_t Q_{t+s} \left[ \tilde{p}_t Y_{t+s|t} - \psi(Y_{t+s|t}) \right] \]  

(22)

where \( \psi(\cdot) \) is the cost function and \( Q \) represents the stochastic discount factor.

A first-order Taylor expansion of the price-changing firms’ first-order condition yields, after some algebra, a New Keynesian Phillips Curve,

\[ \pi_t \simeq \beta E_t \pi_{t+1} + \frac{(1 - \theta)(1 - \beta\theta)}{\theta} \hat{MC}_t \]  

(23)

where \( \hat{MC}_t \) represents the percentage deviation of real marginal cost from its steady state value, \( \frac{1}{\mu} \).

2.5 Closing the Model

Two additional conditions are needed to close the model. The first is the market-clearing condition that

\[ C_t^m = Y_t \]  

(24)

Note that this definition of GDP excludes home production, as does its counterpart in the national income and product accounts.

The other condition is a description of monetary policy. The monetary authority is assumed to follow the canonical “Taylor Rule” where nominal interest rates respond to inflation and the output gap, \( y^{gap} \), which is defined as the log
difference between flexible-price and actual output. That is,

\[ i_t = -\ln \beta + \phi_\pi \pi_t - \phi_y y_t^{\text{gap}} + \nu_t^M \]  

(25)

where \( -\ln \beta \) is the equilibrium natural rate of interest in a zero-inflation steady state, and \( \phi_\pi \) and \( \phi_y \) govern the monetary policy responses to inflation and the output gap, respectively. In order to investigate the implications of changing matching efficiency due to hysteresis, the flexible-price level of output is calculated from the model with no price or real wage rigidities, but the level of matching efficiency, \( A \), is assumed to evolve as under sticky prices and real wage rigidity. The monetary policy shock, \( \nu_t^M \), follows an AR(1) process,

\[ \nu_t^M = \rho_M \nu_{t-1}^M + \varepsilon_t^M \]  

(26)

3 Solution and Results

The model is log-linearized and solved using Dynare (Adjemian et al., 2011). The parameterization, summarized in Table 1, is quarterly and largely follows that of Ravenna and Walsh (2011) and, otherwise, employs mainstream values in the literature.

Following Ravenna and Walsh (2011), \( \bar{A} \) and \( \kappa \) are set so that steady state employment, \( \bar{N} \), is 0.94, and the hiring rate in the steady state is 0.9. Productivity in home production of unemployed workers, \( z^h \), is also taken from Ravenna and Walsh (2011); although the model does not feature unemployment benefits, it can also be interpreted as being akin to the replacement ratio from unemployment insurance. The real wage adjustment parameter follows Blanchard and Gali (2010); Fuia (2008) uses a similar value (0.6). The persistence of the discount shocks, 0.7, is set to be in line with results from estimated DSGE
models such as Sala, Söderström and Trigari (2008), Justiniano, Primiceri and Tambalotti (2010) and Garín, Lester and Sims (2016). The persistence of the monetary shocks corresponds with the experiment of a “moderately persistent” shock discussed in Galí (2008, ch. 3).

Fig. 4 shows the median duration of unemployment spells (in quarters) and matching efficiency, calculated as \( H/(V^{0.5}U^{0.5}) \) where \( H \) and \( U \) are the quarterly average level of hires and unemployment and \( V \) is the quarterly average level of openings, which is calculated with a 1-month lag because the JOLTS openings series is end-of-month. The inverse relationship apparent in the figure is consistent with the model: rise in the duration of unemployment spells around the 2008-09 recession corresponds with a decrease in matching efficiency, and, as duration has fallen since then, efficiency has begun to rise. A regression of efficiency on duration yields a coefficient of 0.42, which is the basis for setting \( \gamma = 0.4 \) for the cases with hysteresis below.

Impulse responses are generated for one standard deviation negative productivity shocks, negative discount (intertemporal preference) shocks, and positive (contractionary) monetary shocks under two cases: hysteresis (\( \gamma = 0.4 \)) and no hysteresis (\( \gamma = 0 \)).

The response of unemployment which, measured at the end of period, is \( 1 - L \), to a contractionary productivity shock is shown in Fig. 5. In the absence of hysteresis, firms respond to the shock by sharply reducing vacancies (shown in Fig. 6) which leads to a jump in unemployment. Hysteresis implies that hiring will be more costly in the future, so, in the presence of hysteresis, firms will not cut back their hiring as much in the present. The smoother adjustment of vacancies under hysteresis causes unemployment to rise for several periods before beginning to fall. Note that because matching efficiency is lower after the shock, hysteresis implies firms need to post more vacancies in the later periods. The
introduction of hysteresis also generates a larger and more persistent increase in the duration of unemployment spells shown in Fig. 7.

Dynamics of inflation and interest rates are shown in Fig. 8. The decrease in productivity is a negative supply shock, and is therefore inflationary; while this also creates an output gap, under the chosen parameters, the net effect through the Taylor rule is an increase in interest rates. In the absence of hysteresis, vacancies drop sharply immediately after the shock, reducing labor market tightness and real marginal cost, so the increases in inflation and interest rates are less in the first period, but then higher in the second period, when vacancies recover quickly and marginal cost rises. With hysteresis, because of the smoother dynamics of unemployment and vacancies, there is more inflation and a larger increase in interest rates in the first period, but not a large jump in the second period. With hysteresis, inflation and interest rates return to their steady states more slowly over time.

A discount shock, where households reduce their weight on present consumption, also leads to an increase in unemployment. As with the productivity shock, the response of unemployment, shown in Fig. 9, is more gradual but more persistent with hysteresis.

The discount shock is deflationary, and monetary rule implies a cut in interest rates. As Fig. 10 shows, the deflationary impact and resulting interest rate response are sharper in the absence of hysteresis because of the immediate increase in unemployment. With hysteresis the initial decreases in inflation and interest rates are smaller, but slightly more persistent.

Similarly, hysteresis also leads to a more persistent unemployment response to a contractionary monetary policy shock, shown in Fig. 11.

Model standard deviations of output, vacancies and unemployment are reported in Table 2. At a quarterly frequency over the period when US JOLTS
data is available, 2001 Q1 - 2015 Q4, the standard deviation of the unemployment rate is 26.8% (or 1.72 pt)\(^6\) and the standard deviation of the vacancy rate is somewhat lower, at 17.0% (0.47 pt). In the absence of hysteresis, the model generates excessive volatility of vacancies relative to unemployment, with a standard deviation of the vacancy rate nearly twice that of unemployment. By reducing the volatility of vacancies while increasing that of unemployment, hysteresis improves the ability of the model to better capture their relative volatilities. Hysteresis also increases slightly the volatility of output.

Table 3 reports autocorrelations of several variables in the model. In data, output, vacancies and unemployment are all highly autocorrelated. The inclusion of hysteresis helps generate persistence in the model that is closer to the data. At quarterly frequency, the autocorrelations of the US job openings and unemployment rates are 0.94 and 0.98, respectively, over 2001 Q1 - 2015 Q4 (going back to 1948 Q1, the autocorrelation of the unemployment rate is 0.97). The effect of hysteresis on model-generated autocorrelations is particularly noteworthy for the vacancy rate. In the absence of hysteresis, since vacancies are a ‘jump’ variable, the model generates a high volatility of the vacancy rate and the dynamics are such that the autocorrelation is negative. Although the model-generated autocorrelation remains considerably less than in the data, the addition of hysteresis moves it closer to its empirical counterpart\(^7\).

The “natural rate” of unemployment can be proxied by the unemployment rate that would prevail under flexible prices and real wages, taking as given the changes in hiring costs due to hysteresis. Figure 12 illustrates that, in response to a negative productivity shock, both the unemployment rate (under hysteresis) and the natural rate (labelled “Flexible Price”) increase as the productivity of

\(^6\)Over the period 1948 Q1 - 2015 Q4, the standard deviation of the unemployment rate is similar: 28.2% (1.64 pt).

\(^7\)Fujita and Ramey (2007) show that another approach to generating more persistence in the vacancy rate is to incorporate a sunk cost for vacancy creation.
employed household members falls relative to those engaged in home production. The increase in unemployment is somewhat smaller when wages and prices are flexible.

Wage and price rigidities are more important for the dynamics in response to discount and monetary shocks. The response to the discount shock is shown in Fig. 13. In the absence of hysteresis (not shown), the natural rate moves very little. With hysteresis, the increase in hiring costs cause the flexible price unemployment rate to rise over time; the unemployment rate with sticky prices converges to the natural rate as the effects of wage and price rigidities die out. Note that flexible price unemployment initially declines slightly as firms would bring forward hiring in anticipation of higher matching costs in the future. The case of the monetary shock (Fig. 14) is similar. The increases in flexible price unemployment illustrate how hysteresis worsens the tradeoff for the monetary authority between low unemployment and stable prices.

4 Conclusion

In the wake of the severe 2007-09 recession, the shift in the Beveridge curve and the increase in unemployment duration are consistent with a model of hysteresis in which labor market matching efficiency deteriorates as the duration of unemployment increases. Integrating such a mechanism into a simple New Keynesian model shows that hysteresis leads to more persistent dynamics of unemployment rates and unemployment duration in response to technology, intertemporal preference, and monetary shocks. Hysteresis also leads to an increase in the natural rate of unemployment, proxied by flexible-price level of unemployment.

The model presented in this paper has been deliberately kept simple in order to illustrate the proposed hysteresis mechanism. Its ability to generate more persistent dynamics of macroeconomic variables in this setting suggests
it might be useful for further study in the context of richer models intended to more closely replicate empirical business cycle behavior.
References


### Table 1: Parameters

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<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
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<td>0.99</td>
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<td>$\sigma$</td>
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<td>$\phi_y$</td>
<td>0.5</td>
<td>Taylor rule output gap response</td>
</tr>
<tr>
<td>$\rho_Z$</td>
<td>0.9</td>
<td>Persistence of productivity shocks</td>
</tr>
<tr>
<td>$\rho_D$</td>
<td>0.7</td>
<td>Persistence of discount shocks</td>
</tr>
<tr>
<td>$\rho_M$</td>
<td>0.5</td>
<td>Persistence of monetary shocks</td>
</tr>
<tr>
<td>$\sigma_{\varepsilon,Z}$</td>
<td>0.01</td>
<td>Standard deviation of productivity shocks</td>
</tr>
<tr>
<td>$\sigma_{\varepsilon,D}$</td>
<td>0.01</td>
<td>Standard deviation of discount shocks</td>
</tr>
<tr>
<td>$\sigma_{\varepsilon,M}$</td>
<td>0.0025</td>
<td>Standard deviation of monetary shocks</td>
</tr>
</tbody>
</table>

### Table 2: Model Standard Deviations

<table>
<thead>
<tr>
<th></th>
<th>Output</th>
<th>Vacancies</th>
<th>Unemployment</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Hysteresis</td>
<td>2.73</td>
<td>18.35</td>
<td>9.40</td>
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<tr>
<td>Hysteresis</td>
<td>2.91</td>
<td>12.24</td>
<td>12.49</td>
</tr>
</tbody>
</table>

### Table 3: Model Autocorrelations

<table>
<thead>
<tr>
<th></th>
<th>Output</th>
<th>Vacancies</th>
<th>Unemployment</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Hysteresis</td>
<td>0.89</td>
<td>-0.20</td>
<td>0.65</td>
</tr>
<tr>
<td>Hysteresis</td>
<td>0.92</td>
<td>0.05</td>
<td>0.92</td>
</tr>
</tbody>
</table>

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Fig. 1: US Beveridge Curve, Dec. 2001 - Jan. 2016

Fig. 2: Duration of US Unemployment Spells, Jul. 1967 - Feb. 2016
Fig. 3: US Beveridge Curve by Duration, Dec. 2001 - Jan. 2016

Fig. 4: Matching Efficiency and Duration, 2001Q1 - 2015Q4
Fig. 5: Unemployment Response to Productivity Shock

Fig. 6: Vacancy Response to Productivity Shock
Fig. 7: Duration Response to Productivity Shock

![Graph showing duration response to productivity shock with two lines: one for Hysteresis and one for No Hysteresis.](image)

Fig. 8: Inflation and Interest Rate Response to Productivity Shock

![Graph showing inflation and interest rate response to productivity shock with multiple lines indicating different scenarios.](image)
Fig. 9: Unemployment Response to Discount Shock

Fig. 10: Inflation and Interest Rate Response to Discount Shock
Fig. 11: Unemployment Response to Monetary Shock

Fig. 12: Unemployment Response to Productivity Shock
Fig. 13: Unemployment Response to Discount Shock

Fig. 14: Unemployment Response to Monetary Shock