Money and Pay-As-you-Go Pension

Yasuoka, Masaya

Kwansei Gakuin University

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Masaya Yasuoka†

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Abstract

In an aging society with fewer children, a pay-as-you-go pension system presents severe difficulties. A decrease in the share of working people among the population raises the burden for pensions per capita to maintain a constant replacement ratio of pensions. This burden reduces capital accumulation. Therefore, income growth is prevented. The analyses in this paper demonstrate that if the replacement rate of pension is high, a decrease in population growth reduces the income growth rate even if a decrease in population growth can raise the income growth rate per capita because the capital stock that the workers can use increases. However, by setting an appropriate monetary policy for decreasing population growth, the income growth is not prevented by an increase in the burdens for pensions. The negative effect of the burden for pensions on income growth can be eliminated by the change of the money supply rate in the long run.

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†Corresponding to: School of Economics, Kwansei Gakuin University, 1-155 Uegahara, Ichiban-Cho, Nishinomiya, Hyogo 662-8501 Japan, Tel.:+81-798-54-6993, E-mail: yasuoka@kwansei.ac.jp

1 Introduction

In an aging society with fewer children, pension sustainability is an important topic. With pay-as-you-go pension systems, incoming funds paid by younger people are distributed to older people as pensions in a single period. Therefore, fewer children implies a decrease in the number of working people in the future. The per-capita pension burden on younger people becomes larger or the per-capita pension benefit for older people becomes smaller.

[Insert Fig. 1 around here.]

Fig. 1 shows the elderly population rate in some OECD countries. As shown in Fig. 1, the elderly population rate continues to increase. Because of an increase in the elderly population rate, younger people must pay for their government to provide a social security system that includes programs such as public pensions and medical services. As shown in Fig. 2, social security contributions continue increasing in some OECD countries. Especially, in Japan, which is highest the elderly population late in the world, social security contributions are steeply increasing.

[Insert Fig. 2 around here.]

To maintain the pension benefit, the government must raise the contribution rate for pensions: the burden for younger people should be raised. However, without increasing the contribution rate, the pension benefit can be raised. Fanti and Gori (2010) report that a decrease in the contribution rate for pensions raises the pension benefit because capital accumulation is facilitated by increased saving. As shown by the overlapping-generations model, a pay-as-you-go pension reduces savings, thereby inhibiting capital accumulation. A decrease in the contribution rate increases capital accumulation. Consequently, the wage rate increases. The government can collect sufficient revenue to provide pension benefits.

Other ways to increase the pension benefit without increasing the burden for younger people might be used. A child-care policy such as a child allowance might be used. Van Groezen, Leers, and Meijdam (2003) derive that child allowance can raise fertility. An increase in fertility increases the future number of working people. Therefore, the pension sustainability is strong. Abío, Mahieu and Patxot (2004) and Fenge and Meier (2005) consider an incentive policy by which the pension benefit depends on the number of children in the household. This policy has a positive effect on population growth.

As these related studies have demonstrated, some means to maintain or raise pension benefits in an aging society with fewer children society exist. However, other policies can be considered for pensions: monetary policies. This paper sets an endogenous growth model with money in the utility and pay-as-you-go pension and examines how a decrease in population growth affects the income growth rate and the inflation rate.

The main results derived by the analyses described in this paper are presented as follows. The effect of a population growth rate decrease depends on the pension contribution rate. With a low contribution rate, a decrease in the population growth rate raises the income growth rate. Even if a decrease in the number of working people impedes capital accumulation because of an increase in the per-capita burden for pension benefits, a decrease in the number of working people raises the per-capita capital stock and raises the rate of productivity. The income growth rate increases if the latter effect is large.

This paper presents examination of the effect of monetary policy on the income growth rate and the inflation rate. An increase in the money stock raises both the income growth rate and the inflation rate on a balanced growth path. An increase in the inflation rate reduces the demand for money stock and increases the demand for investment. An increase in investment raises the income growth rate because of capital accumulation. Moreover, this paper presents derivation of an endogenous monetary policy. Given a high replacement rate of the pension, a decrease in the population growth rate reduces the income growth rate. Nevertheless, by virtue of an increase in money stock, the income growth rate can be constant. Even if a decrease in population growth and the burden for pensions increases, appropriate monetary policy measures can cancel the negative effects on the income growth rate.

Some studies related to monetary policy exist. Mino and Shibata (1995) examine how monetary policy affects the income growth rate. In addition, Chang, Chen and Chang (2013) describe their examination of how monetary policy affects the income growth rate and welfare in an endogenous growth model. Heer and Maußner (2012) refer to the effects on income inequality. Bhattacharya, Haslag, and Martin (2009) derive an optimal monetary policy. Gregorio (1993) also refers to an optimal monetary policy.

As derived herein, Corneo and Marquardt and Ono (2010) show that a pay-as-you-go pension reduces the income growth rate, too. These papers do not consider monetary policy measures.

Yakita (2006) sets an endogenous growth model with monetary policy and examines how increased life expectancy affects the income growth rate and the inflation rate. Depending on the parameter conditions, the effects of increased life expectancy on the income growth rate and the inflation rate are determined.

No report in the relevant literature describes consideration of monetary policy in a pay-as-you-go pension model. This paper presents a derivation showing that the effect of fewer children on the income growth rate depends on the contribution rate for pensions.

The remainder of this paper presents the following. Section 2 explains the model setting. Section 3 derives the equilibrium. Section 4 presents examination of the respective effects of fewer children on the income growth rate and the inflation rate. Section 5 describes derivation of monetary policy measures that might be used to maintain the income growth rate or the inflation rate. The final section concludes this paper.

2 The Model

Three types of agents exist in this model economy: households, firms, and a government. This model economy is assumed as a two-period overlapping-generations model.

2.1 Household

Individuals in the household live in two periods: young and old. In any t period, younger people and the older people exist simultaneously. Younger people supply labor inelastically according to units of time. The individuals in the household care about consumption during the younger period c_{1t}, the consumption in older period c_{2t+1}, and the real money stock per capita m_t. These variables are assumed as real variables. The utility function in this paper is assumed as the following log utility function with money in the utility as assumed by Sidrauski (1967), Walsh (2010) and others

$$u_t = \alpha \ln c_{1t} + (1-\alpha) \ln m_t + \rho \ln c_{t+1},\ 0 < \alpha < 1,\ 0 < \rho < 1. \tag{1}$$

In that equation, α denotes the preference for money. Small α shows that the preference for money is large. Also, ρ denotes the discount factor. During the young period, households supply labor to obtain the real wage income w_t and pay the burden for public pension $\tau_t w_t$, where τ_t denotes the contribution rate for pension.[1] In the older period, the older people can receive the real pension benefit Z_{t+1}. Then,

[1] The analyses in this paper assume an inelastic labor supply and that the wage tax does not distort household allocations.

the budget constraint in the young period is

$$s_t = (1 - \tau_t) w_t - c_{1t} - m_t, \tag{2}$$

where s_t denotes the real savings.[2] The budget constraint in old period is

$$c_{2t+1} = (1 + r_{t+1}) s_t + \frac{m_t}{1 + \pi_t} + Z_{t+1}, \tag{3}$$

where $1+\pi_t$ and $1+r_{t+1}$ respectively denote inflation rate between t and $t+1$ period and the real interest rate.[3] Then, using (2) and (3), lifetime budget constraint can be derived as

$$c_{1t} + \frac{c_{2t+1}}{1 + r_{t+1}} = (1 - \tau_t) w_t - \left(1 - \frac{1}{(1 + r_{t+1})(1 + \pi_t)}\right) m_t + \frac{Z_{t+1}}{1 + r_{t+1}}. \tag{4}$$

The optimal allocations to maximize the utility function (1) subject to budget constraint (4) are derived as

$$c_{1t} = \frac{\alpha}{1 + \rho} \left((1 - \tau_t) w_t + \frac{Z_{t+1}}{1 + r_{t+1}} \right), \tag{5}$$

$$c_{2t+1} = \frac{\rho(1 + r_{t+1})}{1 + \rho} \left((1 - \tau_t) w_t + \frac{Z_{t+1}}{1 + r_{t+1}} \right), \tag{6}$$

$$m_t = \frac{1 - \alpha}{(1 + \rho)\left(1 - \frac{1}{(1 + r_{t+1})(1 + \pi_{t+1})}\right)} \left((1 - \tau_t) w_t + \frac{Z_{t+1}}{1 + r_{t+1}} \right). \tag{7}$$

We assume $(1 + r_{t+1})(1 + \pi_t) > 1$ to hold a positive demand for money stock.

2.2 Firms

This model includes the assumption of the final goods production function as the following equation:

$$Y_t = K_t^{\theta} (A_t L_t)^{1-\theta}, \ 0 < \theta < 1, \tag{8}$$

where Y_t denotes final goods. Final goods Y_t are produced by inputting capital stock K_t and labor L_t. A_t denotes the productivity. Assuming a perfectly competitive market, the real wage rate w_t and the real interest rate $1 + r_{t+1}$ are given as

$$w_t = (1 - \theta) A_t K_t^{\theta} (A_t L_t)^{-\theta}, \tag{9}$$

$$1 + r_{t+1} = \theta K_t^{\theta - 1} (A_t L_t)^{1-\theta}. \tag{10}$$

[2] Assuming P_t as the price level, $P_t s_t = (1 - \tau_t) P_t w_t - P_t c_{1t} - P_t m_t$ is obtained. This real budget constraint in the young period is derived by omitting the equation by P_t.

[3] The nominal budget constraint is $P_{t+1} c_{2t+1} = (1 + i_{t+1}) P_t s_t + P_t m_t + P_{t+1} Z_{t+1}$. Also, i_{t+1} denotes the nominal interest rate. Considering $1 + r_{t+1} = \frac{1+i_{t+1}}{1+\pi_{t+1}}$ (Fisher equation) and omitting P_{t+1}, the budget constraint in the older period is derived.

This paper presents consideration of the Romer type externality given by Romer (1986), i.e., $A_t = a\frac{K_t}{L_t}$ is assumed. This setting is given by Grossman and Yanagawa (1993). a denotes a positive parameter. Then, the wage rate and real interest rate are given as

$$w_t = (1-\theta)a^{1-\theta}k_t, \tag{11}$$

$$1 + r_t = \theta a^{1-\theta}, \tag{12}$$

where $k_t \equiv \frac{K_t}{L_t}$. The capital stock is assumed to be depreciated in a single period.

2.3 Government

The government supplies money at the rate of $\mu(>0)$. Assuming an aggregate money stock as M_t, then the money stock in the next period is $M_{t+1} = (1+\mu)M_t$. m_t is real money stock per capita. we can obtain $m_t = \frac{M_t}{P_t N_t}$, where P_t denotes the price level in t period and N_t denotes the total population size of younger people in t period. Then, the older population size is N_{t-1} in t period. Assuming the population growth rate as $\frac{N_{t+1}}{N_t} = 1+n$, then the dynamics of money stock m_t is

$$m_{t+1} = \frac{1+\mu}{(1+\pi_t)(1+n)} m_t. \tag{13}$$

The government can use seigniorage when issuing currency. The analyses in this paper subsume that the seigniorage is used for non-productive government expenditure.

In addition to the money supply, the government runs a pay-as-you-go pension system. The government collects revenue from younger people at the contribution rate τ_t and provides pension benefit Z_t for older people. Now, considering balanced budget constraint, Z_t is assumed as presented below:

$$Z_t = \epsilon w_t. \tag{14}$$

Therein, ϵ represents the replacement rate of pension $(0 < \epsilon < 1)$. The government collects the tax revenue to provide fixed pension benefit ϵw_t, such as a defined benefit pension. Then, the pension benefit is $N_t \tau_t w_t = N_{t-1}\epsilon w_t$, i.e., the following equation is reduced as

$$\tau = \frac{\epsilon}{1+n}. \tag{15}$$

The analyses in this paper assume that the contribution rate is constant over time. The contribution must be pulled up to provide a fixed pension benefit if low fertility brings about a decrease in population growth rate. This paper presents consideration of the defined benefit pension. Such an increase in the burden might occur in OECD countries. In Japan, the contribution rate for pension continues increasing because of an aging society.[4]

3 Equilibrium

The equilibrium of this model economy can be specified by three market equilibria of the monetary market, labor market, and capital market. Because of the inelastic labor supply, $N_t = L_t$ and (11) provide the labor market equilibrium.

The capital market equilibrium is derived as $K_{t+1} = N_t s_t$, i.e., $(1+n)k_{t+1} = s_t$. s_t is given as $s_t = (1-\tau)w_t - c_{1t} - m_t$. Then, with (5), (7), (11), (12), (14), and (15), the dynamics of capital stock per capita k_t is derived as shown below:

$$\frac{k_{t+1}}{k_t} = \frac{\left(1+\rho-\alpha-\frac{1-\alpha}{1-\frac{1}{(1+r)(1+\pi_t)}}\right)\left(1-\frac{\epsilon}{1+n}\right)(1-\theta)a^{1-\theta}}{(1+n)(1+\rho)+\frac{1-\theta}{\theta}\left(\alpha+\frac{1-\alpha}{1-\frac{1}{(1+r)(1+\pi_t)}}\right)}. \tag{16}$$

This paper presents consideration of the equilibrium of balanced growth path. Now, when the growth rate is defined as $1+g$ ($g>0$), both the growth rate of capital stock per capita $\frac{k_{t+1}}{k_t}$ given by (16) and that of $\frac{m_{t+1}}{m_t}$ given by (13) are equal to $1+g$.[5] Considering (7), (9), (14), and (15), the ratio of $\frac{m_t}{k_t}$ is given as

$$\frac{m_t}{k_t} = \frac{1-\alpha}{(1+\rho)\left(1-\frac{(1+n)}{(1+r)(1+\pi_t)}\right)}\left(\left(1-\frac{\epsilon}{1+n}\right)(1-\theta)a^{1-\theta}+\frac{\epsilon(1-\theta)(1+g)}{\theta}\right). \tag{17}$$

In the balanced growth path with constant inflation rate π, $\frac{m_t}{k_t}$ is also constant. Then, the growth rate of $\frac{m_{t+1}}{m_t}$ is $1+g$.

4 Fewer Children Society

This section presents an examination of how fewer children, i.e., a decrease in population growth rate affects the income growth rate and inflation rate. The income growth rate $1+g$ and the inflation rate π

[4]In Japan, the contribution rate continues increasing to 18.3% by 2017.(Data: Ministry of Health, Labour and Welfare, Japan)

[5]Considering (13) and (16), $1+g = \frac{\left(1+\rho-\alpha-\frac{1-\alpha}{1-\frac{(1+g)(1+n)}{(1+r)(1+\mu)}}\right)\left(1-\frac{\epsilon}{1+n}\right)(1-\theta)a^{1-\theta}}{(1+n)(1+\rho)+\frac{1-\theta}{\theta}\left(\alpha+\frac{1-\alpha}{1-\frac{(1+g)(1+n)}{(1+r)(1+\mu)}}\right)}$ is reduced. The growth rate $1+g$ can be obtained uniquely to hold the equation.

in balanced growth path are given as

$$1+g=\frac{\left(1+\rho-\alpha-\frac{1-\alpha}{1-\frac{1}{(1+r)(1+\pi)}}\right)\left(1-\frac{\epsilon}{1+n}\right)(1-\theta)a^{1-\theta}}{(1+n)(1+\rho)+\frac{1-\theta}{\theta}\left(\alpha+\frac{1-\alpha}{1-\frac{1}{(1+r)(1+\pi)}}\right)}, \tag{18}$$

$$1+g=\frac{1+\mu}{(1+\pi)(1+n)}. \tag{19}$$

Total differentiation of (18) and (19) by g, π and n at the balanced growth path reduces the following equations as[6]

$$\begin{pmatrix} \phi_1 & -\phi_2 \\ 1 & \frac{1+g}{1+\pi} \end{pmatrix}\begin{pmatrix} \frac{dg}{dn} \\ \frac{d\pi}{dn} \end{pmatrix}=\begin{pmatrix} \phi_3 \\ -\frac{1+g}{1+n} \end{pmatrix},$$

where

$$\phi_1=(1+n)(1+\rho)+\frac{1-\theta}{\theta}\left(\alpha+\frac{1-\alpha}{1-\frac{1}{(1+r)(1+\pi)}}\right)>0,$$

$$\phi_2=\frac{1-\alpha}{\left(1-\frac{1}{(1+r)(1+\pi)}\right)^2}\frac{1-\theta}{(1+r)(1+\pi)^2}\left(\frac{1+g}{\theta}+a^{1-\theta}\left(1-\frac{\epsilon}{1+n}\right)\right)>0,$$

$$\phi_3=\left(1+\rho-\alpha-\frac{1-\alpha}{1-\frac{1}{(1+r)(1+\pi)}}\right)\frac{\epsilon(1-\theta)a^{1-\theta}}{(1+n)^2}-(1+\rho)(1+g).$$

The effects of decreased population growth (fewer children) on the income growth rate and inflation rate in the balanced growth path are

$$\frac{dg}{dn}=\frac{\frac{1+g}{1+\pi}\phi_3-\frac{1+g}{1+n}\phi_2}{det}, \tag{20}$$

$$\frac{d\pi}{dn}=\frac{-\frac{1+g}{1+n}\phi_1-\phi_3}{det}, \tag{21}$$

where $det=\frac{1+g}{1+\pi}\phi_1+\phi_2>0$. The signs of these equations are ambiguous because the sign of ϕ_3 is ambiguous. The first term of ϕ_3 presents the positive effect of an increase in population growth on the income growth. An increase in population growth reduces the burden of pensions for younger people. Then, the younger people can increase the savings. Therefore an increase in capital accumulation facilitates income growth in an increase in population growth n. However, the second term of ϕ_3 shows the negative effect of an increase in population growth on the income growth. An increase in population growth makes the population size large and reduces the capital stock per capita. Then, the income growth per capita $1+g$ is reduced. The positive effect on income growth shows the effect of a decrease in burdens for pension. Without ϵ, an increase in population growth brings about negative effects on income growth.

[6] See Appendix for a detail proof.

If $\phi_3 > 0$, i.e., the positive effect on income growth is larger than the negative effect on income growth, then an increase in population growth brings about $\frac{d\pi}{dn} < 0$. However, even if the sign of $\frac{d\pi}{dn}$ is determined, the sign of $\frac{dg}{dn}$ remains ambiguous because of the change in demand for money m_t. This point demonstrates that the inflation rate decreases in an increase in population growth n if $\frac{d\pi}{dn} < 0$ is obtained. A decrease in the inflation rate increases the demand for money, as shown by (7), because of an increase in benefits from holding money. The household decreases savings s_t. Capital accumulation decreases because they hold more cash. Actually, ϕ_2 shows the effect of the demand for money on income growth.

Then, how does a decrease in population growth affect the income growth and the inflation rate with a pay-as-you-go pension? I define ϵ^* to hold $\phi_3 = 0$.[7] $\phi_3 > 0$ and $\frac{d\pi}{dn} < 0$ is obtained if $\epsilon^* < \epsilon < 1$. That is, a decrease in population growth increases the inflation rate. However, the effect on the income growth is ambiguous because $\phi_3 > 0$ and $\phi_2 > 0$.

The sign of $\frac{dg}{dn}$ is always negative because $\phi_3 < 0$ if $\epsilon < \epsilon^*$. Then, a decrease in population growth increases income growth because the workers can use more capital stock per capita. However, the sign of $\frac{d\pi}{dn}$ is ambiguous because $\phi_1 > 0$ and $\phi_3 < 0$. Then, the proof above reduces to the following proposition.

Proposition 1 A decrease in population growth increases the inflation rate if $\epsilon^* < \epsilon < 1$. The effect on the income growth rate is ambiguous. A decrease in population growth raises the income growth rate if $\epsilon < \epsilon^*$. The effect on the inflation rate is ambiguous.

A decrease in population growth has the effect of increasing the capital stock per capita. Such a result is derived in related studies such as that of the Solow (1956) model. However, this paper includes two other effects. One is the effect of the pension burden. A decrease in population growth raises the pension burden for younger people because people of working generations decrease. The burden per capita must be pulled up when the government makes the level of pension benefit for older people constant if the working generations of people decrease. The level of pension is the replacement ratio of pension ϵ. This negative effect of a decrease in population on the income growth is greater if ϵ is larger.

The other effect is the effect via the demand for money stock. As shown by ϕ_2, a decrease in population

[7] See Appendix for a detailed proof.

growth has the effect of an increase in the inflation rate.[8] This effect decreases the demand for money and then the savings to bring about increased capital accumulation. Therefore, an increased pension burden does not necessarily decrease the income growth rate.

Yakita (2006) examines life expectancy effects on the income growth rate and inflation rate. An increase in life expectancy and a decrease in the population growth are substantially equivalent in terms of processes of an aging society with fewer children. Both this paper and Yakita (2006) present the same result: the parameter conditions determine the effects on income growth and inflation. However, this paper presents derivation indicating that the effect is determined by the replacement rate of the pension ϵ.

5 Monetary Policy

This section presents an examination of how monetary policy affects the income growth and inflation rate. Moreover, the following subsection presents examination of endogenous monetary policy. Concretely, we derive the appropriate monetary policy to maintain the income growth rate or the inflation rate in an environment with a decreasing population growth rate. As derived in the preceding section, a decrease in the population growth rate might give negative effects on the income growth rate. This research shows how the government should provide the money stock in a society with fewer children.

5.1 Effect of Monetary Policy

This subsection presents effects of monetary policy on the income growth rate and the inflation rate. Total differentiation of (13) and (16) by g, π, and μ presents the following signs:

$$\frac{dg}{d\mu} = \frac{\phi_2 \frac{1+g}{1+\mu}}{det} > 0, \tag{22}$$

$$\frac{d\pi}{d\mu} = \frac{\phi_1 \frac{1+g}{1+\mu}}{det} > 0. \tag{23}$$

These results are intuitive. An increase in the money supply increases the inflation rate based on the 'quantity theory of money'. An increase in the inflation rate decreases the demand for money stock. Then, households increase savings or investment for goods. Therefore, capital accumulation is facilitated.

[8] Given g and μ at (19), a decrease in population growth n increases the inflation rate π.

5.2 Endogenous Monetary Policy

This subsection derived the endogenous monetary policy to maintain the income growth rate or the inflation rate. The income growth rate and inflation rate might change if the population growth rate decreases. This subsection presents consideration of a policy to maintain the income growth rate or the inflation rate by changing the money supply rate μ with a decreasing population growth rate n.

First, we derive the appropriate monetary policy to maintain the income growth rate $\frac{dg}{d\mu} = 0$. A decrease in the income growth rate reduces the income level in the future period. The welfare of future generations might decrease. Consequently, there exists an appropriate reason to maintain the income growth rate. From total differentiation of (18) and (19) by π, n and μ, the following signs are obtained:

$$\frac{d\mu}{dn} = -\frac{\phi_3}{\phi_2} + \frac{1+\mu}{1+n}, \tag{24}$$
$$\frac{d\pi}{d\mu} = \frac{1+n}{1+\mu}\frac{1}{\frac{1+n}{1+\pi} - \frac{\phi_2}{\phi_3}}. \tag{25}$$

These signs are ambiguous. However, without the burden for pension ϵ, ϕ_3 is invariably negative. Therefore, $\frac{d\mu}{dn} > 0$ and $\frac{d\pi}{d\mu} > 0$. However, considering a pay-as-you-go pension, the sign of $\frac{d\mu}{dn}$ is ambiguous. Now, this paper presents consideration of a case for which $\phi_3 > 0$, by which a decrease in population growth negatively affects the income growth rate by an increase in the per-capita burden for pensions. A decrease in population growth is expected to raise the supply rate of money stock to maintain the income growth rate if $\frac{\phi_3}{\phi_2} > \frac{1+\mu}{1+n}$. A decrease in the population growth negatively affects the income growth rate because of an increase in the pension burden. The inflation rate must be raised to recover the income growth.

However, if $\frac{\phi_3}{\phi_2} < \frac{1+\mu}{1+n}$, then a decrease in population growth can be expected to reduce the supply rate of money stock to maintain income growth. A decrease in population has a positive effect on the inflation rate, as shown by (19) for given $1+g$. A high inflation rate accelerates investment. Then the income growth is higher than the initial growth rate. The supply rate of the money stock should be reduced to prevent income growth. The inflation rate is given as (25) to maintain the income growth rate.

Second, we derive an endogenous monetary policy to maintain the inflation rate $\frac{d\pi}{d\mu} = 0$. An increase in the inflation rate entails higher costs such as shoe costs and menu costs. Therefore, the inflation rate should not be raised. Total differentiation of (18) and (19) by g, n, and μ gives the following signs:

$$\frac{d\mu}{dn} = \frac{\frac{1+n}{1+g}\frac{\phi_3}{\phi_1} + 1}{\frac{1+n}{1+\mu}}, \tag{26}$$

$$\frac{dg}{d\mu} = \frac{\frac{1+g}{1+\mu}}{1 + \frac{1+g}{1+n}\frac{\phi_1}{\phi_3}}. \tag{27}$$

Here, $\phi_3 > 0$ is assumed. Then, both $\frac{dg}{d\mu}$ and $\frac{d\mu}{dn}$ are positive. A decrease in population growth is expected to reduce the supply rate of money stock, as shown by (26). Considering (19), a decrease in population growth increases the inflation rate. The money supply rate is expected to decrease, not to increase the inflation rate. Because of a decrease in the money supply, the income growth rate is reduced because the burden for pension increases. Then, the following proposition is established.

Proposition 2 With a decrease in population growth rate n, the government sets the monetary policy to hold (24) to maintain the income growth rate. Then, the inflation rate is changed, as shown by (25). The monetary policy is set as (26) if the government wants to maintain the inflation rate. Then, the income growth rate is given as (27).

Considering (24)–(27) in $\phi_3 > 0$, the government should adopt some policy to maintain the income growth rate because the inflation rate might decrease. The effects of these policies are determined by the pension replacement ratio ϵ.

6 Conclusion

This paper presents an examination of how a decrease in the population growth as shown in an aging society with fewer children affects the income growth rate and the inflation rate in the balanced growth path in the model with money in the utility and pay-as-you-go pension. A decrease in population growth has a positive effect on income growth because the workers can use more capital stock per capita. However, considering the pension burden, decreased population growth negatively affects income growth. Public pensions have a negative effect on capital accumulation, as shown by many related studies. The per-capita pension burden decreases. Capital accumulation might increase if younger people increase in number as a result of child care policies. The government might decrease the pension burden for younger people and cut benefits for older people to maintain the income growth rate. However, considering monetary policy, even if a decrease in population growth decreases the income growth rate, income growth can be expected to recover by virtue of an appropriate monetary policy.

References

[1] Abío G., Mahieu G. and Patxot C. (2004) "On the optimality of PAYG pension systems in an endogenous fertility setting," *Journal of Pension Economics and Finance* vol.3(1), pp.35-62.

[2] Bhattacharya J., Haslag J. and Martin A. (2009) "Optimal monetary policy and economic growth," *European Economic Review* vol.53, pp.210-221.

[3] Chang W.-Y., Chen Y.-A. and Chang J.-J. (2013) "Growth and welfare effects of monetary policy with endogenous fertility," *Journal of Macroeconomics* vol.35, pp.117-130.

[4] Corneo G. and Marquardt M. (2000) "Public pensions, unemployment insurance, and growth," *Journal of Public Economics* vol.75, pp.293-311.

[5] Fanti L. and Gori L. (2010) "Increasing PAYG pension benefits and reducing contribution rates," *Economics Letters* vol.107, pp.81-84.

[6] Fenge R. and Meier V. (2005) "Pensions and fertility incentives," *Canadian Journal of Economics* vol.38, pp.28-48.

[7] Gregorio J. D. (1993) "Inflation, taxation, and long-run growth," *Journal of Monetary Economics* vol.31, pp.271-298.

[8] Grossman G. M. and Yanagawa, N. (1993) "Asset bubbles and endogenous growth," *Journal of Monetary Economics* vol.31(1), pp.3-19.

[9] Heer B. and Maußner A. (2012) "The burden of unanticipated inflation: Analysis of an overlapping-generations model with progressive income taxation and staggererd prices," *Macroeconomic Dynamics* vol.16, pp.278-308.

[10] Mino K. and Shibata A. (1995) "Monetary policy, overlapping generations, and patterns of growth," *Economica* vol.62, pp.179-194.

[11] Ono T. (2010) "Growth and unemployment in an OLG economy with public pensions," *Journal of Population Economics* vol.23, pp.737-767.

[12] Romer P. M. (1986) “Increasing returns and long-run growth,” *Journal of Political Economy* vol. 94(5), pp.1002-1037.

[13] Sidrauski M. (1967) “Rational Choice and patterns of growth in a monetary economy,” *American Economic Review* vol.57(2), pp.534-544.

[14] Solow R. (1956) “A contribution to the theory of economic growth,” *Quarterly Journal of Economics* vol.70(1), pp.65-94.

[15] van Groezen B., Leers, T. and Meijdam L. (2003) “Social security and endogenous fertility: pensions and child allowances as Siamese twins,” *Journal of Public Economics* vol.87(2), pp.233-251.

[16] Walsh C.E. (2010) ”Monetary theory and policy, third edition,” MIT Press, Cambridge, Massachusetts, London, England.

[17] Yakita A. (2006) “Life expectancy, money, and growth,” *Journal of Population Economics* vol.19, pp.579-592.

Appendix

The derivation of the signs

Total differentiation of (18) and (19) by g, π, n, and μ, the following equations are derived,

$$\phi_1 dg - \phi_2 d\pi = \phi_3 dn,$$
$$dg = -\frac{1+g}{1+n}dn - \frac{1+g}{1+\pi}d\pi + \frac{1+g}{1+\mu}d\mu.$$

Setting $d\mu = 0$, (20) and (21) are derived. Setting $dn = 0$, (22) and (23) are derived. Setting $dg = 0$, (24) and (25) are derived. Setting $d\pi = 0$, (26) and (27) are derived.

The growth rate in balanced growth path

Considering (13), (16), and $1+g = \frac{k_{t+1}}{k_t} = \frac{m_{t+1}}{m_t}$, the following equation is obtained as

$$1+g = \frac{\left(1+\rho-\alpha-\frac{1-\alpha}{1-\frac{(1+g)(1+n)}{(1+r)(1+\mu)}}\right)\left(1-\frac{\epsilon}{1+n}\right)(1-\theta)a^{1-\theta}}{(1+n)(1+\rho)+\frac{1-\theta}{\theta}\left(\alpha+\frac{1-\alpha}{1-\frac{(1+g)(1+n)}{(1+r)(1+\mu)}}\right)}.$$

Defining $L = 1+g$ and $R = \frac{\left(1+\rho-\alpha-\frac{1-\alpha}{1-\frac{(1+g)(1+n)}{(1+r)(1+\mu)}}\right)\left(1-\frac{\epsilon}{1+n}\right)(1-\theta)a^{1-\theta}}{(1+n)(1+\rho)+\frac{1-\theta}{\theta}\left(\alpha+\frac{1-\alpha}{1-\frac{(1+g)(1+n)}{(1+r)(1+\mu)}}\right)}$, L and R has a unique intersect and the growth rate in balanced growth path $1+g^*$ is obtained, as shown in Fig. 3.

[Insert Fig. 3 around here.]

The sign of ϕ_3

This appendix shows the sign of ϕ_3. I define the following equations:

$$\bar{L} = \left(1+\rho-\alpha-\frac{1-\alpha}{1-\frac{1}{(1+r)(1+\pi)}}\right)\frac{\epsilon(1-\theta)a^{1-\theta}}{(1+n)^2}, \tag{28}$$
$$\bar{R} = (1+\rho)(1+g). \tag{29}$$

$\bar{L}$ increases with an increase in ϵ. $\bar{R}$ decreases with an increase in ϵ because an increase in ϵ reduces g as shown in Fig. 1. The sign of ϕ_3 is negative because $\bar{L} < \bar{R}$ if $\epsilon = 0$. I think that the range of ϵ is between 0 to $1+n$ not to be negative saving. $\bar{L} > \bar{R}$ because $\bar{R}$ becomes zero if $\epsilon = 1+n$. I can obtain $\phi_3 > 0$ and ϵ^* to hold $\phi_3 = 0$ in $0 < \epsilon < 1+n$.

[Insert Fig. 4 around here.]

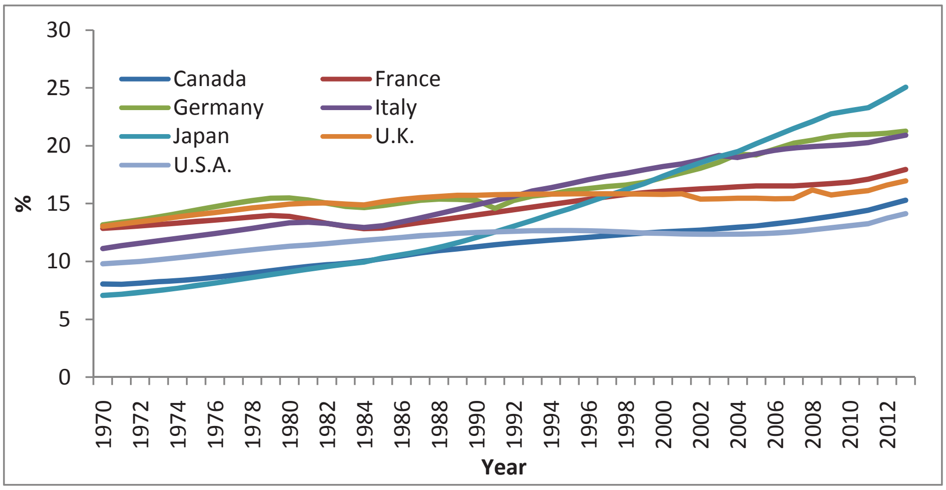

Fig. 1: Elderly population rate (Data: OECD Data, Elderly Population).

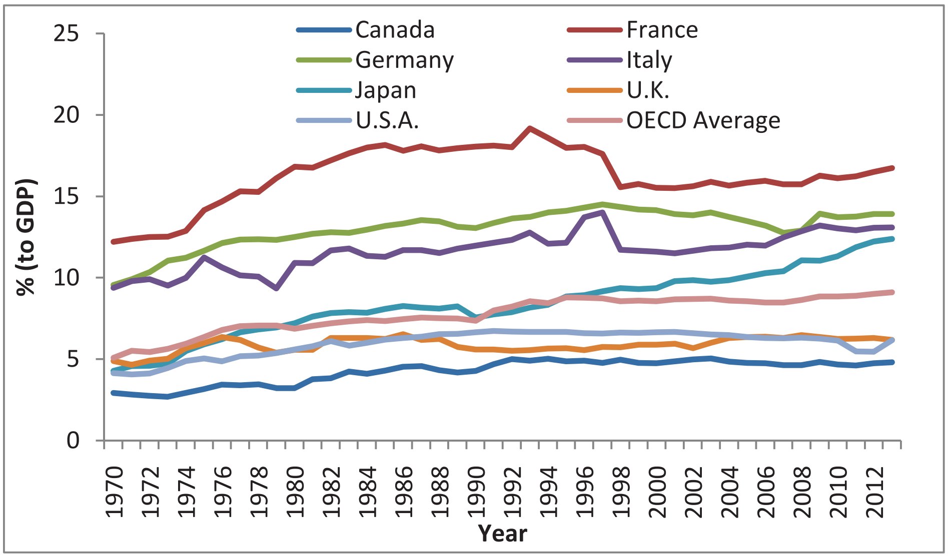

Fig. 2: Social security contributions (data: OECD Data, Social security contributions).

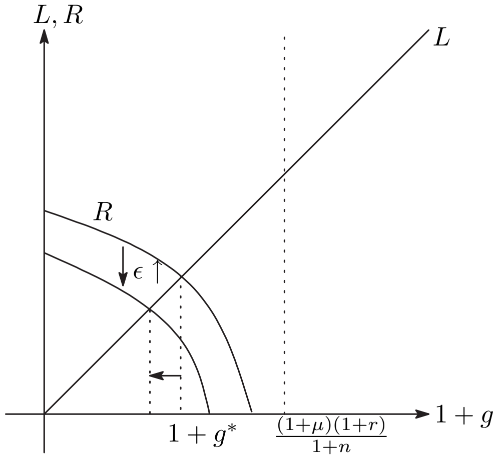

Fig. 3: Growth rate in balanced growth path.

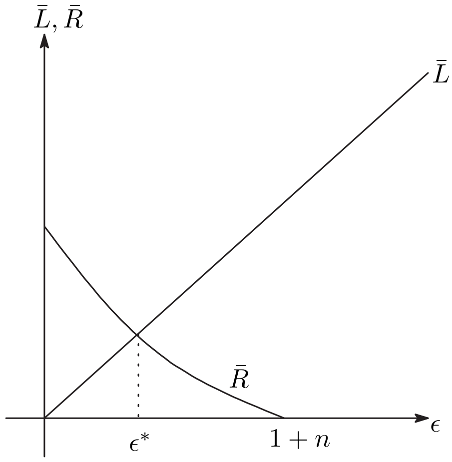

Fig. 4: Range of ϵ.