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Trump, Condorcet and Borda: Voting paradoxes in the 2016 Republican presidential primaries¹

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Abstract. The organization of US presidential elections make them potentially vulnerable to so-called “voting paradoxes”, identified by social choice theorists but rarely documented empirically. The presence of a record high number of candidates in the 2016 Republican Party presidential primaries may have made this possibility particularly latent. Using polling data from the primaries we identify two possible cases: Early in the pre-primary (2015) a cyclical majority may have existed in Republican voters’ preferences between Bush, Cruz and Walker—thereby giving a rare example of the Condorcet Paradox. Furthermore, later polling data (March 2016) suggests that while Trump (who achieved less than 50% of the total Republican primary vote) was the Plurality Winner, he could have been beaten in pairwise contests by at least one other candidate—thereby exhibiting a case of the Borda Paradox. The cases confirm the empirical relevance of the theoretical voting paradoxes and the importance of voting procedures.

Key words: Social choice; Condorcet Paradox; Borda Paradox; US presidential election 2016; Jeb Bush; Chris Christie; Ted Cruz; John Kasich; Marco Rubio; Donald Trump; Scot Walker; voting system.

JEL-codes: D71; D72.

1. Introduction

Since the 1950s social choice theory has questioned the possibility of aggregating individual preferences to straightforward, meaningful collective choices (Arrow [1951] 1963). Most well-known are two so-called “voting paradoxes” named after the French mathematicians Marie-Jean-Antoine-Nicolas de Caritat, Marquis de Condorcet (1743-1794) and Jean-Charles de Borda (1733-1799) (McLean and Urken 1995). The Borda Paradox occurs when a plurality of voters prefer A to B and C , but where B or C (or both) can beat A in pairwise match-ups. The Condorcet Paradox exists when there is a

¹ I am grateful to Daniel Bochsler for useful comments.

so-called “cyclical majority” between three or more alternatives when compared head-to-head, so that *A* beats *B*, and with the latter beating a third, *C*, but where *C* also beats *A*. In both situations there is no stable or consistent choice: Whatever is chosen as the outcome depends as much on the sequence (or form) of voting as on the preferences of the voters. As such both paradoxes deal with situations where there is a seeming discrepancy between what a majority of the voters would prefer and what the actual outcome is.

Since the rediscovery of the paradoxes in the 20th century (Arrow [1951] 1963; Black [1958] 1998), social scientists have devoted considerable efforts to proving their empirical relevance, first and foremost the late William H. Riker (e.g., Riker 1982; Riker 1986). However, the empirical examples identified by Riker and others (cf. Nurmi 1999; Gehrlein 2006; Van Deemen 2014; Kaminski 2015: 371ff) have occasionally been criticized for being anecdotal or empirically weak, etc., and often relying on speculative configurations of preferences (Green and Shapiro 1994; Mackie 2003).

One of the problems of the entire field is that even if voting paradoxes exist in the voter preferences underlying social choices, these may not be easily visible, or even visible at all, given that observers very rarely have detailed information about the decision-makers’ preference orderings over the relevant alternatives. Even when survey data exists it rarely includes pairwise comparisons or complete rankings of all the relevant candidates or policies. For that reason alone, scholars of empirical social choice have very often had to rely on more or less speculative reconstructions of the preferences (Kaminski 2015: 367ff). In other cases where scholars are fortunate to have data on voter preferences, these in reality deal with hypothetical scenarios of modest empirical relevance, e.g., preferences over prime ministers where such are not directly elected (Kurrild-Klitgaard 2001) or over political parties in a system of proportional representation (Kurrild-Klitgaard 2004; Kurrild-Klitgaard 2008).

However, the practical organization of the US presidential election process should make it an eminently suitable setting for the

study of the possible empirical occurrence of voting paradoxes. For both major parties, Democrats and Republicans, the primary process involves various forms of plurality voting (although in different forms at different stages), among the registered voters of the parties (or among broader groups of voters), and where the winner of the individual primary (or caucus) is the candidate with most votes, but with no requirement of an absolute majority of the votes cast. The election process also includes sequential decisions (state by state in the primaries followed by a general election among the voters in all states).

Since voters in US presidential elections usually have more than two alternatives to choose between, both in the parties' primaries and in the general election, the selection of a nominee and subsequently of a president who might be beaten by one or more other candidates if compared in pairwise head-to-head match-ups would seem to be at least a theoretical possibility. There are reasons to believe that this, or even "cyclical majorities" between three or more candidates may have occurred in some US presidential elections (Riker 1982; Riker 1986; cf. Van Deemen 2014; Kurrild-Klitgaard 2014). Furthermore, given that the voters voting in the individual primaries, as well as in the general election, are not identical sets of decision-makers, there is ripe possibility for inconsistent social choices even if the individual preferences are consistent and the individual stages unproblematic (cf. Brams, Kilgour and Zwicker 1998; Kurrild-Klitgaard 2013). And this is even without including the potential problems arising from the existence of the Electoral College (Miller 2011; Miller 2012).

The present note is not meant as an in-depth, academic treatment of the topic but merely as a brief note using some simple polling data from the period leading up to and the beginning of the 2016 US presidential election in order to illustrate the empirical relevance of the theoretical claims of social choice analysis.

2. Voter preferences in the GOP primary field 2015-16

It is really only possible to conclude on the actual presence of voting paradoxes in democratic decisions if there exist voting situations or polls where voters are asked to compare the candidates pairwise or to rank or score them (cf., e.g., Regenwetter et al. 2006; Kurrild-Klitgaard 2014). But even though the use of polling in elections has increased almost exponentially in recent decades, very few polls are conducted using other methods than a form of plurality—i.e., almost all pit all candidates judged to be relevant against each other in one big field and with every respondent given one “vote”. Polls using ranking data are occasionally found in larger election surveys, e.g., through “thermometer” evaluations of parties or politicians (e.g., Feld and Grofman 1992; Kurrild-Klitgaard 2008; Bochsler 2010), but head-to-head comparisons are rarely seen (cf. Van Deemen 2014).

In the lead up to and during the 2016 presidential election in the US there seems to have been only four polls published using head-to-head comparisons between more than the two leading candidates, and only two of these pitted all candidates considered against everyone else in pairwise comparisons thus allowing to draw inferences about social orderings.²

No Condorcet Winner

Among the latter was a Monmouth University Poll, conducted 30 March – 2 April 2015, asking for comparisons of four Republican presidential candidates: fmr. Gov. Jeb Bush, Gov. Chris Christie, Senator Ted Cruz and Gov. Scott Walker. Because the poll was taken so long before the primaries it did not include Donald Trump who only declared his candidacy later that year but focused on four

² The surveys were a) Monmouth University Poll 30 March – 2 April 2015; b) NBC News/Wall Street Journal Poll (Hart Research Associates & Public Opinion Strategies, 14 – 16 February 2016; c) ditto, 3 – 6 March 2016; d) ABC News/Washington Post Poll 3 – 5 March 2016 (all available through pollingreport.com).

declared candidates widely seen to be serious contenders at the time. The poll results are shown in detail in Table 1, with the relevant p -values for differences in vote shares calculated and added.

Table 1. Head-to-head match-ups, four Republican presidential candidates, March-April 2015. T-test.

Candidates (vote shares)		P-value of vote share difference
Jeb Bush (54%)	→****	Chris Christie (28%) 0.0000
Jeb Bush (49%)	→	Ted Cruz (40%) 0.0718
Ted Cruz (55%)	→****	Chris Christie (30%) 0.0000
Ted Cruz (41%)	→	Scott Walker (36%) 0.2830
Scott Walker (46%)	→	Jeb Bush (42%) 0.4219
Scott Walker (58%)	→****	Chris Christie (26%) 0.0000

Note: * $p < 0.05$; ** $p < 0.01$; **** $p < 0.005$; **** $p < 0.001$. Two-tailed.

Source: Monmouth University Poll. 30 March – 2 April, 2015. N=355 registered voters nationwide who are Republicans or lean Republican. (pollingreport.com).

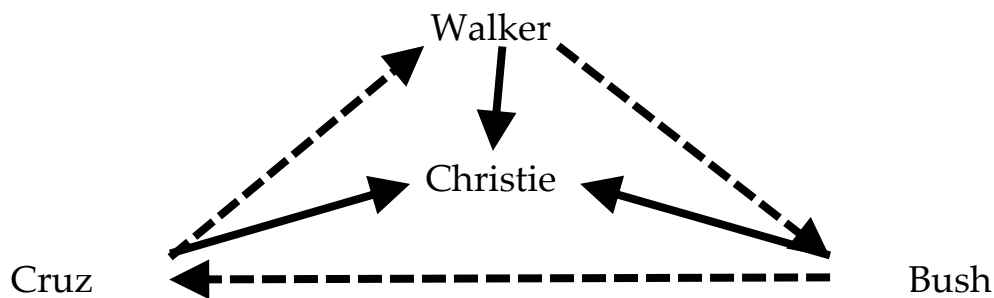
Question: "If the only two candidates for the Republican nomination were ..., who would you choose?"

The poll results suggest that there was a cyclical majority between three of the four candidates, such that Walker would beat Bush, who would beat Cruz who would beat Walker. In other words, there was no Condorcet Winner—someone who could not be beaten by at least one other candidate. However, there was a Condorcet Loser: All other candidates could individually beat Christie in head-to-head match-ups.

However, it should be noted that the sample of voters is so relatively small (N=355) that for several of the pairwise comparisons

we cannot, at conventional levels of statistical significance, say with any certainty what the majority relations would be for the population of GOP voters—i.e., we cannot rule out that the relation identified here is a function of sample composition rather than a reflection of the preferences of the GOP electorate as a whole (cf. Regenwetter, Adams and Grofman 2002). The possible majority relations (those apparent in the poll’s vote shares but not statistically significant) as well as the likely majority relations (those found to be statistically significant) are summarized in Figure 1, where full lines refer to the latter and broken lines to the former. But while we cannot say with conventional degrees of certainty that there was a cycle in the preferences of the GOP electorate, we can say that it appears that was one.

Figure 1. The possible majority cycle between Bush, Christie, Cruz and Walker, March-April 2015.



Source: See Table 1.

Plurality Winner but Condorcet Loser

The second poll allowing for pairwise comparisons of the candidates came almost a year later, in March 2016 just while the primaries were really taking off. An ABC News/Washington Post Poll conducted 3 – 5 March, 2016, pitted the then remaining GOP candidates against each other: Senator Ted Cruz, Gov. John Kasich, Senator Marco Rubio and business tycoon Donald Trump. The poll did not conduct a full set of head-to-head comparisons, but rather first pitted the four candidates against each other in one field, with Trump coming out the Plurality Winner (34%) followed by Cruz (25%), Rubio (18%) and Kasich (13%). Secondly, the survey then

compared Trump in head-to-head contests with Cruz and Rubio respectively, but with Kasich omitted. Here Trump was beaten by both Cruz and Rubio. The results are given in detail in Table 2, with the relevant p -values calculated and added.

Table 2. Poll results, including partial head-to-head match ups, four Republican presidential candidates, March 2016. T-test.

A. Candidate	Vote share	P-value of vote share relative to Plurality Winner
Donald Trump	34%	
Ted Cruz	25%*	0.0188
Marco Rubio	18%****	0.0000
John Kasich	13%****	0.0000
Other	4%	

B.	Candidates (vote shares)	P-value of vote share difference
Ted Cruz (54%)	→**	Donald Trump (41%) 0.0074
Marco Rubio (51%)	→	Donald Trump (45%) 0.2205

Note: * $p < 0.05$; ** $p < 0.01$; **** $p < 0.005$; ***** $p < 0.001$. Two-tailed.

Source: ABC News/Washington Post Poll. 3 – 6 March, 2016. N=400 registered voters nationwide who are Republicans or lean Republican (pollingreport.com).

Question A.: "Who would you like to see win the Republican nomination for president this year: Ted Cruz, John Kasich, Marco Rubio, or Donald Trump?"

Question B.: "What if the choice was just between Trump and Cruz? Who would you like to see win?" or "What if the choice was just between Trump and Rubio? Who would you like to see win?"

Because the survey does not include Kasich and does not compare Cruz and Rubio in head-to-head match-ups, we cannot conclude anything on the possible presence of a cyclical majority. However, we can glean enough from the poll to conclude that it exhibits a case of the Borda Paradox: While Trump beat the three others in a

comparison of all of them simultaneously, he would lose to (at least) two of them in pairwise contests. So, while we do not know who (if any) would have been the Condorcet Winner among GOP voters, we know that it would not have been the same as the one who was the Plurality Winner in both the poll and the actual primary (Trump).

Table 3. Poll results, including partial head-to-head match ups, four Republican presidential candidates, March 2016. T-test.

A. Candidate	Vote share		P-value of vote share relative to Plurality Winner
Donald Trump	30%		
Ted Cruz	27%		0.4286
John Kasich	22%*		0.0267
Marco Rubio	20%***		0.0047
B.	Candidates (vote shares)		P-value of vote share difference
	Ted Cruz (57%)	→****	Donald Trump (40%) 0.0005
	John Kasich (57%)	→****	Donald Trump (40%) 0.0005
	Marco Rubio (56%)	→**	Donald Trump (43%) 0.0090

Note: * $p < 0.05$; ** $p < 0.01$; **** $p < 0.005$; ***** $p < 0.001$. Two-tailed.

Source: NBC News/Wall Street Journal (Hart Research Associates & Public Opinion Strategies). 3 – 6 March, 2016. N=397 Republican primary voters nationwide (pollingreport.com).

Question A.: "And, if the Republican primary for president were being held today, which one of the following candidates would you favor: [see below]?"

Question B.: "Now, if you had to choose between ONLY Donald Trump and [see below] in the Republican primary for president, which one would you favor?"

We are again, as in the previous poll (Table 1), faced with the partial challenge that not all the candidates' vote shares are statistically different from each other at conventional levels of

significance. Trump beats Cruz, Kasich and Rubio with comfortable margins (A.), but in the head-to-head comparisons (B.) the difference in the sample between Rubio (51%) and Trump (45%) is so relatively small, when the number of respondents is considered (N=400), that we cannot with certainty generalize to the Republican electorate as a whole. However, this finding does not undermine the central finding here: Given that Cruz beats Trump comfortably (54% to 41%), we still have a case of the Borda Paradox occurring.

At exactly the same dates as the ABC News/Washington Post Poll (3-6 March 2016) another survey was conducted for NBC News/Wall Street Journal, with an almost identical but more exhaustive method and producing very similar results, albeit much clearer. Section A. of Table 3 again pits all candidates considered against each other, with Trump again being a (relatively narrow) Plurality Winner (30%) against Cruz (27%), this time with Kasich as no. 3 (22%) and Rubio closely thereafter as fourth (20%).

Together the polls provide strong evidence that during the primaries, or at least in March 2016—a very crucial time in the GOP primary—Trump may have simultaneously been the Plurality Winner and the Borda Loser, or at least would have lost to at least one other contender in pairwise contests.

3. Conclusion

Most of the time polls do not allow for social choice analysis, because they are not framed in a way enabling analysts to establish the rankings of the alternatives, and this generally held true for the 2016 US presidential primaries and general election too. All the more interesting is it therefore that of the few polls that did, there seems to be several indications that voting paradoxes may have been present. A cyclical majority between a set of top-ranked candidates may very likely have existed in Republican voter preferences early on in the pre-primary season, and Donald Trump, who went on to win both the GOP nomination and the presidential election itself, could have been beaten in head-to-head contests if

such had been used early on in the primaries. This is all the more interesting and consequential given that Trump won less than 50% of the vote both in the GOP nomination contest and in the presidential election itself.

That Trump could win despite being a Borda Loser is no doubt due to the historically large number of candidates in the GOP field: There seems to be a well-established tendency for larger sets of alternatives to increase the probability of, e.g., cyclical majorities occurring (cf. Gehrlein and Fishburn 1976; Radcliff 1994; Jones et al. 1995). However, the outcome was also due to the process: Had the primary process involved either head-to-head contests or one or more run-offs between the contenders, it is quite plausible that Trump would not have won the nomination and therefore not become president.

As such the present research lends empirical credibility to the hotly debated proposition in political science and political economy that voter preferences collective choices may take such shapes that paradoxical outcomes may prevail. Specifically, that there is an at least non-trivial possibility that the preferences of US voters at the onset of the 2016 presidential election may have looked in manner reminiscent of the problems identified by Condorcet, Arrow and Riker. To the extent this is true, it represents one of the few empirically supported examples of one or more cycles between alternative candidates and yet another example of the Borda Paradox (cf. Gehrlein and Lepelley 2011). In fact, the character of the empirical matter makes the cases considered stronger than most of the historical examples examined by Riker (1982; 1986), which did not rely on “hard” data but mostly on hypotheses about what the preferences may have looked like. Furthermore, it is a clear, non-contrived real-world example demonstrating that the way real social choices are made may end up having significant consequences for the election outcomes. In this way the simple polling data considered demonstrate the potential instability of social choices, and that this is not just a theoretical phenomenon but a genuine empirical possibility—with far-reaching consequences.

References

- Arrow, K. J. ([1951] 1963). *Social choice and individual values* (2. ed.). New Haven: Yale University Press.
- Black, D. ([1958] 1998). *The theory of committees and elections* (2. rev. ed.), I. McLean, A. McMilland, & B. L. Monroe (Eds.). Dordrecht: Kluwer Academic Publishers.
- Bochsler, D. (2010). The Marquis de Condorcet goes to Bern. *Public Choice* 144(1), 119-131.
- Brams, S. J., Kilgour, D. M., & Zwicker, W. S. (1998). The paradox of multiple elections. *Social Choice and Welfare* 15, 211-236.
- Feld, S. L. & Grofman, B. (1992). Who's afraid of the big bad cycle? Evidence from 36 elections. *Journal of Theoretical Politics* 4(2), 231-237.
- Gehrlein, W. V. (2006). *Condorcet's paradox*. Berlin: Springer.
- Gehrlein, W. V. & Fishburn, P. C. (1976). The probability of the paradox of voting: A computable solution. *Journal of Economic Theory* 12(August), 14-25.
- Gehrlein, W. V. & Lepelley, D. (2011). *Voting paradoxes and group coherence*. Berlin: Springer.
- Green, D. P. & Shapiro, I. (1994). *Pathologies of rational choice: A critique of applications in political science*. New Haven: Yale University Press.
- Jones, B., Radcliff, B., Taber, C. S., & Timpone, R. J. (1995). Condorcet winners and the Paradox of Voting: Probability calculations for weak preference orders. *American Political Science Review* 89(1), 137-144.
- Kaminski, M. M. (2015). Empirical examples of voting paradoxes. In J. C. Heckelman & N. R. Miller (Eds.), *Handbook of social choice and voting* (pp. 367-387). Cheltenham: Edward Elgar.
- Kurrild-Klitgaard, P. (2001). An empirical example of the Condorcet paradox of voting in a large electorate. *Public Choice* 107(1-2), 135-145.
- Kurrild-Klitgaard, P. (2004). Voting paradoxes in list systems of proportional representation. In C. K. Rowley & F. Schneider (Eds.), *The encyclopedia of public choice* (pp. 599-602), Vol. 2. Dordrecht: Kluwer.
- Kurrild-Klitgaard, P. (2008). Voting paradoxes under proportional representation: Evidence from eight Danish elections. *Scandinavian Political Studies* 31(3), 242-267.
- Kurrild-Klitgaard, P. (2013). Election inversions, coalitions and proportional representation: Examples of voting paradoxes in Danish government formations. *Scandinavian Political Studies* 36(2), 121-136.
- Kurrild-Klitgaard, P. (2014). Empirical social choice: An introduction. *Public Choice* 158(3-4), 297-310.
- Mackie, G. (2003). *Democracy defended*. Cambridge: Cambridge University Press.

- McLean, I. & Urken, A. B. (Eds.) (1995). *Classics of social choice*. Ann Arbor: University of Michigan Press.
- Miller, N. R. (2011). Why the Electoral College is good for political science (and public choice). *Public Choice* 150, 1-25.
- Miller, N. R. (2012). Election inversions by the U.S. Electoral College. In D. S. Felsenthal & M. Machover (Eds.), *Electoral systems: Paradoxes, assumptions and procedures* (pp. 93-127), Studies in Choice and Welfare. Berlin: Springer.
- Nurmi, H. (1999). *Voting paradoxes and how to deal with them*. Berlin: Springer.
- Radcliff, B. (1994). Collective preferences in presidential elections. *Electoral Studies* 13(1), 50-57.
- Regenwetter, M., Adams, J., & Grofman, B. (2002). On the (sample) Condorcet efficiency of majority rule: An alternative view of majority cycles and social homogeneity. *Theory and Decision* 53, 153-186.
- Regenwetter, M., Grofman, B., Marley, A. A. J., & Tsetlin, I. M. (2006). *Behavioral social choice: Probabilistic models, statistical inference, and applications*. Cambridge: Cambridge University Press.
- Riker, W. H. (1982). *Liberalism against populism: A confrontation between the theory of democracy and the theory of social choice*. San Francisco: W.H. Freeman.
- Riker, W. H. (1986). *The art of political manipulation*. New Haven: Yale University Press.
- Van Deemen, A. M. A. (2014). On the empirical relevance of Condorcet's paradox. *Public Choice* 158(3), 311-330.