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Abstract

We investigate public infrastructure investment that reduces production costs in oligopoly markets. The government decides on its public investment based on cost/benefit analysis that estimates the benefit as a reduction in production costs. In the short run, equilibrium investment falls short of the social optimum level (i.e. underinvestment) because it neglects the welfare gain of the subsequent production expansion. In the long run, equilibrium investment may exceed the social optimum level (i.e. overinvestment), depending on the demand and cost functions. This simple cost/benefit measure is thus conservative in the short run, but may not be from the long-run viewpoint.

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1 Introduction

We investigate public infrastructure investment that reduces production costs in oligopoly markets. We propose a simple cost/benefit analysis that estimates the benefit as a reduction in production costs and investigate the welfare consequence of this rule. We find that in the short run, equilibrium investment falls short of the social optimum level (i.e. underinvestment) because it neglects the welfare gain of the subsequent production expansion. In the long run, however, equilibrium investment may exceed the social optimum level (i.e. overinvestment), depending on the demand and cost functions. These results suggest that this simple cost/benefit measure is thus conservative in the short run, but may not be from the long-run viewpoint.

Developing countries still require a huge amount of public investment. Institutions such as the Asian and African Development Banks as well as the newly established Asian Infrastructure Investment Bank have been created to meet demand for such investment.

Even in developed countries, public infrastructure investment is still high. For example, the US, British, Japanese, and Korean governments spent between 2% and 5% of GDP on public investment in 2011.[1] While much public investment aims to improve the welfare of inhabitants and consumers, some such investments like the constructions of industrial parks, roads, ports, and network facilities as well as public R&D investment at least partially aim to support businesses by directly reducing production costs. In this study, we focus on this latter type of cost-reducing public investment.

The reduction in production costs increases firms' profits by the same amount if the price remains unchanged. If the price falls according to the reduction in production costs, it increases the consumer surplus, too. In both cases, the reduction in production costs

[1] See http://www5.cao.go.jp/j-j/wp/wp-je13/h05_hz030303.html.

directly increases total social surplus by the same amount. We refer to these benefits as 'technological benefits' or 'cost-saving benefits'. Technological benefits are calculated once the government knows the effect of the public investments in production costs and can be easily introduced into their cost/benefit analysis.[2]

However, cost-reducing public investment may generate additional effects, especially in imperfectly competitive markets, because the expansion of production caused by public investment may reduce the deadweight loss due to the underprovision of goods. These additional benefits can be summarized as the welfare gain of the public investment minus technological benefits. Generally, the estimation of such additional benefits is far more difficult than that of cost-saving benefits because it requires the government to obtain a lot of additional information.[3] For example, the government must obtain accurate information on the demand curve and/or price-cost margin to estimate the magnitude of this effect. However, the reliable estimation of the demand function and/or price-cost margin requires rich datasets, while it is also difficult to estimate market conditions in not near-future markets.[4]

Because of the difficulties of evaluating these additional benefits, their estimation is likely to become arbitrary and overly optimistic owing to cognitive limitations and bureaucratic incentives.[5] Moreover, in much public infrastructure investment, the cost of overinvestment

[2]See Diewert (1986) for more discussion on the cost-saving benefits. In most developed countries, cost/benefit analysis is required before executing public investment and thus the guidelines for cost/benefit analysis are well developed.

[3]Although how to measure the cost-saving effect is intensively discussed in the literature, Boardman *et al.* (2010) pointed out that that the measurement of additional effects is difficult under oligopoly. Indeed, these authors provided no way in which to evaluate them under oligopoly.

[4]If the social discount rate is high, the conditions for far-future markets may not matter. However, in many developed countries, the social discount rate is low. For example, the guideline rates in Germany, the United Kingdom, and Japan are 3%, 3.5%, and 4%, respectively. In addition, the social discount rate should equal the yield on long gilts, and even these rates may be too high given the recent low interest rates in these countries.

[5]This 'optimistic bias' is noted in many government guidelines (e.g. https://www.treasury.qld.gov.au/publications-resources/project-assessment-framework/paf-cost-benefit-analysis.pdf (the guideline

exceeds that of underinvestment because investments are sunk costs and the government cannot sell excessive facilities. Therefore, conservative estimation is more suitable for public investment.

To convey the policy implication in the clearest manner, we consider the situation in which the government adopts a conservative attitude to its cost/benefit analysis. Boardman *et al.* (2010) suggested that the worst-case scenario should be adopted in sensitivity analysis.[6] In this context, the worst-case scenario might be to assume that the additional benefits are zero.[7] We thus suppose that the government only considers technological benefits and chooses its investment level to maximize the cost-saving benefits minus the investment cost.

We compare this equilibrium investment level with the socially optimal one that reflects all the welfare effects of the public investment.[8] We then discuss whether this cost/benefit analysis in fact yields conservative public investment.

Because the government neglects the additional effects of public investment, we naturally expect that the equilibrium level falls short of the social optimum, and thus, the above rule is conservative. We show that this is indeed true in the short run (when the number of firms is given exogenously). However, it is not always true in the long run (when the number of firms are determined as a free entry condition).[9]

In the long run, the equilibrium investment is socially optimal when demand is linear

of Queensland, Australia)). See also Boardman *et al.* (2010).

[6]Considering the worst-case scenario is actually required by a number of government guidelines. See, for example, http://www.mlit.go.jp/tec/hyouka/public/090601/shishin/shishin090601.pdf (Japanese national guideline), https://www.treasury.qld.gov.au/publications-resources/project-assessment-framework/paf-cost-benefit-analysis.pdf (the guideline of Queensland, Australia), and the references therein.

[7]If negative additional effects such as environmental damage are apparent, neglecting them does not imply conservative investment and such negative externality effects should be incorporated into the cost/benefit analysis. In this study, we do not consider such technological externalities.

[8]We ignore all interventions in the product market; thus, we consider the second-best investment level to be the social optimum one.

[9]In many contexts, a free entry market often results in contrasting implications. See Cato and Matsumura (2013), Etro (2004, 2007), Ino and Matsumura (2012), Lahiri and Ono (1995, 2007), and Matsumura and Kanda (2005).

and the effect of the cost reduction is proportional to the output level (we call this the double linear case). By contrast, in the double concave case (i.e. when demand is concave, the cost-saving effect is concave with respect to output and at least one of the two is strictly concave), the equilibrium investment level exceeds the social optimum. Cost-reducing public investment thus increases the number of entering firms, resulting in additional distortion. This distortion effect is so significant that investments become excessive when demand and the cost-saving effect are concave. That is, public investment may increase the deadweight loss in the long run. However, in the double convex case (i.e. demand is convex, the cost-saving effect is convex with respect to output, and at least one of these two is strictly convex), the equilibrium investment level falls short of the social optimum. In brief, the additional benefits of public investment can be both positive and negative in the long run, although they are always positive in the short run. Thus, cost/benefit analysis that ignores these additional effects under imperfect competition does not always yield conservative public investment.

Because the public infrastructure expiration date is usually long, the long-run evaluation of public investment is relevant. Our result clearly points out that the long-run efficiency of public infrastructure investment should not be evaluated in the same way as short-run efficiency. In the short run, assuming no additional benefits is a conservative approach because it overlooks some of the positive effects. However, it may suffer from excessive investment in the long run. In other words, only considering the cost-saving benefits may not be conservative and may induce overinvestment in the long run. Our result suggests that cost/benefit analysis should distinguish between short-run and long-run evaluations, even though this point has generally been ignored in existing research and government guidelines on cost/benefit analysis.[10]

[10]Some academic papers have considered firms' entry in the context of cost/benefit analysis (e.g. Holtz-

Our analysis is closely related to the excess entry theorem of Mankiw and Whinston (1986) and Suzumura and Kiyono (1987).[11] In the long run, cost-reducing public investment stimulates new entries, and thereby, harms welfare. We should note, however, that the welfare gain of public investment can exceed the cost-saving benefits (i.e. public investment can be insufficient) in spite of its entry-enhancing effect.

The remainder of the paper is organized as follows. Section 2 presents the model of production cost-reducing public investment. Section 3 analyzes the case with a fixed number of firms as a benchmark. Section 4 investigates the free entry model. Section 5 examines the model of entry cost-reducing public investment. Section 6 concludes.

2 The Model

There are infinitely many potential new entrants. Each potential new entrant has cost function $c(x, I) + F$, where $c(x, I) : \mathbb{R}^2_+ \mapsto \mathbb{R}_+$ is the production cost, $x \in \mathbb{R}_+$ is the output of the firm, $I \in \mathbb{R}_+$ is the public investment, and $F \in \mathbb{R}_{++}$ is the fixed entry cost. The public investment is assumed to reduce the marginal costs. We assume that $c(x, I)$ is three times differentiable, $c_x \geq 0$, $c_{xx} \geq 0, c_{xI} < 0, c_{xII} > 0 \ \forall x \geq 0$ (the subscript denotes the derivative, for example, $c_x = \partial c/\partial x$ and $c_{xx} = \partial^2 c/\partial x^2$). In addition, we assume that $c_I(0, I) = 0$ (i.e. if a firm does not produce, the public investment does not benefit the firm).

Let n (≥ 1) be the number of entering firms. We define $g(x, I) := -c_I(x, I)$ and $G(n, x, I) := ng(x, I)$, where $G(n, x, I)$ is the direct marginal gain (cost-saving benefit)

Eakin and Lovely, 1996; Rouwendal, 2012). However, their focus was not on the additional effect on the deadweight loss under imperfect competition, as discussed throughout the present paper. Moreover, they ignored the welfare loss due to public investment inducing more entries.

[11] See also Konishi *et al.* (1990) and Okuno-Fujiwara and Suzumura (1993). We discuss the relationship of our result with another important work on excess entry presented by Lahiri and Ono (1988) in Section 4.

of the public investment. We consider three cases, that is $g(x, I)$ is strictly concave, convex, or linear with respect to x. An example of the cost function of the linear case is $c(x) = C(x) - k(I)x$, which is often used in the literature on cost-reducing investment.[12]

Let X be total output in the market. The (inverse) demand function is given by $p(X) : \mathbb{R}_+ \mapsto \mathbb{R}_+$, where $p(X)$ is twice differentiable and $p'(X) < 0$ for all X as long as $p > 0$.

The game runs as follows. In the first stage, the government chooses public investment I. In the second stage, after observing I, potential new entrants choose whether they enter the market. In the third stage, after observing the number of new entrants n, each new entrant i ($i = 1..., n$) independently chooses x_i. We assume that demand is sufficiently large and/or F is sufficiently small that $n \geq 1$ holds in all relevant subgames.

3 Benchmark: Short-Run Analysis

In this section, we discuss a case with a fixed number of firms n as a benchmark. There is no entry stage (second stage) and the number of firms n (≥ 1) is given exogenously.

In the last stage, n-symmetric firms face Cournot competition. The first-order condition of firm i is

$$p + p'x_i - c_x = 0. \tag{1}$$

We assume that the strategies in the production stage are strategic substitutes (i.e. $p' + p''x < 0$) or c_{xx} is sufficiently large.[13]

We restrict our attention to the symmetric equilibrium in which all firms choose the

[12]The example of the non-linear (non-uniform) case is as follows. Suppose that small producers mainly use coal plants and large producers require natural gas plants because environmental regulations are stricter for heavier polluters. If public investment is made in ports mainly used for importing coal (natural gas), $g(x, I)$ is concave (convex) with respect to the output level.

[13]If the strategies are strategic complements (i.e. $p' + p''x > 0$) and c_{xx} is small, neither the second-order condition nor the stability condition are satisfied.

same output level. Let $x^S(n, I)$ and $X^S := nx^S(n, I)$ denote the equilibrium output of each firm and total output in this short-run game, respectively.

The government expects x correctly, estimates the marginal cost-saving benefits of public investment $G(n, x, I)$, and maximizes the benefits minus investment cost given x.[14] Needless to say, $G(n, x, I)$ is not the exact marginal welfare gain of the public investment. For the reasons discussed in the Introduction, we assume that the government regards the cost-saving benefits as the welfare gain. The first-order condition of the government is

$$G(n, x, I) = 1.$$

Let I^{ES} be the equilibrium investment level in the short run.

Next, we discuss social welfare. Total social surplus is given by

$$W = \int_0^{X^S} p(q)dq - n(c(x, I) + F) - I. \tag{2}$$

Consider the social optimum investment level given the Cournot competition in the last stage (the second-best investment level). The first-order condition is

$$\frac{dW}{dI} = (p - c_x)\frac{\partial X^S}{\partial I} + G - 1 = 0. \tag{3}$$

From (1), $p > c_x$ (the price exceeds the marginal production cost under imperfect competition). Therefore, from (3), we find that the marginal welfare gain of public investment exceeds the marginal investment cost when $I = I^{ES}$ if and only if $\partial X^S/\partial I > 0$.

[14] If the government overestimates G, overinvestment may take place.

From (1) and the equation $X^S = nx^s$, we obtain

$$\frac{dX^S}{dI} = \frac{nc_{xI}}{(n+1)p' + np''x - c_{xx}}. \tag{4}$$

This is positive because $c_{xI} < 0$, $p' < 0$, and $p' + p''x < 0$ (strategic substitutes) or c_{xx} is sufficiently large.

The following proposition summarizes the above discussions.

Proposition 1 *When $n \geq 1$ is given exogenously, the public investment level falls short of the efficient one if the government regards the cost-saving benefits only.*

As long as the public investment increases total output, $G(n, x, I)$ underestimates the marginal welfare gain. Therefore, the proposed measure is conservative for deciding the level of public investment.

4 Long-Run Analysis: Free Entry Equilibrium

We now discuss the long-run effect of public investment. We solve the three-stage game with an entry decision stage by backward induction.

In the third stage, firm i $(i = 1, ..., n)$ in the market simultaneously chooses x_i to maximize its profit, given I. The first-order condition is given by (1). We again assume the symmetric equilibrium in this stage.

In the second stage, infinitely many potential new entrants decide whether to enter the market. The number of entrants n is given by the zero-profit condition:

$$px - c - F = 0. \tag{5}$$

Equations (1) and (5) determine n and x given I. We denote the number of entrants and output of each firm by n^L and x^L, respectively. Let $X^L := n^L x^L$.

We now present a supplemental result on the relationship between I and total output X^L.

Lemma 1 X^L *is strictly increasing in* I *(i.e. an increase in the public investment increases total output).*

Proof See Appendix.

In the first stage, the government chooses I. The government expects n and x and chooses I such that $G(n, x, I) = 1$ as in the short-run case. Let I^{EL} be the equilibrium investment level in the long run.

We now discuss the exact welfare gain of I. Consider the social optimum investment level given the behavior of the second and third stages discussed above. The first-order condition is

$$\begin{aligned} \frac{dW}{dI} &= (p - c_x)\frac{\partial x^L}{\partial I} + (px^L - c - F)\frac{\partial n^L}{\partial I} + G - 1 \\ &= (p - c_x)\frac{\partial x^L}{\partial I} + G - 1 = 0, \end{aligned} \tag{6}$$

where we use (5).

We now present our main result.

Proposition 2 *Suppose that* n *is endogenously determined by the zero-profit condition. Suppose also that the government regards the benefits of the public investment as the cost-saving benefits only.*

(i) The equilibrium investment level exceeds (falls short of, is equal to) the efficient one if per-firm output is decreasing in (increasing in, independent of) the public investment.

(ii) The equilibrium investment is efficient for welfare in the double linear case (i.e. both demand function p and cost-reducing gain function G are linear with respect to output).
(iii) The equilibrium investment is excessive if both p and G are concave and at least one of them is strictly concave with respect to output.
(iv) The equilibrium investment is insufficient if both p and G are convex and at least one of them is strictly convex with respect to output.

Proof See Appendix.

From the short-run perspective, the supposed measure of the marginal gain in public investment $G(n, x, I)$ always yields insufficient investments (Proposition 1). In this sense, the proposed measure is conservative. However, in the long run, it can be exact (in the double linear case) or overestimated (in the double concave case). In contrast to the short-run case, an increase in the public investment induces additional entries in the long run. Similar to the short-run case, an increase in the public investment increases total output (Lemma 1) but owing to the rise in the number of entering firms, not the increase in per-firm output. In the double concave case, an increase in the public investment reduces the output of each entrant and decreases production efficiency.[15] For this reason, the cost-saving benefits of public investment G can be larger than the exact welfare gain.

To obtain the exact welfare gain of the public investment, the government must know the precise demand and cost-reduction curves. However, it can judge whether only considering the cost-saving benefit is conservative if it knows whether these curves are concave or convex. Hence, we should recognize that the additional effect of the public investment can be negative

[15]In the double concave case, an increase in the public investment increases total output and reduces the output of each entrant. This finding implies that an increase in the public investment yields production substitution from firms that entered the market before the increase in the public investment to the new entrants who would not enter without it. Because the marginal cost of firms that have already entered the market is smaller than the price, while the average cost of new entrants is equal to the price, the above production substitution reduces production efficiency and harms welfare. For a discussion on welfare-reducing production substitution, see Lahiri and Ono (1988).

in the long run.

5 Entry Cost-Reducing Public Investment

In the previous sections, we assumed that the public investment reduces production costs. In this section, we briefly discuss the case in which it reduces entry cost F. Constructing industrial parks is an example of such public investment. Suppose that c is independent of I. Let $F(I)$ be the entry cost function. We assume that $F' < 0$ and $F'' > 0$. We also assume that F'' is sufficiently large so that the relevant second-order conditions are satisfied. The game is the same as that formulated in Section 2, except that F, not c, depends I.

Again, the total output of the firms is strictly increasing in I.

Lemma 2 X^L *is strictly increasing in* I *(i.e. an increase in the public investment increases total output).*

Proof See Appendix.

Let $G(n, I) := -nF'$ be the marginal cost-saving benefits of public investment I. The government expects n and chooses I such that $G(n, I) = 1$. Let I^{EL} be the equilibrium investment level.

We now discuss the exact welfare gain of I. Consider the social optimum investment level given the behavior of the second and third stages discussed above. Again, the first-order condition is

$$\begin{aligned} \frac{dW}{dI} &= (p - c_x)\frac{\partial x^L}{\partial I} + (px^L - c - F)\frac{\partial n^L}{\partial I} + G - 1 \\ &= (p - c_x)\frac{\partial x^L}{\partial I} + G - 1 = 0, \end{aligned} \tag{7}$$

where we use (5).

Proposition 3 *Suppose that n is endogenously determined by the zero-profit condition. Suppose also that the government regards the benefit of the public investment as the cost-saving effect only. If the public investment affects entry cost F rather than production cost c, the equilibrium investment level exceeds (falls short of) the efficient one if the strategies in the production stage are strategic substitutes (complements).*

Proof See Appendix.

In contrast to Proposition 2, as long as the strategies in the final stage are strategic substitutes, the cost-saving benefits of the public investment are always larger than the true welfare gain, resulting in an overinvestment in social welfare. An increase in the public investment stimulates new entries, whereas it does not stimulate the production of each entering firm, as opposed to the production cost-reducing investment. Further, the public investment accelerates the inefficiency caused by excessive entries and reduces some of the cost-saving benefits.

In reality, the public investment may reduce both entry and production costs. In this case, Proposition 2 (iii) is strengthened by introducing the entry cost-reducing effect because both effects have the same direction.[16] In other words, if both the demand and the marginal cost-saving gain functions are concave, the welfare gain of the cost-reducing public investment falls short of the cost-saving gain.

By contrast, Proposition 2 (iv) and Proposition 3 have different directions as long as the strategies are strategic substitutes. Therefore, even if both the demand and the direct marginal gain functions are convex, the true welfare gain of the cost-reducing public investment may be smaller than the cost-saving gain when the public investment reduces F as well as c.

[16]Note that if the demand function is concave, the strategies are always strategic substitutes.

6 Concluding Remarks

In this study, we investigate public infrastructure investment that reduces the costs of firms. First, we investigate production cost-reducing public investment. We demonstrate that the welfare gain of cost-reducing public investment exceeds the cost-saving benefits in the short run. Thus, the investment level derived from cost/benefit analysis that only considers the cost-saving benefits falls short of the social optimum one. In the long run, however, the cost-saving benefit is either larger or smaller than the true welfare gain. Therefore, the equilibrium investment level can exceed or fall short of the optimal one in the long run. We present a sufficient condition for the equilibrium investment being optimal, insufficient, and excessive.

Next, we investigate entry cost-reducing public investment. We show that the welfare gain of cost-reducing public investment falls short of the cost-saving benefit regardless of the demand condition as long as the strategies in the production stage are strategic substitutes.

In this study, we focus on the cost-reducing effect of public investment. We can show that our analysis applies to situations in which they increase product value for consumers. Public investment that reduces per-firm output, but not total output, exceeds the optimum investment. However, our analysis may not apply to the case in which public investment aims to reduce negative externalities such as carbon emissions. If serious negative externalities exist and appropriate environmental policies internalizing these externalities are not adopted, per-firm output can be too large from the viewpoint of social welfare. Under such a circumstance, public investment that restricts both the number of entering firms and per-firm output may be desirable. Incorporating environmental problems into our analysis remains for future research.

Appendix

Proof of Lemma 1

We show that $dX^L/dI = n^L(dx^L/dI) + (dn^L/dI)x^L > 0$. By differentiating (1) and (5), we obtain

$$\begin{pmatrix} x^L p' + (x^L)^2 p'' & (n^L+1)p' + n^L x^L p'' - c_{xx} \\ (x^L)^2 p' & n^L x^L p' + p - c_x \end{pmatrix} \begin{pmatrix} dn^L \\ dx^L \end{pmatrix} = \begin{pmatrix} c_{xI} \\ c_I \end{pmatrix} dI. \tag{8}$$

Using (1), $n^L x^L p' + p - c_x = (n^L - 1)x^L p'$. It follows that

$$\det \begin{pmatrix} x^L p' + (x^L)^2 p'' & (n^L+1)p' + n^L x^L p'' - c_{xx} \\ (x^L)^2 p' & n^L x^L p' + p - c_x \end{pmatrix}$$
$$= -(x^L)^2 p'(2p' + x^L p'' - c'_{xx}) < 0 \tag{9}$$

because the second-order condition of (1) ensures $2p' + x^L p'' - c_{xx} < 0$.

By applying Cramer's rule to (8), we obtain

$$\frac{dx^L}{dI} = \frac{p' x^L (c_I - x^L c_{xI}) + (x^L)^2 c_I p''}{-(x^L)^2 p'(p' + x^L p'' + p' - c'_{xx})} \tag{10}$$
$$\frac{dn^L}{dI} = \frac{(n^L-1)x^L p' c_{xI} - (n^L+1)p' c_I - n^L x^L p'' c_I + c_{xx} c_I}{-(x^L)^2 p'(p' + x^L p'' + p' - c'_{xx})}. \tag{11}$$

By using (10) and (11), we obtain

$$\frac{dX^L}{dI} = n^L \frac{dx^L}{dI} + \frac{dn^L}{dI} x^L$$
$$= \frac{-(x^L)^2 p' c_{xI} - c_I p' x^L + x^L c_{xx} c_I}{-(x^L)^2 p'(2p' + x^L p'' - c'_{xx})}. \tag{12}$$

The numerator of (12) is negative since $(x^L)^2 p' c_{xI} > 0, c_I p' x^L > 0$, and $x^L c_{xx} c_I < 0$. Moreover, the denominator is negative from (9). Thus, $dX^L/dI > 0$. Q.E.D.

Proof of Lemma 2

As in the proof of Lemma 1, we show that $dX^L/dI = n^L(dx^L/dI) + (dn^L/dI)x^L > 0$. By differentiating (1) and (5), with $c(x, I)$ and F being replaced with $c(x)$ and $F(I)$, respectively, we obtain

$$\begin{pmatrix} x^L p' + (x^L)^2 p'' & (n^L+1)p' + n^L x^L p'' - c_{xx} \\ (x^L)^2 p' & n^L x^L p' + p - c_x \end{pmatrix} \begin{pmatrix} dn^L \\ dx^L \end{pmatrix} = \begin{pmatrix} 0 \\ F' \end{pmatrix} dI. \tag{13}$$

By applying Cramer's rule to (13), we obtain

$$\frac{dx^L}{dI} = \frac{F' x^L (p' + x^L p'')}{-(x^L)^2 p'(2p' + x^L p'' - c'_{xx})} \tag{14}$$

and

$$\frac{dn^L}{dI} = \frac{F'(-(n^L+1)p' - n^L x^L p'' + c_{xx})}{-(x^L)^2 p'(2p' + x^L p'' - c'_{xx})}. \tag{15}$$

Therefore,

$$\begin{aligned} \frac{dX^L}{dI} &= n^L \frac{dx^L}{dI} + \frac{dn^L}{dI} x^L \\ &= \frac{F'(-x^L p' + x^L c_{xx})}{-(x^L)^2 p'(2p' + x^L p'' - c'_{xx})} > 0. \end{aligned}$$

Q.E.D.

Before showing Proposition 2, we present a supplemental result that is useful for the proof of this proposition.

Lemma 3 $c_I - x^L c_{xI} < \ (>, =)\ 0$ *if* G *is strictly concave (strictly convex, linear) with respect to* x*.*

Proof

$$c_I(x^L.I) - x^L c_{xI}(x^L, I) = \frac{1}{n^L} \int_0^{x^L} (G_x(n^L, x^L, I) - G_x(n^L, x, I))dx \tag{16}$$

where we use $G(0, I) = -n^L c_I(0, I) = 0$. Therefore, (17) is negative (positive, zero) if $G_{xx} < \ (>, =)\ 0$. Q.E.D.

We now prove Proposition 2.

Proof of Proposition 2 (i)

From (1), we obtain $p > c_x$. Therefore, from (6), we find that the marginal welfare gain of public investment exceeds the marginal investment cost when $I = I^{EL}$ if and only if $\partial x^L/\partial I > 0$. Q.E.D.

Proof of Proposition 2 (ii)–(iv)

From Proposition 2(i), we focus on the sign of $\partial x^L/\partial I$. Since the denominator of (10) is negative from (9), the sign of dx^L/dI is determined by those of $c_I - x^L c_{xI}$ and p''.

From Lemma 3, $dx^L/dI = 0$ if G is linear in x and demand function p is linear (i.e. $p'' = 0$). This implies Proposition 2 (ii). Similarly, $dx^L/dI < (>)0$ if G is concave (convex) in x, the demand function p is concave (convex), and at least one of them is strictly concave (convex). Thus, Proposition 2 (iii) and (iv) are obtained. Q.E.D.

Proof of Proposition 3

From (7), we find that the marginal welfare gain of public investment exceeds the marginal investment cost when $I = I^{EL}$ if and only if $\partial x^L/\partial I > 0$. We show that $dx^L/dI < 0$ if the strategies in the last stage are strategic substitutes (i.e. $p' + x^L p'' < 0$).

Equation (14) shows that the sign of $dx^L/dI < 0$ depends on that of $p' + x^L p''$. Because the denominator and F' are negative, $dx^L/dI < 0$ if and only if $p' + x^L p'' < 0$. Q.E.D.

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